

1. How many different ways are there to pay a \$9.75 bill if only dimes and quarters are available?

(a) 39 (b) 19 (c) 20 (d) 40 (e) None of the above

SOLUTION (c): We need to find the number of non-negative integer solutions of the equation $10x + 25y = 975$, or $2x + 5y = 195$, or $2x = 195 - 5y$. Because the right hand side is divisible by 5, x must also be divisible by 5, so $x = 5d$ for some non-negative integer d . Then the equation becomes $10d = 195 - 5y$ or $2d = 39 - y$. So the number of solutions will be the number of non-negative even integers less than or equal to 39. There are $\frac{39+1}{2} = 20$ such numbers.

2. How many different ways are there to pay a \$9.75 bill if only dimes and quarters are available?

(a) 39 (b) 19 (c) 20 (d) 40 (e) None of the above

3. The digits of the whole numbers from 1 to 99 are concatenated in order to form the number N :

$$N = 1234567891011121314 \dots 979899$$

Which of the following is true?

- (a) N is divisible by 3 but not by 6 and 9
 (b) N is divisible by 3 and 6 but not by 9
 (c) N is divisible by 3 and 9 but not by 6
 (d) N is not divisible by any of 3, 6 or 9
 (e) None of the above

SOLUTION (c): The sum of the digits on N is $10(1 + 2 + 3 + \dots + 9) + 10(1 + 2 + 3 + \dots + 9) = 20 \cdot 45 = 900$ which is divisible by 3 and 9, so N is divisible by both 3 and 9.

N is not divisible by 2 since it ends in 9, so N cannot be divisible by 6.

4. A circular table has exactly 60 chairs around it. There are N people seated around the table. The next person coming to the table will have to be seated next to an occupied seat. Find the smallest possible value of N .

(a) 15 (b) 20 (c) 30 (d) 40 (e) 58

SOLUTION (b): For the next person to have to sit next to an occupied seat, there cannot be three consecutive chairs currently unoccupied (otherwise the next person would simply sit in the middle of the three empty chairs). Therefore for every three consecutive chairs at least one of them has to be occupied. Since we are looking for the smallest N , exactly one of the three will have to be occupied, and each two people will have to have two empty seats between them. Therefore the number of people sitting at the table is $1/3$ of the number of seats, or 20 people.