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Today

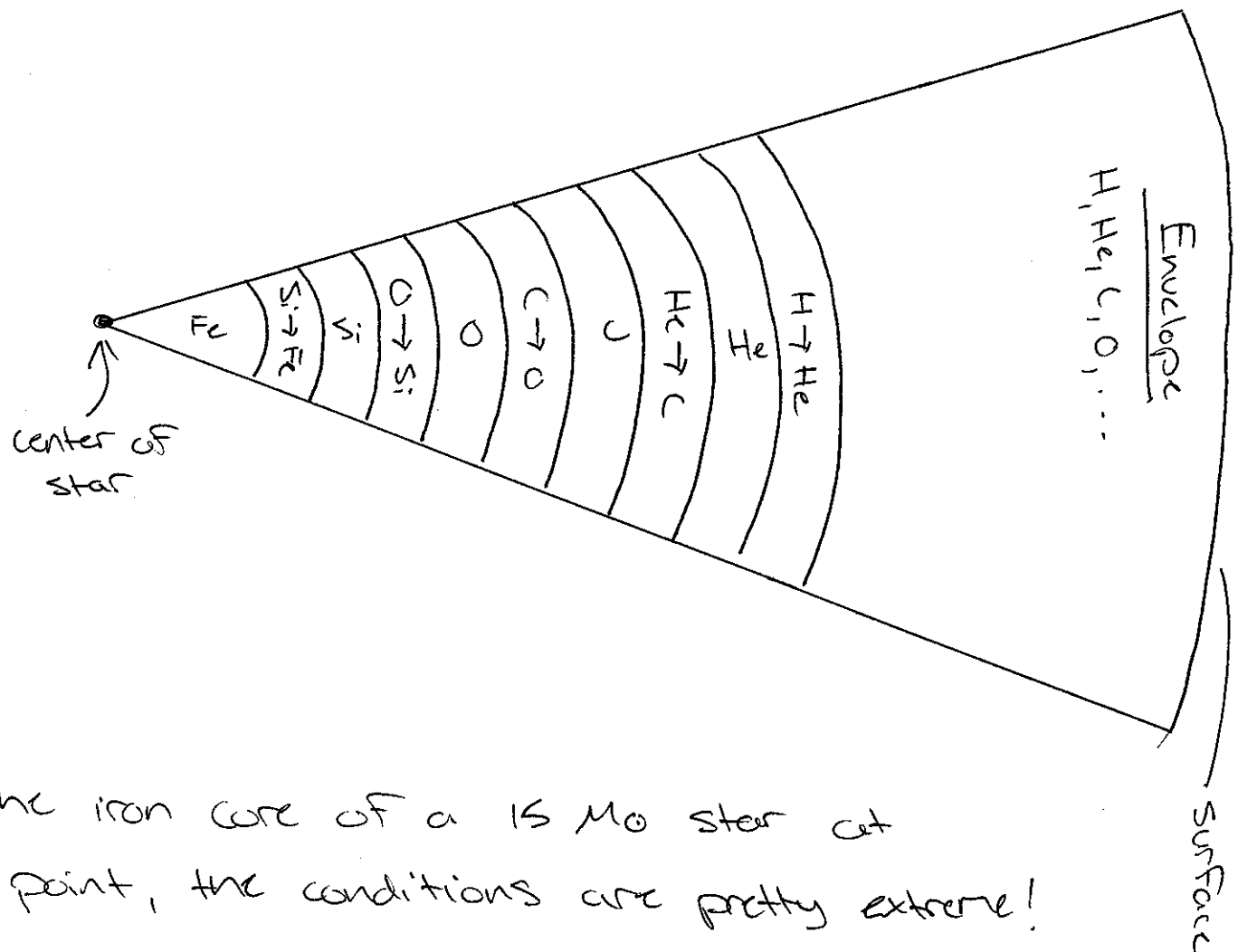
- Massive Stellar Evolution
- Supernovae
- Neutron Stars
- Black Holes
- Star Clusters

Today we will finish up our discussion of Stellar Evolution.

## - Massive Stellar Evolution

High mass stars have a similar early evolution as low mass stars. IF a star is massive enough ( $M_* \gtrsim 8 M_\odot$ ) it retains enough mass during earlier stages of evolution that after the helium core is depleted, helium shell burning adds mass to a contracting carbon core. In this post-AGB phase, the contracting carbon core becomes hot enough that carbon burning can proceed. Eventually after a rapid series of core and shell burning, Silicon burning takes place which builds up an iron core. At this point any further core burning would be endothermic.

At this point, the internal structure of the star is onion-like

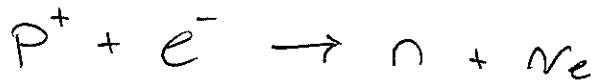
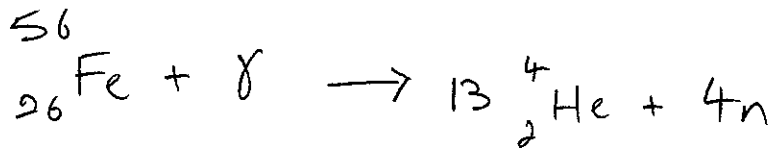


In the iron core of a 15  $M_{\odot}$  star at this point, the conditions are pretty extreme!

$$T_c \approx 8 \times 10^9 \text{ K}$$

$$\rho_c \approx 10^{10} \text{ g/cm}^3$$

Eventually, a runaway process of photo-disintegration occurs.



This process of photo-disintegration and neutronization has two effects.

- ① endothermic consumption of energy reduces core pressure
- ② removal of electrons removes electron degeneracy pressure as a source of support.

The end result is dramatic!

- core collapse releases heat which speeds neutronization
- enormous cooling via neutrinos
- core is almost entirely converted into neutrons.

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At this point two things can happen.

①  $M_{*} \gtrsim 25 M_{\odot}$

neutron degeneracy pressure fails  
and a fraction of the star collapses  
into a black hole

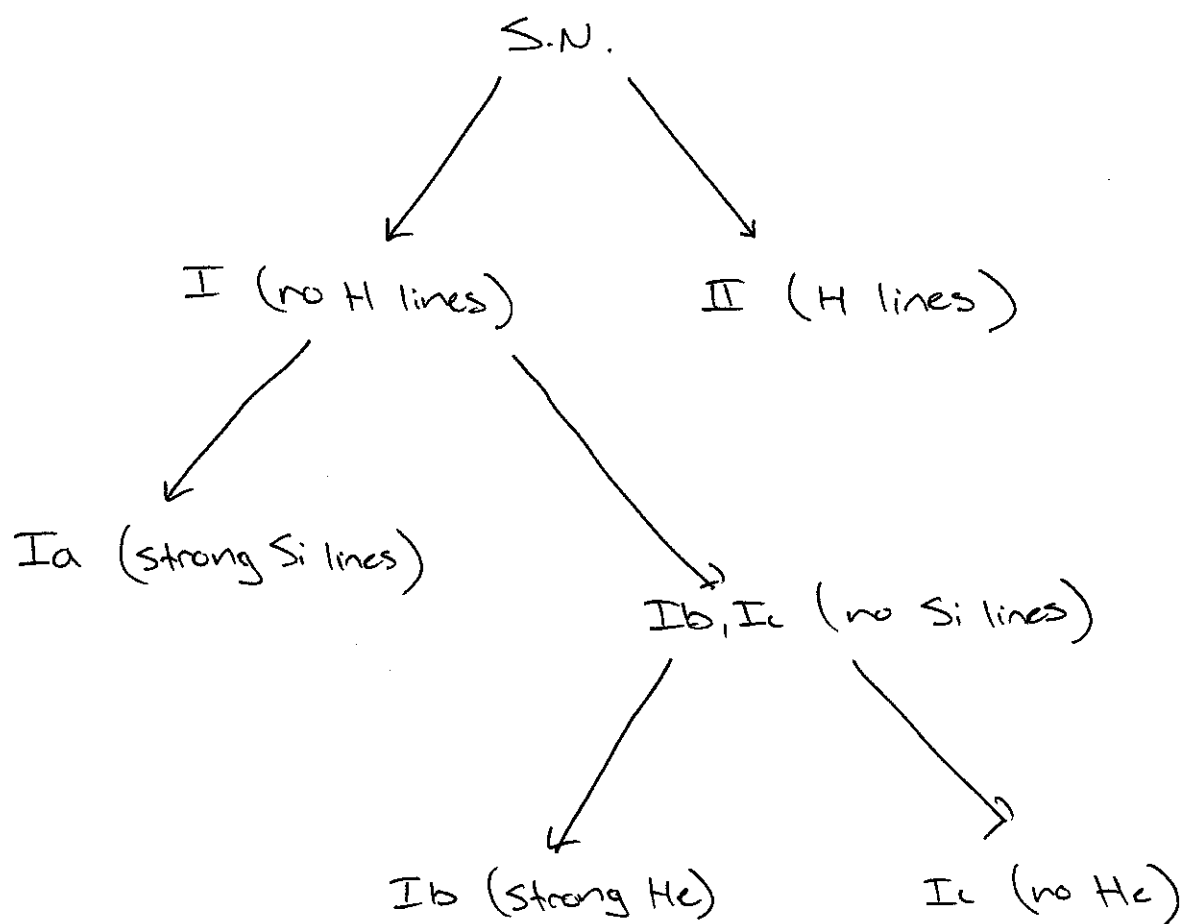
②  $8 \lesssim M_{*} \lesssim 25 M_{\odot}$

the core is supported by neutron  
degeneracy pressure

In both cases, the material above the core  
bounces off the (perhaps temporarily)  
degenerate core. A shock wave is driven  
through the outer layers of the star. The  
result is a supernova explosion.

## - Supernova Explosions

Astronomers have classified S.N. by their spectral features.



Understanding when/where/why each of these S.N. types occur is the result of a lot of ongoing research.

- \* Type Ia are found in all environments
- \* Type II, Ib, Ic only occur in active star forming regions

Massive stars (since they are short lived) are also only found in active star forming regions. This is taken as evidence that II, Ib, Ic are core collapse S.N. from the deaths of massive stars.

Type Ia are thought to be the thermonuclear detonation of  $1.4 M_{\odot}$  white dwarfs.

## - Neutron Stars

Last time we saw that a star (or stellar core) that is supported by degenerate electron pressure has a maximum mass:

$$M_{\text{ch}} = (1.4 M_{\odot}) \left( \frac{Z}{A} \right)_{0.5}^2$$

We also derived the mass-radius relation for a polytrope.

$$R^{(3-n)/n} M^{(n-1)/n} = \frac{K_p}{G N_n}$$

$$\Rightarrow R^{(3-n)/n} \propto M^{-(n-1)/n}$$

Recall that for a non-relativistic electron gas  $n = 3/2$ . This implies that,

$$R \propto M^{-1/3}$$



What happens as the electrons become relativistic? Let's parameterize the polytropic index as

$$n = (1 + \epsilon) \left( \frac{3}{2} \right)$$

$\epsilon = 0 \rightarrow$  non-relativistic

$\epsilon = 1 \rightarrow$  ultra-relativistic

The mass radius relation is now

$$R \propto M^{(1-n)/(3-n)}$$

~~Now we can~~

$$R \propto M^{1 - 3\epsilon / (3(\epsilon - 1))}$$

For  $\epsilon = 0$  (non-relativistic) we recover our original result. As  $\epsilon \rightarrow 1$ , we see that

$$R \propto \frac{1}{M^\infty}$$

As the electrons become relativistic, pressure

Support fails. So, what happens next?

Recall that the condition for degeneracy for electrons was given by,

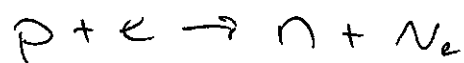
$$\frac{T}{n_e^{2/3}} \sim \frac{\hbar^2}{3Km_e} (3\pi^2)^{2/3}$$

The same condition holds for other fermions, e.g. protons and/or neutrons.

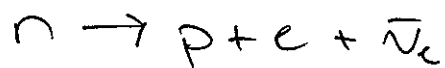
The condition then is found by replacing electron density and mass with e.g. neutron density and mass. It should

be clear that heavier particles become degenerate after (higher density/lower temp) electrons. Because stars are generally electrically neutral, when electron degeneracy pressure fails, the electrons have to "go" somewhere.

The answer: electrons combine with protons at high densities.



The reverse reaction (neutron decay)



is suppressed because there are no quantum states for the electron to go into.

The equation of state for degenerate neutron matter is much more complicated (and an open research question) because unlike an electron gas, it can't be assumed that the neutrons don't interact. Nuclear forces become important.

However, rough estimates for the properties of neutron stars can be estimated using the results for electron degenerate stars (with neutron properties).

The radius of a non-relativistic white dwarf is given by,

$$R_{wd} = (2.3 \times 10^9 \text{ cm}) \left( \frac{Z}{A} \right)^{5/3} \left( \frac{M}{M_0} \right)^{-1/3}$$

For a carbon white dwarf  $\frac{Z}{A} = 0.5$ .

A  $1 M_0$  white dwarf then has a size of  $\sim 14,000 \text{ km}$ . With this same formalism, the size of a neutron star is,

$$R_{ns} \approx (2.3 \times 10^9 \text{ cm}) \left( \frac{M_0}{m_n} \right) \left( \frac{Z}{A} \right)^{5/3} \left( \frac{M}{M_0} \right)^{-1/3}$$

For a neutron star,  $\frac{Z}{A} = 1$  is appropriate (all particles participate).

$$R_{NS} \approx (11 \text{ km}) \left( \frac{M}{1.4 M_{\odot}} \right)^{-1/3}$$

The density of a  $1.4 M_{\odot}$  neutron star is  $\rho \sim 10^{14} \text{ g/cm}^3$ . This is similar to nuclear densities. In some sense, a neutron star is an atomic nucleus with  $A \sim 10^{57}$

To see how extreme these objects are, we can calculate the escape velocity from the surface,

$$v_{esc} = \left( \frac{2GM}{R} \right)^{1/2}$$

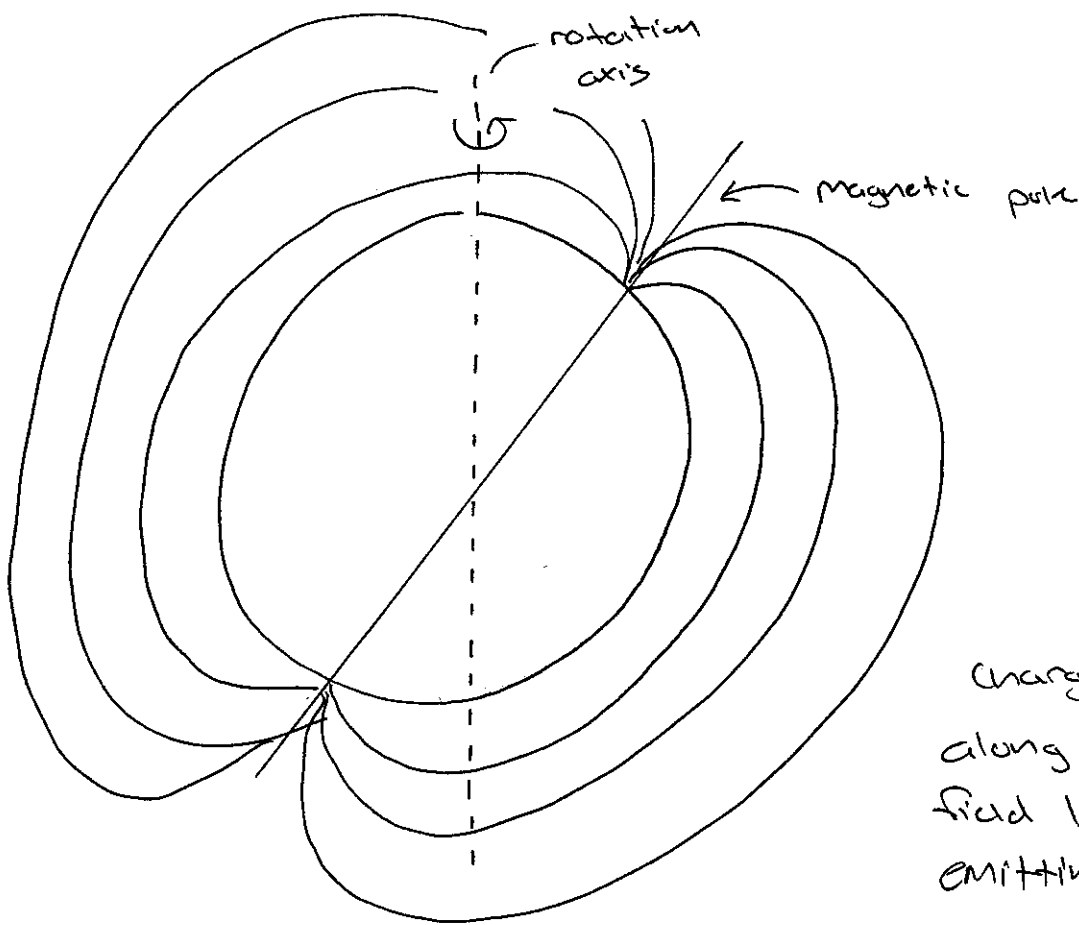
For  $M \sim 1.4 M_{\odot}$  and  $R \sim R_{NS}$

$$v_{esc} = 1.8 \times 10^8 \text{ km/s} \sim 0.6 c$$

The upper mass limit of neutron stars is not known in detail. A similar calculation to white dwarfs results in a mass that is too large. A hard upper limit is found to be  $\sim 3.2 M_{\odot}$ , but  $\sim 2 M_{\odot}$  is more likely. Most neutron stars are found to have masses slightly above  $1.4 M_{\odot}$ .

Neutron stars are thought to be the source of a class of objects called pulsars. Pulsars are radio sources which give out pulses of radiation on  $\sim 1$  s timescales.

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The cores that eventually become neutron stars undergo such dramatic contraction ( $r \sim 1 R_{\odot} \rightarrow \sim 10 \text{ km}$ ) that any non-zero rotation or magnetic field is vastly amplified. In general, the magnetic and rotation axis do not need to be aligned.



Charged particles move along the magnetic field lines relativistically emitting synchrotron emission primarily along the magnetic axis.

IF the magnetic pole rotates such that it is periodically aligned with the earth, we observe a pulsar. Typical pulsation periods are  $10^{-3} \rightarrow 1$  second.

What is the evidence that pulsars are neutron stars?

Consider that the fastest a star can spin is set by the balance of centrifugal forces and gravitational force at the surface.

$$\frac{GMm}{R^2} > m\omega^2 R$$

$\nwarrow$  mass of star       $\nwarrow$  angular speed  
 $\nearrow$  radius of star       $\nwarrow$  test mass

$$\Rightarrow \frac{M}{R^3} > \frac{\omega^2}{G}$$

$$\Rightarrow \bar{\rho} > \frac{3\omega^2}{4\pi G}$$



The pulsar in the Crab Nebula has an angular frequency of pulsations of:

$$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{3 \text{ ms}} = 190 \text{ Hz}$$

$$\Rightarrow \bar{\rho} > (1.3 \times 10^{11} \text{ g/cm}^3)$$

This is  $\sim 10^5$  times denser than a white dwarf.

## - Black Holes

The escape velocity from the surface of a neutron star is  $\sim 0.5 c$ . If a star collapses further, eventually  $v_{esc} = c$  (or larger). In this case nothing can escape such an object.

This critical radius is called the Schwarzschild radius. A hokey derivation can be done by setting  $v_{esc} = c$

$$\Rightarrow c = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow R_s = \frac{2GM}{c^2}$$

A black hole is thought to be the final remanant of massive stars ( $M \sim 25 M_\odot$ ).