Pricing strategy optimization considering customer sensitivity with Monte Carlo simulations

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Abstract

Pricing is a fundamental problem in the banking sector and is closely related to many financial products, such as insurance or credit scoring.

While setting a price for a given customer and product can be solved with standard regression models like Generalized Lineal Models (GLM), the pricing strategy optimization deals with two main components: on the one hand, the pricing model and, on the other, the customer's degree of acceptance of a given price (sensitivity to the price). Its main goal is to deal with two central objectives: (a) increase retention (probability of acceptance of a given price) and (b) increase revenue.

The purpose of this research is to provide a simple framework to calculate the Pareto frontier of several pricing strategies through Random Optimization (RO) driven by a probabilistic model, maximum decay points (MDP) and business constraints. This methodology yields expected retentions and revenues and allows for the testing of new prices.

In the experimental section we compare the results obtained in several scenarios taking into account the set of different options presented in the proposed framework (e.g., distributions used by RO, best expected retention).

We also focus on computational aspects such as parallel computation which provides the advantage to independently compute different pricing scenarios through RO.

Keywords: Technical pricing, pricing strategy optimization, Random Optimization, Insurance

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1. Introduction

The main goal of *pricing* is to set prices at which a company will sell its products and services [1]. The margin between the price and the total cost of the product or service is the revenue. If a company wants to adjust the price taking into account customer characteristics, product features, market status, risk, price sensitivity, etc., the *pricing strategy optimization* framework defines the techniques, methods and process to achieve this goal. As we show in the next sections, this goal is mainly divided into optimizing revenue and optimizing retention. Moreover, pricing is a dynamic process (dynamic pricing [2]) because of the high competitiveness in the markets and the continuous changes in customer needs. Thus, it is mandatory to receive feedback from customers in order to keep the strategies and needs of companies and markets aligned.

However, the pricing strategy optimization is not only related to solving a mathematical problem. We must consider business constraints in order to support coherence between the markets and the business strategy, i.e., pricing should hold regulatory terms[1] or include pricing constraints for a given segment of customers [3, 4, 5]. This last point is of prime importance as, from a business perspective, a company might be interested in keeping prices low for a given segment which can accept higher prices in order to ensure the retention. The proposed framework provides a general approach to choosing among several simulation options taking into account these business constraints.

The remainder of the paper is organized as follows: A brief definition of the pricing context and introduction to pricing strategy optimization are given in sections 2 and 3 respectively. In section 4 a mathematical definition of the problem is presented. Section 5 presents simulation, distributions and business constraints. Section 6 describes in detail experiments and results. The calculation of the maximum decay point (MDP) is discussed in section 7. Finally, the paper is concluded in section 8.

2. Pricing context

Generalized Lineal Model (GLM) is the standard in the pricing industry [6] as it is easy to fit and interpret. It is supported by the majority of software tools. From a mathematical point of view, pricing can be solved using GLM as a general approach. We can define a GLM [7, 8] as a relation between dependent and independent variables which takes the following form:

$$\varphi = g(\mu) = \mathbf{X}\boldsymbol{\beta} \tag{1}$$

[1] National Association of Insurance Commissioners (US) http://www.naic.org/

where $g(\mu) \equiv \mathbb{E}(Y)$, g is the link function, $\mathbf{X}$ is the model matrix and $\boldsymbol{\beta}$ is the vector of unknown parameters, with $\mathbf{Y}$ being independent and distributed as some exponential family distribution.

Insurance is one example of an industry which deals with pricing. The insurance industry encompasses several kinds of sectors mainly divided in life or non-life. Each sector defines a specific pricing methodology (customer life expectancy, home and car claims, etc.). In this paper we use the car insurance sector as an example. More specifically, the car insurance sector (among others [9]) commonly deals with two GLM models (frequency-severity method). One model is for the frequency of claims, assuming Poisson distribution $N \sim \mathcal{P}(\lambda)$, and another is for the cost of a claim (severity), assuming Gamma distribution $Z \sim \mathcal{G}(\alpha, \theta)$. In this specific case, if $g(\mu_c)$ represents the model for the number of claims and $g(\mu_s)$ the model for the severity of claims, and assuming independence between distributions, the final pricing model is defined as:

$$\varphi = g(\mu_c) * g(\mu_s) \tag{2}$$

where $\varphi \in \mathbb{R}$.

There are other approaches such as General Additive Models (GAM) [10] and their extensions like GAMLSS [11] to deal with non-lineal relations. Other studies deal with modeling loss costs by means of Tweedie Distributions [12], taking into account the frequency and severity of claims. We will focus on GLM as the industry standard.

3. Pricing strategy optimization

In the previous section pricing has been defined. Solving the pricing problem leads us to find one specific scenario in which prices are set for each customer. However, the customer sensitivity to a given price is not taken into account, and several pricing scenarios cannot be tested. Customer sensitivity represents a valuable piece of information to correctly adjust the price and simulate several probabilistic scenarios. As the grade of acceptance of a given price can be represented as a probability function, a GLM with logistic link function [13] is able to represent the probability of renewal for a given price.

The next Equation represents the *logit* function [14]:

$$\eta = \ln\left(\frac{\pi}{1-\pi}\right) = \mathbf{X}\boldsymbol{\beta} \tag{3}$$

where η is the logit or log-odd function and π is the probability of renewal, $\mathbf{X}$ is the model matrix and $\boldsymbol{\beta}$ is the vector of unknown parameters. Knowing the GLM model parameters, we can obtain the probability of renewal directly from the above equation:

$$\pi = \frac{\mathrm{e}^{\mathbf{X}\boldsymbol{\beta}}}{1+\mathrm{e}^{\mathbf{X}\boldsymbol{\beta}}} = \frac{1}{1+\mathrm{e}^{-\mathbf{X}\boldsymbol{\beta}}} \tag{4}$$

This renewal ratio decrements the price when $v \in [0.0, 1.0]$ or increments the price when $v \in (1.0, 2.0]$. Notice that when v is equal to 1.0 it represents neither incremental nor decremental, i.e., the same price of renewal. Next Figure represents the probability of acceptance of two customers for a given renewal ratio (v).

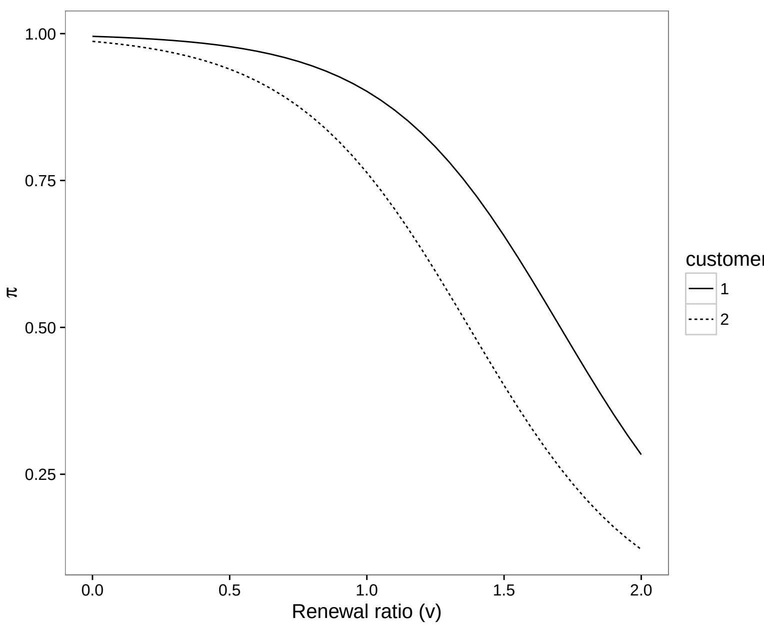

Figure 1: Example of acceptance of renewal ratios of two customers

The sensitivity of each customer can be defined as the decay ratio ($\frac{\partial\pi}{v}$, $\frac{\partial^2\pi}{v^2}$, $\frac{\partial^3\pi}{v^3}$, ...) of the customer's probability for a given range of prices, where v is the variable related to the renewal ratio in the model matrix. More formally we define the sensitivity function of one customer as:

$$\frac{\partial^k \pi}{v^k} = \frac{\partial^k}{v^k}\left[\frac{1}{1+e^{-(v\boldsymbol{\beta}_v+\mathbf{Y}\boldsymbol{\beta}_y)}}\right] \tag{5}$$

Where k is the order of the partial equation, v is the renewal ratio and y is the rest of the variables included into the model matrix (excluded v). Some examples of $\frac{\partial^k \pi}{v^k}$ are as follows:

$$
\begin{aligned}
\psi' &= \frac{\partial \pi}{\partial v} = \frac{\boldsymbol{\beta}_v \mathrm{e}^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y}}{(\mathrm{e}^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} + 1)^2}, \\
\psi'' &= \frac{\partial^2 \pi}{\partial v^2} = -\frac{{\boldsymbol{\beta}_v}^2 (e^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} - 1) e^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y}}{\left(e^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} + 1\right)^3}, \\
\psi''' &= \frac{\partial^3 \pi}{\partial v^3} = \frac{{\boldsymbol{\beta}_v}^3 \mathrm{e}^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} (e^{2v\boldsymbol{\beta}_v + 2\mathbf{Y}\boldsymbol{\beta}_y} - 4\mathrm{e}^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} + 1)}{(\mathrm{e}^{v\boldsymbol{\beta}_v + \mathbf{Y}\boldsymbol{\beta}_y} + 1)^4}, \\
\psi'''' &= \ldots.
\end{aligned}
\tag{6}
$$

The next Figure represents the function ψ'' for both customers represented in Figure 1 for a given renewal ratio v.

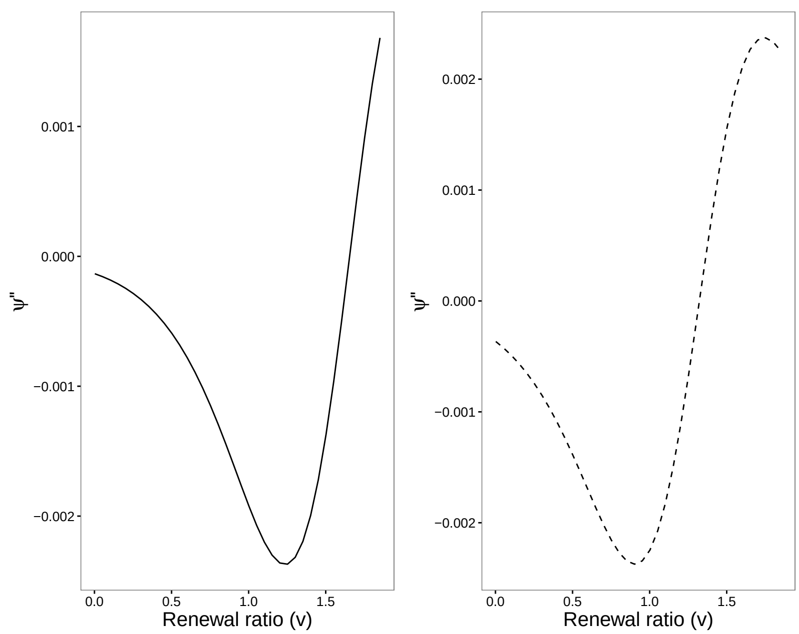

Figure 2: On the left: ψ'' for customer 1. On the right: ψ'' for customer 2.

As can be observed in the above Figure, interesting properties can be extracted from the sensitivity functions, i.e., some points in which the functions change faster, have minimum or maximum or are equal to 0. Identifying change points in behaviours or trends in the financial sector represents information which is highly actionable and valuable [15, 16, 17, 18].

In the present framework, several options to choose these valuable points are provided. As an example, we use the $min\{\psi''\}$ to measure how the rate of change in the probability π is itself changing. Therefore the point where the rate of change of the probability π is minimum is targeted as a maximum decay point (MDP). Later new prices are simulated close to these MDPs. In the last section we present and justify the relation between $min\{\psi''\}$ and the expected revenue.

The next Proposition provides the proof of the existence of two extremes in ψ'' function. These extremes are related to $min\{\psi''\}$ and $max\{\psi''\}$.

Proposition 3.1. *Let the sigmoid function $f(x) = \frac{1}{1+e^{-x}}$ (related to the Equation 4 used to model the price sensitivity); the extremes of $\frac{\partial^2 f(x)}{x^2}$ are located at the unit roots of $\frac{\partial^3 f(x)}{x^3}$.*

Proof of Proposition 3.1. We find the unit roots solving the next Equation:

$$\frac{\partial^3}{\partial x^3}\left[\frac{e^x(e^{2x} - 4e^x + 1)}{(e^x + 1)^4}\right] = 0 \rightarrow x = log(2 \pm \sqrt{3}) \tag{7}$$

Two solutions are calculated, a positive one located at $log(2 + \sqrt{3})$ that corresponds to $min\{\psi''\}$, and a second one located at $log(2 - \sqrt{3})$ corresponding to $max\{\psi''\}$.

4. Mathematical definition of the problem

As we showed before, we need two main components to define a pricing strategy (for the whole portfolio of customers): the pricing model represented by Equation 1 (φ) and the probability model represented by Equation 3 (π). Also, MDPs ($min\{\psi''\}$) previously calculated from Equation 5 and business constraints C (ranges in which each renewal ratio can be chosen) are taken into account. For simplicity we define a set of n customers as:

$$\mathcal{D} = \{(p_i, min\{\psi''_i\}, \mathrm{C}_i, Cost_i)|i = 1, ..., n\} \tag{8}$$

where $p_i = \varphi_i + \varphi_i * v_i$, $\underset{1\times 2}{\mathrm{C}_i}$, $\forall_i \mathrm{C}_{i0} < \mathrm{C}_{i1}$ and $Cost_i$ represents the current customer cost.

Therefore, the pricing strategy optimization is defined as:

$$\arg\max_{\mathbb{v}}(f_1(\mathbb{v}), f_2(\mathbb{v})) \tag{9}$$

where $\mathbb{V} = \bigtimes_{i=1}^{n} [C_{i0}, C_{i1}]$ represents the set of combinations of possible renewal ratios, $\mathbb{v} \subset \mathbb{V}$. The revenue function is represented by $f_1 \in \mathbb{R}$ and $f_2 \in [0, 1]$

represents the retention function. These two functions are defined as:

$$f_1(\mathbb{v}) = \sum_{i \in \mathbb{v}} \mathcal{B}(\pi_i(v_i))p_i - Cost_i \tag{10}$$

where $\mathcal{B}$ represents the Bernoulli distribution.

$$f_2(\mathbb{v}) = \frac{1}{n} \sum_{i \in \mathbb{v}} \pi_i(v_i) \tag{11}$$

where $\forall_i p_i \in [\mathrm{C}_{i0}, \mathrm{C}_{i1}]$ for both equations. As long as π and φ are known, we need to find the best set of $\mathbb{v}$ that maximizes Equation 9.

5. Simulation, distributions and business constraints

The last problem to be solved in the proposed framework is to find the best prices $\mathbb{v}$ that maximize Equation 9.

Since the main goal is multi-objective (Equations 10, 11) we can apply the family of Monte Carlo algorithms. Monte Carlo methods (Random Optimization, Metaheuristics [19], etc.) can solve the problem of finding an optimized price $\mathbb{v}$ that maximizes revenue for several retentions. We choose Random Optimization for its simplicity and its interesting properties from the engineering point of view. Since Random Optimization needs a large number of simulations to explore and find several scenarios [20, 21, 22], we can distribute the computation of $\{\mathbb{v}_1 \subset \mathbb{V}, ..., \mathbb{v}_k \subset \mathbb{V}\}$ in k machine processing cores. As we will demonstrate later, this parallelization leads us to reduce the calculation time with the objective of improving the results.

Two scenarios are possible in an insurance pricing application. On the one hand, a scenario in which renewal prices from optimization have been applied to the customers in the past, and on the other, a scenario in which renewal prices from business rules have been applied. In the former, the distributions of renewal prices can be used to simulate new prices, in the latter, we propose the usage of the MDPs as a direct input for simulations.

The choice of the correct distribution is a difficult one [23] where prior knowledge and previous data distributions generated from the process should be taken into account. The proposed framework has great flexibility since several distributions or assumptions can easily be applied. For instance, MDPs can be used as a direct input for Normal distributions in the following form:

$$\mathbb{v} \sim \mathcal{N}(\mu = min\{\psi''_i\}, \sigma = K) \tag{12}$$

where K is a constant that can be chosen to reduce or increase the simulation variability. Besides, in most applications of practical value we should include business constraints in order to include the global strategy as a part of the optimization. In this case, the Triangular distribution can be used to simulate prices (largely used in the finance sector when some prior knowledge is available [24]). It is defined as:

$$\mathbb{v} \sim \mathcal{T}(l = \mathrm{C}_{i0}, m = min\{\psi''_i\}, u = \mathrm{C}_{i1}) \tag{13}$$

where m is the mode represented by the MDP and l, u are the lower and the upper values to be simulated. Otherwise, the Uniform distribution $\mathbb{v} \sim \mathcal{U}(\mathrm{C}_{i0}, \mathrm{C}_{i1})$ can be used to simulate scenarios if there is no prior knowledge. In this case MDPs are not included. Moreover, it is possible that MDPs are both inside or outside the range defined by constraints. In this case, the Triangular distribution $\mathcal{T}$ could be used to test prices biased to the MDPs along the whole constraints range as shown here:

$$\mathbb{v} \sim \begin{cases} \mathcal{T}(l = m = \mathrm{C}_{i0}, u = \mathrm{C}_{i1}), & \text{if } min\{\psi''_i\} < \mathrm{C}_{i0} \\ \mathcal{T}(l = \mathrm{C}_{i0}, m = u = \mathrm{C}_{i1}), & \text{if } min\{\psi''_i\} > \mathrm{C}_{i1} \\ \mathcal{T}(l = \mathrm{C}_{i0}, m = min\{\psi''_i\}, u = \mathrm{C}_{i1}), & \text{otherwise} \end{cases} \tag{14}$$

6. Discussion and results

In order to conduct experiments on the proposed framework a synthetic data set is created.

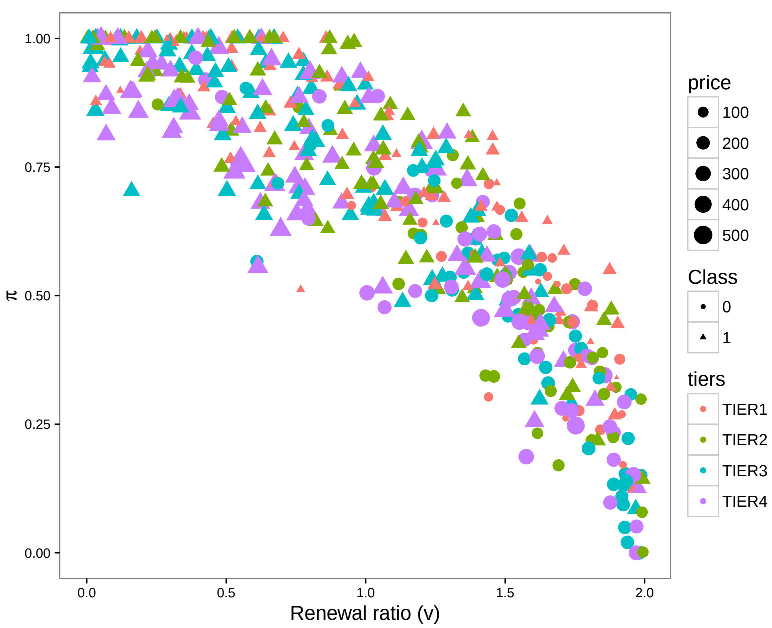

Figure 3: Synthetic data set with *price*, *tier* and *probability*

We take into account the main characteristics of real datasets to generate the synthetic set. Therefore, in this synthetic dataset we simulate three customer features: *price*, *tier* and *probability*. *Tier* is a well known feature in the insurance sector that indicates customer risk (the greater the value of tier, the greater the risk), *price* is the price given from Equation 1 and *probability* is an artificially created probability of customer acceptance for experimentation purposes (from this *probability* a renewal acceptance related class is calculated through $\sim \mathcal{B}(probability)$). Figure 3 graphically represents the synthetic data set including these three features.

Note that higher prices are applied to customers with higher risk. Both variables *price* and *probability* exhibit a non-lineal increment.

We conducted four experiments to show the flexibility of the framework and we compared results on different scenarios. We performed 10000 simulations in each scenario. We also proposed a **baseline** scenario that corresponds to the direct multiplication between the renewal ratio (v) and the probability of each $\pi(v)$ given by the proposed MDP ($min\{\psi''_i\}$) when no simulation is performed. The next table shows the main results of each one:

Table 1: Simulation scenarios

Scenario	$\mathbb{E}[f_1]$	$\sigma[f_1]$	$\mathbb{E}[f_2]$	$\sigma[f_2]$
baseline	**63719**		**0.7828**	
(I) $\mathcal{U}(\mathrm{C}_{i0}, \mathrm{C}_{i1})$	43580	2323	0.7433	1.22
(II) $\mathcal{N}(min\{\psi''_i\}, 0.2)$	60650	1962	0.7657	0.54
(III) $\mathcal{N}(min\{\psi''_i\}, 0.5)$	49470	2280	0.7162	1.14
(IV) $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi''_i\}, \mathrm{C}_{i1})$	**63580**	1812	**0.7820**	0.11

where, in scenario (I): $\mathrm{C}_{i0} = 0.0, \mathrm{C}_{i1} = 1.9$ and in scenario (IV): $\mathrm{C}_{i0} = min\{\psi''_i\} - 0.1, \mathrm{C}_{i1} = min\{\psi''_i\} + 0.1$

We also include the best $\mathbb{v}$ in each scenario:

Table 2: Best $\mathbb{v}$ in each scenario

Scenario	$\mathbb{E}[f_1]$	$\mathbb{E}[f_2]$
(I) best $\mathbb{v}$	44912	0.7445
(II) best $\mathbb{v}$	69095	0.7665
(III) best $\mathbb{v}$	57284	0.7056
(IV) best $\mathbb{v}$	64569	0.7863

As we can see, several simulation options can be applied in this framework to generate different scenarios $\mathbb{v}$. The different obtained μ and σ are directly related to the distribution used in each scenario.

As an example, we get lower value of the best $\mathbb{v}$ in scenario (I) than in scenario (II), because scenario (I) has greater space search $\sigma[f_1] = 2323$ than scenario (II) $\sigma[f_1] = 1962$. Also, with the distribution used in scenario (I) we search in a wide range of f_2 in contrast to scenario (II) ($\sigma[f_2]$). Besides this specific synthetic data set we found that scenario (IV) has a great performance obtaining values of $\mathbb{E}[f_1], \mathbb{E}[f_2]$ similar to the baseline.

Next plots represent graphically each proposed scenario. Notice that different shapes are related to the different simulation distributions and given options.

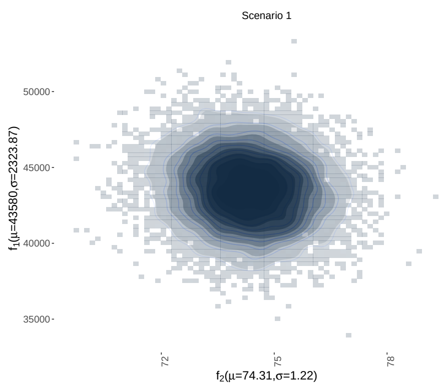

Figure 4: $\mathcal{U}(\mathrm{C}_{i0}, \mathrm{C}_{i1})$

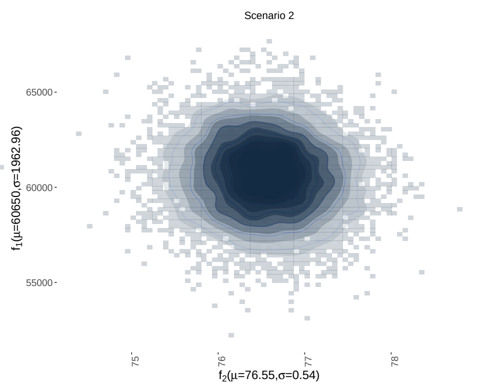

Figure 5: $\mathcal{N}(min\{\psi''_i\}, 0.2)$

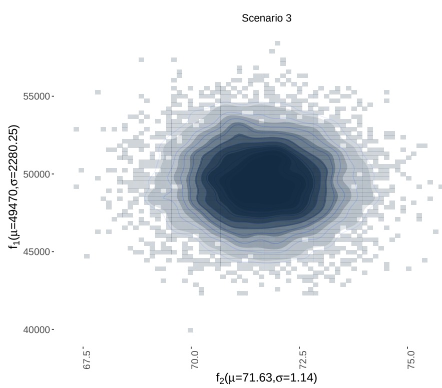

Figure 6: $\mathcal{N}(min\{\psi''_i\}, 0.5)$

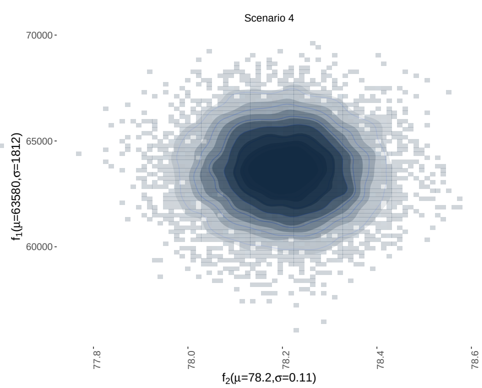

Figure 7: $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi''_i\}, \mathrm{C}_{i1})$

Simulating several scenarios v also has the advantage of having similar expected revenues but with different retention points in some. This concept is related to the Pareto frontier of solutions. As a consequence, if $\mathbb{v}^k$ represents the k-best scenario in expected revenue terms, i.e. $f_1(\mathbb{v}^1) \sim f_1(\mathbb{v}^2)$, the analyst could choose the scenario with higher probability taking into account the degree of similarity in f_1. Regardless of the final decision of each analyst, several $\mathbb{v}$ can be chosen as optimized pricing strategy for the whole portfolio of customers.

From a computational point of view, another interesting result (Table 3) derives from the parallel computation of several price scenarios $\mathbb{v}$. The next experiment runs 100000 simulations in order to achieve the best theoretical $\mathbb{E}[f_1]$ for a given $\mathbb{E}[f_2]$. It can be observed that the elapsed time decreases with the number of cores that we use. This occurs with up to 20 cores (we used machine AWS[2] $m4.10xlarge$). The overhead of launching processes and context changes are getting slower and offset the improvement of parallelization. Even so, the very positive effect of parallelization is remarkable.

Table 3: Parallel computation performance (in seconds)

Number of threads	User time	System time	Elapsed time
1	359.436	1.944	9.444.352
4	357.584	2.208	2.543.240
8	362.004	5.520	1.771.168
12	375.436	2.608	1.719.724
16	390.460	6.472	1.773.876
20	417.084	4.224	1.822.883

7. Relation between ψ, $\mathbb{E}(f_1)$ and $\mathbb{E}(f_2)$

Since any point related to retention can be chosen by the analyst, we propose the MDP calculated by $min\{\psi''\}$. To justify the usage of $min\{\psi''\}$ we performed several simulations (with values $\sim \mathcal{N}(min\{\psi''\}-k, 0.01)$ to reduce variability) on different scenario to show the relation between $\mathbb{E}(f_1)$ and $\mathbb{E}(f_2)$. The scenarios are generated with a constant displacement k.

$$\begin{aligned} \{min\{\psi''\} - k * 10, ..., min\{\psi''\} - k, \\ min\{\psi''\}, \\ min\{\psi''\} + k, ..., +min\{\psi''\} + k * 9\} \end{aligned} \tag{15}$$

where $k = 0.05$. The next Figure represents the simulations performed taking into account the MDPs calculated by the previous Equation 15, i.e., point 1 = $min\{\psi''\} - 0.5, ...$, point 10 = $min\{\psi''\}, ...$, point 20 = $min\{\psi''\} + 0.45$:

[2] Amazon Web Services

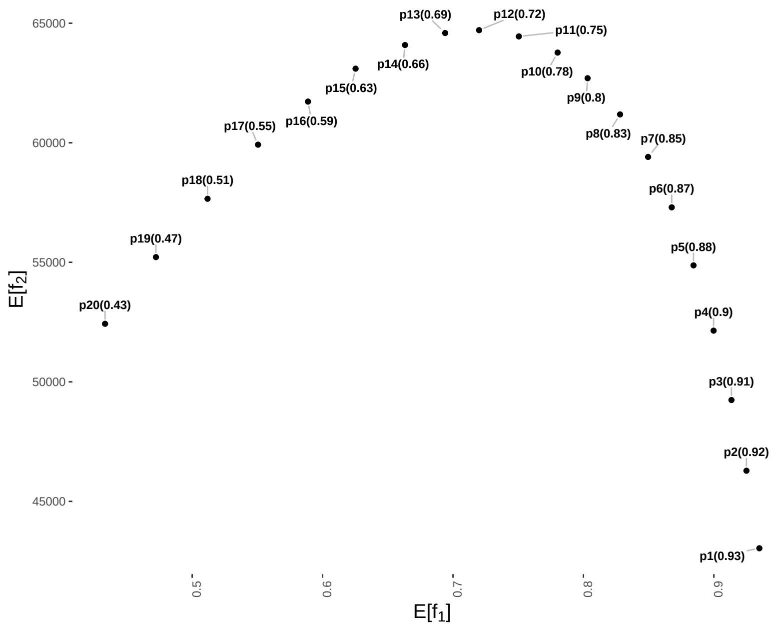

Figure 8: Relation between $\mathbb{E}[f_1]$ and $\mathbb{E}[f_2]$ along several MDPs

The above Figure 8 shows the convex relation between $\mathbb{E}[f_1]$ and $\mathbb{E}[f_2]$. Thus, taking into account this relation and the MDP targeted by $min\{\psi''\}$, we are able to find the point where the loss of retention represented by $\mathbb{E}[f_1]$ decays faster and hence $\mathbb{E}[f_2]$ becomes slow. Notice that although the most optimized scenario is achieved at point $p12$, it is only if you look at it in terms of $\mathbb{E}[f_2]$, the scenario at point $p10$ provides the best $\mathbb{E}[f_2]$ integrating the ratio of change in the sensitivity of customers calculated by $min\{\psi''\}$.

We graphically represent this relation through two main components:

I) A *scaled* expected revenue related Equation 10, defined as:

$$\hat{f}_1(\mathbb{v}) = \sum_{i \in \mathbb{v}} \pi_i(p_i) p_i \tag{16}$$

II) Two slopes (these slopes can be calculated from simple lineal model Equation 1) defined as:

$$\beta_A = \frac{\Delta \mathbb{E}[\hat{f}_1]}{\Delta \mathbb{E}[f_2]}, \beta_B = \frac{\Delta \mathbb{E}[f_2]}{\Delta \mathbb{E}[\hat{f}_1]} \tag{17}$$

The next Figure represents the results in four scenarios, in which the scaled expected revenue $\hat{f}_1$ and the β_A (transformed into a line) are included:

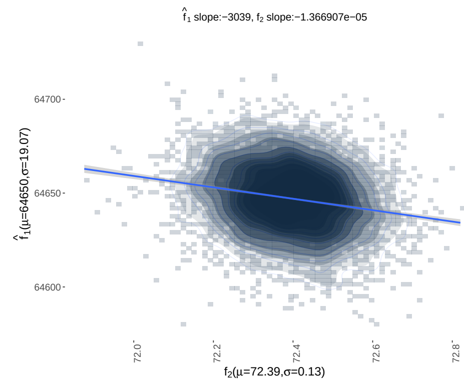

Figure 9: (I) $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi'''\} + 2k, \mathrm{C}_{i1})$

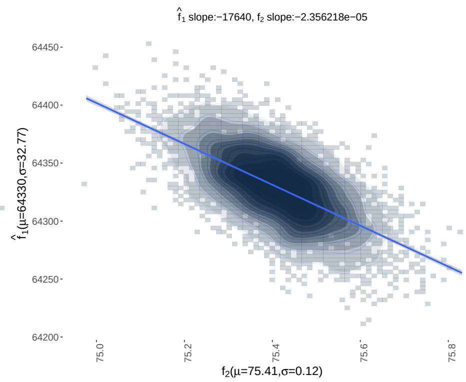

Figure 10: (II) $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi'''\} + k, \mathrm{C}_{i1})$

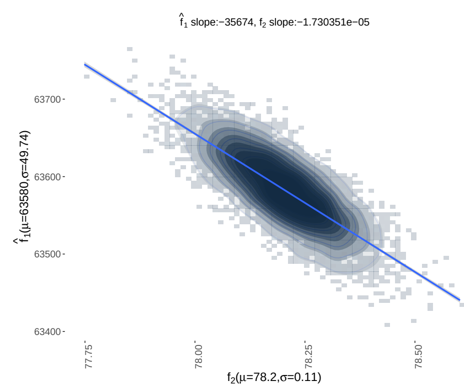

Figure 11: (III) $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi'''\}, \mathrm{C}_{i1})$

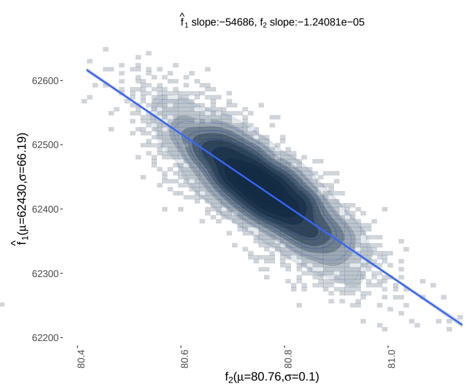

Figure 12: (IV) $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi'''\} - k, \mathrm{C}_{i1})$

We can observe a significant change in β_A in scenario (II). Numerically, we can represent this significant change through the ratio between the previously described slopes β_A and β_B. Next Figure shows the drastic drop in the expected revenue $\mathbb{E}[\hat{f}_1]$ in comparison with the expected retention $\mathbb{E}[f_2]$.

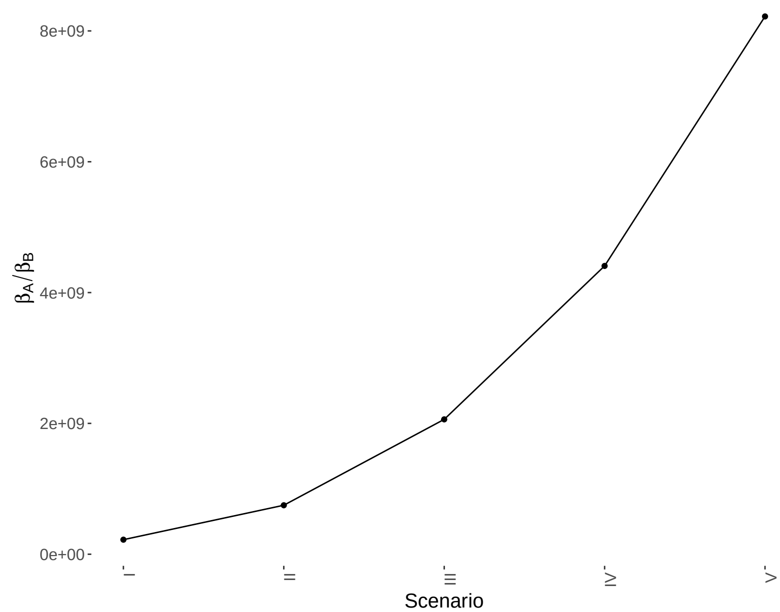

Figure 13: Geometrical relation between $\mathbb{E}[\hat{f}_1]$ and $\mathbb{E}[f_2]$

We can clearly observe at scenario II, a drastic reduction of the ratio $\frac{\beta_A}{\beta_B}$ pointing out a significant drop in $\mathbb{E}[\hat{f}_1]$ and rise in $\mathbb{E}[f_2]$ (we have included one more scenario (V) representing $\mathcal{T}(\mathrm{C}_{i0}, min\{\psi'''\} - 2k, \mathrm{C}_{i1})$ to provide a clearer tendency in the plot).

8. Conclusions

In this paper, 1) we present a general framework to deal with pricing strategy optimization. We define the main components in the framework such us pricing model, renewal model, business constrains and Maximum Decay Point. All these components are used to finding a scenario in which the revenue is maximized. Because the price sensitivity (and therefore the MDPs) is calculated from the renewal model, we need to ensure a robust model.
2) Several probabilistic distributions such as Normal ($\mathcal{N}$), Triangular ($\mathcal{T}$), Uniform ($\mathcal{U}$), etc. are used to generate renewal prices in order to test new scenarios.
3) We identify the Maximum Decay Point (MDP) as the highest valuable point. At this point the ratio of change of price sensitivity drops significantly in relation between revenue and retention. However, as a general framework, several retention points can be chosen in simulation.
4) By means of Random Optimization, we provide a Pareto Frontier of optimal pricing strategies.

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