Modeling Structural and Magnetic Field Stability in Magnetars

Sam Frederick, Dr. Kristen Thompson, Dr. Michelle Kuchera

Department of Physics, Davidson College, NC 28035

Abstract

Possessing the strongest magnetic fields (~ 10^{15} gauss) in the Universe, a group of pulsars known as magnetars represent an extremum of our understanding of physical phenomena. Recent research has sought to explain the structure of these magnetic fields, their evolution, and their stability. In theory, the strength of such fields induces topological deformations which give way to a time-varying quadrupole, leading to the formation of gravitational waves. With the advent of gravitational wave astronomy following the detection of multiple black hole and neutron star mergers, magnetars represent a source of continuous gravitational waves (GWs). Although recent work regarding the study of magnetars has sought to improve the accuracy of magnetic field models interior and exterior to the stellar surface by taking into consideration the various structural zones of the star, little work has been completed towards estimating the wave strain, or intensity, of GWs resulting from such magnetars. Here, we present a study of model stability for stellar structure and magnetic field models pertaining to magnetars. We base our structural model on an n = 1 polytropic equation of state (EOS) and verify that our model satisfies the condition of hydrostatic equilibrium. Our magnetic field model combines toroidal and poloidal field components. We show that such a mixed-field configuration is necessary for field stability. This work represents a first step in modelling magnetars, which we intend to further refine with a more physically representative EOS for the purpose of providing upper-limit estimates for the wave strain of potential GWs resulting from known magnetar sources.

Introduction

First discovered in 1979, magnetars join the classification of pulsars, and are distinguished by their exceptional magnetic field strength (~ 10^{15} gauss), giving these objects the strongest magnetic fields in the universe.

The strength of these magnetic fields alter the structure of magnetars via the Lorentz force. If the star's magnetic field axis is not aligned with the rotational axis, these deformations may lead to the production of continuous gravitational waves (GWs).

This ongoing work seeks to develop a computational model from which we intend to determine potential GW signal strength for known magnetar sources.

Research Goals

We intend to achieve the following:

- Describe how state variables such as pressure vary interior to the stellar model.
- Achieve hydrostatic equilibrium in balancing the structural pressure gradient with the force due to gravity.
- Verify stability of chosen magnetic field model configuration.
- Verify that additional phenomena in our simulation have firm physical justification.

Stellar Model

For our model to be structurally stable, we require that our model satisfies the following relation guaranteeing hydrostatic equilibrium:

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$

To form equations for state variables such as pressure, we choose a polytropic equation of state (EOS), conveniently directly relating density and pressure:

$$P(\rho) = K\rho^{\gamma}$$

The adiabatic index, γ is based on the following relationship:

$$\gamma = \frac{n+1}{n}$$

where the choice of n = 1 serves as a fair approximation of neutron star structure such that we can completely describe the magnetar's stellar structure by the following equations:

$$P_{n=1}(r) = K\rho(r)^2 \qquad \rho(r) = \rho_c \frac{\sin(\pi r/R)R}{r\pi} \qquad r < R$$

To fully satisfy the equation for hydrostatic equilibrium, we construct a gravitational potential by determining solutions to the spherically symmetric form of Poisson's equation.

In order to satisfy boundary conditions, our resulting potential is a three-part piecewise continuous function:

- $\varphi(r = 0)$
- $\varphi(r)$ for $0 < r < 1.0$
- $\varphi(r)$ for $r \geq 1.0$

Magnetic Field Model

Stable magnetic field configurations for pulsars require mixed poloidal and toroidal field components (Flowers and Ruderman, 1977). **Figure 1** below shows stable and unstable field configurations. Stability is maintained as poloidal and toroidal components act in tandem; as one component wanes, the other strengthens.

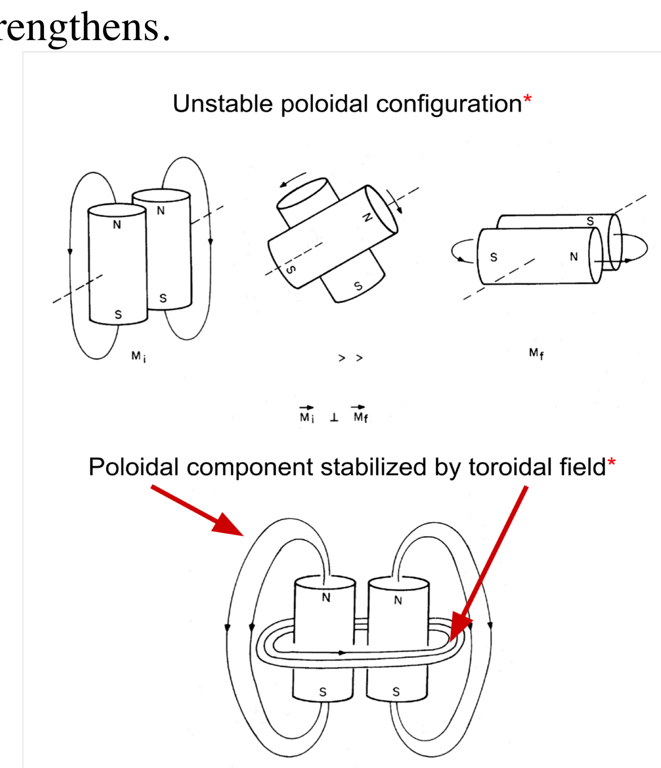

Figure 1: Image of unstable and stable field configurations. Image credit (Flowers and Ruderman, 1977)

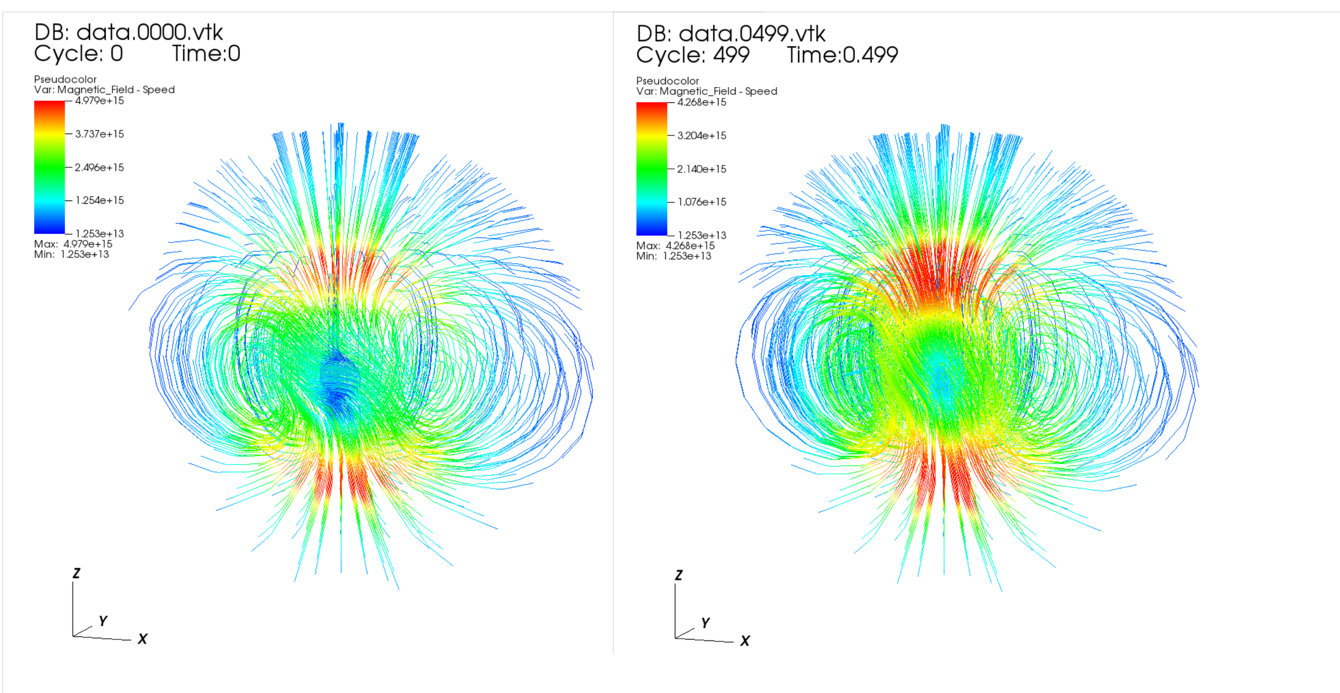

Figure 2: Magnetic field configuration in simulations.

Computational Methods

For the purposes of simulation we implement the following software:

- PLUTO – Computational package for implementing structural and magnetic field equations and evolving magnetohydrodynamic (MHD) phenomena.
- VisIt – Open-source visualization and graphical analysis software.

We implement a spherical computational domain. Figure 4 shows a hemisphere of this domain, detailing higher radial resolution interior to $r < 1.0$ (where radius is in terms of normalized radial units, $r = R_{star} \equiv 1.0$). Higher resolution interior to the star minimizes computational artifacts across regions where pressure and density rapidly change.

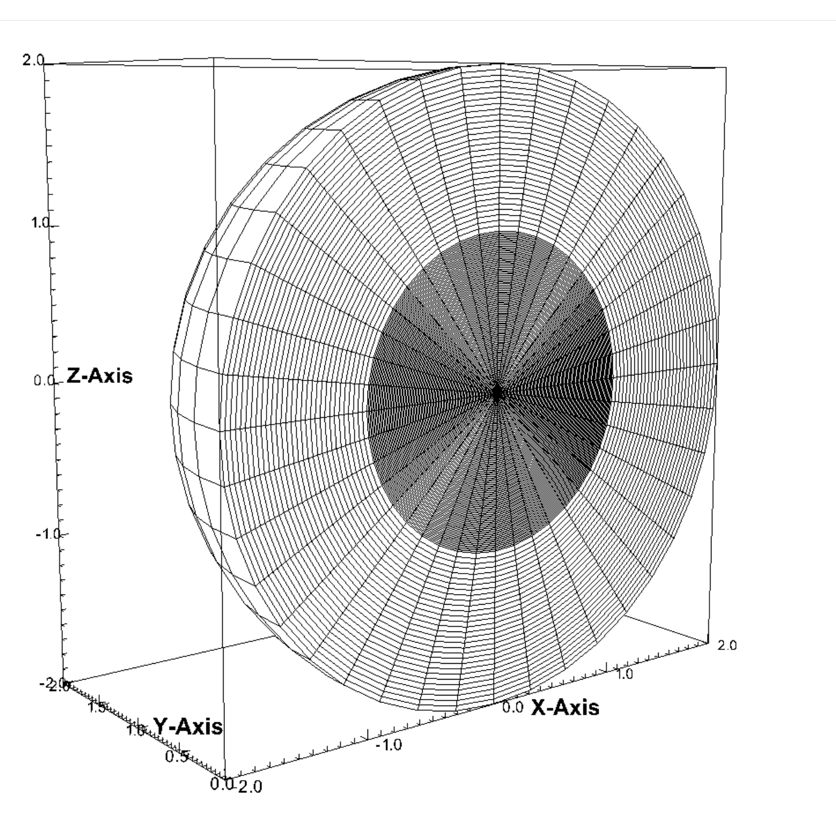

Figure 3: A hemisphere displaying the grid mesh composing the computational domain.

Results

We verify that our model is in hydrostatic equilibrium:

- Pressure and density profiles remain nearly constant throughout simulation; stellar composition remains consistent.

We confirm stability of chosen magnetic field model:

- **Figures 2** and **4** display stable field evolution in simulations.

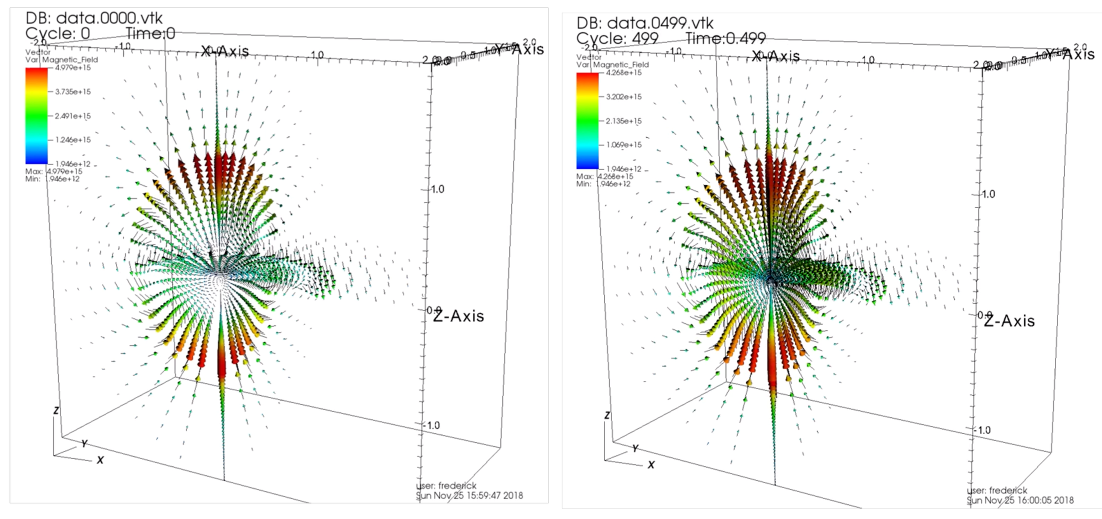

Figure 4: Three-slice vector plot for the magnetic field.

Under evolution, velocity perturbations settle near the equatorial plane of the star as shown in **Figure 5**.

- This plane is also referred to as the neutral line, where the magnetic field is minimized. The Lorentz force in this region is also minimized, resulting in less acceleration due to the magnetic field.

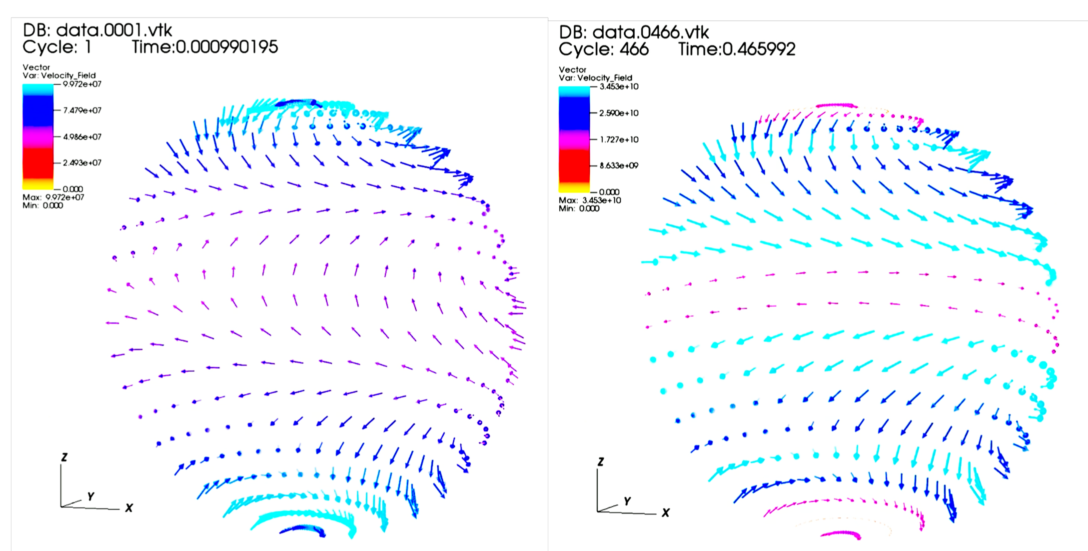

Figure 5: Vector plots for velocity field near the surface of the star. Notice that velocity is minimized near the equatorial plane.

Future Work

We seek to improve this model with the following implementations:

- Relativistic EOS: Tolman-Oppenheimer-Volkoff (TOV) equation (replaces Newtonian formulation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{m(r)c^2}\right)\left(1-\frac{2Gm(r)}{rc^2}\right)^{-1}$$

  - Such an EOS will be a function of many variables:

$$P = P(\rho, T, x_p, \ldots)$$

- A change of EOS will require the choice of a new magnetic field model (Fujisawa & Kisaka 2014) via the SLy EOS.

Acknowledgements

Appreciation is extended to research advisors Dr. Kristen Thompson and Dr. Michelle Kuchera for continued support of this research. Additional thanks to the Society of Physics Students for providing financial support for this presentation.

References

1. Kuhn, E. "Magnetic Field Evolution in Magnetars with Gravitational Wave Applications." Undergraduate Thesis, Duke University (2016).
2. E G. Flowers and M A. Ruderman. "Evolution of pulsar magnetic fields". In: The Astrophysical Journal 215 (June 1977), pp. 302–310.
3. Kotaro Fujisawa and Shota Kisaka. "Magnetic field configurations of a magnetar throughout its interior and exterior core, crust and magnetosphere." In: Monthly Notices of the Royal Astronomical Society. 445 (Sept. 2014).