FX Forward Invariance & Discounting with CSA Collateral

Nicholas Burgess

July 1, 2017

Abstract

In what follows we outline briefly the Credit Support Annex and how it impacts securities pricing. We then proceed to discuss synthetic forward rate calculation and the FX forward invariance relationship from which we show how to calculate CSA collateral adjusted discount factors using GBP curves with a EUR CSA collateral as an illustration.

Keywords: FX Forward Invariance, CSA, Standard CSA, Native CSA, Non-Standard CSA, Yield Curve, Collateralization, Discount Factors

Electronic copy available at: https://ssrn.com/abstract=3009281

Standard & Non-Standard CSAs

A Credit Support Annex or CSA is a legal document, which supplements the ISDA Master Agreement and provides credit protection by setting forth the rules governing collateral posting between two counterparties. It specifies amongst other things the collateral currency to be used[1], eligible securities with cash being the standard, minimum transfer amounts MTA, credit thresholds et al.

A standard or native CSA indicates that collateral is being posted in the currency native to the trade e.g. counterparties agree to post GBP collateral for GBP swap trades. A non-standard CSA indicates that collateral will be posted in a currency other than that of the underlying trade e.g. collateral posting in GBP for a EUR swap trade.

The CSA has an impact on the pricing of interest swaps and other financial products. Cashflows under a non-standard CSA should be discounted with discount factors adjusted for CSA collateral currency choice.

FX Forward Rates

Forward FX rates can be implied from spot FX rates and discount factors. Synthetically we can replicate an FX forward rate by trading spot FX and making a simultaneous loan and deposit to flatten the spot position and replicate a forward FX rate. Mathematically we can express this as[2]

$$F(t,T)^{GBP/EUR} = S \underbrace{\frac{P(t,T)^{GBP\_USDCSA}}{P(t,T)^{EUR\_USDCSA}}}_{\textbf{From Xccy Curves}} = S\frac{P(t,T)^{GBP\_EURCSA}}{P(t,T)^{EUR\_EURCSA}} \quad (1)$$

where F(t,T) is the forward FX from time t to T, P(T,T) the discount factor S represents spot FX and we note that $P(t,T)^{EUR\_EURCSA}$ represents a OIS discount factor under a native CSA.

FX Forward Invariance

The ratio of discount factors is invariant for any given CSA collateral currency and can be expressed as

$$\underbrace{\frac{P(t,T)^{GBP\_USDCSA}}{P(t,T)^{EUR\_USDCSA}}}_{\textbf{From Xccy Curves}} = \frac{P(t,T)^{GBP\_EURCSA}}{P(t,T)^{EUR\_EURCSA}} \quad (2)$$

We typically work with a quotient term with a USD CSA for liquidity purposes. Discount factors consistent with a USD CSA are typically derived from liquid Xccy basis quotes.

[1] Typically a single currency is specified

[2] Note The CSA collateral currency for each quotient must be the same

Electronic copy available at: https://ssrn.com/abstract=3009281

CSA Curves & Discount Factors

Given a set of OIS discount factors derived under a native CSA we can imply CSA collateral adjusted discount factors from (1), whereby we can choose to imply CSA adjusted discount factors from either Forward FX rates

$$F(t,T)^{GBP/EUR} = S\frac{P(t,T)^{GBP\_EURCSA}}{P(t,T)^{EUR\_EURCSA}} \tag{3}$$

simple rearrangement gives

$$P(t,T)^{GBP\_EURCSA} = \left(\frac{1}{S}\right) F(t,T)^{GBP/EUR} \times \underbrace{P(t,T)^{EUR\_EURCSA}}_{Native\ OIS\ Disc\ Factor} \tag{4}$$

or alternatively why can imply CSA discount factors from Xccy Basis quotes[3]

$$\frac{P(t,T)^{GBP\_USDCSA}}{P(t,T)^{EUR\_USDCSA}} = \frac{P(t,T)^{GBP\_EURCSA}}{P(t,T)^{EUR\_EURCSA}} \tag{5}$$

leading to

$$P(t,T)^{GBP\_EURCSA} = \left(\frac{P(t,T)^{GBP\_USDCSA}}{P(t,T)^{EUR\_USDCSA}}\right) \times \underbrace{P(t,T)^{EUR\_EURCSA}}_{Native\ OIS\ Disc\ Factor} \tag{6}$$

Conclusion

In conclusion we have reviewed standard and non-standard CSAs, outlined the construction of synthetic FX forward rates which led to FX forward invariance relationship. The FX forward invariance principle allows us to deduce discount factors adjusted for CSA collateralization.

[3]Note the Spot FX terms S cancel