CUNY IS 622 Week 15 Homework

Ben Arancibia

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11.1.6

For the matrix of Exercise 11.1.4:

```
matrix <- matrix(c(1,1,1,1,2,3,1,3,5), ncol=3, byrow=TRUE)
matrix
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1    2    3
## [3,]    1    3    5
```

(a) Starting with a vector of three 1's, use power iteration to find an approximate value of the principal eigenvector.

```
x <- matrix(c(1,1,1))
tolerance <- 0.0007

#page 409
frobenius.norm <- function(M) {
  return(sqrt(sum(apply(M, c(1, 2), function(x) {x*x}))));
}

#pages 409-410
powerm_nr <- function(M, x, tolerance){
 diff = 1
  x_new <- x
  count <- 0;
  while(diff > tolerance && count < 10000) {
    x_old <- x_new
    x_new <- (M %*% x_old) / frobenius.norm(M %*% x_old)
    if(x_new[which.max(abs(x_new))] < 0) {
      x_new <- -1 * x_new;
    }
    diff <- frobenius.norm(x_new - x_old)
    count <- count + 1;
  }
  return(x_new);
}


evector <- powerm_nr(matrix, x, tolerance)
evector
```

```
##           [,1]
## [1,] 0.2185605
## [2,] 0.5216310
## [3,] 0.8247014
```

(b) Compute an estimate the principal eigenvalue for the matrix.

```r
eigen_value <- function(M, evector) {
  return(as.double(t(evector) %*% M %*% evector))
}
eigen.one <- eigen_value(matrix, evector)
eigen.one
```

```
## [1] 7.162278
```

(c) Construct a new matrix by subtracting out the effect of the principal eigenpair, as in Section 11.1.3.

```r
#substract out the effect of the principal eigenpair
matrixc <- matrix - eigen.one * evector %*% t(evector)
```

(d) From your matrix of (c), find the second eigenpair for the original matrix of Exercise 11.1.4.

```r
evector.two <- powerm_nr(matrixc, x, tolerance)
evector.two
```

```
##            [,1]
## [1,]  0.8861741
## [2,]  0.2471062
## [3,] -0.3919616
```

```r
eigen.two <- eigen_value(matrixc, evector)
eigen.two
```

```
## [1] -2.711861e-16
```

(e) Repeat (c) and (d) to find the third eigenpair for the original matrix.

```r
matrixe <-  matrixc- eigen.two * evector.two %*% t(evector.two)

evector.three <- powerm_nr(matrixe, x, tolerance)
evector.three
```

```
##            [,1]
## [1,]  0.8861741
## [2,]  0.2471062
## [3,] -0.3919616
```

```r
eigen.three <- eigen_value(matrixe, evector.three)
eigen.three
```

```
## [1] 0.8377228
```

11.3.2

Use the SVD from Fig. 11.7. Suppose Leslie assigns rating 3 to Alien and rating 4 to Titanic, giving us a representation of Leslie in "movie space" of [0, 3, 0, 0, 4]. Find the representation of Leslie in concept space. What does that representation predict about how well Leslie would like the other movies appearing in our example data?

$$
\underset{M}{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix}} = \underset{U}{\begin{bmatrix} .14 & 0 \\ .42 & 0 \\ .56 & 0 \\ .70 & 0 \\ 0 & .60 \\ 0 & .75 \\ 0 & .30 \end{bmatrix}} \underset{\Sigma}{\begin{bmatrix} 12.4 & 0 \\ 0 & 9.5 \end{bmatrix}} \underset{V^{\mathrm{T}}}{\begin{bmatrix} .58 & .58 & .58 & 0 & 0 \\ 0 & 0 & 0 & .71 & .71 \end{bmatrix}}
$$

Figure 11.7: SVD for the matrix M of Fig. 11.6

```r
vt <- matrix(c(0.58, 0.58, 0.58, 0, 0, 0, 0, 0, 0.71, 0.71), ncol = 5, byrow = TRUE)

leslie <- c(0, 3, 0, 0, 4)

leslie %*% t(vt)
```

```
##      [,1] [,2]
## [1,] 1.74 2.84
```

She will rank the Titanic higher than Alien.