## SUPPLEMENTARY MATERIAL

## Results for the Branch-And-Cut using the original and the updated parameters

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August 1, 2017

## **Abstract**

In this document we show supplementary results of part of the experiments for the paper An efficient evolutionary algorithm for the orienteering problem. We compare the performance of B&C algorithm when it is implemented with the original parameter values and with the updated values, both of which use the new LP solver CPLEX 12.5.

Table 1: Generation 1,  $n \leq 400$ 

| Original)      | h-&-Cut ( | Brancl            | (Updated) | h-&-Cut | Branc             |                   |          |               |                 |
|----------------|-----------|-------------------|-----------|---------|-------------------|-------------------|----------|---------------|-----------------|
| time           | gap       | best              | time      | gap     | best              | $_{ m opt}$       | $\alpha$ | d0            | instance        |
| 0.00           | *         | 31                | 0.00      | *       | 31                | 31                | 0.50     | 5314          | att48           |
| 0.00           | *         | 31                | 0.00      | *       | 31                | 31                | 0.50     | 2523          | gr48            |
| 0.00           | *         | 30                | 0.00      | *       | 30                | 30                | 0.50     | 5731          | hk48            |
| 0.00           | *         | 29                | 0.00      | *       | 29                | 29                | 0.50     | 213           | eil51           |
| 0.00           | *         | 37                | 0.00      | *       | 37                | 37                | 0.50     | 3771          | berlin52        |
| 0.00           | *         | 46                | 0.00      | *       | 46                | 46                | 0.50     | 12698         | brazil58        |
| 0.10           | *         | 43                | 0.10      | *       | 43                | 43                | 0.50     | 338           | st70            |
| 0.00           | *         | 47                | 0.10      | *       | 47                | 47                | 0.50     | 269           | eil76           |
| 0.10           | *         | 49                | 0.10      | *       | 49                | 49                | 0.50     | 54080         | pr76            |
| 0.10           | *         | 64                | 0.10      | *       | 64                | 64                | 0.50     | 27605         | gr96            |
| 0.40           | *         | 52                | 0.40      | *       | 52                | 52                | 0.50     | 606           | rat99           |
| 0.40           | *         | 56                | 0.40      | *       | 56                | 56                | 0.50     | 10641         | kroA100         |
| 72.80          | *         | 58                | 95.40     | *       | 58                | 58                | 0.50     | 11071         | kroB100         |
| 0.50           | *         | 56                | 0.40      | *       | 56                | 56                | 0.50     | 10375         | kroC100         |
| 0.20           | *         | 59                | 0.10      | *       | 59                | 59                | 0.50     | 10647         | kroD100         |
|                |           |                   | 159.20    | *       | 57                | 57                | 0.50     | 11034         | kroE100         |
| 0.20           | *         | 61                | 0.20      | *       | 61                | 61                | 0.50     | 3955          | rd100           |
| 0.10           | *         | 64                | 0.10      | *       | 64                | 64                | 0.50     | 315           | eil101          |
| 0.30           | *         | 66                | 0.30      | *       | 66                | 66                | 0.50     | 7190          | lin105          |
| 0.30           | *         | 54                | 0.30      | *       | 54                | 54                | 0.50     | 22152         | pr107           |
| 0.10           | *         | 75                | 0.10      | *       | 75                | 75                | 0.50     | 3471          | gr120           |
| 0.40           | *         | 75                | 0.30      | *       | 75                | 75                | 0.50     | 29515         | pr124           |
| 0.30           | *         | 103               | 0.30      | *       | 103               | 103               | 0.50     | 59141         | bier127         |
| 1.70           | *         | 71                | 1.40      | *       | 71                | 71                | 0.50     | 48386         | pr136           |
| 1.50           | *         | 81                | 1.50      | *       | 81                | 81                | 0.50     | 34927         | gr137           |
| 1.70           | *         | 77                | 1.30      | *       | 77                | 77                | 0.50     | 29269         | pr144           |
| 204.60         | *         | 86                | 175.40    | *       | 86                | 86                | 0.50     | 13262         | kroA150         |
| 1.40           | *         | 87                | 1.20      | *       | 87                | 87                | 0.50     | 13065         | kroB150         |
| 1.60           | *         | 77                | 1.40      | *       | 77                | 77                | 0.50     | 36841         | pr152           |
| 3.60           | *         | 93                | 3.40      | *       | 93                | 93                | 0.50     | 21040         | u159            |
| 3.10           | *         | 102               | 2.60      | *       | 102               | 102               | 0.50     | 1162          | rat195          |
| 4.90           | *         | 123               | 3.20      | *       | 123               | 123               | 0.50     | 7890          | d198            |
| 1.50           | *         | 117               | 1.20      | *       | 117               | 117               | 0.50     | 14684         | kroA200         |
| 1.50           |           | 111               | 14.10     | *       | 119               | 119               | 0.50     | 14719         | kroB200         |
| 12.80          | *         | 145               | 12.70     | *       | 145               | 145               | 0.50     | 20080         | gr202           |
| 12.00          |           | 140               | 10216.30  | *       | 124               | 124               | 0.50     | 63322         | ts225           |
| 151.40         | *         | 129               | 94.40     | *       | 129               | 129               | 0.50     | 1958          | tsp225          |
| 433.30         | *         | 126               | 166.20    | *       | 126               | 126               | 0.50     | 40185         | pr226           |
| 0.90           | *         | 176               | 0.90      | *       | 176               | 176               | 0.50     | 67301         | gr229           |
| 1.10           | *         | 158               | 0.90      | *       | 158               | 158               | 0.50     | 1189          | gil262          |
| 25.20          | *         | 132               | 21.20     | *       | 132               | 132               | 0.50     | 24568         | pr264           |
| 25.20<br>14.10 | *         | $\frac{132}{147}$ | 13.60     | *       | $\frac{132}{147}$ | $\frac{132}{147}$ | 0.50     | 1290          | a280            |
| 129.50         | *         | 162               |           | *       | 162               | 162               | 0.50     |               |                 |
| 24.60          | *         | 162<br>205        | 111.50    | *       | 162<br>205        | $\frac{162}{205}$ | 0.50     | 24096 $21015$ | pr299<br>lin318 |
|                | *         |                   | 22.40     | *       |                   |                   |          |               |                 |
| 40.70          |           | 239               | 37.40     | **      | 239               | 239               | 0.50     | 7641          | rd400           |

Table 2: Generation 1, n > 400

|          |        |          |             | Branch | n-&-Cut | (Updated) | Branch | Branch-&-Cut (Origin |        |  |
|----------|--------|----------|-------------|--------|---------|-----------|--------|----------------------|--------|--|
| instance | d0     | $\alpha$ | $_{ m opt}$ | best   | gap     | time      | best   | gap                  | time   |  |
| fl417    | 5931   | 0.50     | 228         | 228    | *       | =         |        |                      |        |  |
| gr431    | 85707  | 0.50     | 350         | 350    | *       | 139.90    | 350    | *                    | 138.10 |  |
| pr439    | 53609  | 0.50     | 313         | 313    | *       | 833.30    |        |                      |        |  |
| pcb442   | 25389  | 0.50     | 251         | 251    | *       | 14.90     | 251    | *                    | 18.60  |  |
| d493     | 17501  | 0.50     | 320         | 320    | *       | 347.30    | 320    | *                    | 203.80 |  |
| att532   | 13843  | 0.50     | 363         | 363    | *       | 593.00    |        |                      |        |  |
| pa561    | 1382   | 0.50     | 356         | 356    | *       | 2103.60   |        |                      |        |  |
| u574     | 18453  | 0.50     | 354         | 354    | *       | 61.40     | 354    | *                    | 63.80  |  |
| rat575   | 3387   | 0.50     | 322         | 322    | *       | 59.50     | 322    | *                    | 72.60  |  |
| p654     | 17322  | 0.50     | 327         | 327    | *       | _         | 327    | *                    | -      |  |
| d657     | 24456  | 0.50     | 386         | 386    | *       | 715.70    | 386    | *                    | 763.30 |  |
| gr666    | 147179 | 0.50     | 503         | 503    | *       | 634.20    | 503    | *                    | 696.70 |  |
| u724     | 20955  | 0.50     | 439         | 439    | *       | 1077.10   |        |                      |        |  |
| rat783   | 4403   | 0.50     | 438         | 438    | *       | 594.30    |        |                      |        |  |
| pr1002   | 129523 | 0.50     | 604         | 604    | *       | _         | 449    | 25.66                | -      |  |
| vm1084   | 119649 | 0.50     | 777         | 777    | *       | 4927.40   |        |                      |        |  |
| rl1323   | 135100 | 0.50     | 811         | 811    | *       | _         |        |                      |        |  |
| fl1400   | 10064  | 0.50     | 922         | 909    | 1.41    | _         | 922    | *                    | -      |  |
| d1655    | 31064  | 0.50     | 798         |        |         |           | 798    | *                    | -      |  |
| vm1748   | 168278 | 0.50     | 1168        | 873    | 25.26   | =         | 1168   | *                    | _      |  |
| rl1889   | 158268 | 0.50     | 890         | 890    | *       | _         | 890    | *                    | -      |  |
| pr2392   | 189016 | 0.50     | 1140        | 1140   | *       | _         | 1140   | *                    | -      |  |
| fl3795   | 14386  | 0.50     | 1470        |        |         |           | 1470   | *                    | -      |  |

Table 3: Generation 2,  $n \le 400$ 

| Original | -&-Cut ( | Branch | Updated) | &-Cut ( | Branch-              |             |          |               |                 |
|----------|----------|--------|----------|---------|----------------------|-------------|----------|---------------|-----------------|
| tim      | gap      | best   | time     | gap     | best                 | $_{ m opt}$ | $\alpha$ | d0            | instance        |
| 0.1      | *        | 1717   | 0.00     | *       | 1717                 | 1717        | 0.50     | 5314          | att48           |
| 0.2      | *        | 1761   | 0.20     | *       | 1761                 | 1761        | 0.50     | 2523          | gr48            |
| 0.1      | *        | 1614   | 0.10     | *       | 1614                 | 1614        | 0.50     | 5731          | hk48            |
| 0.4      | *        | 1674   | 0.40     | *       | 1674                 | 1674        | 0.50     | 213           | eil51           |
| 92.1     | *        | 1897   | 93.40    | *       | 1897                 | 1897        | 0.50     | 3771          | berlin52        |
| 0.1      | *        | 2220   | 0.10     | *       | 2220                 | 2220        | 0.50     | 12698         | brazil58        |
| 21.4     | *        | 2286   | 19.40    | *       | 2286                 | 2286        | 0.50     | 338           | st70            |
| 0.1      | *        | 2550   | 0.10     | *       | 2550                 | 2550        | 0.50     | 269           | eil76           |
| 0.4      | *        | 2708   | 0.40     | *       | 2708                 | 2708        | 0.50     | 54080         | pr76            |
| 1.7      | *        | 3396   | 1.70     | *       | 3396                 | 3396        | 0.50     | 27605         | gr96            |
| 0.9      | *        | 2944   | 0.90     | *       | 2944                 | 2944        | 0.50     | 606           | rat99           |
| 0.8      | *        | 3212   | 0.90     | *       | 3212                 | 3212        | 0.50     | 10641         | kroA100         |
| 7.5      | *        | 3241   | 6.70     | *       | 3241                 | 3241        | 0.50     | 11071         | kroB100         |
| 77.2     | *        | 2947   | 85.60    | *       | 2947                 | 2947        | 0.50     | 10375         | kroC100         |
| 48.0     | *        | 3307   | 45.00    | *       | 3307                 | 3307        | 0.50     | 10647         | kroD100         |
| 327.1    | *        | 3090   | 230.10   | *       | 3090                 | 3090        | 0.50     | 11034         | kroE100         |
| 0.2      | *        | 3359   | 0.20     | *       | 3359                 | 3359        | 0.50     | 3955          | rd100           |
| 63.5     | *        | 3655   | 153.00   | *       | 3655                 | 3655        | 0.50     | 315           | eil101          |
| 71.0     | *        | 3544   | 67.30    | *       | 3544                 | 3544        | 0.50     | 7190          | lin105          |
| 0.6      | *        | 2667   | 0.60     | *       | 2667                 | 2667        | 0.50     | 22152         | pr107           |
| 72.3     | *        | 4371   | 35.80    | *       | 4371                 | 4371        | 0.50     | 3471          | gr120           |
| 0.7      | *        | 3917   | 0.50     | *       | 3917                 | 3917        | 0.50     | 29515         | pr124           |
| 59.4     | *        | 5383   | 58.80    | *       | 5383                 | 5383        | 0.50     | 59141         | bier127         |
| 2.1      | *        | 4309   | 2.10     | *       | 4309                 | 4309        | 0.50     | 48386         | pr136           |
| 82.6     | *        | 4286   | 196.90   | *       | 4286                 | 4286        | 0.50     | 34927         | gr137           |
| 93.2     | *        | 4003   | 90.40    | *       | 4003                 | 4003        | 0.50     | 29269         | pr144           |
| 172.8    | *        | 4918   | 241.40   | *       | 4918                 | 4918        | 0.50     | 13262         | kroA150         |
| 29.0     | *        | 4869   | 24.80    | *       | 4869                 | 4869        | 0.50     | 13065         | kroB150         |
| 2.3      | *        | 4279   | 2.20     | *       | 4279                 | 4279        | 0.50     | 36841         | pr152           |
| 97.5     | *        | 4960   | 192.20   | *       | 4960                 | 4960        | 0.50     | 21040         | u159            |
| 63.6     | *        | 5791   | 128.80   | *       | 5791                 | 5791        | 0.50     | 1162          | rat195          |
| 54.3     | *        | 6670   | 74.20    | *       | 6670                 | 6670        | 0.50     | 7890          | d198            |
| 72.6     | *        | 6547   | 68.70    | *       | 6547                 | 6547        | 0.50     | 14684         | kroA200         |
| 74.6     | *        | 6419   | 34.70    | *       | 6419                 | 6419        | 0.50     | 14719         | kroB200         |
| 14.0     |          | 0410   | 85.70    | *       | 7789                 | 7789        | 0.50     | 20080         | gr202           |
| 8.1      | *        | 6834   | 6.60     | *       | 6834                 | 6834        | 0.50     | 63322         | ts225           |
| 0.1      |          | 0004   | 174.50   | *       | 6987                 | 6987        | 0.50     | 1958          | tsp225          |
| 265.2    | *        | 6662   | 74.10    | *       | 6662                 | 6662        | 0.50     | 40185         | pr226           |
| 200.2    |          | 0002   | 182.60   | *       | 9177                 | 9177        | 0.50     | 67301         | gr229           |
| 75.4     | *        | 8321   | 89.60    | *       | 8321                 | 8321        | 0.50     | 1189          | gil262          |
| 27.1     | *        | 6654   | 23.00    | *       | 6654                 | 6654        | 0.50     | 24568         | pr264           |
| 105.8    | *        | 8428   | 103.80   | *       | 8428                 | 8428        | 0.50     | 1290          | a280            |
| 105.8    |          | 0440   | 426.50   | *       | 9182                 | 9182        | 0.50     | 24096         | pr299           |
|          | •        | •      | 862.40   | *       | $\frac{9182}{10923}$ | 10923       | 0.50     | 24096 $21015$ | pr299<br>lin318 |
|          |          |        | 002.40   | *       | 13652                | 13652       | 0.50     | 7641          | rd400           |

Table 4: Generation 2, n > 400

| Original) | &-Cut ( | Branch- | Updated) | &-Cut ( | Branch- |             |          |         |          |
|-----------|---------|---------|----------|---------|---------|-------------|----------|---------|----------|
| time      | gap     | best    | time     | gap     | best    | $_{ m opt}$ | $\alpha$ | d0      | instance |
|           |         |         | _        | *       | 11894   | 11894       | 0.50     | 5931    | fl417    |
|           |         |         | 969.50   | *       | 18318   | 18318       | 0.50     | 85707   | gr431    |
|           |         |         | 1298.30  | *       | 16171   | 16171       | 0.50     | 53609   | pr439    |
|           |         |         | 6259.10  | *       | 14484   | 14484       | 0.50     | 25389   | pcb442   |
|           |         |         | _        | *       | 19598   | 19598       | 0.50     | 13843   | att532   |
|           |         |         | 2099.70  | *       | 21954   | 21954       | 0.50     | 101170  | ali535   |
|           |         |         | 1487.10  | *       | 19576   | 19576       | 0.50     | 1382    | pa561    |
|           |         |         | 612.50   | *       | 19351   | 19351       | 0.50     | 18453   | u574     |
| 1311.50   | 0.01    | 18250   | 931.10   | *       | 18251   | 18251       | 0.50     | 3387    | rat575   |
| _         | *       | 17224   | _        | 0.37    | 17160   | 17224       | 0.50     | 17322   | p654     |
|           |         |         | 2682.40  | *       | 21503   | 21503       | 0.50     | 24456   | d657     |
|           |         |         | 5830.50  | *       | 24223   | 24223       | 0.50     | 20955   | u724     |
|           |         |         | _        | *       | 35772   | 35772       | 0.50     | 9329844 | dsj1000  |
| _         | *       | 27066   | _        | *       | 27066   | 27066       | 0.50     | 129523  | pr1002   |
|           |         |         | _        | *       | 40687   | 40687       | 0.50     | 119649  | vm1084   |
| _         | *       | 42181   |          |         |         | 42181       | 0.50     | 126474  | rl1304   |
| _         | 0.30    | 43216   | _        | *       | 43347   | 43347       | 0.50     | 135100  | rl1323   |
| _         | *       | 54255   | _        | 1.90    | 53222   | 54255       | 0.50     | 10064   | fl1400   |
| _         | *       | 44528   |          |         |         | 44528       | 0.50     | 31064   | d1655    |
| _         | *       | 62925   |          |         |         | 62925       | 0.50     | 168278  | vm1748   |
| _         | *       | 52047   | _        | *       | 52047   | 52047       | 0.50     | 158268  | rl1889   |
| _         | *       | 53976   | _        | *       | 53976   | 53976       | 0.50     | 32127   | u2152    |
| _         | *       | 72790   | _        | *       | 72790   | 72790       | 0.50     | 117128  | u2319    |
| _         | *       | 64577   | _        | *       | 64577   | 64577       | 0.50     | 189016  | pr2392   |
| -         | *       | 83951   | =        | *       | 83951   | 83951       | 0.50     | 68847   | pcb3038  |
| _         | *       | 89089   |          |         |         | 89089       | 0.50     | 14386   | fl3795   |

Table 5: Generation 3,  $n \le 400$ 

| Original) | &-Cut ( | Branch-        | (Updated) | &-Cut | Branch-        |                |                |                      |          |
|-----------|---------|----------------|-----------|-------|----------------|----------------|----------------|----------------------|----------|
| time      | gap     | best           | time      | gap   | best           | $_{ m opt}$    | $\alpha$       | d0                   | instance |
| 41.30     | *       | 1049           | 38.50     | *     | 1049           | 1049           | 0.50           | 5314                 | att48    |
| 0.20      | *       | 1480           | 0.20      | *     | 1480           | 1480           | 0.50           | 2523                 | gr48     |
| 0.00      | *       | 1764           | 0.00      | *     | 1764           | 1764           | 0.50           | 5731                 | hk48     |
| 0.20      | *       | 1399           | 0.20      | *     | 1399           | 1399           | 0.50           | 213                  | eil51    |
| 139.40    | *       | 1036           | 124.70    | *     | 1036           | 1036           | 0.50           | 3771                 | berlin52 |
| 0.00      | *       | 1702           | 0.00      | *     | 1702           | 1702           | 0.50           | 12698                | brazil58 |
| 0.50      | *       | 2108           | 0.40      | *     | 2108           | 2108           | 0.50           | 338                  | st70     |
| 0.40      | *       | 2467           | 0.40      | *     | 2467           | 2467           | 0.50           | 269                  | eil76    |
| 0.30      | *       | 2430           | 0.20      | *     | 2430           | 2430           | 0.50           | 54080                | pr76     |
| 39.90     | *       | 3170           | 61.50     | *     | 3170           | 3170           | 0.50           | 27605                | gr96     |
| 4.40      | *       | 2908           | 4.90      | *     | 2908           | 2908           | 0.50           | 606                  | rat99    |
| 68.50     | *       | 3211           | 63.30     | *     | 3211           | 3211           | 0.50           | 10641                | kroA100  |
| 0.90      | *       | 2804           | 0.60      | *     | 2804           | 2804           | 0.50           | 11071                | kroB100  |
| 1.80      | *       | 3155           | 1.50      | *     | 3155           | 3155           | 0.50           | 10375                | kroC100  |
| 11.90     | *       | 3167           | 10.70     | *     | 3167           | 3167           | 0.50           | 10647                | kroD100  |
| 1.70      | *       | 3049           | 1.50      | *     | 3049           | 3049           | 0.50           | 11034                | kroE100  |
| 105.30    | *       | 2926           | 113.20    | *     | 2926           | 2926           | 0.50           | 3955                 | rd100    |
| 31.50     | *       | 3345           | 29.80     | *     | 3345           | 3345           | 0.50           | 315                  | eil101   |
| 56.00     | *       | 2986           | 51.90     | *     | 2986           | 2986           | 0.50           | 7190                 | lin105   |
| 294.60    | *       | 1877           | 660.90    | *     | 1877           | 1877           | 0.50           | 22152                | pr107    |
| 1.80      | *       | 3779           | 1.50      | *     | 3779           | 3779           | 0.50           | 3471                 | gr120    |
|           |         |                | 1021.50   | *     | 3557           | 3557           | 0.50           | 29515                | pr124    |
| 122.10    | *       | 2365           | 79.90     | *     | 2365           | 2365           | 0.50           | 59141                | bier127  |
| 95.20     | *       | 4390           | 86.70     | *     | 4390           | 4390           | 0.50           | 48386                | pr136    |
| 8.80      | *       | 3954           | 8.60      | *     | 3954           | 3954           | 0.50           | 34927                | gr137    |
| 337.30    | *       | 3745           | 112.60    | *     | 3745           | 3745           | 0.50           | 29269                | pr144    |
| 001.00    |         | 0140           | 330.70    | *     | 5039           | 5039           | 0.50           | 13262                | kroA150  |
| 61.90     | *       | 5314           | 107.60    | *     | 5314           | 5314           | 0.50           | 13065                | kroB150  |
| 01.30     |         | 3314           | 1122.40   | *     | 3905           | 3905           | 0.50           | 36841                | pr152    |
| 62.10     | *       | 5272           | 52.20     | *     | 5272           | 5272           | 0.50           | 21040                | u159     |
| 51.10     | *       | 6195           | 49.90     | *     | 6195           | 6195           | 0.50           | 1162                 | rat195   |
| 01.10     |         | 0133           | 286.10    | *     | 6320           | 6320           | 0.50           | 7890                 | d198     |
| 133.70    | *       | 6123           | 122.30    | *     | 6123           | 6123           | 0.50           | 14684                | kroA200  |
| 43.10     | *       | 6266           | 40.10     | *     | 6266           | 6266           | 0.50           | 14084                | kroB200  |
| 45.10     |         | 0200           | 224.80    | *     | 8616           | 8616           | 0.50           | 20080                | gr202    |
| 110.80    | *       | 7575           | 171.20    | *     | 7575           | 7575           | 0.50           | 63322                | ts225    |
| 110.60    |         | 1313           | 150.30    | *     | 7740           | 7740           | 0.50           | 1958                 | tsp225   |
| 26.00     | *       |                | 32.60     | *     | 6993           |                |                |                      |          |
| 36.80     | *       | 6993           |           | *     |                | 6993           | 0.50           | 40185                | pr226    |
| 15.90     | *       | 6328           | 10.20     | *     | 6328           | 6328           | 0.50           | 67301                | gr229    |
| 57.40     | *       | $9246 \\ 8137$ | 133.40    | *     | $9246 \\ 8137$ | $9246 \\ 8137$ | $0.50 \\ 0.50$ | $\frac{1189}{24568}$ | gil262   |
| 21.40     | *       |                | 20.70     | *     |                |                |                |                      | pr264    |
| 192.90    |         | 9774           | 213.30    | *     | 9774           | 9774           | 0.50           | 1290                 | a280     |
| FOF 00    | *       | 10000          | 363.60    | *     | 10343          | 10343          | 0.50           | 24096                | pr299    |
| 507.20    | *       | 10368          | 534.80    | *     | 10368          | 10368          | 0.50           | 21015                | lin318   |
| 266.00    | 77      | 13223          | 293.20    | ~     | 13223          | 13223          | 0.50           | 7641                 | rd400    |

Table 6: Generation 3, n > 400

| Original) | &-Cut ( | Branch- | (Updated) | -&-Cut | Branch |             |          |        |          |
|-----------|---------|---------|-----------|--------|--------|-------------|----------|--------|----------|
| time      | gap     | best    | time      | gap    | best   | $_{ m opt}$ | $\alpha$ | d0     | instance |
|           |         |         | 6227.60   | *      | 14220  | 14220       | 0.50     | 5931   | fl417    |
|           |         |         | 1046.90   | *      | 10911  | 10911       | 0.50     | 85707  | gr431    |
|           |         |         | _         | *      | 15160  | 15160       | 0.50     | 53609  | pr439    |
|           |         |         | _         | *      | 14819  | 14819       | 0.50     | 25389  | pcb442   |
|           |         |         | _         | *      | 25167  | 25167       | 0.50     | 17501  | d493     |
| 849.50    | *       | 15498   | 933.20    | *      | 15498  | 15498       | 0.50     | 13843  | att532   |
|           |         |         | 10543.80  | *      | 14482  | 14482       | 0.50     | 1382   | pa561    |
|           |         |         | 1409.30   | *      | 20064  | 20064       | 0.50     | 18453  | u574     |
|           |         |         | 1426.50   | *      | 20109  | 20109       | 0.50     | 3387   | rat575   |
|           |         |         | _         | *      | 24492  | 24492       | 0.50     | 17322  | p654     |
|           |         |         | 4053.30   | *      | 24562  | 24562       | 0.50     | 24456  | d657     |
|           |         |         | _         | *      | 17020  | 17020       | 0.50     | 147179 | gr666    |
|           |         |         | 5870.60   | *      | 28348  | 28348       | 0.50     | 20955  | u724     |
|           |         |         | 7232.30   | *      | 27566  | 27566       | 0.50     | 4403   | rat783   |
| _         | *       | 39508   | _         | 0.15   | 39449  | 39508       | 0.50     | 129523 | pr1002   |
|           |         |         | _         | *      | 37653  | 37653       | 0.50     | 119649 | vm1084   |
| -         | *       | 30106   | _         | *      | 30106  | 30106       | 0.50     | 25401  | d1291    |
| -         | *       | 41385   | _         | 2.19   | 40478  | 41385       | 0.50     | 126474 | rl1304   |
| _         | 7.90    | 40948   | _         | *      | 44458  | 44458       | 0.50     | 135100 | rl1323   |
| -         | *       | 39435   |           |        |        | 39435       | 0.50     | 28319  | nrw1379  |
| _         | 0.91    | 54293   | _         | *      | 54792  | 54792       | 0.50     | 10064  | fl1400   |
| -         | *       | 51168   | _         | *      | 51168  | 51168       | 0.50     | 31064  | d1655    |
| _         | 1.56    | 67905   | -         | *      | 68979  | 68979       | 0.50     | 168278 | vm1748   |
| -         | *       | 52186   | _         | *      | 52186  | 52186       | 0.50     | 28601  | u1817    |
| _         | *       | 43374   | -         | *      | 43374  | 43374       | 0.50     | 158268 | rl1889   |
| _         | *       | 76035   | _         | *      | 76035  | 76035       | 0.50     | 40225  | d2103    |
| -         | *       | 52091   | -         | *      | 52091  | 52091       | 0.50     | 32127  | u2152    |
| -         | 0.16    | 79221   | -         | *      | 79351  | 79351       | 0.50     | 117128 | u2319    |
| -         | *       | 60225   | -         | *      | 60225  | 60225       | 0.50     | 189016 | pr2392   |
| -         | *       | 96356   | -         | *      | 96356  | 96356       | 0.50     | 68847  | pcb3038  |

Table 7: Generation 4,  $n \le 400$ 

| Original) | &-Cut ( | Branch- | (Updated) | &-Cut | Branch-          |               |                |                |                 |
|-----------|---------|---------|-----------|-------|------------------|---------------|----------------|----------------|-----------------|
| time      | gap     | best    | time      | gap   | best             | $_{ m opt}$   | $\alpha$       | d0             | instance        |
| 167.20    | *       | 1870    | 106.00    | *     | 1870             | 1870          | 0.65           | 6909           | att48           |
| 23.20     | *       | 2264    | 22.40     | *     | 2264             | 2264          | 0.80           | 4037           | gr48            |
| 0.20      | *       | 2177    | 0.20      | *     | 2177             | 2177          | 0.80           | 9169           | hk48            |
| 64.60     | *       | 2490    | 82.10     | *     | 2490             | 2490          | 0.90           | 384            | eil51           |
| 124.50    | *       | 2089    | 115.00    | *     | 2089             | 2089          | 0.60           | 4526           | berlin52        |
| 74.60     | *       | 2070    | 132.00    | *     | 2070             | 2070          | 0.45           | 11428          | brazil58        |
|           |         |         | 127.70    | *     | 3316             | 3316          | 0.85           | 574            | st70            |
| 41.50     | *       | 3646    | 45.10     | *     | 3646             | 3646          | 0.85           | 458            | eil76           |
|           |         |         | 1047.70   | *     | 3361             | 3361          | 0.70           | 75712          | pr76            |
|           |         |         | 212.30    | *     | 4851             | 4851          | 0.95           | 52449          | gr96            |
| 8.00      | *       | 3502    | 16.00     | *     | 3502             | 3502          | 0.60           | 727            | rat99           |
|           |         |         | 187.10    | *     | 4999             | 4999          | 0.95           | 20218          | kroA100         |
| 37.10     | *       | 2935    | 34.40     | *     | 2935             | 2935          | 0.45           | 9964           | kroB100         |
| 283.10    | *       | 1962    | 261.60    | *     | 1962             | 1962          | 0.35           | 7263           | kroC100         |
| 13.40     | *       | 1212    | 11.80     | *     | 1212             | 1212          | 0.20           | 4259           | kroD100         |
|           |         |         | 203.40    | *     | 4635             | 4635          | 0.80           | 17655          | kroE100         |
| 164.90    | *       | 3815    | 164.60    | *     | 3815             | 3815          | 0.60           | 4747           | rd100           |
| 110.50    | *       | 4308    | 90.80     | *     | 4308             | 4308          | 0.65           | 409            | eil101          |
|           |         |         | 1020.60   | *     | 2455             | 2455          | 0.35           | 5033           | lin105          |
| 158.20    | *       | 2072    | 159.00    | *     | 2072             | 2072          | 0.30           | 13291          | pr107           |
|           |         |         | 236.70    | *     | 5830             | 5830          | 0.85           | 5901           | gr120           |
|           |         |         | 163.80    | *     | 2036             | 2036          | 0.30           | 17710          | pr124           |
| 168.80    | *       | 5068    | 278.40    | *     | 5068             | 5068          | 0.45           | 53227          | bier127         |
|           |         |         | 6303.60   | *     | 2860             | 2860          | 0.35           | 33871          | pr136           |
| -         | -       | -       | 203.10    | *     | 6523             | 6523          | 0.80           | 55883          | gr137           |
| 230.10    | *       | 5641    | 357.90    | *     | 5641             | 5641          | 0.70           | 40976          | pr144           |
|           |         |         | 415.90    | *     | 6858             | 6858          | 0.75           | 19893          | kroA150         |
| -         | -       | -       | 303.00    | *     | 7023             | 7023          | 0.80           | 20904          | kroB150         |
|           |         | •       | 483.60    | *     | 5823             | 5823          | 0.70           | 51578          | pr152           |
|           |         | •       | 1145.20   | *     | 3147             | 3147          | 0.35           | 14729          | u159            |
|           |         | •       | 205.40    | *     | 9753             | 9753          | 0.95           | 2207           | rat195          |
|           |         | •       | 492.70    | *     | 4661             | 4661          | 0.40           | 6312           | d198            |
|           |         | •       | 340.30    | *     | 9892             | 9892          | 0.90           | 26432          | kroA200         |
|           |         |         | 253.20    | *     | 9849             | 9849          | 0.90           | 26494          | kroB200         |
|           |         | •       | 376.10    | *     | 1071             | 1071          | 0.15           | 6025           | gr202           |
|           |         |         | 3524.60   | *     | 11002            | 11002         | 0.10           | 113979         | ts225           |
|           |         |         | 706.70    | *     | 1002             | 1002          | 0.90           | 3525           | tsp225          |
|           |         |         | 1183.10   | *     | 4893             | 4893          | 0.40           | 32148          | pr226           |
|           |         | •       | 563.10    | *     | 11482            | 11482         | 0.90           | 121142         | gr229           |
| •         | •       | •       | 1770.50   | *     | 2031             | 2031          | 0.30           | 357            | gil262          |
|           | •       | •       | 277.50    | *     | 10253            | 10253         | 0.13           | 34395          | pr264           |
| 214.70    | *       | 12064   | 351.80    | *     | 10253 $12064$    | 10255 $12064$ | 0.75           | 1935           | a280            |
| 214.70    |         | 12004   | 7771.90   | *     | 14986            | 14986         | 0.75           | 45782          | pr299           |
|           |         |         | 1111.90   | *     | $14986 \\ 15132$ | 15132         | $0.95 \\ 0.85$ | 45782<br>35725 | pr299<br>lin318 |
| •         | •       |         | 5093.10   | *     |                  |               |                |                | rd400           |
|           |         |         | 5093.10   | 7     | 20107            | 20107         | 0.95           | 14517          |                 |

Table 8: Generation 4, n > 400

| &-Cut (Updated) Branch-&-Cut | k-Cut (U | Branch- |        |          |         |          |
|------------------------------|----------|---------|--------|----------|---------|----------|
| gap time best gap            | gap      | best    | opt    | $\alpha$ | d0      | instance |
| *                            | *        | 20496   | 20496  | 0.85     | 10082   | fl417    |
| * <u>-</u>                   | *        | 13976   | 13976  | 0.30     | 51425   | gr431    |
| * 3936.10                    | *        | 19613   | 19613  | 0.70     | 75052   | pr439    |
| *                            | *        | 5839    | 5839   | 0.20     | 10156   | pcb442   |
| 0.02 - <b>21745</b> *        | 0.02     | 21740   | 21745  | 0.70     | 24502   | d493     |
| *                            | *        | 26728   | 26728  | 0.95     | 26302   | att532   |
| * 15739.60                   | *        | 13520   | 13520  | 0.20     | 40468   | ali535   |
| *                            | *        | 27712   | 27712  | 0.90     | 2487    | pa561    |
| *                            | *        | 28823   | 28823  | 0.95     | 35060   | u574     |
| * <u>-</u>                   | *        | 28364   | 28364  | 0.90     | 6096    | rat575   |
| * <u>-</u>                   | *        | 31814   | 31814  | 0.80     | 27715   | p654     |
| * 13485.10                   | *        | 32548   | 32548  | 0.90     | 44021   | d657     |
| *                            | *        | 21013   | 21013  | 0.35     | 103026  | gr666    |
| *                            | *        | 34988   | 34988  | 0.85     | 35624   | u724     |
| * _                          | *        | 7829    | 7829   | 0.15     | 1321    | rat783   |
| *                            | *        | 27357   | 27357  | 0.35     | 6530891 | dsj1000  |
| 0.10 - <b>23551</b> *        | 0.10     | 23527   | 23551  | 0.35     | 90666   | pr1002   |
| * _                          |          | 51768   | 51768  | 0.85     | 190480  | u1060    |
| *                            | *        | 38678   | 38678  | 0.45     | 107684  | vm1084   |
| *                            | *        | 55954   | 55954  | 0.85     | 48359   | pcb1173  |
| * 2335.60 4029 *             | *        | 4029    | 4029   | 0.10     | 5081    | d1291    |
| * - 57643 0.24               | *        | 57782   | 57782  | 0.75     | 189711  | rl1304   |
| * _                          | *        | 65476   | 65476  | 0.90     | 243180  | rl1323   |
| *                            | *        | 69119   | 69119  | 0.95     | 53807   | nrw1379  |
| * _                          | *        | 70476   | 70476  | 0.90     | 18115   | fl1400   |
| *                            | *        | 54540   | 54540  | 0.60     | 91783   | u1432    |
| * _                          | *        | 22191   | 22191  | 0.35     | 7788    | fl1577   |
| * _                          | *        | 29920   | 29920  | 0.35     | 21745   | d1655    |
| 0.53 - 82216 *               | 0.53     | 81778   | 82216  | 0.75     | 252417  | vm1748   |
| * - 31800 *                  |          | 31800   | 31800  | 0.35     | 20021   | u1817    |
| * - 71527 *                  | *        | 71527   | 71527  | 0.75     | 237402  | rl1889   |
| * - 31029 0.05               | *        | 31045   | 31045  | 0.30     | 24136   | d2103    |
| * - 48472 *                  | *        | 48472   | 48472  | 0.45     | 28914   | u2152    |
| 0.43 - 111472 *              | 0.43     | 110995  | 111472 | 0.80     | 187405  | u2319    |
| * _                          |          | 45407   | 45407  | 0.35     | 132312  | pr2392   |
| * - 91831 *                  | *        | 91831   | 91831  | 0.55     | 75732   | pcb3038  |
| * - 71328 *                  | *        | 71328   | 71328  | 0.40     | 11509   | fl3795   |
| * - 84098 *                  | *        | 84098   | 84098  | 0.30     | 54770   | fnl4461  |
| * _                          | *        | 279116  | 279116 | 0.85     | 480701  | rl5915   |
| * - 106131 *                 | *        | 106131  | 106131 | 0.25     | 5815182 | pla7397  |