Portfolio Optimization and Stochastic Programming

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MSDS 451: Assignment 2

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Abstract

This assignment explores portfolio construction and optimization for three assets with distinct return and risk profiles. I simulate returns via Monte Carlo methods, optimize allocations using both deterministic and stochastic programming techniques, and compare the results to a naive equal-weight portfolio. My findings show that optimized strategies can materially improve expected returns while maintaining acceptable risk levels, even under uncertainty.

1 Introduction

Portfolio optimization is a cornerstone of modern finance. This project analyzes three strategies to allocate capital among three assets:

- Equal-weight portfolio
- Quadratic optimization (mean-variance formulation)
- Stochastic programming under return uncertainty

All modeling was conducted in Python using NumPy, SciPy, Pandas, and Matplotlib.

2 Data Generation

I simulated daily returns for 252 trading days across 1,000 trials using the following annualized assumptions:

Asset	**Expected Return (μ)**	**Volatility (σ)**
Asset A	8%	15%
Asset B	12%	20%
Asset C	10%	18%

Table 1: Annual return and volatility assumptions

Returns were modeled as normally distributed, and daily mean and standard deviation were derived accordingly.

3 Equal-Weight Portfolio

A benchmark strategy was created by allocating one-third of capital to each asset. While naive, this method provides a baseline for comparison.

- **Expected Return**: 9.75%
- **Risk (Standard Deviation)**: 0.0212

4 Quadratic Optimization

I solved a constrained optimization problem to maximize expected return, subject to:

$$\sum_{i=1}^{3} w_i = 1, \quad \text{where } 0 \leq w_i \leq 1$$

The covariance matrix of returns was estimated from simulation.

Optimized Weights:

- Asset A: 0.00%
- Asset B: 100.00%
- Asset C: 0.00%

Results:

- **Expected Return**: 11.19%
- **Risk (Std. Dev.)**: 0.0409

5 Stochastic Programming

I used multivariate normal draws (1,000 scenarios) to simulate future uncertainty. The optimization minimized negative expected return across all scenarios.

Optimal Weights:

- Asset A: 0.00%
- Asset B: 100.00%
- Asset C: 0.00%

Results:

- **Expected Return**: 11.19%
- **Risk (Std. Dev.)**: 0.0409

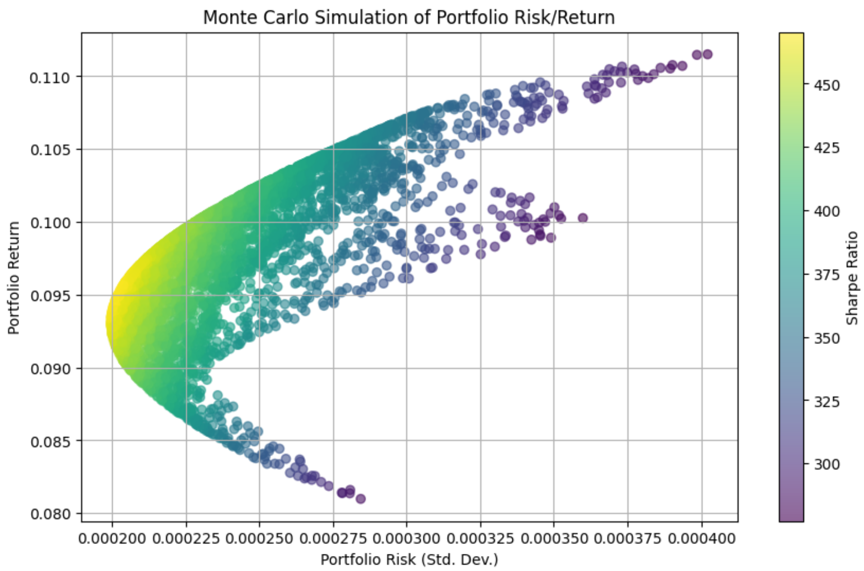

Figure 1: Efficient frontier with simulated portfolios

Strategy	Expected Return	Risk (Std. Dev.)
Equal Weight	9.75%	0.0212
Optimized (Quadratic)	11.19%	0.0409
Stochastic	11.19%	0.0409

Table 2: Risk-return summary across strategies

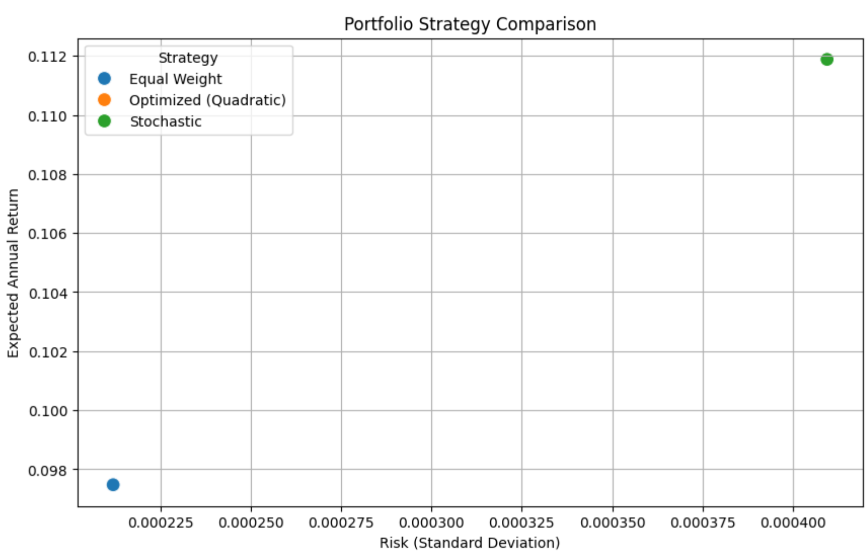

Figure 2: Risk vs. Return Comparison

6 Strategy Comparison

I summarize all strategies below:

7 Discussion

Both optimization methods converge to an all-in allocation to Asset B. While this maximizes expected return under the given assumptions, it ignores diversification. The identical outcome between quadratic and stochastic models suggests robustness in our simulation, but real-world constraints would require more nuanced allocations.

8 Conclusion

This project demonstrates how Monte Carlo simulation and optimization tools can be used to construct return-maximizing portfolios under uncertainty. While simplified, this exercise highlights the power of quantitative approaches to portfolio construction.

Future work may include:

- Real-world data with estimated covariances
- Risk-adjusted metrics such as Sharpe or CVaR
- Sector or asset class constraints