IS 622 Week 11 Homework

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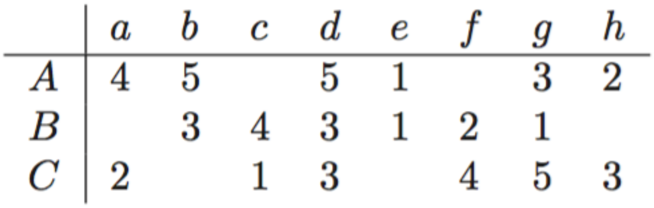
9.3.1

Figure 9.8 is a utility matrix, representing the ratings, on a 1–5 star scale, of eight items, a through h, by three users A, B, and C. Compute the following from the data of this matrix.



	a	b	c	d	e	f	g	h
A	4	5		5	1		3	2
B		3	4	3	1	2	1	
C	2		1	3		4	5	3

(a) Treating the utility matrix as boolean, compute the Jaccard distance between each pair of users.

```
df <- data.frame(a = c(4, NA, 2),
                 b = c(5, 3, NA),
                 c = c(NA, 4, 1),
                 d = c(5, 3, 3),
                 e = c(1, 1, NA),
                 f = c(NA, 2, 4),
                 g = c(3, 1, 5),
                 h = c(2, NA, 3))

rownames(df) <- c("A", "B", "C")
cn <- colnames(df)
user.pairs <- as.data.frame(t(combn(rownames(df), 2)))
colnames(user.pairs) <- c("UserPair1", "UserPair2")
```

```
library(Matrix)
jaccard <- function(m) {
    A = tcrossprod(m)
    im = which(A > 0, arr.ind=TRUE)
    b = rowSums(m)
    Aim = A[im]

    ## Jacard formula: #common / (#i + #j - #common)
    J = sparseMatrix(
          i = im[,1],
          j = im[,2],
          x = Aim / (b[im[,1]] + b[im[,2]] - Aim),
          dims = dim(A)
    )
    return( J )
}

jaccard(df)
```

```
##   UserPair1 UserPair2 Jaccard.Distance
## 1         A         B              0.5
## 2         A         C              0.5
## 3         B         C              0.5
```

(b) Repeat Part (a), but use the cosine distance.

```
cos.sim <- function(ix)
{
    A = X[ix[1],]
    B = X[ix[2],]
    return( sum(A*B)/sqrt(sum(A^2)*sum(B^2)) )
}
n <- nrow(X)
cmb <- expand.grid(i=1:n, j=1:n)
C <- matrix(apply(cmb,1,cos.sim),n,n)

cos.sim(df)
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.6010408
## 2         A         C       0.6149187
## 3         B         C       0.5138701
```

(c) Treat ratings of 3, 4, and 5 as 1 and 1, 2, and blank as 0. Compute the Jaccard distance between each pair of users.

```
binrating <- function(i) {
  ifelse(i %in% c(3, 4, 5), TRUE, FALSE)
  }
jaccard <- function(m) {
    A = tcrossprod(m)
    im = which(A > 0, arr.ind=TRUE)
    b = rowSums(m)
    Aim = A[im]

    ## Jacard formula: #common / (#i + #j - #common)
    J = sparseMatrix(
          i = im[,1],
          j = im[,2],
          x = Aim / (b[im[,1]] + b[im[,2]] - Aim),
          dims = dim(A)
    )

    return( J )
}

jaccard(binrating)
```

```
##   UserPair1 UserPair2 Jaccard.Distance
## 1         A         B        0.4000000
## 2         A         C        0.3333333
## 3         B         C        0.1666667
```

(d) Repeat Part (c), but use the cosine distance.

```
cos.sim <- function(ix)
{
    A = X[ix[1],]
    B = X[ix[2],]
    return( sum(A*B)/sqrt(sum(A^2)*sum(B^2)) )
}
n <- nrow(X)
cmb <- expand.grid(i=1:n, j=1:n)
C <- matrix(apply(cmb,1,cos.sim),n,n)

cos.sim(binrating)
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.5773503
## 2         A         C       0.5000000
## 3         B         C       0.2886751
```

(e) Normalize the matrix by subtracting from each nonblank entry the average value for its user.

```
 normalized <- t(apply(df, 1, function(i) {
  i - mean(i, na.rm=TRUE)
}))

normalized
```

```
##            a         b         c         d         e          f          g
## A  0.6666667 1.6666667        NA 1.6666667 -2.333333         NA -0.3333333
## B         NA 0.6666667  1.666667 0.6666667 -1.333333 -0.3333333 -1.3333333
## C -1.0000000        NA -2.000000 0.0000000        NA  1.0000000  2.0000000
##           h
## A -1.333333
## B        NA
## C  0.000000
```

(f) Using the normalized matrix from Part (e), compute the cosine distance between each pair of users.

```
normalized[is.na(normalized)] <- 0

cos.sim <- function(ix)
{
    A = X[ix[1],]
    B = X[ix[2],]
    return( sum(A*B)/sqrt(sum(A^2)*sum(B^2)) )
}
```

```
n <- nrow(X)
cmb <- expand.grid(i=1:n, j=1:n)
C <- matrix(apply(cmb,1,cos.sim),n,n)

cos.sim(normalized)
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.5843065
## 2         A         C      -0.1154701
## 3         B         C      -0.7395740
```