Joe Holbrook Memorial Invitational Competition

7th Grade

March 20, 2022

General Rules

- You will have **90 minutes** to solve **16 questions**. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam and their score will be disqualified.

- You may use the following aids:
  – Pencil or other writing utensil
  – Eraser
  – Blank scrap paper

- You may not use the following aids:
  – The Internet
  – Books or other written sources
  – Other people
  – Calculator or other computing device
  – Compass
  – Protractor
  – Ruler or straightedge

Other Notes

- Please input your answers into the Google form provided by your proctor.

- **All answers are integers.** Please enter them with no spaces in between. For example, enter -7 not - 7.

- Do not include commas in your answers. For example, the number one thousand is to be entered 1000 not 1,000.

- You do not need to write units in your answers.

- Make sure you do not make any typing mistakes, as you will not be given credit if you do so.

- Ties will be broken by the number of correct responses to the last 5 problems. Further ties will be broken by the number of correct responses to the previous 5 problems, etc.

- Keep in mind that the JHMIC is a difficult contest and very different from school assessments. If you even get a few questions right, you should feel proud of yourself!

1. Rachel leaves Central Perk at 9:00 AM driving 30 mph. Ross leaves Central Perk at 10:00 AM traveling in the same direction as Rachel at 45 mph. At 10:40 AM, what will the distance between them be?

2. Find the smallest positive integer $x > 1$ such that the smallest prime factor of x is greater than the smallest prime factor of $x + 6$.

3. In Catsville, there are two types of cats: lazy cats and hard-working cats. Lazy cats work at a constant speed and hard-working cats work at another constant speed. If three lazy cats take ten days to complete a job and two hard-working cats take five days to complete the same job, then how many lazy cats can complete the job in the same amount of time a team of three hard-working cats can?

4. The constant term in the expansion of $(x^3 + \frac{1}{3x^5})^8$ can be written as $\frac{a}{b}$, where a and b are relatively prime integers. Find $a + b$.

5. Let the letters in the alphabet correspond to consecutive numbers. In other words, if n is a positive integer, $A = n$, $B = n + 1$, $C = n + 2$, $\cdots$, $Z = n + 25$. Find the smallest possible value of n such that $J + H + M + M + C$ is divisible by 2021.

6. Jaiden the corn farmer and Nikhil the potato farmer are having a land dispute and decide to settle it with a sequence of fair coin flips. Out of 5 flips, if there are 2 or fewer heads, Nikhil wins and Jaiden wins otherwise. However, Jaiden rigs the coin so that it now has a $\frac{3}{4}$ chance of landing heads. By rigging the coin, Nikhil's chances of winning have decreased by $\frac{m}{n}$ for relatively prime positive integers m and n. What is $m + n$?

7. At 7:00 AM, Aminah's alarm buzzes. Whenever Aminah hears her alarm, she has a 5% chance of getting up from bed and a 95% chance of hitting the snooze button, in which case the alarm resumes buzzing exactly one minute later. The expected time that Aminah gets up is n minutes after 7:00 AM. Find n.

8. Two circles O_1 and O_2 of radius r are externally tangent to each other and internally tangent to a circle O_3 of radius $2r$. In addition, circle O_4 of radius 1 is externally tangent to O_1 and O_2 and internally tangent to O_3. The area of the circle with radius r can be expressed as $\frac{a}{b}\pi$, where a and b are relatively prime positive integers. Find $a + b$.

9. Rapunzel is celebrating her birthday and wants to put a large square banner inside an arc of her castle. If the arc forms a parabola $y = -4x^2 + 48x - 114$, what is the largest possible area she can make the banner if it cannot go below $y = 0$?

10. Jaiden loves writing troll problems so much that he creates a weekly routine called *Troll Tuesday*, but he skips the Tuesday that is Groundhog's Day. Thus, there are 51 total Troll Tuesdays in 2021. On the nth Troll Tuesday of the year, Jaiden writes n troll problems. If I choose 2 (not necessarily different) Troll Tuesdays in 2021, the probability that 17 divides the sum of the number of troll problems written on those days is $\frac{m}{n}$ for relative prime positive integers m and n. What is $m + n$?

11. David's calculator broke! All the buttons for even digits function properly, but the buttons for the odd digits are completely nonfunctional. Luckily, he has a plan. If he wants to input an integer, like 2479, which would require him to use a broken button, he will instead input the closest possible integer containing only even digits. For example, instead of 2479, David would input 2480. Similarly, instead of 357, David would input 400. In the event that two alternative inputs are equally good approximations, David will input either one of them. David inputted $2000 + 600 + 40 + 8$ into his calculator. He knows that the displayed result will be at most k away from his intended result. What is k?

12. Let $ABCD$ be a trapezoid with $AB \parallel CD$. The diagonals meet at point E such that $AC \perp BD$ and $AE = 3$, $CE = 5$. If the area of the trapezoid is 96, compute AB^2.

13. Let $p_1, p_2, \ldots, p_6$ be an increasing arithmetic sequence of primes. Find the smallest possible value of p_6.

14. Nikhil is very hungry and visits the Infinite Cafe. Nikhil is randomly given a lunchbox, each containing either a Broccoli, Carrot, or Potato meal. Nikhil is curious and wants to try each of the meals, so he constantly sneaks back into line; every time Nikhil receives a meal, he is equally likely to get each type. On average, Nikhil has to get $\frac{m}{n}$ lunchboxes to try each type, for relatively prime positive integers m and n. What is $m + n$?

15. Define the function f to be $f(x) = \lfloor x \rfloor + \lceil x \rceil$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. The sum of the real numbers x that satisfy the equation $f(x) + f(2x) + f(3x) = 100$ can be written in the form $\dfrac{m}{n}$ for relatively prime positive integers m and n. Compute $m + n$.

16. Nikhil has a circle with 10 evenly spaced points on it. He draws 3 random chords (not necessarily distinct), by picking 2 distinct points and drawing the chord between them. The probability that the circle is cut into 5 regions can be written as $\dfrac{a}{b}$ in lowest terms. What is $a + b$?