# Joe Holbrook Memorial Math Competition

## 7th Grade

### October 15, 2017

**General Rules**

- You will have **75 minutes** to solve **40 questions**. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam.

- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.

- You may not use the following aids:
  - Calculator or other computing device
  - Compass
  - Protractor
  - Ruler or straightedge

  In addition, you must use the scrap paper supplied by the proctors.

**Other Notes**

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.

- Fractions should be written in lowest terms. Please convert all mixed numbers into improper fractions.

- For constants such as $e$ or $\pi$, do not approximate your answer: for example, if the answer to a question is $7\pi$, then you should not write 22 or 21.99.

- You do not need to write units in your answers.

- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.

- Ties will be broken by the number of correct responses to questions 31 through 40. Further ties will be broken by the number of correct responses in the last five questions.

1. It is currently 12:24pm and Jonathan's mom won't let him go swimming until 3:00pm (on the same day). How many minutes does he have to wait until he can go?

2. BCA Math Team loves donuts. At the last practice, there were 300 donuts. 20% of the donuts had sprinkles, 10% of the donuts had chocolate, and 37% of the donuts had jelly. The rest were plain. How many plain donuts were there?

3. Bill's phone has 40% battery left, so he decides to start charging it. The battery charges at 2% per minute if he is not using it. But, he starts using his phone, which discharges it at 0.5% per minute. How long will it take for his battery to charge fully?

4. Jaylen is trying to improve his basketball shooting percentage at the gym, and he refuses to go home until he has made at least 70% of the shots he has taken. So far, he has taken 15 shots and made only 7 of them. Assuming that from now on, he never misses a shot, what is the minimum number of shots that he has to make consecutively to reach his goal?

5. The entire BCA Math Team are trying to cross the Hackensack River and we have a bunch of boats. We realize that if we have another boat, there will be exactly 6 people on each boat; if we take away a boat, there will be exactly 9 people on each boat. How many people are on the team?

6. Haneul and Julia are at opposite ends of the country. In order to communicate regularly, they agree to video-call each other from 4:00pm to 4:30pm on every perfect square date of the month, and to talk on the phone from 5:00pm to 6:00pm on any day that is a perfect power of 2 (if they did not already video-call on this day). How many hours do they spend communicating in the month of July?

7. An obtuse triangle has side lengths of 4, 7, and $x$. If $x$ is a positive integer, how many possible values of $x$ are there?

8. A basketball player makes 60 baskets in one game. One quarter of his shots are worth 2 points, 20 shots are worth 3 point, and the rest are only worth 1 point. What was the average number of points per shot?

9. Sarah is making a sandwich for lunch. In her fridge, she has 5 types of meat: ham, salami, turkey, baloney, and roast beef, as well as 3 types of cheese: swiss, american, and provolone. She wants her sandwich to have 2 different types of meat and 1 type of cheese, but absolutely hates the taste of salami with american cheese or roast beef with baloney. How make types of sandwiches can Sarah make for lunch that she will actually like?

10. An evil goat spelled the name *Tifany* wrong. The goat had written her name as *Tiffany*. How many more ways are there to rearrange the letters of *Tiffany* than *Tifany*?

11. John multiplies all the factors of 42 together, and to his surprise, finds that this is $42^n$. What is $n$?

12. If the least common multiple of positive integers $a, b$ is 20 and the greatest common divisor of $a, b$ is 10, find the product $a \cdot b$.

13. Let $a\Delta b = \dfrac{2ab}{a+b}$. Find $((2\Delta 6)\Delta 3)\Delta(0\Delta(3\Delta 7))$.

14. The Fibonacci sequence is a sequence of numbers where any term after the second is the sum of the previous two terms. Among the first 100 terms of the sequence, how many are even? The first 8 terms are given here: 1, 1, 2, 3, 5, 8, 13, 21.

15. What is the maximum number of intersections between 17 circles and 17 lines?

16. The average, median, and unique mode of 5 positive numbers is 5. What is the maximum possible value of the largest number in those positive numbers?

17. In the world of Tri, the inhabitants have three feet. Trigo, an inhabitant of Tri, has 6 red socks, 6 blue socks, and 6 green socks. In the early morning, Trigo grabs 3 random socks and puts them on without looking. What is the probability that all three are of the same color?

18. Rhombus $ABCD$ has side length 6 and $\angle ABC = 120°$. What is the area of rhombus $ABCD$?

19. The difference of two 3-digit numbers is 894. Find the product of the digits of the 2 numbers.

20. Square $ABCD$ has sidelength 1. Points $M$ and $N$ are on sides $BC$ and $CD$, respectively, such that the perimeter of $\triangle CMN$ is 2. Find the degree measure of $\angle MAN$.

21. Ben, David, Haneul, Ivy, and Julia go to the movies. Julia does not want to sit next to neither Ben nor David and Ivy insists that she sits next to Haneul. How many ways are there for the five friends to sit?

22. Find the largest prime divisor of $3^{10} - 1$.

23. David is standing 6 feet away from a tree. He sees the top of the tree at an angle of 60 degrees to the ground. If David spots a bug crawling along the tree at an angle of 30 degrees to the ground, how far above the ground was the bug at that instant? (Note: David's height is negligible)

24. A coin is flipped 5 times. What is the probability that at least three consecutive heads are flipped?

25. Triangle $ABC$ has circumcircle $\Omega$ with center $O$ and radius 2. If line $BO$ meets $\Omega$ again at a point $P$ other than $A$ with $PC = 3$, find the length of side $BC$.

26. How many of the factors of 1200 are not perfect squares?

27. Let $ABCD$ be a rectangle with $AB = 8$ and $BC = 6$. If $M$ is the midpoint of $BD$ and $N$ is the foot of the perpendicular from $A$ to $BD$, find $MN$.

28. Parallelogram $ABCD$ has a perimeter of 75. When side $BC$ is the base, the height is 14. When side $CD$ is the base, the height is 16. Find the area of parallelogram $ABCD$.

29. Neb was told to draw an equilateral triangle with side length 6. However, instead Neb draws a regular hexagon instead. Edwin notes that the area of the regular hexagon Neb drew has area exactly twice that of the equilateral triangle he was supposed to draw. What is the side length of the hexagon?

30. How many positive integers divide at least two of the numbers 84, 120, and 126?

31. Define a *lattice point* to be a point in the coordinate plane whose coordinates are both integers. Bessie selects a lattice point $(x, y)$ whose coordinates satisfy $1 \le x, y \le 10$ uniformly and at random, and draws a line connecting the chosen point with the origin. Compute the probability that the slope of this line is a positive integer.

32. Badville is a nation filled with bad highways. There are 20 cities, and every pair of cities is connected by a unique highway, all of which need to be renovated. What is the maximum number of highways the government can close such that there is a path through open highways between any two cities?

33. What is the units digit of $2017^{2016^{2015}}$?

34. Suppose you have a circle with center $O$ and diameter 15. Point $P$ is outside the circle and $A$ is a point such that $PA$ is a tangent. Extend $A$ through $O$ to get $C$ on the circle and let $B$ be the intersection of $PC$ and the circle. $PB = 16$. Find $PC$.

35. Haneul has two distinct fair six-sided dice. If she rolls two numbers $x$ and $y$ and computes $\binom{x}{y}$, what is the expected value of $\binom{x}{y}$? ($\binom{n}{k} = 0$ if $k < n$).

36. Cube $ABCD - EFGH$ has edge length 1. Find the distance from $BH$ to $EG$.

37. If $x$ satisfies $x^2 + x + 1 = 0$, compute $x^{200} + x^{100} + 1$.

38. Circles of radii $8, 8, 9$ and $X$ are all externally tangent to each other. What is $X$?

39. In right triangle $ABC$, $BC = 24$ and $\angle B = 90°$. A circle of radius 5 intersects sides $AB, BC$, and $CA$ at $P$ and $Q$, $R$ and $S$, and $T$ and $U$, respectively, such that $PQ = RS = TU = 6$. What is the length of leg $AB$?

40. Haneul has a fair six-sided die, and she rolls the numbers $a, b, c$ and $d$ in that order. Let $P$ be the polynomial of degree at most 3 such that $P(1) = a + b, P(2) = ab, P(3) = c + d$, and $P(4) = cd$. What is the expected value of $P(5)$?