$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \ell$$

$$\iff \forall \epsilon > 0,\ \exists \delta > 0 \text{ s.t. if } 0 < |\Delta x - 0| < \delta, \text{ then } \left| \frac{\Delta y}{\Delta x} - \ell \right| < \epsilon$$

$$\iff \forall \epsilon > 0,\ \exists \delta > 0 \text{ s.t. if } 0 < |x - x_0| < \delta, \text{ then } \left| \frac{f(x) - f(x_0)}{x - x_0} - \ell \right| < \epsilon$$