Physics 777: Introduction to Astrophysics

Problem Set 4
Due: May 1, 2020

1 Galaxies

1. The modern Hubble galaxy morphology sequence consists of:

   - Elliptical: E0, E1, E2, E3, E4, E5, E6, E7
   - Lenticular: $S0_1$, $S0_2$, $S0_3$
   - Normal Spiral: Sa, Sb, Sc, Sd
   - Barred Spiral: SBa, SBb, SBc, SBd
   - Dwarf and Irregular: dE, dSph, Sm, Im, Ir

   (a) Describe the differences between elliptical (E) and lenticular (S0) galaxies.

   (b) Describe the differences between the dwarf elliptical (dE) and the dwarf spheroidal (dSph) galaxies.

   (c) Describe the differences between the Sa, Sb, and Sc spiral galaxies.

   (d) Describe the differences between the Sm, Im, and Ir galaxies.

2. It is typical to approximate massive components of galaxies (e.g. stellar bulges, stellar haloes, dark matter haloes, etc.) using power-law density models (over some range in radius). The density profile for a power-law distribution is given by,

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha} \tag{1}$$

   Show that the circular velocity for this generalized density profile is given by,

$$v_c^2(r) = \frac{4\pi G \rho_0 r_0^\alpha}{3-\alpha} r^{2-\alpha} \tag{2}$$

   Plot the density and circular velocity profile for:

   - the singular isothermal sphere, $\alpha = 2$
   - the homogeneous sphere, $\alpha = 0$

   Compare these profiles to the double power-law NFW profile which is used to model dark matter haloes (you can look this up in your textbooks). In order to make an insightful comparison, try setting the constants to unity and using a log-log scaling for your plots.

3. Classical bulges in spiral galaxies and globular clusters are often modelled using a Plummer model. Derive the circular velocity profile due to the Plummer potential:

$$\Phi(r) = -\frac{GM}{\sqrt{b^2 + r^2}} \tag{3}$$

   where b is a constant. Plot the results. Derive the density profile which gives rise to this potential. Finally, show that M is the total mass of the system.

2 Cosmology

1. The "standard" cosmological model is called a flat ΛCDM model because curvature is negligible ($\Omega_0 \approx 1$), the mass-energy density today is dominated by the cosmological constant ($\Omega_{\Lambda,0} \approx 0.73$), and matter ($\Omega_{\mathrm{m},0} \approx 0.27$) is mostly in the form of cold dark matter (CDM), i.e. $\Omega_{\mathrm{b},0}/\Omega_{\mathrm{m},0} << 1$.

   (a) Give a physical explanation for why the age of a flat ΛCDM universe is larger than that of an Einstein-de-Sitter ($\Omega_{m,0} = 1,\ \Omega_{\Lambda,0} = 0$) universe when the current expansion rate is the same in both cases (i.e. H_0 is the same).

   (b) Modern observations find that the current total density, Ω_0, may deviate from unity by only $\pm\, 0.005$. Derive an expression for the total density $\Omega_{\mathrm{total}}(a)$ as a function of the scale factor a. Briefly discuss how $\Omega_{\mathrm{total}}(a)$ compares to unity in the early Universe (i.e. in the radiation-dominated era) in order to have the current value be close to unity.

   (c) The Universe is currently undergoing accelerated expansion because of some mysterious dark energy. Show that the scale factor, $a(t)$, will increase exponentially deep in the Λ era.

   (d) For exponential growth of the form $a(t) \propto \exp(t/\tau)$, there is a characteristic time τ for the expansion. What is the characteristic time for the currently accepted model and how does it compare with the age of the Universe? Comment on the likelihood of living in an era when $\Omega_\Lambda(a)$ and $\Omega_{\mathrm{m}}(a)$ are approximately equal.

2. Estimate the gas pressure in the Universe at the epoch of recombination, $z \sim 1100$. For hydrogen gas, apply the Saha equation to calculate the ionization fraction as a function of temperature (plot your result). At what temperature does the ionization fraction transition from $\sim 1 \rightarrow 0$?

3. At recombination, the primordial cosmic microwave background (CMB) has a blackbody spectrum because radiation and matter were in thermodynamic equilibrium in the early Universe. The current temperature of the CMB radiation is 2.726 K.

   (a) Show that in an expanding Universe the CMB retains a blackbody spectrum form but with an evolving temperature.

   (b) List five examples of what we can determine by knowing the current CMB temperature (along with knowledge of other cosmological parameters).

4. Show that the equation of state for the non-relativistic matter component of the Universe can be written as,

$$P = w\rho c^2 \tag{4}$$

   where,

$$w = w(T) = \frac{k_B T}{\mu m_p c^2}\left[1 + \frac{1}{\gamma - 1}\frac{k_B T}{\mu m_p c^2}\right]^{-1} \tag{5}$$

   What is the "dust approximation" in this context?