Astro 519A; Problem Set 2

Due Oct 30 by 5pm (in McQuinn's mailbox), unless you are submitting an NSF proposal!

Please use cgs units for all calculations.

1.) The Eddington Limit luminosity is thought to more or less cap the luminosity of astrophysical sources. Provide an expression for the Eddington Limit luminosity in the case where the radiation is coming from a disk of material of uniform surface brightness (rather than the point source case as derived in question 1.4 of RL). For this calculation assume that (1) the disk has radius R and is optically thick, (2) the accreting optically-thin blob of plasma is pure hydrogen that is completely ionized such that Thomson scattering is the major source of opacity, (3) the blob lies a distance z along the normal line that pierces the the center of the disk at position 0, and (4) the gravitational force attracting the the blob is just due to a small (point mass) at the center of the disk, with mass M. Neglect the mass in the disk.

How does your expression differ from the spherical case derived in RL, $L_{edd} = 4\pi G M m_H c/\sigma_T$?

2.) Imagine a spherical dust-free HII region in which the ionized gas has electron/proton density $n = 10^4$ cm^{-3} out to a radius 1 pc, where the HII bubble ends. This HII region is at a distance of 1 kpc from the earth.

a) What is the flux of Lyα photons from the region [photons cm^{-2} s^{-1} cm^{-2}]? You may use that the recombination rate coefficient of hydrogen is $\alpha_B = 4 \times 10^{-13}$cm^3 s^{-1} such that the rate density of recombinations is $\alpha_b n^2$ and that every recombination results in 0.66 of a Lyα photon. Note that in the absence of dust, Lyα photons can be scattered but they cannot be destroyed.

(The flux in other lines in the hydrogen recombination cascade can be calculated by using a different number than 0.66 if the region is optically thin in those lines [as generally they can be destroyed by absorptions]. The flux in different lines can be used as a diagnostic that the radiation is indeed recombination radiation.)

b) Assume there is a central star and that all of the photons it emits that can ionize the hydrogen are absorbed within the HII region. What is the ionizing luminosity of the star in photons per second? Assume the star's emission has temperature $50,000$K, the highest temperature the the most massive POPI/II stars can achieve. From the ionizing luminosity, estimate the stellar luminosity [erg s^{-1}]. Does such a luminosity seem plausible for a single star?

c) The gas in the HII region is at temperature $T = 10^4$K. (Ionized gas tends to be at this temperature.) In the optical through radio, thermal free-free emission dominates the emission from the HII region. This emission process has isotropic power emitted per unit volume per frequency of

$$P_\nu = 6.8 \times 10^{-38} n^2 T^{-1/2} e^{-h\nu/kT} \text{erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}. \tag{1}$$

What is the brightness temperature of the free-free emission through the cloud center? You can assume that we are observing $h\nu \ll kT$ such that the Rayleigh-Jeans law applies, and that the sightline does not intersect a dense star. [Hint: Remember that the radiation is thermal and so P_ν not only gives the emission coefficient but also the absorption coefficient. It also might be most helpful to solve the RT equation for T_b (eq. 1.61 in RL) rather than I_ν, although of course both approaches will work.]

d) WMAP used the spatial distribution of Hα emission of hydrogen (which conveniently falls in the optical) as a template to subtract galactic free-free emission in the microwave in order to isolate the cosmic microwave background. Why could they do this?

3.) A dipole antennae measures the electric field of incoming radio waves as a function of time, $E(t)$. To calculate the spectrum of the electromagnetic wave it breaks the measurement into intervals of duration T and takes a Fourier transform, i.e.

$$\tilde{E}_T(\nu) = \int_{t_i - T/2}^{t_i + T/2} dt\, e^{2\pi i\nu(t - t_i)} E(t) \tag{2}$$

a) Calculate the approximate bandwidth of the receiver using two different methods.

i) using the uncertainty principle $2\pi d\nu dt > 1$.

ii) by writing equation (2) as a convolution of a window function, $\tilde{W}$, with $\tilde{E}_\infty(\nu) = \lim_{T\to\infty} \tilde{E}_T(\nu)$ and, then, solving for the full width half maximum of the window function. Hint:

$$\int_{-\infty}^{\infty} dt\, e^{2\pi i\nu(t-t_i)} X(t) Y(t) = (2\pi)^2 \int d\nu' \tilde{X}(\nu')\tilde{Y}(\nu - \nu')$$

where tildes denote Fourier transforms such that $\tilde{X}(\nu) = \int_{-\infty}^{\infty} dt\, e^{2\pi i\nu(t-t_i)} X(t)$. The window function should be defined such that $\tilde{W}(0) = 1$.

b) From $E(t)$ write an expression for the incident electromagnetic flux, F, and specific flux, F_ν, on the antennae.

c) Are there any constraints on maximum physical size of the dipole if the aim is to measure radiation with wavelength λ?