Particle-in-cell plasma simulation with OmpSs-2

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Outline

Introduction

What exactly is a plasma?
Talk about 2D only.
Introduce species.
Plasma must be neutral.

Vlaslov equation

The Vlaslov equation describes the evolution of a plasma, with long range iteration between particles.

$$\frac{\partial f_j}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_j}{\partial \boldsymbol{x}} + \frac{q_j}{m_j} \left(\boldsymbol{E} + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} \right) \cdot \frac{\partial f_j}{\partial \boldsymbol{v}} = 0 \tag{1}$$

Where $f_j(\boldsymbol{x}, \boldsymbol{v}, t)$ is the distribution function of the specie j, and the fields electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ are determined by Maxwell equations.

Maxwell equations

The evolution of the fields is governed by the Maxwell equations

$$
\begin{aligned}
\nabla \cdot \boldsymbol{E} &= \frac{\rho}{\epsilon_0} & \qquad \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \cdot \boldsymbol{B} &= 0 & \qquad \nabla \times \boldsymbol{B} &= \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)
\end{aligned} \tag{2}
$$

Where ρ is the charge density distribution, $\boldsymbol{J}$ the current field and ϵ_0 and μ_0 are constants.

Electrostatic approximation

In our model, we will only have a **strong fixed magnetic field** $\boldsymbol{B}_0$ and the Maxwell equations can be simplified.

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As the electric field is irrotational, we can introduce a scalar field ϕ, the electric potential, and rewrite $\boldsymbol{E}$ as a gradient

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}, \quad \boldsymbol{E} = -\nabla\phi \tag{4}$$

This approximation is called **electrostatic** as opposed to the **electromagnetic** case, when $\boldsymbol{B}$ varies with time.

The particle-in-cell method

Solving the Vlaslov equation directly is very **computationally expensive**. The current solution is to introduce a spatial grid where the Maxwell equations are solved.

The distribution function f_j is then modeled by macro-particles which interact with the grid by an interpolation method.

This method is known as **particle-in-cell** (PIC).

The particle-in-cell method

The space of length $\boldsymbol{L}$ is discretized into grid points with a separation $\Delta\boldsymbol{x}$.

The scalar fields ρ and ϕ are now matrices of size $\boldsymbol{G} = (G_x, G_y)$

The vector field $\boldsymbol{E}$ is decomposed into E_x and E_y, which are now matrices of the same size.

The particle-in-cell method

The PIC method has four main steps

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- **Interpolation of electric field**: The electric field is interpolated back to the particle positions.
- **Particle motion**: The force is computed from the electric field at the particle position and the particle is moved accordingly.

Charge accumulation

At each grid point g at $\boldsymbol{x}$, the charge of each particle p at $\boldsymbol{x}_p$ is accumulated using an interpolation function W.

$$\rho(\boldsymbol{x}) = \sum_p q\, W(\boldsymbol{x} - \boldsymbol{x}_p) + \rho_0 \tag{5}$$

Using linear interpolation, we can define W for two dimensions as

$$W(\boldsymbol{x}) = \begin{cases} \left(1 - \dfrac{|x|}{\Delta x}\right)\left(1 - \dfrac{|y|}{\Delta y}\right) & \text{if } -\Delta\boldsymbol{x} < \boldsymbol{x} < \Delta\boldsymbol{x} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Charge accumulation

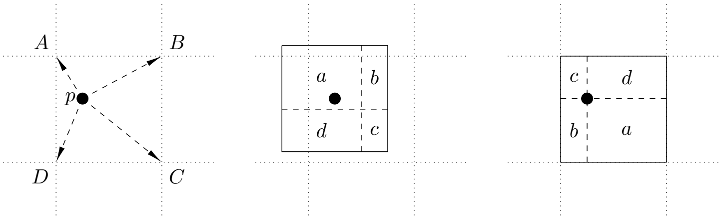

Figure: Interpolation of particle p charge into the four grid points A to D.

Field equations

End

Thanks for your attention.