MSDA 605 - Fundamentals of Computational Mathematics Final Exam

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This final exam consists of three parts. The three parts are 1) Essential Concepts, 2) Coding, 3) Small Project.

Essential Concepts

1) What is the rank (number of lineraly indepdent rows) of the following matrix:

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    1    3    5
## [2,]    2   -1    5    7
## [3,]    6  -10   -1    3
```

First step is multiply first row by -2, results in the following matrix:

```
##      [,1] [,2] [,3] [,4]
## [1,]    2   -2   -6  -10
## [2,]    2   -1    5    7
## [3,]    6  -10   -1    3
```

Second step is subtract first row from second row:

```
##      [,1] [,2] [,3] [,4]
## [1,]    1   -1   -3   -5
## [2,]    0    1   11   17
## [3,]    6  -10   -1    3
```

Third step multiply row 1 by 6 then subtract first row from third row.

```
##      [,1] [,2] [,3] [,4]
## [1,]    1   -1   -3   -5
## [2,]    0    1   11   17
## [3,]    0   -4   17   33
```

Fourth step multiply row 1 by -1 and then second row by -4.

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    1    3    5
## [2,]    0   -4  -44   68
## [3,]    0   -4   17   33
```

Last step subtract second row from third row.

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    1    3    5
## [2,]    0    1   11   17
## [3,]    0    0   61  101
```

The matrix rank is 3. All rows are linearly indepdent.

2) What is the determinant of the following matrix:

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    1    3    5
## [2,]    2   -1    5    7
## [3,]    6  -10   -1    3
```

It is not possible to calculate the determinant of the matrix because it is not a square matrix.

3)Define orthonormal basis vectors. Please write down at least one orthonormal basis for the 5-dimensional vector space R5.

An Orthonormal Basis vector is when a orthogonal vector divided by its length = 1. An orthogonal vector is when

$$q1, ..., qn$$

have dot products equal to zero (

$$qi * qj$$

). Divide each vector by its length and the vectors become orthogonal unit vectors. The lengths are one.

A five dimenial orthonormal basis is the following:

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    0    0
## [2,]    0    1    0    0    0
## [3,]    0    0    1    0    0
## [4,]    0    0    0    1    0
## [5,]    0    0    0    0    1
```

(this is just the standard basis)

4)Given the following matrix, what is its characteristic polynomial?

```
##      [,1] [,2] [,3]
## [1,]    2   -1    4
## [2,]   -1   -2    6
## [3,]    1    0   -3
```

Characteristic polynomial of a square matrix is a polynomial, which is invariant under matrix similarity and has the eigenvalues as roots.

$$det(A - XI)$$

```r
q4.1 = matrix(c(NA, -1, 1, -1, NA, 0, 4, 6, NA), nrow=3, ncol=3) # NA = X in this matrix
q4.1
```

```
##      [,1] [,2] [,3]
## [1,]   NA   -1    4
## [2,]   -1   NA    6
## [3,]    1    0   NA
```

Characteristic Polynomial =

$$-x^3 - 3x^2 + 9x + 17$$

5) What are its eigenvectors and eigenvalues of the following matrix?

```
##      [,1] [,2] [,3]
## [1,]    2   -1    4
## [2,]   -1   -2    6
## [3,]    1    0   -3
```

Characteristic Polynomial =

$$-x^3 - 3x^2 + 9x + 17$$

derived from question 4. Find the roots for the characterisitic polynominal.

Eigenvalue of x1 (2-x) = 2.691 Eigenvalue of x2 (-2-x) = -4.18 Eigenvalue of x3 (-3-x) = -1.511

With those values if you plug them in the following are the results eigenvectors: x1: (86.43 , 1 , 15.18) x2: (0.36 , 1 , -0.30) x3: (0.16 , 1 , 0.11)

6)When would a model be said to have a high bias and when would it be said to have a high variance?