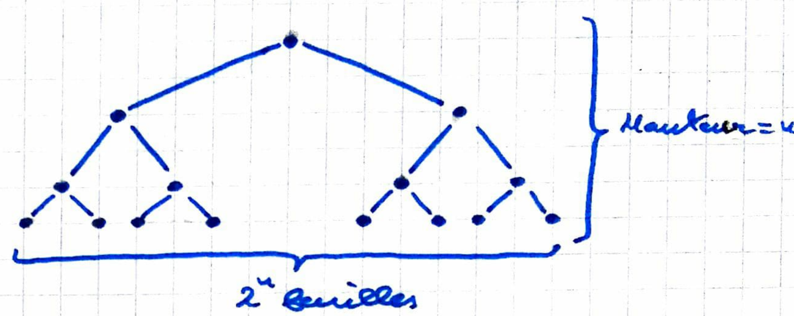
Evidence Pastel...

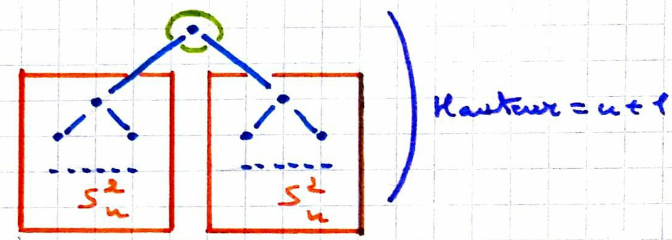
$$\forall n \in \mathbb{N}_{\geq 1},\ \forall q \in \mathbb{N}_{\geq 2},\ \underbrace{\sum_{k=0}^{n} q^k}_{S_n^q \text{ dans la suite...}} = \frac{q^{n+1}-1}{q-1}.$$

$\boxed{q=2}$

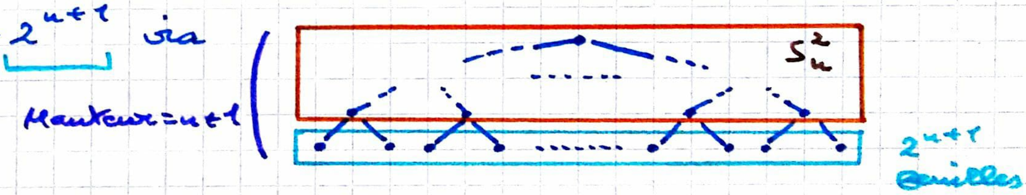
$$S_n^2 := \sum_{k=0}^{n} 2^k \longleftrightarrow$$



$$S_{n+1}^2 = \underbrace{1} + \underbrace{2S_n^2} \quad \text{via}$$



$$S_{n+1}^2 = \underbrace{S_n^2} + \underbrace{2^{n+1}} \quad \text{via}$$



On arrive à : $1 + 2S_n^2 = S_n^2 + 2^{n+1}$

$$\Longrightarrow S_n^2 = 2^{n+1} - 1$$

$\boxed{q \geq 3}$

$$1 + qS_n^q = S_n^q + q^{n+1}$$

$$\Longrightarrow (q-1)S_n^q = q^{n+1} - 1$$

$$\Longrightarrow S_n^q = \frac{q^{n+1}-1}{q-1}$$

Joli!

ⓘ Via les polynômes, on passe de $q \in \mathbb{N}_{\geq 2}$ à $q \in \mathbb{R} - \{1\}$.