Noodling On Pulsars

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1 The Current Loop

1.1 A Current Loop in the XY-Plane

The model I will use is simply a loop of constant current I with radius k centered on the origin which is initially inclined such that the normal vector lies along the xz-plane and forms an angle θ with the z axis.
We start by defining a current loop with no angle, thus a loop in the xy-plane. The current is nonzero only at points $\vec{r} = (x, y, z)$ where $z = 0$ and $|\vec{r}| = k$. We can parameterize this loop with parameter λ as

$$\vec{r}(\lambda) = (k \cos \lambda, k \sin \lambda, 0)$$

for λ from 0 to 2π. The current vector along this loop is

$$\vec{I}(\lambda) = (-I \sin \lambda, I \cos \lambda, 0)$$

1.2 Incline and Rotation

To make this model more interesting, we use rotation matrices to change the plane in which the current loop lies. Consider

$$\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Then $\boldsymbol{R}_y(\theta)\vec{r}(\lambda)$ is our inclined current loop with current $\boldsymbol{R}_y(\theta)\vec{I}(\lambda)$.
Similarly, we will have the rotation over time about the z-axis perfomed by

$$\boldsymbol{R}_z(t) = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus our time-varying parameterized loop and current are simply

$$\boldsymbol{R}_z(t)\boldsymbol{R}_y(\theta)\vec{r}(\lambda), \quad \boldsymbol{R}_z(t)\boldsymbol{R}_y(\theta)\vec{I}(\lambda)$$

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