# SUBMODULE 1.3
# INTRODUCTION TO MONTE CARLO METHODS

## A. RANDOM NUMBERS

We'll begin by exploring random number generation. Here's some tasks to perform.

1. Create a large set of random numbers from $[0, 1)$. How you do this is up to you; you can use either standard Python or NumPy.
2. Are those numbers really random? Plot them and see.
3. A better test is to bin the random set of numbers into, say, 100 bins, and plot a bar graph of the number of random numbers in each bin. Does it look like it should?

## B. GAUSSIAN RANDOM NUMBERS

In the above, we sampled our random numbers from a uniform distribution. In general, this probability density function looks like

$$p(x) = \frac{1}{b-a},$$

where we used $b = 1$ and $a = 0$ above. However, we might want some other distribution of random numbers; one common distribution is the Gaussian or Normal, given by

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2},$$

where $\mu$ is the mean and $\sigma$ the standard deviation.

Although we could generate our own random number generator to give us random numbers with this probability distribution, NumPy provides one, too. Give it a try, and test it by binning your numbers as above.

## C. MMM, PI

Now for something different. Using a uniform distribution, calculate the value of $\pi$, correct to 4 digits (!).

## D. MONTE CARLO INTEGRATION

Suppose we want to integrate a function $g(x)$ from $a$ to $b$:

$$I = \int_a^b g(x)\, dx$$

(we'll keep things one dimensional for simplicity, but the real power of Monte Carlo methods is best shown in higher-dimensional integrals). We'll rewrite the integral as

$$I = \int_a^b \frac{g(x)}{f(x)} f(x)\, dx = \int_a^b h(x) f(x)\, dx.$$

If we suppose $f(x)$ is a probability density function, then this looks like the expectation value for $h(x)$:

$$I = \langle \frac{g(x)}{f(x)} \rangle.$$

For example, if $f(x)$ is the uniform probability density function, then we have

$$I \approx \frac{b-a}{N} \sum_{n=1}^{N} g(x_n),$$

where $x_n$ is a random number pulled from $f(x)$.

So, try to integrate the following:

$$\int_0^\infty \frac{x^3}{e^x - 1}\, dx.$$