SUBMODULE 1.4
MONTE CARLO RADIATIVE TRANSFER

Handling radiative transfer with Monte Carlo techniques is very different than we've seen in lecture. We essentially are now going to follow, individually, a large number of photons as they progress through an atmosphere. As in class, we'll assume a plane parallel atmosphere, of height z_{max}, with all photons originating at $x = 0$, $y = 0$, $z = 0$. We'll also assume that the only interaction the photon can have as it goes is to be scattered – no absorption allowed (although it's not that difficult to extend our analysis here to include it). Finally, we'll take the density ρ to be constant.

In terms of moving the photon through the atmosphere, the optical depth is the important quantity (rather than physical depth z). As we've seen, it's related to the mean free path of a photon; in fact, the probability that a photon travels an optical depth τ without having an interaction is $e^{-\tau}$. That means the probability a photon scatters prior to τ is

$$P(\tau) = 1 - e^{-\tau}.$$

Once scattered, the photon goes off in an entirely new (random) direction. One quick note about the direction before we get to the algorithm, though. We'll use the usual spherical coordinates (θ, ϕ), but it's customary to use the cosine of θ, $\mu = \cos(\theta)$, along with ϕ, rather than the angle itself.

Since we have constant density, we can relate the optical depth to the physical depth by

$$s = \frac{\tau}{\tau_{\text{max}}},$$

where τ_{max} is the total vertical optical depth.

For one photon, the algorithm that creates and then moves the photon to the surface goes like this:

1. Create a photon at $x = 0$, $y = 0$, $z = 0$.
2. Pick a direction for the photon to go in; this should be isotropic (i.e., any direction with equal probability). To do that, you need to choose the angles θ and ϕ from spherical coordinates:

   $$\mu = \cos(\theta) = 2\xi - 1, \quad \phi = 2\pi\chi,$$

   where ξ and χ are random numbers uniformaly drawn from $(0, 1)$. (Why like this? See `http://corysimon.github.io/articles/uniformdistn-on-sphere/` for details.)
3. Assume the photon travels in that direction until it scatters; the optical depth is goes should be chosen from

   $$\tau = -\log(1 - \xi),$$

   where ξ is another uniform random number between $(0, 1)$. The actual distance it goes is given by s, though, so the new position of the photon before it scatters is given by

   $$x_{i+1} = x_i + s\sin(\theta)\cos(\phi), \quad y_{i+1} = y_i + s\sin(\theta)\sin(\phi), \quad z_{i+1} = z_i + s\cos(\theta).$$

   We repeat the process (scatter, move the photon, scatter, etc) until one of two things happen:

   (a) The photon reaches the surface ($z > z_{\text{max}}$).

   (b) The photon turns around and goes back into the star ($z < 0$). In this case, just throw it away and start again.

As a first stab at this game, let's follow just one photon as it makes it's way to the surface. For concreteness, let's all use

$$z_{\text{max}} = 1, \quad \tau_{\text{max}} = 10.$$

Move the photon through the atmosphere, and plot it's journey. This is a 3D code, so maybe just plot (y, z). Remember, ignore the photons that go back into the star.