

$$\sum (2k+1) \Leftrightarrow \sum k$$

$$\bullet \sum_{k=0}^p (2k+1) = (p+1)^2$$

$$\hookrightarrow n = 2p+1 : \sum_{k=0}^n k$$

$$\begin{aligned} &= \sum_{k=0}^p (2k+1) + \sum_{k=0}^p (2k+1-1) \\ &= 2(p+1)^2 - (p+1) \\ &= (2p+2-1)(p+1) \\ &= n \times \frac{n+1}{2} \end{aligned}$$

Pairs vers impaires

$$\hookrightarrow n = 2p+2 : \sum_{k=0}^n k$$

$$\begin{aligned} &= \sum_{k=0}^p (2k+1) + \sum_{k=0}^{p+1} (2k+1-1) \\ &= (p+1)^2 + (p+2)^2 - (p+2) \\ &= (p+1)^2 + (p+2)(p+1) \\ &= (p+1)(p+1+p+2) \\ &= \frac{n}{2} (n+1) \end{aligned}$$

$$\bullet \sum_{k=0}^n k = \frac{n(n+1)}{2}$$

$$\begin{aligned} \hookrightarrow \sum_{k=0}^p (2k+1) &= 2 \sum_{k=0}^p k + p+1 \\ &= p(p+1) + p+1 \\ &= (p+1)^2 \end{aligned}$$