$$|G/\mathrm{Stab}\,x| = |\mathrm{Orb}\,x|$$

si G agit sur $X \ni x$.

- On note $g_1 = e, g_2, \dots, g_k$ tq $g_i\,\mathrm{Stab}\,x \neq g_j\,\mathrm{Stab}\,x$ si $i \neq j$, ie tq $g_i\,\mathrm{Stab}\,x \cap g_j\,\mathrm{Stab}\,x = \emptyset$.

En effet, $h \in g_i\,\mathrm{Stab}\,x \cap g_j\,\mathrm{Stab}\,x$ donne $h = g_i s_i = g_j s_j$, puis ~~$g_i g_j^{-1} = s_j s_i^{-1}$~~ $g_j^{-1} g_i = s_j s_i^{-1} \in \mathrm{Stab}\,x$, et donc $g_i\,\mathrm{Stab}\,x = g_i\,\mathrm{Stab}\,x$.

- On note $\mathrm{Stab}\,x = \{ s_1 = e, s_2, \dots, s_\ell \}$.

$$\begin{array}{l}
g_1 s_1 \searrow \\
g_1 s_2 \rightarrow \; g_1 x \\
\vdots \\
g_1 s_\ell \nearrow \\[1em]
g_2 s_1 \searrow \\
g_2 s_2 \rightarrow \; g_2 \cdot x \\
\vdots \\
g_2 s_\ell \nearrow \\
\vdots \qquad\quad \vdots
\end{array}$$

Mieux ! $g \cdot x = h \cdot x \iff g = h \cdot s_i$ ⚠

Approche découvreuse.

Clairement, $|G| = |\mathrm{Stab}\,x| \times |\mathrm{Orb}\,x|$

COMME LA PREUVE STANDARD, MAIS ICI ON VISUALISE TOUT.