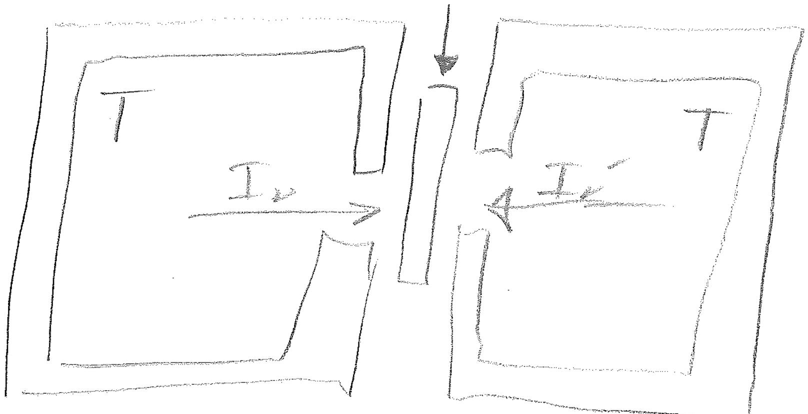
Thermal Emission (Reading: Skim deKoter Ch 7 + read deKoter Ch 8)

Even before Planck derived the form for thermal radiation it was known that it must satisfy a certain brightness

Imagine two enclosures w/ small hole & same T



- Because photons can be created at walls, must reach thermal state (no conservation of particle number)
- The radiation emerging from hole must have certain form & cannot depend on enclosure or there would be energy transfer, violating 2nd law of thermodynamics which requires no heat transfer between bodies at same temperature

Thus $I_\nu = I_\nu' = B_\nu(T)$ universal function [Kirkoff's law] 1859]

Imagine material in cavity. Must be in detailed balance w/ cavity so does not heat up

Cannot have $S_\nu > B_\nu$ or for optically thick amount $I_\nu > B_\nu$. Thus

Kirkoff's laws: $\boxed{S_\nu = B_\nu(T)}$ thermal radiation

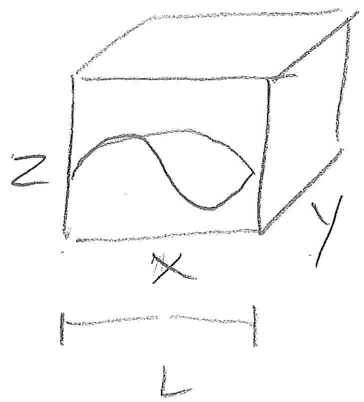
$\boxed{j_\nu = \alpha_\nu B_\nu(T)}$

In optically thick limit $I_\nu \longrightarrow B_\nu$

Blackbody!

Planck Spectrum

Consider a conducting cubic box of size L + temperature T. It is filled w/ radiation.

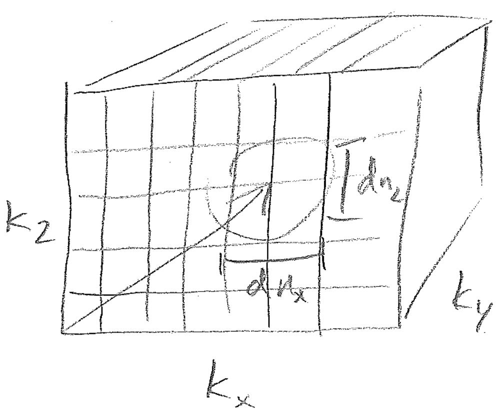


We can write the electric field in box as

$$E(x) = \sum_{n_x, n_y n_z} \hat{E}(k) \sin(k_x x) \times \sin(k_y y) \times \sin(k_z z)$$

where $\vec{k} = \frac{\pi}{L} \vec{n}$

In Fourier space (which is ν space as $\nu = \frac{c}{\lambda} = \frac{c}{\pi} \vec{k}$)



We can write the density of states around some wavevector

$$d^3 n = \left(\frac{L}{\pi}\right)^3 d^3 k$$
$$= \left(\frac{L}{\pi}\right)^3 k^2 \, dk \, d\Omega$$
$$= \left(\frac{L}{c}\right)^3 \nu^2 \, d\nu \, d\Omega$$

# of states per $d\nu \, d\Omega \, dV = 2\left(\frac{L}{c}\right)^3 \nu^2$

The "2" is because there are two polarizations.

[Note: above is so far purely classical]

The energy density per $d\nu\, d\Omega\, dV$ is then

$$u_\nu(\Omega) = \underset{\substack{\uparrow \\ \text{average} \\ \text{energy of state}}}{\langle \varepsilon_\nu \rangle} \times \underbrace{\frac{2\nu^2}{c^3}}_{\substack{\text{number of} \\ \text{states}}}$$

In purely classical picture, any ε_ν was allowed w/ probability $\exp[-\frac{\varepsilon_\nu}{kT}]$ s.t. $\langle \varepsilon_\nu \rangle = kT$. So this is what people guessed. It leads to an "ultraviolet catastrophe". This also does not describe BB at $h\nu > kT$

Planck, 1901

Energy in field is quantized s.t. $\varepsilon_\nu = m h\nu$ where $m = 0, 1, 2, \ldots$

$$\langle \varepsilon_\nu \rangle = \frac{\sum_{m=0}^{\infty} m h\nu\, e^{-\frac{m h\nu}{kT}}}{\sum_{m=0}^{\infty} e^{-\frac{m h\nu}{kT}}} \underset{\substack{\uparrow \\ \text{some} \\ \text{tricks}}}{=} \frac{h\nu}{\exp[\frac{h\nu}{kT}] - 1}$$

Because $h\nu$ is energy of each photon, the "occupation number" (typical # of photons in a state) is

$$n_\nu = \frac{1}{\exp[\frac{h\nu}{kT}] - 1}$$

$h\nu \ll kT$ $\quad n_\nu = \frac{kT}{h\nu} \gg 1$ classical regime

$h\nu \gg kT$ $\quad n_\nu \gg 1$ quantum mechanics

Thus,

$$u_\nu(\Omega) = \frac{2h\nu^3/c^3}{\exp\left[\frac{h\nu}{kT}\right] - 1} = \frac{B_\nu}{c}$$

since we found $\frac{I_\nu}{c} = u_\nu(\Omega)$

$$\left[I_\nu = \frac{c}{4\pi} u_\nu \right]$$

This is famous Planck law!!

Limits:

Rayleigh-Jeans law $(h\nu \ll kT)$

$$I_\nu = \frac{2\nu^2}{c^2} kT \quad \text{[Classical limit]}$$

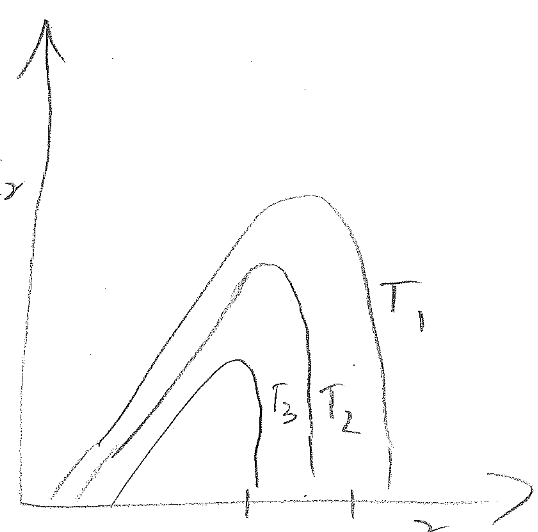
Wien law $(h\nu \gg kT)$

$$I_\nu = \frac{2h\nu^3}{c^2} e^{-\frac{h\nu}{kT}}$$

Properties:

$$B_\nu(T_1) > B_\nu(T_2)$$

if $T_1 > T_2$



Motivates "brightness temperature"
[often used in radio astronomy & generally for non-thermal sources]

$$T_b \equiv \frac{c^2}{2\nu^2 k_B} I_\nu$$

Yields actual temperature for thermal source in

Properties (cont):

The peak frequency ν_{max} is given by

$$h\nu_{max} = 2.82\, kT$$

The energy density in thermal equilibrium is

$$u = \frac{4\pi}{c}\int d\nu\, B_\nu(T) = \underset{\substack{\uparrow \\ \text{radiation} \\ \text{constant}}}{a}\, T^4$$

$$\boxed{a = \frac{8\pi^5 k_B^4}{15 c^3 h^3}}$$

Flux at surface:

$$F = \int d\nu\, F_\nu = \underbrace{\pi \int d\nu\, B_\nu(T)}_{\substack{\text{Flux at surface of} \\ \text{uniform brightness} \\ \text{is } \pi B}} = \sigma T^4$$

$$\sigma = \frac{ac}{4}$$

Radiation is "thermal" if it has $S_\nu = B_\nu$. It is a black body if $I_\nu = B_\nu$.