

$$a \cos t + b \sin t = A \cos(t + \varphi) ?$$

$$\begin{aligned} & a \cos t + b \sin t \\ &= \frac{1}{2} (a e^{it} + a e^{-it} - i b e^{it} + i b e^{-it}) \\ &= \frac{1}{2} ((a - ib) e^{it} + (a + ib) e^{-it}) \end{aligned}$$

$$\text{Notons } a - ib = d e^{i\theta}.$$

$$\begin{aligned} & a \cos t + b \sin t \\ &= \frac{1}{2} (d e^{i\theta} e^{it} + d e^{-i\theta} e^{-it}) \\ &= d \cdot \frac{e^{i(\theta+t)} + e^{-i(\theta+t)}}{2} \\ &= d \cos(t + \theta) \end{aligned}$$

$$A = d = \sqrt{a^2 + b^2}$$

$$\varphi = \theta = \arg(a - ib) \rightarrow \pm \frac{\pi}{2} \text{ ou } \arctan\left(-\frac{b}{a}\right) \\ \text{ou } \arctan\left(-\frac{b}{a}\right) + \pi$$

On peut résumer en posant $\arctan\left(\frac{b}{a}\right) = \frac{\pi}{2}$ si $b \neq 0$.