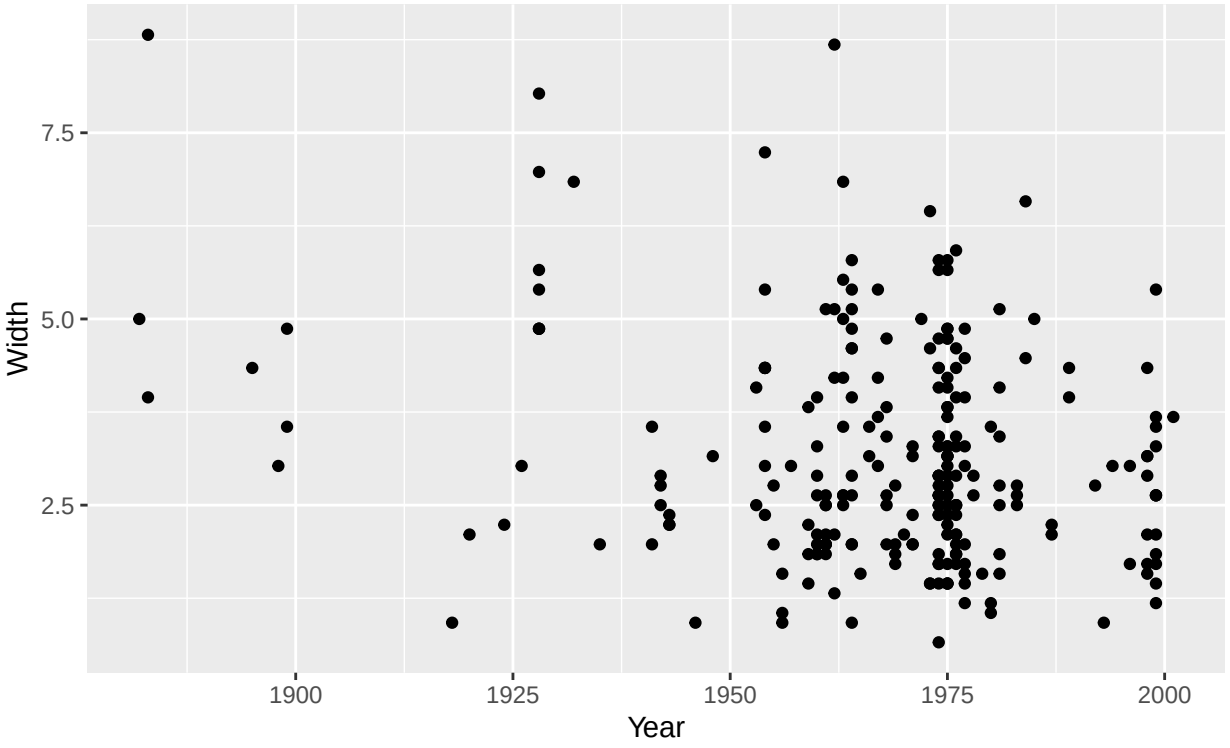
SDS 291 - Multiple Regression - Day 02

January 29, 2020

Biologists know that the leaves on plants tend to get smaller as temperatires rise. The dataset `LeafWidth` has fata on samples of leaves from the species *Dodonaea viscosa* subsp, *angustissima*, which have been collected in a certain region of South Australia for man years.

The variable `Width` is the average width, in mm, of leaves, taken at their widest points, that wer collected in a given year.

1. Scatterplot of Leaf Width (mm) and Year



Describe the scatterplot in words.

There is a negative, moderately weak, shallow magnitude, linear pattern with a few unusual observations, especially in the older years.

2. Find the least squares regression line for predicting leaf width based on year

```
##
## Call:
## lm(formula = Width ~ Year, data = LeafWidth)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.1214 -1.1253 -0.3136  0.9320  5.4144
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 37.723091   8.574977   4.399 1.61e-05 ***
## Year        -0.017560   0.004358  -4.029 7.43e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.424 on 250 degrees of freedom
## Multiple R-squared:  0.06098,    Adjusted R-squared:  0.05723
## F-statistic: 16.24 on 1 and 250 DF,  p-value: 7.425e-05
```

Write the fitted regression model

The fitted regression model is $Y = \hat{\beta}_0 + \hat{\beta}_1 Year$ or $Y = 37.723 - 0.018(Year)$

Interpret the value of the slope for the fitted model in the context of this setting.

Over time, leaf widths got smaller by, on average, -0.018mm each year.

3. Assessing the model

What leaf width would the fitted model predict a leaf in 1994 would have?

$$\hat{y} = 37.723 + (-0.018 \cdot 1994)$$

$$\hat{y} = 37.723 - 35.01464$$

$$\hat{y} = 2.70836$$

This model predicted that a leaf in 1994 would have a width of 2.71 mm.

Find the residual

```
##      Width   Length  LWRatio    Area Year
## 1 3.026316 61.44737 20.30435 119.027 1994
```

We see from above that the actual width (y) was 3.026316.

For a given observation ($_i$), the residual is:

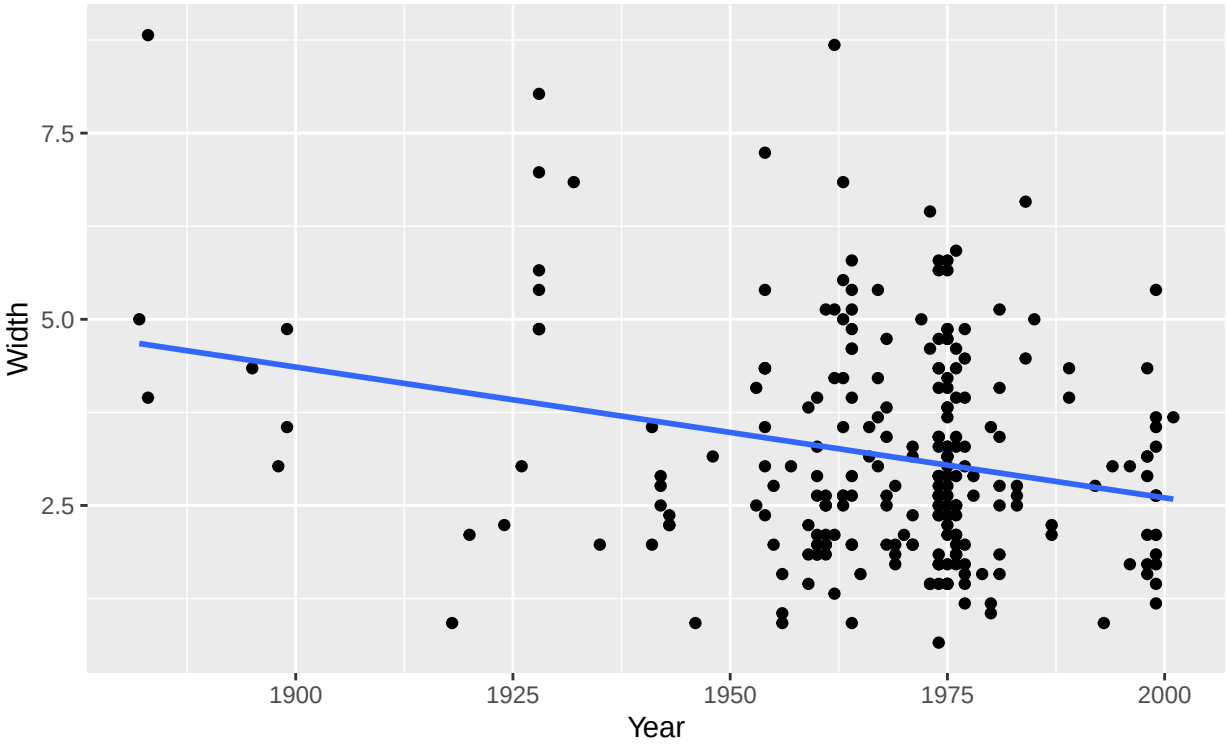
$$Residual_i = y_i - \hat{y}_i$$

Thus the residual for this particular year was 3.026316-2.70836 = 0.317956.

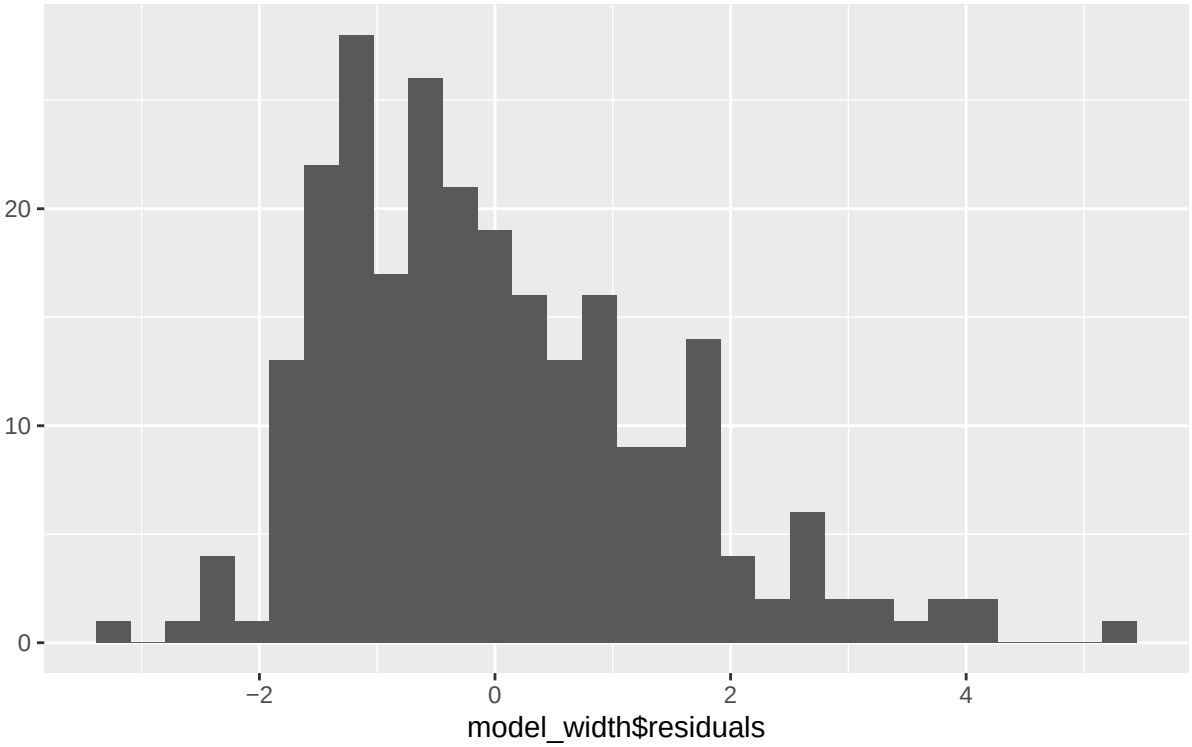
Since this is a positive residual, we know that the model *underestimated* the width by 0.32 mm; that the actual value was larger than what the model predicted it would have been.

4. Visualize the model

Make a scatterplot that includes the regression line

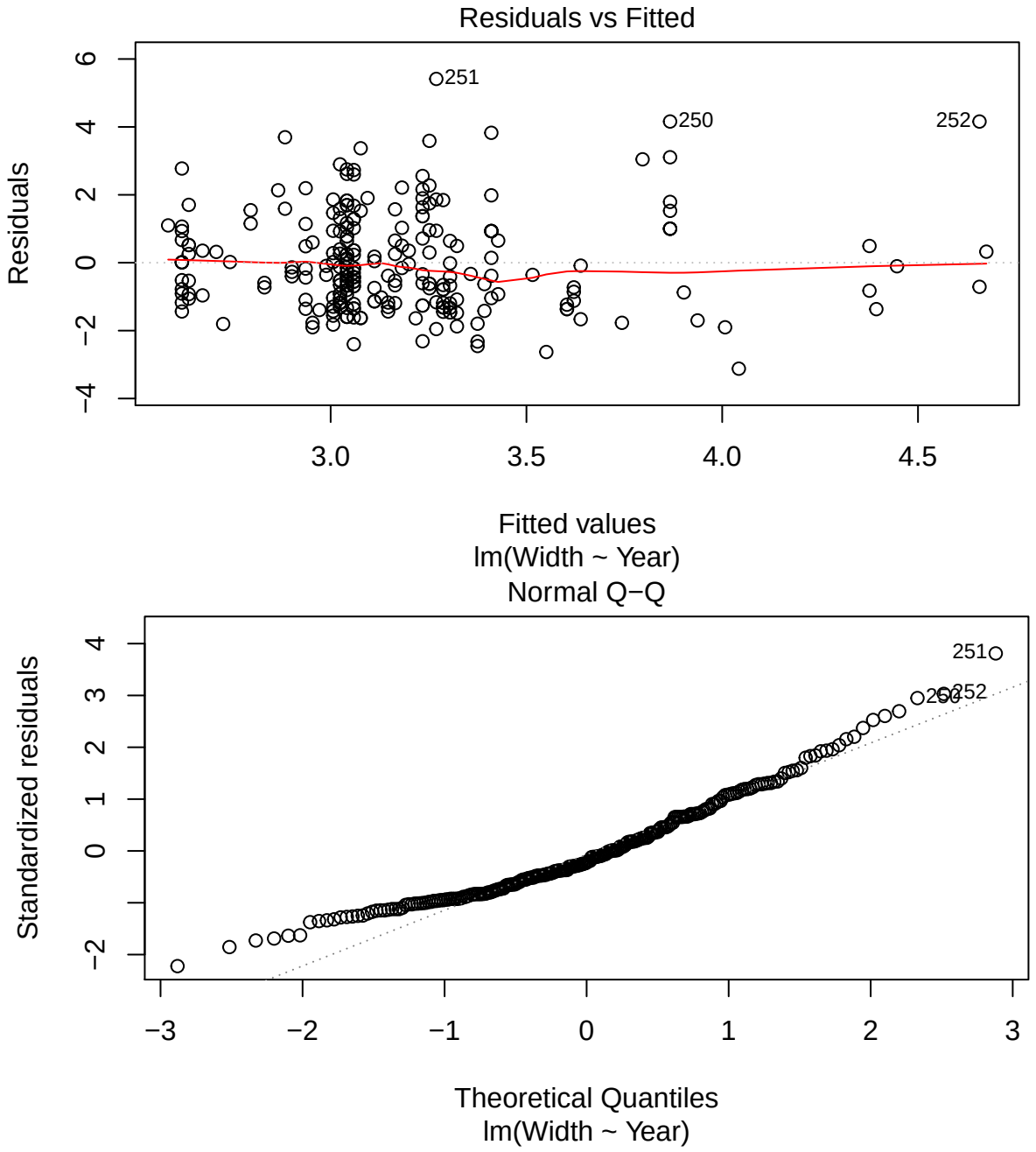


Make a histogram of the residuals



Here, we see that the residuals are not normally distributed, that the mean value may be 0, but that there are large positive residuals that create a skewed distribution of the residuals. We can see visually from the scatterplot with the regression model above that there are many more points substantially above the line – especially before 1950 or so.

Make a probability plot and residual fitted plot



This **first** plot illustrates a number of the key regression assumptions, especially:

- Zero Mean
- Linearity
- Constant Variance
- Normality (roughly)

We see that zero mean is met because the points are evenly distributed above and below the horizontal line at 0 on the y-axis.

Linearity appears to be met since there is no persistent shape left to the data.

Constant variance appears to be met in that there is not an apparent fan shape where the model is doing a better job fitting the data at some points (i.e., in older years) than at other points. We note that this distribution isn't symmetric – the y-axis points tends to be bound between -2 and 4, which is what we saw in the histogram above; if it were symmetric, it should be more evenly spread between, say, -2 and 2 or -4 and 4.

The **second** plot is another way of considering normality, other than the histogram. A normal distribution would have all of the points more or less on the diagonal line, since that would suggest a perfect 1-to-1 correspondence between the theoretical quantiles of a normal distribution (x-axis) and the actual, standardized residuals on the y-axis. Instead, curves in the bottom and top part of the line suggest that the data are not normally distributed. For more details on interpreting a QQ plot, see the OpenIntro textbook (especially p.95-99) here.