

ASSIGNMENT 5

IS 605 FUNDAMENTALS OF COMPUTATIONAL MATHEMATICS - 2014

1. PROBLEM SET 1

In this problem set we'll work out some properties of the *least squares* solution that we reviewed in the weekly readings. Consider the unsolvable system $\mathbf{Ax} = \mathbf{b}$ as given below:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 8 \\ 19 \end{bmatrix} \quad (1)$$

- Write R markdown script to compute $\mathbf{A}^T \mathbf{A}$ and $\mathbf{A}^T \mathbf{b}$.
- Solve for $\hat{\mathbf{x}}$ in R using the above two computed matrices.
- What is the squared error of this solution?
- Instead of $\mathbf{b} = [0; 8; 8; 19]$, start with $\mathbf{p} = [1; 5; 13; 17]$ and find the exact solution (i.e. show that the error goes down to zero for this system of equations)
- Show that the error $\mathbf{e} = \mathbf{b} - \mathbf{p} = [-1; 3; -5; 2]$.
- Show that the error \mathbf{e} is orthogonal to \mathbf{p} and to each of the columns of \mathbf{A} .

Your code should be able to print all of the above requested quantities. Please include enough comments to make it easy to follow your R markdown document.

2. PROBLEM SET 2

Consider the modified auto-mpg data (obtained from the UC Irvine Machine Learning dataset). This dataset contains 5 columns: *displacement*, *horsepower*, *weight*, *acceleration*, *mpg*. We are going to model *mpg* as a function of the other four variables.

Write an R markdown script that takes in the auto-mpg data, extracts an \mathbf{A} matrix from the first 4 columns and \mathbf{b} vector from the fifth (*mpg*) column. Using the *least squares* approach, your code should compute the best fitting solution. That is, find the best fitting equation that expresses *mpg* in terms of the other 4 variables. Finally, calculate the fitting error between the *predicted* error of your model and the actual *mpg*. Your script should be able to load in the 5 column data set, extract \mathbf{A} and \mathbf{b} , and perform the rest of the calculations. Please have adequate comments in your code to make it easy to follow your work.

Please complete both problem set 1 & problem set 2 in one R markdown document and upload it to the site. You don't have to attach the auto-mpg data. Just write your markdown document in such a way that it expects and loads the auto-mpg data file from

the current working directory. When I evaluate the assignment, I'll place a copy of the data in the same directory where I have your markdown document and run your script.