Homework 1 Solutions

Hansen 2.2

$$\begin{aligned}\mathrm{E}[YX] &= \mathrm{E}[X\mathrm{E}[Y|X]] \\ &= \mathrm{E}[X(a+bX)] \\ &= \mathrm{E}[aX+bX^2] \\ &= a\mathrm{E}[X]+b\mathrm{E}[X^2]\end{aligned}$$

where the first equality holds by the law of iterated expectations, the second equality holds by the expression for $\mathrm{E}[Y|X]$ in the problem, and the remaining two equalities are just algebra/basic properties of expectations.

Hansen 2.3

$$\begin{aligned}\mathrm{E}[h(X)e] &= \mathrm{E}[h(X)\mathrm{E}[e|X]] \\ &= \mathrm{E}[h(X)0] \\ &= 0\end{aligned}$$

where the first equality holds by the law of iterated expectations, and the second equality holds because $\mathrm{E}[e|X]=0$.

By the way, the extra condition in the problem just guarantees that the moments in the problem exist (i.e., aren't positive or negative infinity)

Hansen 2.19

Let's start by expanding the expression for $d(b)$. That is, notice that,

$$d(b) = \mathrm{E}[(m(X)^2 - 2m(X)X'b + \beta' XX'\beta)]$$

Now, taking the derivative of $d(b)$ with respect to b and setting it equal to 0 implies that

$$0 = -2\mathrm{E}[m(X)X] + 2\mathrm{E}[XX']\beta$$

which is a $k \times 1$ vector. Solving the previous equation for β, we get that

$$\begin{aligned}\beta &= \mathrm{E}[XX']^{-1}\mathrm{E}[Xm(X)] \\ &= \mathrm{E}[XX']^{-1}\mathrm{E}[X\mathrm{E}[Y|X]] \\ &= \mathrm{E}[XX']^{-1}\mathrm{E}[XY]\end{aligned}$$

where the first equality holds by rearranging terms from the previous equation, the second equality holds by the definition of $m(X)$, and the last equality holds by the law of iterated expectations (in the opposite direction than how we used it in previous problems).

Extra Question 1

```r
load("fertilizer_2000.RData")


# part (a)
nrow(fertilizer_2000)
```

```
## [1] 68
```

```r
# part (b)
fertilizer_2000[21,]$country
```

```
## [1] "Gambia, The"
```

```r
# part (c)
mean_gdp <- mean(fertilizer_2000$avgdppc)
mean_gdp
```

```
## [1] 4291.377
```

```r
# part (d)
above_avg_gdp <- subset(fertilizer_2000, avgdppc > mean_gdp)
mean(above_avg_gdp$prec)
```

```
## [1] 1391.391
```

Extra Question 2

```r
# part (a)
fibonacci <- function(n) {
  # handle n=1 or 2
  if (n==1) return(0)
  if (n==2) return(1)

  fib <- c(0,1) # set first two values
  for (i in 3:n) {
    fib[i] <- fib[i-1] + fib[i-2]
  }
  fib[n]
}

# check that it works
fibonacci(5)
```

```
## [1] 3
```

```r
fibonacci(8)
```

```
## [1] 13
```

```r
# part (b)
alt_seq <- function(a,b,n) {
  if (n==1) return(a)
```

```
  if (n==2) return(b)

  alt_fib <- c(a,b)
  for (i in 3:n) {
    alt_fib[i] <- alt_fib[i-1] + alt_fib[i-2]
  }
  alt_fib[n]
}

# check that it works
alt_seq(a=3,b=7,n=4)
```

```
## [1] 17
```