Joe Holbrook Memorial Math Competition

5th Grade

October 9th, 2016

General Rules

- You will have **90 minutes** to solve **50** questions. Your score is the number of correct answers.

- Only answers recorded on the answer sheet will be graded.

- This is an individual test. Anyone caught communicating with another student will be removed from the exam.

- Scores will be posted on the website. Please do not forget your ID number, as that will be the sole means of identification for the scores.

- You may **not** use the following aids:

    - Calculator or other computing device
    - Compass
    - Protractor
    - Ruler or straightedge

    In addition, you must use the scrap paper supplied by the proctors.

Other Notes

- Write legibly. If the graders cannot read your answer, you will be given no credit for that question.

- Fractions should be written in **lowest terms**. Please convert all mixed numbers into **improper fractions**.

- For constants such as e or π, do not approximate your answer: for example, if the answer to a question is 7π, then you should **not** write 22 or 21.99.

- You do not need to write units in your answers.

- Rationalize all denominators. In addition, numbers within a square root must be squarefree, e.g. $\sqrt{63}$ should be written as $3\sqrt{7}$.

- Ties will be broken by the number of correct responses to questions 41 through 50. Further ties will be broken by the number of correct responses in the last five questions.

1. Kelvin the Frog's favorite song is 6 minutes long. How many times can he listen to the song in half an hour?

2. What is $-17 \times -16 \times -15 \times \cdots \times 15 \times 16 \times 17$?

3. Compute $2 + (0 - (1 \times 6 \times (2^{0 \times (\frac{1}{6})})))$.

4. Alex the Kat has written 25 questions for the JHMMC. If he wants to write 40 in total. how many does he have left to write?

5. The Bergen Tech soccer team won a match 5 to 2. How many goals were scored in all?

6. How many more sides does a hexagon have than a triangle?

7. What fraction of months start with a J?

8. What is the probability of getting an even number when a standard six-sided die is rolled?

9. If Mr. Pinyan grades 150 tests at the end of each of the three trimesters, how many tests does he grade in a year?

10. If Yousun is 5 feet tall and Youjung is 6 inches taller than Yousun, how tall is Youjung in inches?

11. A phone has a maximum battery life of 10 hours. There is 12% battery left. How many minutes are left until the phone dies?

12. Two regular polygons have angles of 135 degrees and 108 degrees. Find the sum of the number of sides on both polygons.

13. Find the greatest common factor of 2016 and 2772.

14. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{6}$?

15. What is $\frac{5}{55}$ of 5555?

16. What is the smallest integer n such that $2^n > 2016$?

17. When Kelvin the Frog was a tadpole, he took the JHMMC. His score improved by the same number of questions every year. If he got 31 questions right in 4th grade and 43 questions right in 8th grade, how many did he get right in 7th grade?

18. Harry Hounini, the famous magician, invents a magic trick, and he hopes you will test it out for him. This is how the trick works: first think of any number. Now add that number to 2016, and multiply the sum by 4. Now subtract 12 from the product, and divide the result by 4. Lastly, subtract off the original number. Incredibly, Hounini knows exactly what that number is. What is it?

19. What is $\frac{1}{2} \cdot \frac{8}{16} \cdot \frac{17}{34} \cdot \frac{50}{100}$?

20. The number of lilypads in Kelvin the Frog's pond doubles every day. If there were 48 lilypads on Saturday, on what day of the week did he have an odd number of lilypads?

21. What is $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + ... + 63$?

22. Jake bought a bad batch of eggs, and some of his cakes have gone bad. There was a total of 132 cakes, but two out of every three were bad. If he threw out all of the bad cakes, how many does he have left?

23. David and June went to a pizza restaurant together. David ordered 3 slices of plain pizza and 2 slices of pepperoni pizza. June ordered 2 slices of plain pizza and 4 slices of pepperoni pizza. If David and June paid $25 in total and a slice of pepperoni pizza costs 50 cents more than a slice of plain pizza, how much did David pay?

24. David Song is writing songs. If one of his songs is 3 minutes and 45 seconds long, what fraction of an hour is it?

25. Jake goes to the grocery store because he needs to buy milk, eggs, and butter for a cake recipe. If there are 3 different brands of milk, 2 different brands of eggs, and 4 different brands of butter, and Jake only wants to buy one of each item, how many different combinations of milk eggs and butter can he buy?

26. The Bergen County Academies Admissions Exam had 40 questions. Of three students that were accepted, their scores were 34, 35, and 39. Of three students that were not accepted, their scores were 23, 14, and 17. What is the difference between the average scores of the accepted students and the students who were not accepted?

27. What is the sum of the roots of $x^2 - 2x + 3$?

28. Suppose 3 flips are worth 5 flops and 9 flops are worth 14 flaps. How many flaps are equal to 54 flips?

29. Letter blocks with the letters "A", "B", and "C" are in a bag. If you take out a letter at a time, what is the probability that they come out in the order spelling "BCA"?

30. Arthur and Sunny are running a 100 meter race against each other. For the first 5 seconds, Arthur runs at 8 m/s. Tired, he slows down to 3 m/s for the rest of the race. Meanwhile, Sunny opened the first 8 seconds of the race running only at 4 m/s. If he wants to at least tie Arthur, what is the minimum speed he must run at for the rest of the race?

31. In how many distinct words (strings of letters) can be formed by permuting all of the letters of $JHMMC$?

32. Compute the units digit of $2^{2016} + 3^{2016}$.

33. Let A, B, and C be the number of positive factors of 2015, 2016, and 2017, respectively. Find the average of A, B, and C.

34. A square on the Cartesian plane has adjacent vertices on the origin and at $(1, 0)$. It is spun $180°$ about the origin. What is the total area that is covered by the square during the turn?

35. Two similar right triangles have areas of $40cm^2$ and $360cm^2$ respectively. If the smaller has a hypotenuse of length $15cm$, what is the length, in centimeters, of the hypotenuse of the larger triangle?

36. What is the largest integer value for x such that $\dfrac{x}{x+2} < \dfrac{61}{64}$?

37. Hannah and Julia wish to sit next to each other at the movie theater and they also want to sit with the 5 other friends that came with them. In how many ways is this possible?

38. What is the (simplified) common fraction equivalent to the repeating decimal expansion $3.\overline{703} = 3.703703703\ldots$?

39. Given that $a^2 + 3b^2 + 5c^2 + 7d^2 + 9e^2 + 11f^2 = 20$ and $3a^2 + 5b^2 + 7c^2 + 9d^2 + 11e^2 + 13f^2 = 40$, find $5a^2 + 7b^2 + 9c^2 + 11d^2 + 13e^2 + 15f^2$.

40. Two circles centered at points A and B, are tangent to each other at a point C. A common external tangent to circles A and B is tangent at X and Y, respectively. Find $\angle XCY$.

41. A bucket contains 10 green balls and 6 blue balls. A second bucket contains 8 green balls and N blue balls. A single ball is drawn at random from each bucket. The probability that both balls are of the same color is 0.575. Find N.

42. What is the value of $\dfrac{(5^{2016})^2 - (5^{2014})^2}{(5^{2015})^2 - (5^{2013})^2}$?

43. Find $\sqrt{(2016)(2017)(2018)(2019) + 1}$.

44. Find all real solutions to $(x+2)(x+4)(x+6)(x+8) = (x+2)^2 + (x+4)^2 + (x+6)^2 + (x+8)^2 + 4$.

45. The quantity $\sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}}$ can be expressed in the form $a\sqrt{b}$, where a and b are positive integers. Find $a + b$.

46. Let polynomial $P(x) = x^{10} + 2x^9 + 4x^8 + \cdots + 512x + 1024$. What is the coefficient of the term with degree 7 of $P(x-2)$?

47. Let roots of the polynomial $x^3 - 6x^2 - x + 30$ are p, q, and r. If the monic cubic polynomial with roots pq, pr, and qr is expressed as $x^3 + bx^2 + cx + d$, what is $b + c + d$?

48. Compute the infinite sum $\dfrac{1}{2} + \dfrac{3}{4} + \dfrac{5}{8} + \dfrac{7}{16} + \ldots$.

49. Let all k positive divisors of 2016^2 be $d_1, d_2, \ldots d_k$. What is the sum $\dfrac{1}{d_1 + 2016} + \dfrac{1}{d_2 + 2016} + \cdots + \dfrac{1}{d_k + 2016}$?

50. In orthodiagonal quadrilateral $ABCD$, $AC = 12$ and $BD = 18$. Points M and N are the midpoints of AB and CD, respectively. If the projection from point M onto CD intersects CD at P and $NP = 6$, find MP.