Astro 519A; Problem Set 5

Due Dec 11 by 5pm (in McQuinn's mailbox).

Please use cgs units for all calculations.

1.) Suppose relativistic electrons of fixed energy γ_0 are being introduced at a constant rate into a medium with (tangled) magnetic field B. These electrons will emit synchrotron radiation, and, as a result, lose energy. After a sufficiently long time, the electrons will settle into a steady state energy distribution $N(\gamma)$, which we would like to determine. [$N(\gamma)d\gamma$ is the total number of electrons having energies between γ and $\gamma + d\gamma$.]
a) Show that the rate of change of energy of an individual electron of energy γ is given by

$$\dot{\gamma} = \frac{d\gamma}{dt} = -A\gamma^2, \quad \text{where} \quad A = \frac{4e^4 B^2}{9m^2c^3}. \tag{1}$$

b) Show that $-\dot{\gamma}N(\gamma)dt$ is equal to the number of electrons whose energies pass through energy γ during time interval dt.
c) Argue that in steady-state $\dot{\gamma}N(\gamma)$ must be independent of γ and thus that $N(\gamma) \propto \gamma^{-2}$ up to a cutoff $\gamma = \gamma_0$.
d) Characterize the synchrotron emission from this medium. (You can assume that the medium is optically thin.)

2.) A millisecond pulsar lies behind a cloud of 10^4K photoionized hydrogen gas. Assume the cloud has a single density. We measure at 1 GHz and 1.1 GHz the timing and duration of a radio pulse from the pulsar. The pulse arrives at 1 GHz a time 10 milliseconds after the pulse arrives in the 1.1 GHz channel. Each pulse's angle of linear polarization is measured to be 0.1 radians larger at 1 GHz than at 1.1 GHz. In between pulses, the intensity of bremsstrahlung emission from the cloud in the direction of the source is measured to be 10^{-7}erg s^{-1} Hz^{-1} cm^{-2} sr^{-1}.

Through the cloud, what is the density, depth, and average line-of-sight magnetic field?

3.) What column of HI λ1216Å and of OVI $\lambda\lambda$1032, 1038Å[1] that has optical depth unity at line center, assuming pure thermal broadening for gas with $T = 10^4$K? You may use that the oscillator strengths for the lines are respectively 0.42, 0.52, and 0.36 (see *Allen's Astrophysical Quantities*). These are some of the most observable ground state absorption lines for diffuse gas.

[1] This "doublet" owes to spin-orbit coupling.

Practice final exam questions. **These are not part of the fifth problem set.** Questions 2 and 3 on the problem set are at the level that may appear on the exam. (Except I would likely keep everything symbolic so you do not have to plug in numbers.) Here are a couple other questions:

1.) Is $(E^2, B_x^2, B_y^2, B_z^2)$ a four vector? Please explain.

2.) The intensity of some unspecified optically thin thermal emission process scales as $I_\nu \sim \nu^2/(1+\exp[h\nu/kT])$. Draw a progression of I_ν curves showing how the emission of this process would look like as the emission becomes more and more optically thick.

3.) Explain how an optical image is distorted by our atmosphere, leading to the "seeing" of a telescope being much worse than the diffraction limit. How does an adaptive optics system correct for this?