CHAPTER 2

METHODS

2.1 Overview

A detailed neuromechanical model of the cat hindlimb with posture matched to the kinematics of a cat standing in its preferred stance was used to simulate the dynamic behavior of the system driven by muscle patterns that were tested for functional stability. Five muscle synergy patterns that produced each of the five corresponding synergy force vectors found from a previous experimental study were identified with static optimization using minimum energetic expenditure as the cost function. These minimum effort muscle synergy patterns were then used as a constant input to the muscles in the model. Forward dynamic simulations were run with joint configurations which varied around the original posture. The resulting behavior of the system in response to each perturbation was subjected to the criteria we present for assessing the relationship between muscle activation patterns and functional stability for postural control of a cat. We then tested whether the eigenvalues and eigenvectors of the system linearzed about an equilibrium point at the original posture could predict each the Lyapunov stability and the functional stability of the perturbed system.

2.2 Hindlimb muculoskeletal model

In order to assess the functional stability of a muscle activation pattern, we used a realistic model of the cat hindlimb (Fig. 1). The three-dimensional model of the right hindlimb was originally built based on the anatomical measurements of muscle attachments and mechanical identification of joint locations (Burkholder and Nichols, 2000 and 2004). Briefly, the model used in this study has seven rotational degrees of freedom at the anatomical joints: hip flexion (HF), hip adduction (HA), hip rotation (HR), knee extension (KE), knee adduction (KA), ankle extension (AE) and ankle adduction (AA). The axes of rotation are orthogonal to each other only at the hip where axes at the knee and ankle are non-orthogonal non-intersecting. The pelvis is fixed to the ground for all six degrees of freedom and the limb endpoint contacting the ground, defined as the metatarsophalangeal (MTP) joint location, is modeled as a pin joint constraining the three degrees of freedom in translation (Fig. 1B). The default joint configuration was adjusted to match the normal preferred stance-like posture of the cat *Russl* measured from a previous postural balance experiment (Jacobs and Macpherson 1996, Ting and Macpherson, 2005; McKay et al., 2007). The coordinate frame was defined as X axis being anterior-posterior (AP) direction, Y axis the vertical direction and Z axis the medial-lateral (ML) direction (Fig. 1A).

The equation of motion, in a matrix-vector form, that describes the dynamic behavior of the hindlimb system in generalized coordinate system of the joint angles $\vec{q} = [q_{\mathrm{HF}}, q_{\mathrm{HA}}, q_{\mathrm{HR}}, q_{\mathrm{KE}}, q_{\mathrm{KA}}, q_{\mathrm{AE}}, q_{\mathrm{AA}}]^T$ can be given as,

$$\mathbf{M}(\vec{q})\ddot{\vec{q}} = \mathbf{R}(q)\vec{F}_M(\vec{q}, \dot{\vec{q}}, \vec{a}) - \mathbf{J}(\vec{q})^{\mathbf{T}}\vec{F}_{End} - \vec{V}(\vec{q}, \dot{\vec{q}}) - \vec{G}(\vec{q}) \tag{1}$$

where $\dot{\vec{q}}$ and $\ddot{\vec{q}}$ are the joint velocity and acceleration vector respectively; $\vec{a}$ is the muscle activation pattern; **M** is the inertia matrix; **R** is the moment arm matrix; **J** is the endpoint Jacobian; $\vec{F}_M$ is the vector of muscle forces; $\vec{F}_{End}$ is the vector of endpoint forces; $\vec{V}$ is the vector of Coriolis terms; and $\vec{G}$ is the gravitational torque vector. However, in order to examine independently the production of the endpoint forces generated by specific muscle activation patterns, the gravitational term was ignored.

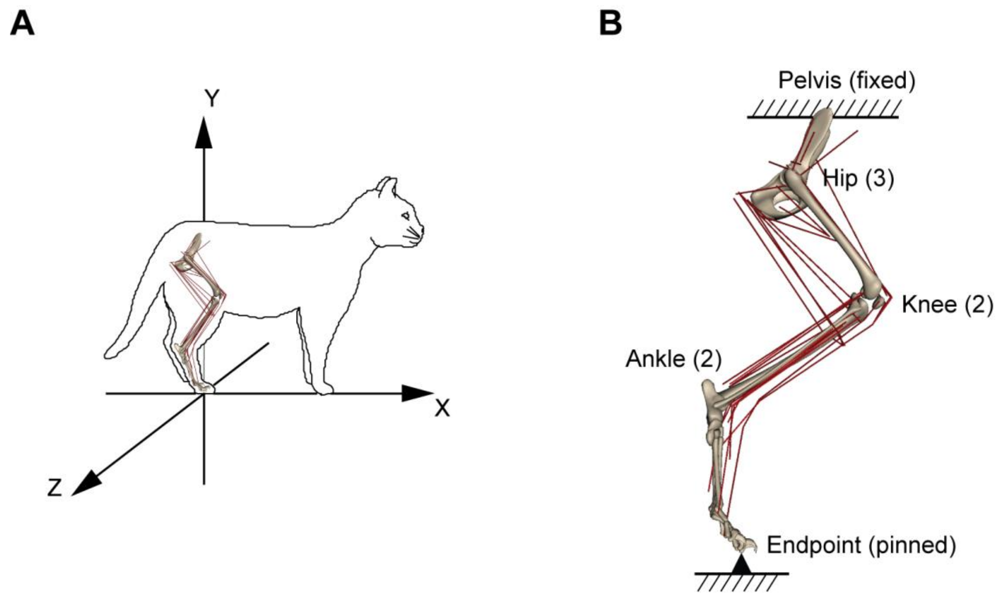

Figure 1. Schematic of the hindlimb musculoskeletal model. **A:** Coordinate frame is defined as positive X, Y, and Z directions being in the anterior, vertical up, and medial direction respectively. **B:** The model has seven degrees of freedom at the anatomical joints: three at the hip, two at each knee and ankle. The pelvis is fixed to the ground where the limb endpoint is modeled as pin joint.

For the 31 muscles in the hindlimb model (list and abbreviations in Table 1), Hill-type muscle model with inelastic tendons and angle of pennation (Zajac, 1989) was used (Fig. 2A). In particular, muscle force is composed of active and passive components, both based on the current state of the musculo-tendon length (MTL) and velocity (MTV) each normalized to the optimal fiber length (L_F/L^0_F) and maximum fiber shortening velocity (V_F/V^{max}_F) respectively. Generated active muscle force is proportional to point on the nonlinear curve for MTL and MTV, which is then linearly scaled by the level of muscle activation and maximum isometric contractile force specified for each muscle (Eq. 2). Since muscle fiber length and velocity are determined by the posture, intrinsic stiffness in terms of resisting force with respect to change in posture is introduced by the characteristic force versus fiber length and velocity curve (Fig. 2B). At given configuration, stiffness property of both the joint and the whole limb is determined by the muscle activation because muscle activation linearly scales the force producing characteristics of the muscle. In this study, each muscle was set to have fiber lengths at 65% of its optimum in the default posture by specifying a specific value for the tendon slack length. With the value 65%, intrinsic stiffness is defined to be about $3F_{max}/L^0_F$ which is near the maximal stiffness that can be found from the force-tension relationship curve (Gordon et al., 1966).

$$F_M = F_{Max}[afl(\mathrm{L_F/L^0_F}) \cdot afv(\mathrm{V_F/V^{max}_F}) \cdot a + pfl(\mathrm{L_F/L^0_F}) + \eta(\mathrm{V_F/V^{max}_F})]\cos\alpha_{penn} \quad (2)$$

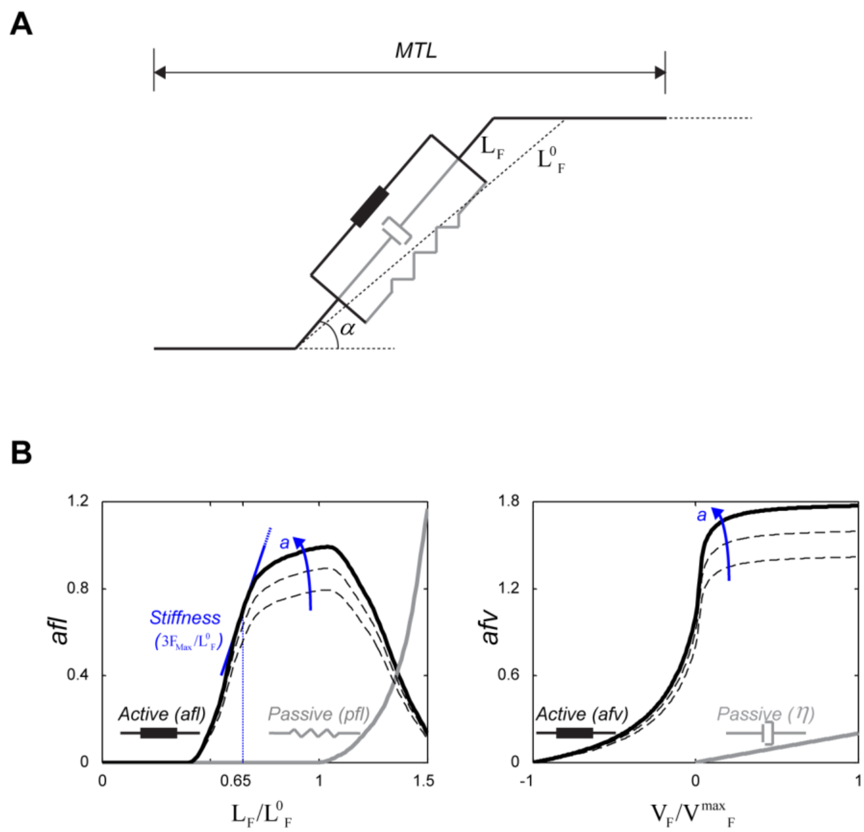

Figure 2. Schematic of the muscle model and active muscle force generation. **A:** The model takes into account of pennation angle of each muscle and has passive and active contractile component. **B:** Muscle force generation as a function of fiber length, fiber velocity and muscle activation. Muscle activation scales (blue arrow) the active force generated given a state of fiber length and velocity. Force-length relationship provides intrinsic stiffness to the change in musculo-tendon length (MTL) which can be represented with the slope at any point on the curve. Fiber lengths were set to be at 65% of its optimum for all muscles providing stiffness about $3F_{max}/L^0_F$ at the default posture (blue line).

Table 1. Muscles included in the hindlimb model and abbreviations.

Name	Abbreviation	Name	Abbreviation
Adductor femoris	ADF	*Plantaris*	PLAN
Adductor lounges	ADL	*Psoas minor*	PSOAS
Biceps femoris anterior	BFA	*Peroneus tertius*	PT
Biceps femoris posterior	BFP	*Pyriformis*	PYR
Extensor digitorum longus	EDL	*Quadratus femoris*	QF
Flexor digitorum longus	FDL	*Rectus femoris*	RF
Flexor hallucis longus	FHL	*Sartorius*	SART
Gluteus maximus	GMAX	*Semimembranosus*	SM
Gluteus medius	GMED	*Soleus*	SOL
Gluteus minimus	GMIN	*Semitendinosus*	ST
Gracilis	GRAC	*Tibialis anterior*	TA
Lateral gastrocnemius	LG	*Tibialis posterior*	TP
Medial gastrocnemius	MG	*Vastus intermedius*	VI
Peroneus brevis	PB	*Vastus lateralis*	VL
Pectineus	PEC	*Vastus medius*	VM
Peroneus longus	PL		