Extra Practice Questions for Midterm 1, Solutions

1. (a) What would be the result from running the following code?

```
all(c(1,2,3,4,5) > 0)
```

Answer: TRUE, this checks if all elements of the vector 1,2,3,4,5 are greater than 0. Since they all are, it returns TRUE

(b) Consider the following function

```
a_function <- function(n) {
  out <- 0
  for (i in 1:n) {
    out <- out + i^2
  }
  out
}</pre>
```

If you run the following code, what will it output?

```
a_function(5)
```

Answer: It will output 55 which comes from adding up 1 + 4 + 9 + 16 + 25

- 2. Suppose there are two random variables X and Y.
 - a) If you know that X and Y are independent, do you know what their covariance is equal to? Explain. If yes, what is the covariance equal to?

Answer: Yes, their covariance is 0. Independent random variables have 0 covariance.

b) If you know that cov(X,Y) = 0, are X and Y independent? Explain.

Answer: Not necessarily, random variables can have 0 covariance without being fully independent.

c) If you know that cov(X,Y) = 1, are X and Y independent? Explain.

Answer: X and Y are not independent in this case. If they were, their covariance would be equal to 0.

- 3. Suppose that X_1 and X_2 are two random variables such that $\mathbb{E}[X_1] = 0$, $\mathbb{E}[X_2] = 5$, $\text{var}(X_1) = 1$, $\text{var}(X_2) = 10$ and $\text{cov}(X_1, X_2) = -1$. Suppose that $Y = X_1 + X_2$.
 - a) What is $\mathbb{E}[Y]$?

Answer:

$$\mathbb{E}[Y] = \mathbb{E}[X_1 + X_2]$$
$$= \mathbb{E}[X_1] + \mathbb{E}[X_2]$$
$$= 0 + 5 = 5$$

where the second equality holds because expectations can pass through sums

b) What is var(Y)?

Answer:

$$var(Y) = var(X_1 + X_2)$$

$$= var(X_1) + var(X_2) + 2cov(X_1, X_2)$$

$$= 1 + 10 + 2(-1)$$

$$= 9$$

4. Consider a random variable Y that is equal to a firm's profits (in thousands of dollars) and another random variable X that is equal to firm's number of employees. Suppose you know that

$$\mathbb{E}[Y|X=x] = 50 + 10x$$

a) Explain how to interpret $\mathbb{E}[Y|X=x]$.

Answer: This is the conditional expectation of Y given X takes the particular value x. In the context of the problem, it is the mean profit's of firms that have x number of employees. The value can change for different numbers of employees.

b) What is $\mathbb{E}[Y|X=10]$?

Answer: $\mathbb{E}[Y|X=10] = 50 + 10(10) = 150$

c) Suppose that var(Y) = 40, $\mathbb{E}[X] = 30$, and var(Y) = 20, calculate $\mathbb{E}[Y]$.

Answer:

$$\mathbb{E}[Y] = \mathbb{E}[50 + 10X]$$

$$= 50 + 10\mathbb{E}[X]$$

$$= 50 + 10(30)$$

$$= 350$$

- 5. Suppose that we have a random sample of n observations of X and Y.
 - a) Suppose that you want to estimate the covariance between X and Y using the data that we have. Propose an estimator for the covariance. **Hint:** Try using the analogy principle and the expression $cov(X,Y) = \mathbb{E}[XY] \mathbb{E}[X]\mathbb{E}[Y]$.

Answer:

$$\widehat{\text{cov}}(X,Y) = \frac{1}{n} \sum_{i=1}^{n} X_i Y_i - \left(\frac{1}{n} \sum_{i=1}^{n} X_i\right) \left(\frac{1}{n} \sum_{i=1}^{n} Y_i\right)$$

b) Alternatively, the definition of covariance is $cov(X,Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])]$. Propose an estimator for the covariance based on this expression. Would you expect this to give you the same estimate of the covariance as in part a?

Answer: To conserve on notation, let \bar{X} and \bar{Y} denote the sample averages of X and Y, respectively. Then,

$$\widehat{\operatorname{cov}}(X,Y) = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$$