$$\begin{split} &\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \ell \\ \iff \forall \epsilon > 0, \ \exists \delta > 0 \text{ s.t. if } 0 < |\Delta x - 0| < \delta, \ \text{then } \left| \frac{\Delta y}{\Delta x} - \ell \right| < \epsilon \\ \iff \forall \epsilon > 0, \ \exists \delta > 0 \text{ s.t. if } 0 < |x - x_0| < \delta, \ \text{then } \left| \frac{f(x) - f(x_0)}{x - x_0} - \ell \right| < \epsilon \end{split}$$