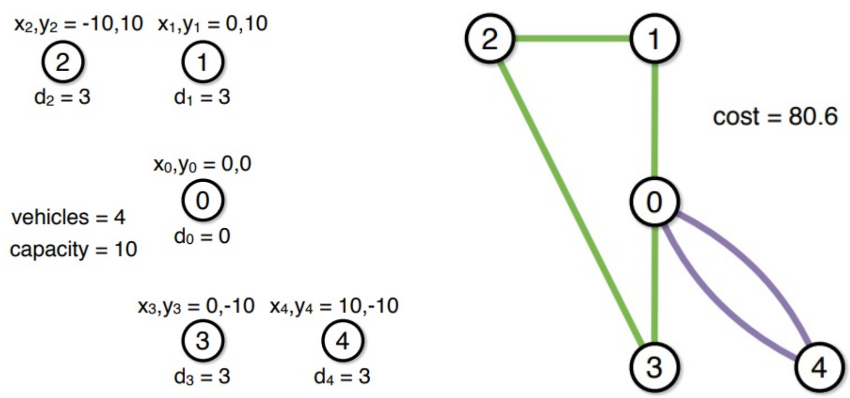
Discrete Optimization Assignment 3

Vehicle Routing Problem

1 Problem Statement

In this assignment, you are asked to design an algorithm to solve the Vehicle Routing Problem (VRP). A delivery company needs to deliver goods to many different customers. The deliveries are made by dispatching a fleet of vehicles from a centralized warehouse. The goal of this problem is to design a route for each vehicle (similar to traveling salesman tours) so that all of the customers are served by exactly one vehicle and the travel distance of the vehicles is minimized. The vehicles have a fixed storage capacity, and the customers have different demands.

Figure 1 illustrates a small VRP and a feasible solution to that problem. The customers are labeled from 0 to 4, with 0 being the warehouse. The solution uses two vehicles which are indicated by different colored routes.



(a) The VRP input data

(b) A solution to the VRP using 2 vehicles

Figure 1: A Vehicle Routing Example

2 Assignment

Write an algorithm to solve the VRP. The problem is mathematically formulated in the following way: We are given a list of locations $N = \{0, \ldots, n-1\}$, where by convention, location 0 is the location of the warehouse. <u>All vehicles start and end their routes at the warehouse.</u> The remaining locations are customers. Each location is characterized by three values (d_i, x_i, y_i), where $i \in N$, i.e., a demand d_i and a point (x_i, y_i). The fleet of vehicles $V = \{0, \ldots, v-1\}$ is fixed, and each vehicle has a limited capacity c. The sum of the demands assigned to a vehicle cannot exceed its capacity c. For each vehicle $i \in V$, let T_i be the sequence of customer deliveries made by that vehicle, and let $dist(m_1, m_2)$ be the Euclidean distance between two customers. Assume that the vehicles can travel in straight lines between each pair of locations. Then, the vehicle routing problem is formalized as the following optimization problem:

Minimize:

$$\sum_{i \in V} \left(dist(0, T_{i,0}) + \sum_{\langle j,k \rangle \in T_i} dist(j,k) + dist(T_{i,|T_i|-1}, 0) \right)$$

(warehouse to the first customer)
+ (between every customer pair on this sequence)
+ (getting back to the warehouse)

subject to:

$$\sum_{j \in T_i} d_j \leq c \qquad (i \in V)$$

(sum of the demands of the customers picked for this sequence should not exceed this vehicle's capacity)

$$\sum_{i \in V} (j \in T_i) = 1 \qquad (j \in N \setminus \{0\})$$

(each location j visited exactly once, when summed over all vehicles)

$$dist(j,k) = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2}$$

(Euclidean distance between two customers)

3 Input and Output Data Format

The input file consists of $|N|+1$ lines. The first line contains 3 numbers: The number of customers $|N|$, the number of vehicles $|V|$, and the vehicle capacity c. It is followed by $|N|$ lines, each line represents a location triple (d_i, x_i, y_i), with a demand $d_i \in N$ and a point $(x_i, y_j) \in \mathbb{R}$.

Input format:

```
|N| |V| c
d_0 x_0 y_0
d_1 x_1 y_1
...
d_|N|-1 x_|N|-1 y_|N|-1
```

The output has $|V|+1$ lines. The first line contains a single value obj. This value obj is the length of all of the vehicle routes (i.e., the objective value) as a real number. The following $|V|$ lines represent the vehicle routes T encoding the solution. Each vehicle line starts with warehouse identifier 0 followed by the identifiers of the customers serviced by that vehicle and ends with the warehouse identifier 0. Each vehicle line can contain between 2 and $|N|+2$ values depending on how many customers that vehicle services. Each customer identifier must appear in one of these vehicle lines.

Output Format:

```
obj
0 t_0_1 t_0_2 ... 0
0 t_0_1 t_1_2 ... 0
...
0 t_|V|-1_1 t_|V|-1_2 ... 0
```

Examples (based on Figure 1)

Input Example:

```
5 4 10
0 0 0
3 0 10
3 -10 10
3 0 -10
3 10 -10
```

Output Example 1:

```
80.6
0 1 2 3 0
0 4 0
0 0
0 0
```

This output represents the following routes for each vehicle:

- Vehicle 0: $\{0 \to 1, 1 \to 2, 2 \to 3, 3 \to 0\}$
- Vehicle 1: $\{0 \to 4, 4 \to 0\}$
- Vehicle 2: $\{0 \to 0\}$
- Vehicle 3: $\{0 \to 0\}$

Note the following equivalent solution using the same routes with different vehicles:

Output Example 2:

```
80.6
0 4 0
0 0
0 1 2 3 0
0 0
```

4 Instructions

For now, please start to work on your computer locally. For uploading to the test system, see the file "instructions.pdf".