Physics 777: Introduction to Astrophysics

Problem Set 2
Due: Monday March 9, 2020

1 Radiative Transfer

1. Amongst problems 1.1 - 1.10 in "Radiative Processes in Astrophysics" by Rybicki & Lightman, choose three to solve. You are encouraged to work with your classmates. Regardless, each student should reproduce and understand the solution to each problem.
2. In a plane-parallel stellar atmosphere, calculate the anisotropy of the radiation field ($I_{\rm in}/I_{\rm out}$) as a function of optical depth. Explicitly state all assumptions and approximations. At what depth is the radiation field isotropic to within $\sim 1\%$?

2 Stellar Atmospheres

1. Plot the ionization state of hydrogen ($N_{\rm II}/N_{\rm I}$) in stellar atmospheres as a function of effective temperature, $T_{\rm eff}$. Along the way, you will find that the partition function for hydrogen diverges–justify a solution to this "problem"[1].
2. As discussed in class, the stellar spectral types, OBAFGKM, constitute a surface temperature sequence. Using the Saha and Boltzmann equations, identify which stellar type should have the "strongest" Hydrogen Balmer absorption lines.
3. Identify the spectral type for a set of stars from the MaNGA Stellar Library (MaStar). Scripts to download, read, and plot the spectra are provided with this problem set. Justify your conclusions. The Object IDs are:
   (a) 7-17509232
   (b) 3-108800468
   (c) 3-112591633
   (d) 7-27095238
   (e) 3-149418784

3 Stellar Interiors

1. According to the current solar models, the center of the Sun has a temperature of $T_c \sim 1.56 \times 10^7$ K, a density of $\rho_c \sim 1.48 \times 10^5$ kg m^{-3}, and a chemical composition given by $X = 0.64$, $Y = 0.34$, and $Z_{\rm CNO} = 0.015$.
   (a) Calculate the amount of power that is generated per unit mass at the center of the Sun due to the PP chain and the CNO cycle. At what temperature would the two reactions be equal?
   (b) Estimate the size of the hydrogen core by approximately calculating the mass and radius of the region where the total (pp + CNO) energy production per unit time is equal to the solar luminosity. Why does this estimate differ from the more accurate value of $R_{\rm core} \sim 0.2\ R_\odot$ (See figure 11.5 in Carroll & Ostlie).
2. Derive approximate power-law scalings for the nuclear energy generation rates in stellar cores for the pp-chain, the CNO cycle, and the triple-α process. Please give your solutions in the form, $\epsilon \propto \rho^\beta (T/T_0)^\alpha$. Justify your choice of T_0 for each process.

[1] It may be useful to know that the electron pressure in stellar atmospheres ranges between $0.1 - 1000$ N/m^2.