Astro 519A; Problem Set 1

Due Oct 16 by 5pm (in McQuinn's mailbox)

Please use cgs units for all calculations.

1. Imagine a homogeneous disk with height H and radius R that is emitting thermally with a single temperature T. A skewer with impact parameter $r < R$ parallel to the disk axis has optical depth τ_ν . The disk is observed at an angle θ w.r.t. the disk axis at a distance $d \gg R$. Calculate the following:
a) the observed I_ν for sightlines that intersect the disk (and not an edge)
b) the observed F_ν (again ignoring the contribution from edges)
c) the optically thick and thin limits of F_ν from part (b)

2. The temperature in a star varies with depth. Assume that we can approximate the source function S_ν of a star as $S_\nu = a_\nu + b_\nu \tau_\nu$, where τ_ν is the optical depth from the surface to some depth in the star.
a) What is the intensity seen at the surface of the star and how does this relate to the source function evaluated at $\tau_\nu = 1$?
b) Apply the result in part (a) to stellar lines versus the stellar continuum. A higher optical depth is achieved at line center to a given depth in the star. How would the intensity at line center differ from the continuum intensity? Please discuss both the case where the temperature decreases with increasing radial coordinate and the case where there is an inversion such that it increases with increasing radial coordinate.

3.) The visible light emitted by planets is star-light diffusely reflected from their surfaces. To calculate this accurately can be complicated, but some insight can be gained by using a simple approximation know as "Lambert's law". In this approximation, a fraction a (the "albedo") of the radiation incident on a small area element of the surface is assumed to be diffusely reflected with uniform brightness in the outward hemisphere.

Show that the diffusely reflected flux seen at earth due to one such surface element dA on the planet is

$$dF = \frac{La \cos\theta_0 \cos\theta_1 dA}{4\pi^2 D^2 d^2},$$

where D is the distance of the planet from the star, and θ_0 is the angle of the star's rays w.r.t. the outward normal of dA. Likewise d is the distance of the planet from earth, and θ_1 is the angle between the earth-planet direction and the outward normal of dA. L is the stellar luminosity.

Next, write an integral for the total reflected flux received at earth. You do *not* have to evaluate this integral.

4. A graduate student was upset about the quality of the pizza at Friday Pizza Lunch. He proceeded to throw all the pizza slices on the ground in a manner that was more or less random so that any area on the ground had an equal probability to be covered in pizza. Each pizza slice has area P, and the student threw down N slices in a room of area A. Assume that, once a pizza slice is on the floor, it does not affect the probability of a pizza slice landing on the same area.
a) What is the "optical depth" in pizza slices between the ground and someone looking down?
b) Using mathematics similar to the radiative transfer equation, calculate the fraction of the ground that is still visible and not covered by a pizza slice? Please justify your approach.
c) What fraction of the ground is covered by exactly two pizza slices? (Hint: First derive the part (b) result using the Poisson probability distribution.)

5.) Most of the energy in radiation in the Universe is in the cosmic microwave background (CMB), a nearly perfect blackbody with temperature 2.73 K today.
a) At what frequency does the CMB specific intensity, I_ν, obtain a maximum? At what wavelength does $I_\lambda = |d\nu/d\lambda| I_\nu$ obtain a maximum? How are the two related?
b) Calculate the number density and energy density of CMB photons. For the energy density, in addition to cgs units, express it in eV cm^{-3}. How does this energy density compare to the energy density in magnetic fields in the Milky Way using that in the Milky Way $B \sim 5\mu$G? (The energy density in cosmic rays is similar to that in magnetic fields; both pressurize the Milky Way disk.)
c) The CMB spectrum changes with time as CMB photons are stretched as $\nu \propto (1+z)$ with the expansion of the Universe. Show that this stretching results in a Planck function being maintained but with T scaling as $(1+z)$. Use that the physical size of some comoving region in the Universe scales as $(1+z)^{-1}$.
d) At what redshift, z, was the energy density in the CMB equal to the rest mass energy density in baryons, which have $n = 2 \times 10^{-7}$ cm^{-3} today? (You can assume all baryons are hydrogen.)