Modeling Stability of Magnetic Fields in Magnetars

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Background

Neutron Stars - Remnant core of massive star

- ➔ Composed almost completely of neutrons
- ➔ Core densities ~ 10^{15} g cm^{-3}

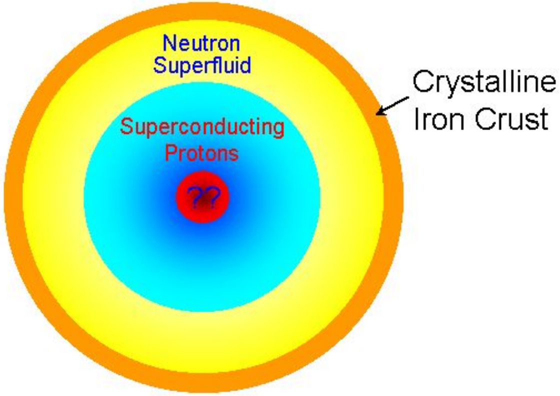


Image credit: Ohio State University

Pulsars - Rotating neutron star with radio beam and B-field

- ➔ B-field ~ 10^{12} - 10^{13} gauss
- ➔ Average rotational period: ~ 0.7 s

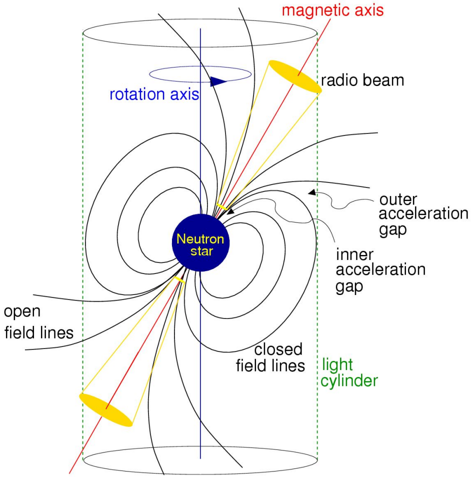


Image credit: NRAO

Magnetars - Highly magnetic pulsars

- B-field ~ 10^{15} gauss, *strongest in universe*
- Fields alter structure of star via Lorentz force.

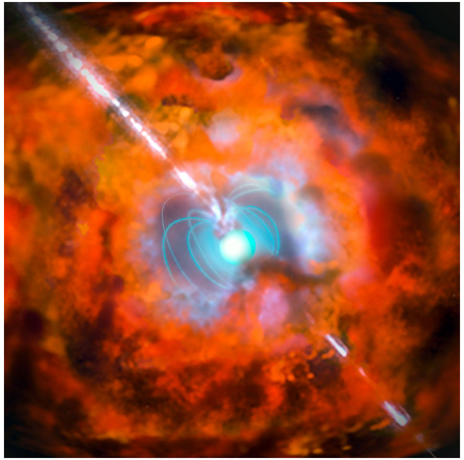
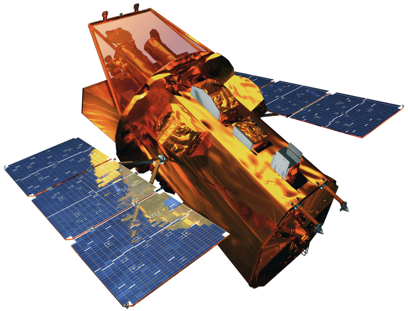


Image credit: ESO

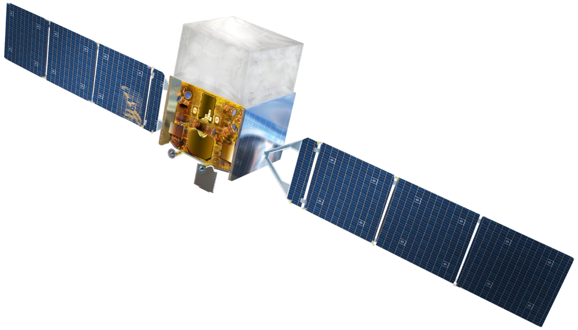
Why Study Magnetars?

- ➔ Rapidly growing field: Rate of detection dramatically increased following launch of Swift and Fermi space telescopes.
  - ◆ First detection in 1979.
- ➔ Potential sources of gravitational waves (GWs):
  - ◆ If the magnetic axis is not aligned with the rotational axis, deformation due to B-field will not rotate symmetrically, causing a time-varying quadrupole, forming GWs.
- ➔ Detection of GWs from Magnetars would inform, set boundaries for models of neutron star composition.



Swift Space Telescope: Launched November, 2004

NASA E/PO, Sonoma State University/Aurore Simonnet



Fermi Gamma-ray Space Telescope: Launched June, 2008

NASA E/PO, Sonoma State University/Aurore Simonnet

Research Goals

- Model stellar structure:
  - Describe how state variables such as pressure vary
  - Achieve stable equilibrium between stellar pressure and force of gravity
- Model magnetic field structure:
  - Verify stability of particular model configurations
  - Show stability is maintained through evolution in time
- Verify that additional phenomena within simulations have physical justifications.

Stellar Model: Structural Equations

➔ Hydrostatic Equilibrium: $\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$ *

➔ Polytropic equation of state (EOS): $P(\rho) = K\rho^{\gamma}$

- Determine γ via choice of polytropic index '*n*' via the relationship: $\gamma = \frac{n+1}{n}$

➔ The choice of n = 1 suits density profile of neutron stars well.

Based on our choice of EOS, we can completely describe stellar structure via the following equations

$$\boxed{P_{n=1}(r) = K\rho(r)^2 \qquad \rho(r) = \rho_c \frac{\sin(\pi r/R)R}{r\pi}}$$

$$r < R$$

ρ_c is core density ~ 10^{15} g cm^{-3}

* M_r , known as the *interior mass*, is the mass interior to the sphere of radius r

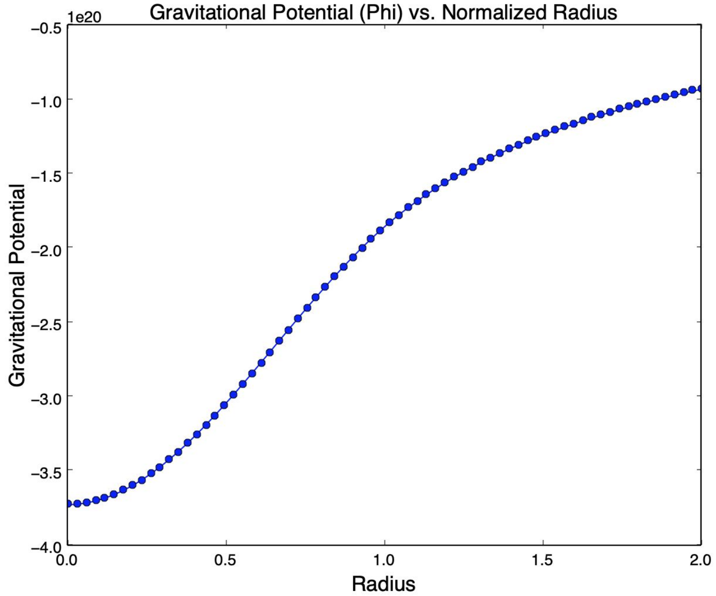
Stellar Model: Gravitational Potential

- Hydrostatic equilibrium requires a balancing force opposing gravity. We seek solutions to the spherically symmetric form of Poisson's equation for gravitational potential energy:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = 4\pi G\rho$$

- φ(r) in three equations:
  - φ(r = 0)
  - φ(r) for 0 < r < 1.0
  - φ(r) for r ≥ 1.0

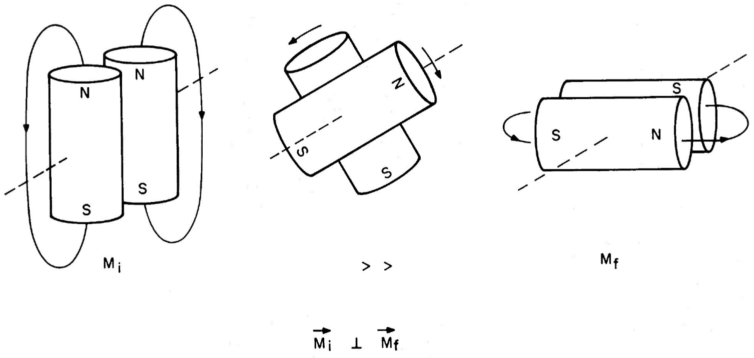
- We require φ(r) to be piecewise continuous across the computational domain:



Magnetic Field Structure

- Flowers & Ruderman (1977): Stable field configurations require poloidal and toroidal field components.
  - Poloidal and Toroidal components act in tandem; when one wanes the other strengthens, allowing dynamic stability despite perturbations.
- Toroidal component is non-zero only interior to r = R_{star}, Poloidal component exists everywhere.

Unstable poloidal configuration*



Poloidal component stabilized by toroidal field*

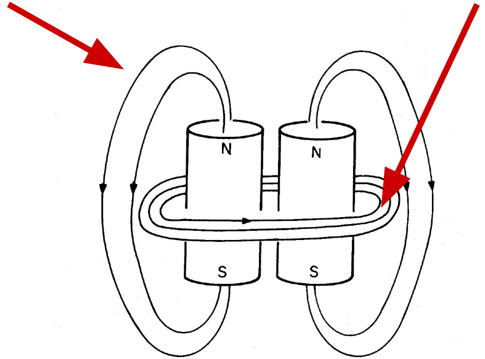


*Image Credit: E G. Flowers and M A. Ruderman. "*Evolution of pulsar magnetic fields*". In: The Astrophysical Journal 215 (June 1977), pp. 302–310.

Magnetic Field Model

- Via Haskell et al. 2008, specific to the n = 1 polytrope model
- Stable as mixed poloidal-toroidal field

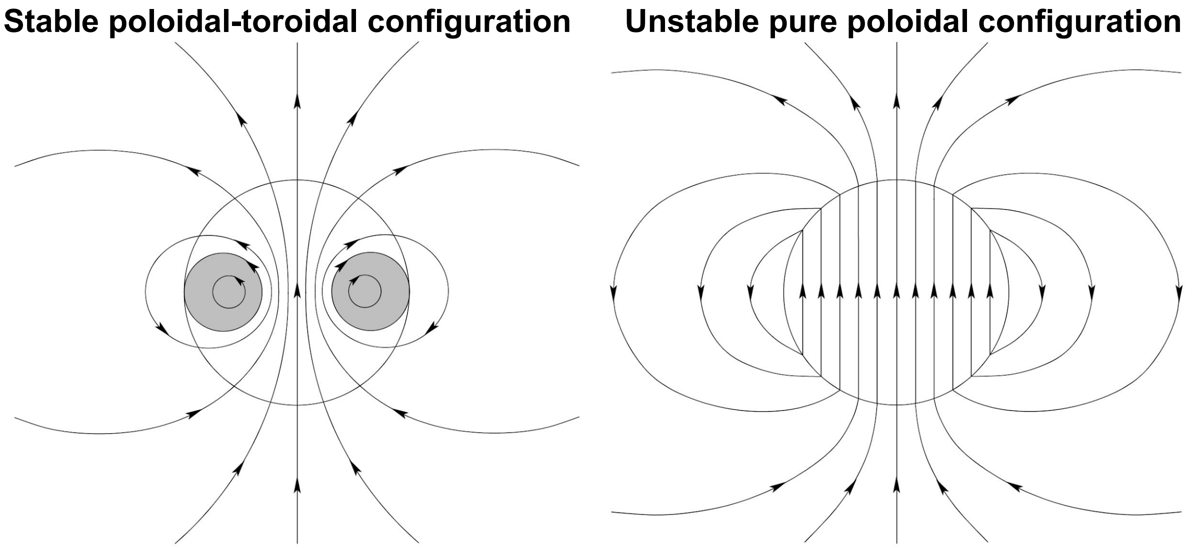


Image credit: J. Braithwaite. "*The stability of poloidal magnetic fields in rotating stars*". In: Astronomy and Astrophysics (November 2018)

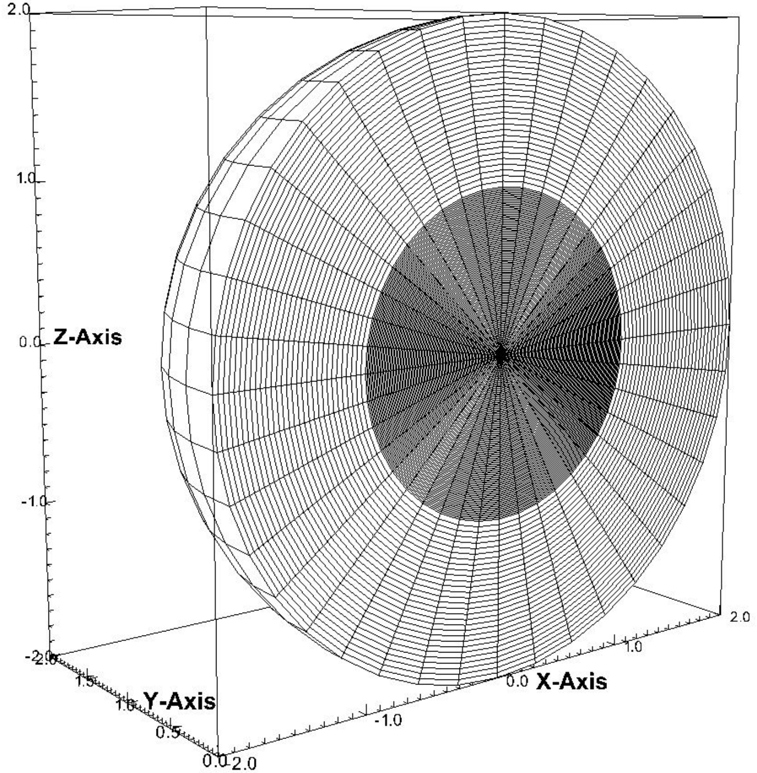
Methods

Software:

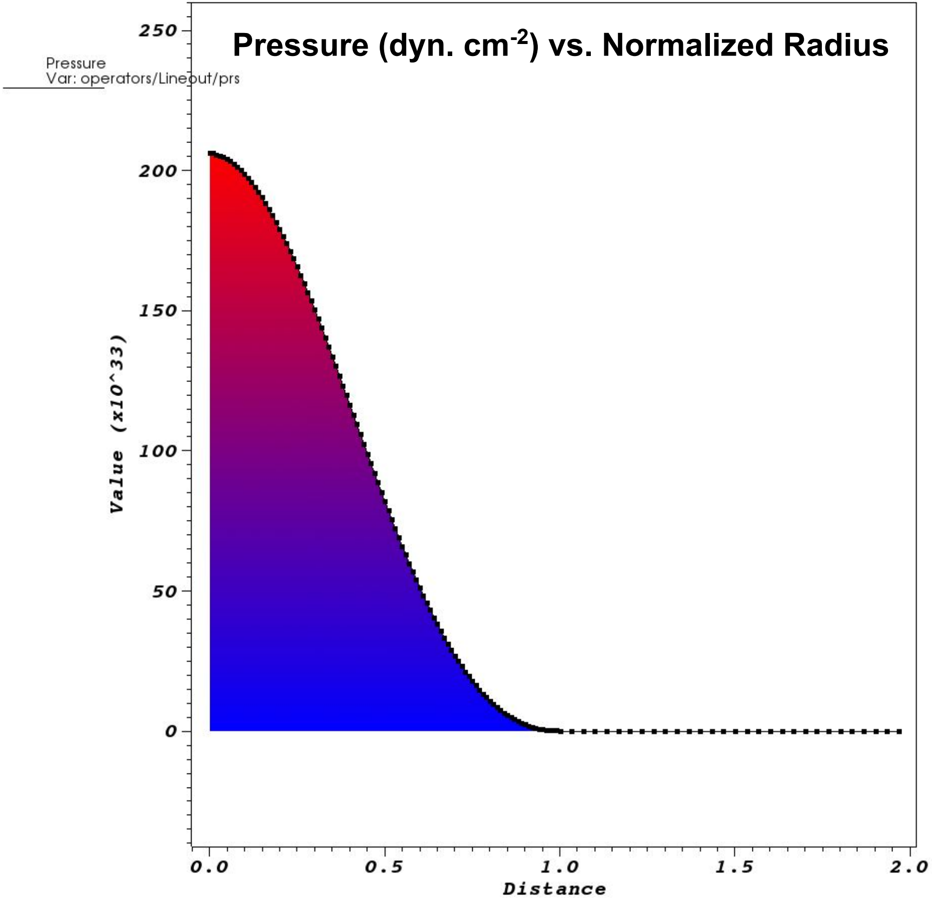
➔ PLUTO - Computational package for evolving computation
➔ VisIt - Open source visualization and graphical analysis software

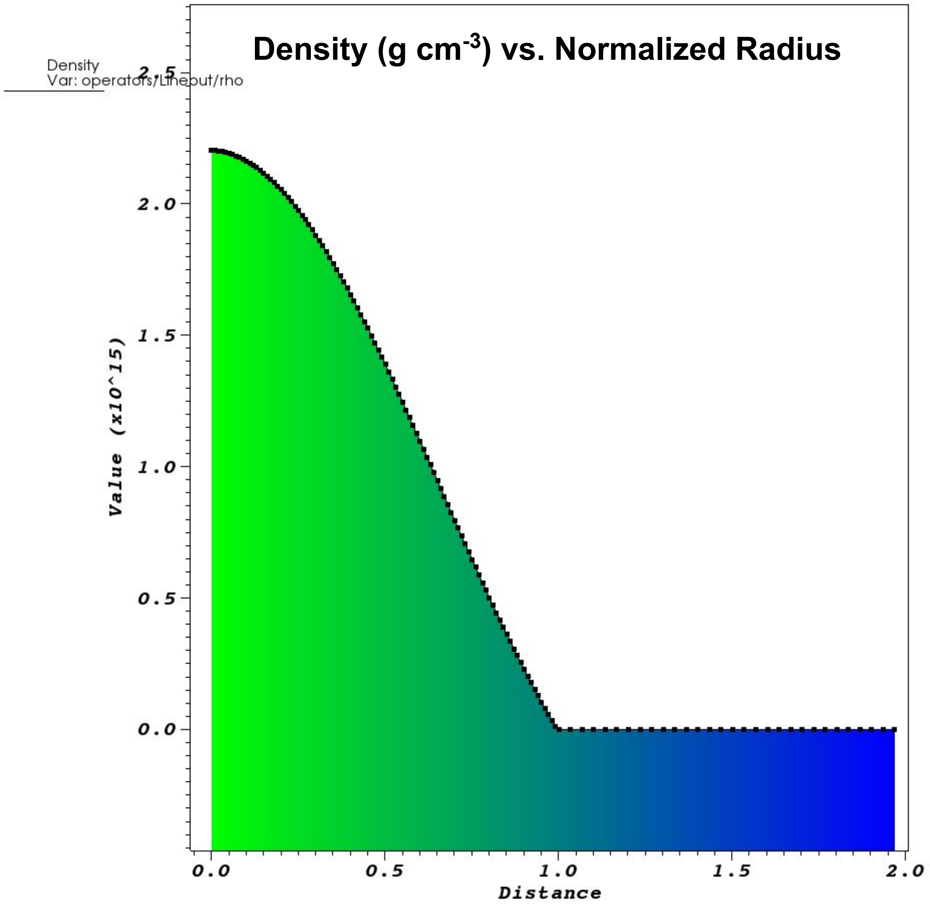
The Computational Domain:

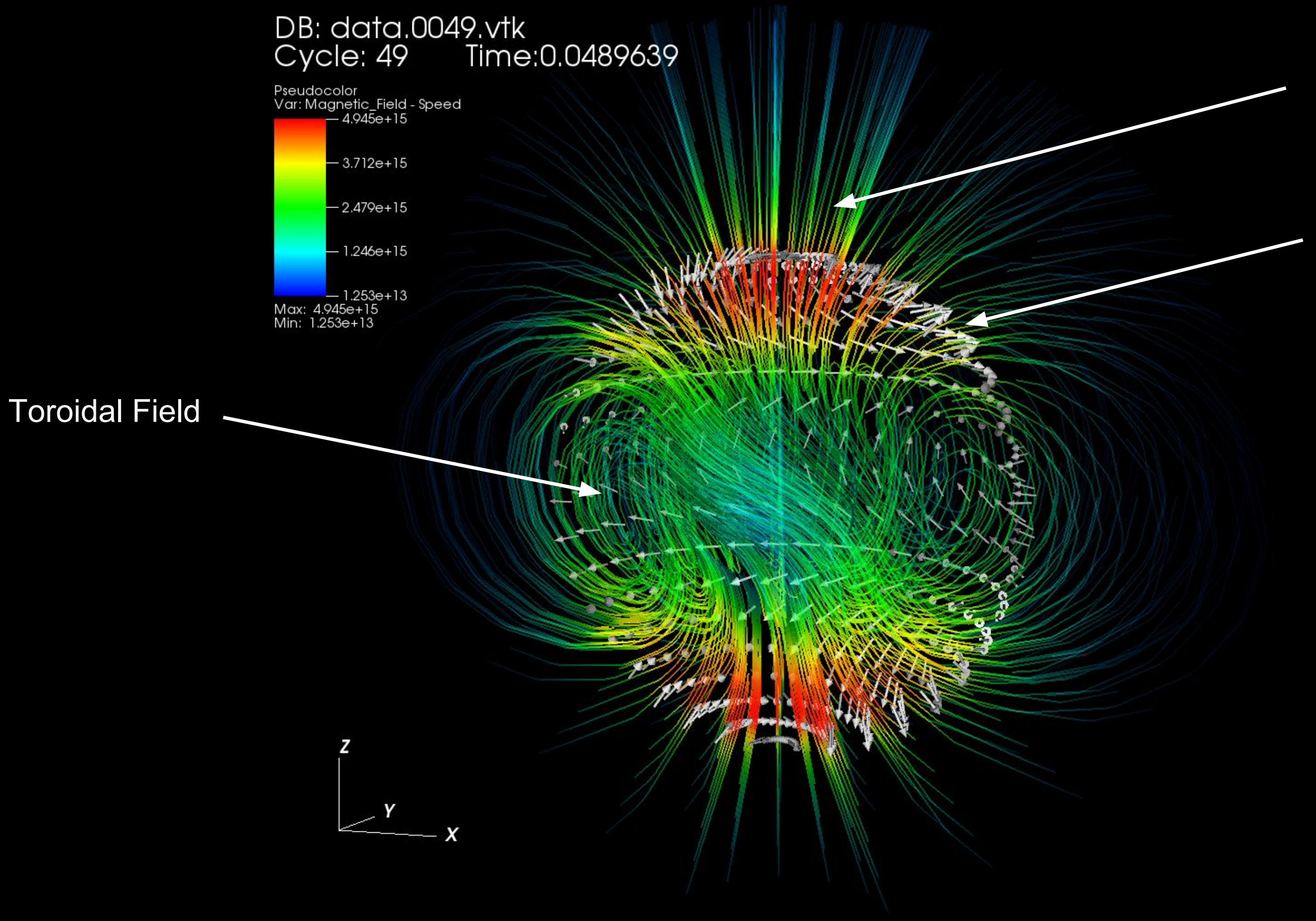
- Spherical domain for evolving simulations.
- 100 equally spaced points for $0 \leq r \leq 1.0$, 30 equally spaced points for $1.0 < r \leq 2.0$
- 20 equally spaced points in both the domains of Θ and φ.

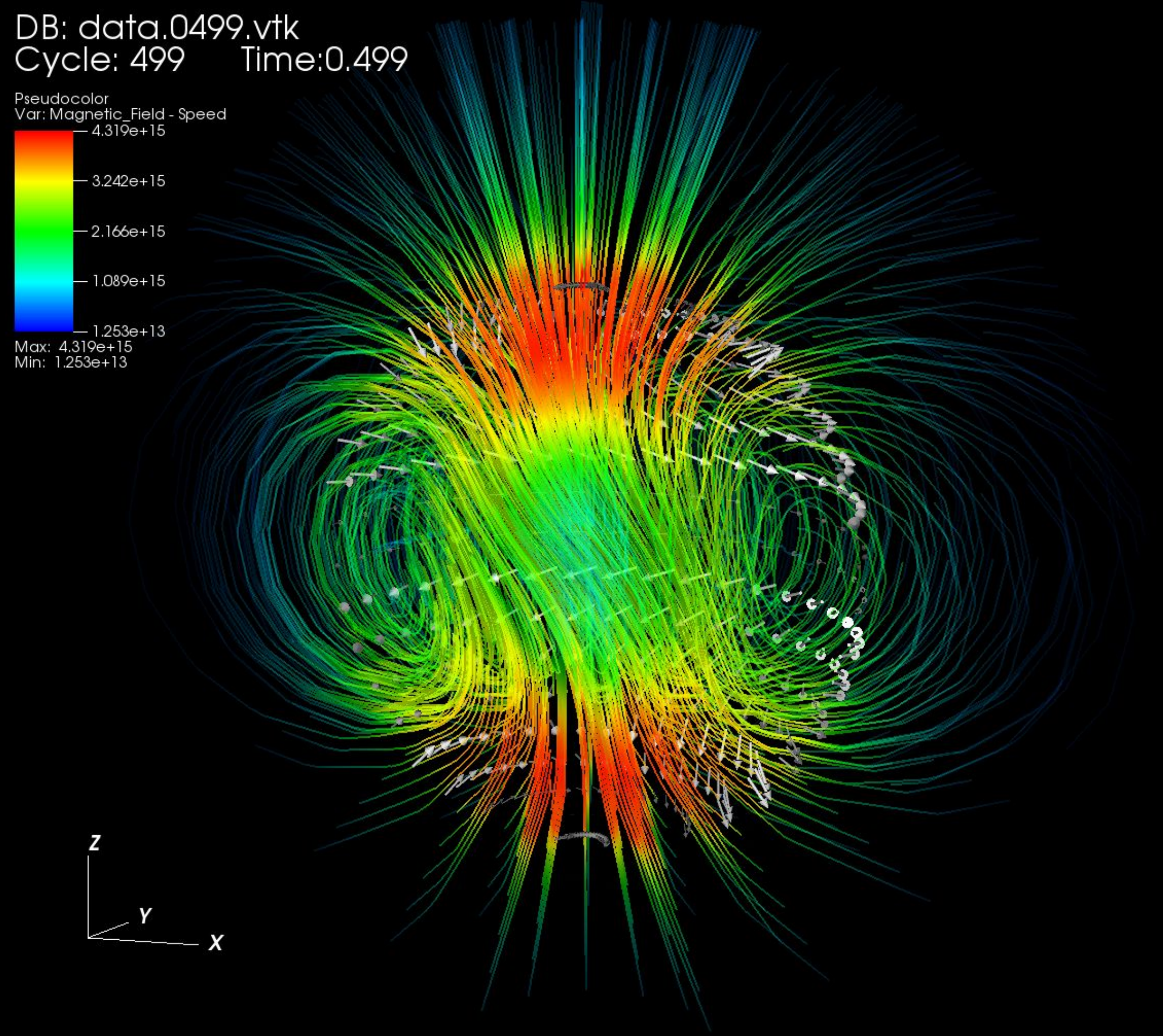


A hemisphere of the computational domain


Pressure (dyn. cm^-2) vs. Normalized Radius
Pressure
Var: operators/Lineout/prs
Value (x10^33)
250
200
150
100
50
0
0.0
0.5
1.0
1.5
2.0
Distance

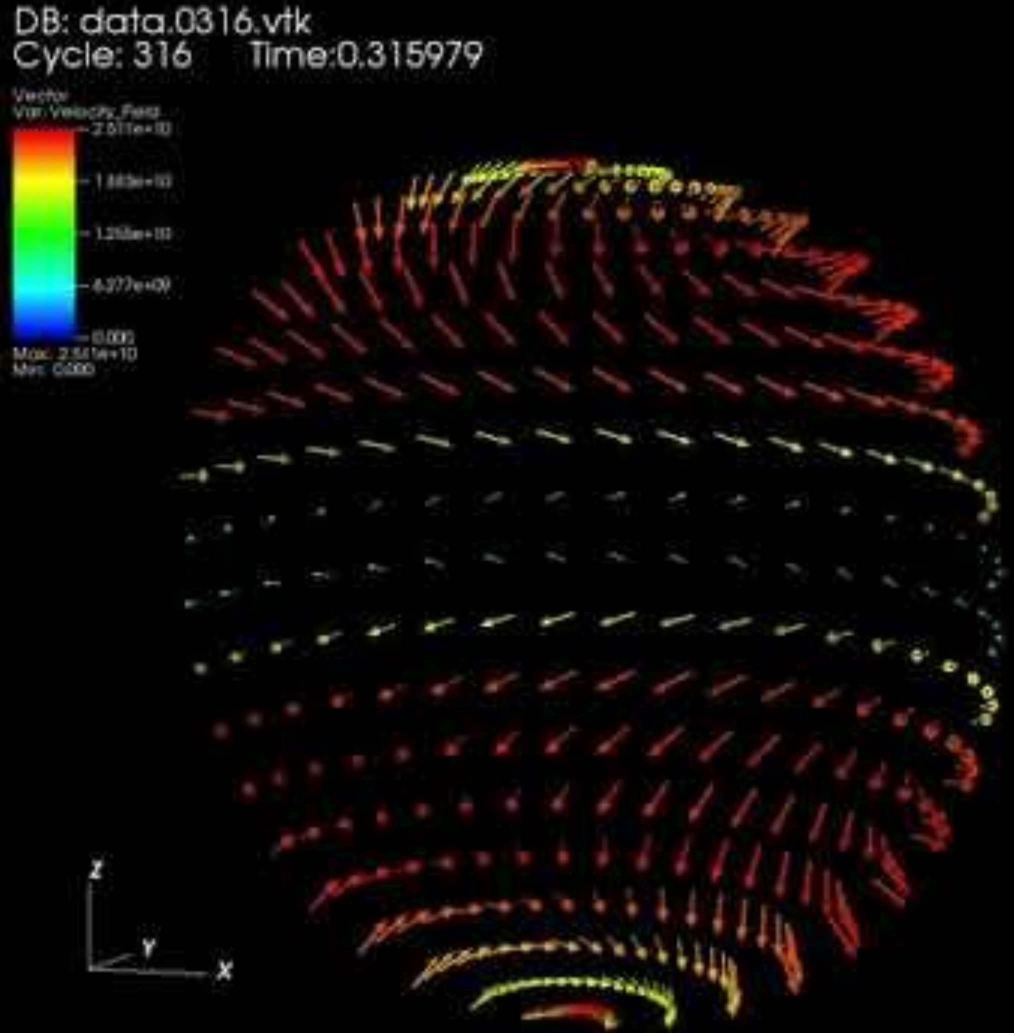

Density (g cm^-3) vs. Normalized Radius
Density
Var: operators/Lineout/rho
Value (x10^15)
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
Distance


DB: data.0049.vtk
Cycle: 49 Time:0.0489639
Pseudocolor
Var: Magnetic_Field - Speed
4.945e+15
3.712e+15
2.479e+15
1.246e+15
1.253e+13
Max: 4.945e+15
Min: 1.253e+13
Poloidal Field
Velocity Vector Field
Toroidal Field
Z
Y
X



DB: data.0499.vtk
Cycle: 499 Time:0.499
Pseudocolor
Var: Magnetic_Field - Speed
4.319e+15
3.242e+15
2.166e+15
1.089e+15
1.253e+13
Max: 4.319e+15
Min: 1.253e+13
Z
Y
X

Variable:
Velocity Vector
Field



Verification and Validation

- Verify model in hydrostatic equilibrium:
    - Gravitational potential balances pressure gradient.
    - Mass does is not spontaneously ejected from stellar surface.
- Confirm stability of magnetic field model:
    - Poloidal and toroidal field components self-regulate, evolve with stability under time.
- Velocity vector field suggests over time, velocity perturbations settle near equatorial plane of star.
    - This plane is also referred to as the neutral line, where the magnetic field is minimized.
    - This behavior is validated given the Lorentz force in this region is also minimized

Future Work: Improving our Model

- Relativistic Equation of State:
    - Tolman-Oppenheimer-Volkoff (TOV) equation (replaces newtonian formulation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}$$

    - EOS as a function of many variables:

$$P = P(\rho, T, x_p, \ldots)$$

- As we change our EOS, we must choose a new magnetic field model:
    - Fujisawa & Kisaka (Sept. 2014) via SLy EOS.

Acknowledgements

- I wish to extend appreciation and thanks to my co-mentors Dr. Michelle Kuchera and Dr. Kristen Thompson.
- I also thank former Duke undergraduate-student Emily Kuhn for providing an important starting point in informing my research.

Additional Photo Credits

- Slide 2
  - Neutron star structure: http://www.astronomy.ohio-state.edu/~pogge/Ast162/Unit3/extreme.htm
  - Pulsar schematic: https://www.cv.nrao.edu/course/astr534/Pulsars.htmll
  - Artists rendition of magnetar: http://www.eso.org/public/images/eso1527a/
- Slide 3
  - Swift Telescope:http://swift.sonoma.edu/resources/multimedia/images/
  - Fermi Telescope: https://science.nasa.gov/toolkits/spacecraft-icons

Requirements & Assumptions

- Our assumption of a polytropic EOS neglects the influence of temperature and composition on pressure, and a more realistic EOS will be a function of many variables:

$$P = P(\rho, T, x_p, ...)$$

- Temperature plays an important role in young neutron stars, adding a buoyancy force. Additionally, young neutron stars develop separate superfluids for neutrons and protons, which can be accounted for by proton fraction 'x_p' dependence in the EOS.

- Our polytropic EOS follows from the ideal gas thermal EOS given we require an adiabatic relationship between pressure and density.
- Model does not account for rotation, justified by relatively slow rotation of magnetars: mean rotational period of known magnetars ~ 6.7 s