# IS 622 Week 11 Homework

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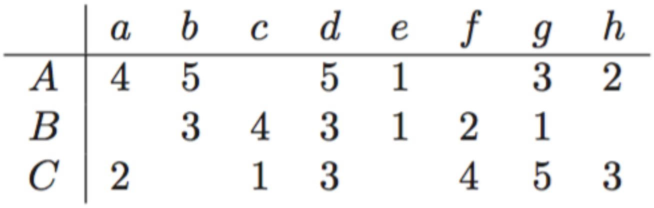
### 9.3.1

Figure 9.8 is a utility matrix, representing the ratings, on a 1–5 star scale, of eight items, a through h, by three users A, B, and C. Compute the following from the data of this matrix.



|  | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ |
|---|---|---|---|---|---|---|---|---|
| $A$ | 4 | 5 |  | 5 | 1 |  | 3 | 2 |
| $B$ |  | 3 | 4 | 3 | 1 | 2 | 1 |  |
| $C$ | 2 |  | 1 | 3 |  | 4 | 5 | 3 |

(a) Treating the utility matrix as boolean, compute the Jaccard distance between each pair of users.

```
df <- data.frame(a = c(4, NA, 2),
                 b = c(5, 3, NA),
                 c = c(NA, 4, 1),
                 d = c(5, 3, 3),
                 e = c(1, 1, NA),
                 f = c(NA, 2, 4),
                 g = c(3, 1, 5),
                 h = c(2, NA, 3))

rownames(df) <- c("A", "B", "C")
cn <- colnames(df)
user.pairs <- as.data.frame(t(combn(rownames(df), 2)))
colnames(user.pairs) <- c("UserPair1", "UserPair2")
```

```
library(Matrix)
jaccard <- function(m) {
    A = tcrossprod(m)
    im = which(A > 0, arr.ind=TRUE)
    b = rowSums(m)
    Aim = A[im]

    ## Jacard formula: #common / (#i + #j - #common)
    J = sparseMatrix(
          i = im[,1],
          j = im[,2],
          x = Aim / (b[im[,1]] + b[im[,2]] - Aim),
          dims = dim(A)
    )
    return( J )
}
```

```
##   UserPair1 UserPair2 Jaccard.Distance
## 1         A         B              0.5
## 2         A         C              0.5
## 3         B         C              0.5
```

(b) Repeat Part (a), but use the cosine distance.

```
len <- function(v) {
  sqrt(sum(v**2))
  }

cosine.dist <- function(v1, v2) {
  v1[is.na(v1)] <- 0
  v2[is.na(v2)] <- 0
  (as.numeric(v1) %*% as.numeric(v2)) / (len(v1) * len(v2))
}


cd <- user.pairs
cd["Cosine.Distance"] <-
  apply(user.pairs, 1, function(i) {
    cosine.dist(df[i[1], ], df[i[2], ])
})


cd
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.6010408
## 2         A         C       0.6149187
## 3         B         C       0.5138701
```

(c) Treat ratings of 3, 4, and 5 as 1 and 1, 2, and blank as 0. Compute the Jaccard distance between each pair of users.

```
binrating <- function(i) {ifelse(i %in% c(3, 4, 5), TRUE, FALSE)}

# Apply distance function
jd.c <- user.pairs
jd.c["Jaccard.Distance"] <-
  apply(user.pairs, 1, function(i) {
    s1 <- cn[sapply(df[i[1], ], binrating)]
    s2 <- cn[sapply(df[i[2], ], binrating)]
    jaccard(s1, s2)
})

jd.c
```

```
##   UserPair1 UserPair2 Jaccard.Distance
## 1         A         B        0.4000000
## 2         A         C        0.3333333
## 3         B         C        0.1666667
```

(d) Repeat Part (c), but use the cosine distance.

```
cd.d <- user.pairs
cd.d["Cosine.Distance"] <-
  apply(user.pairs, 1, function(i) {
    v1 <- binrating(df[i[1], ])
    v2 <- binrating(df[i[2], ])
    cosine.dist(v1, v2)
  })

# Format output
cd.d
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.5773503
## 2         A         C       0.5000000
## 3         B         C       0.2886751
```

(e) Normalize th ematrix by subtracting from each nonblank entry the average value for its user.

```
df.normalized <- t(apply(df, 1, function(i) {
  i - mean(i, na.rm=TRUE)
}))

df.normalized
```

```
##            a         b         c         d         e          f          g
## A  0.6666667 1.6666667        NA 1.6666667 -2.333333         NA -0.3333333
## B         NA 0.6666667  1.666667 0.6666667 -1.333333 -0.3333333 -1.3333333
## C -1.0000000        NA -2.000000 0.0000000        NA  1.0000000  2.0000000
##           h
## A -1.333333
## B        NA
## C  0.000000
```

(f) Using the normalized matrix from Part (e), compute the cosine distance between each pair of users.

```
df.normalized[is.na(df.normalized)] <- 0

# Apply distance function
cd.f <- user.pairs
cd.f["Cosine.Distance"] <-
  apply(user.pairs, 1, function(i) {
    v1 <- df.normalized[i[1], ]
    v2 <- df.normalized[i[2], ]
    cosine.dist(v1, v2)
  })
```

```
# Format output
cd.f
```

```
##   UserPair1 UserPair2 Cosine.Distance
## 1         A         B       0.5843065
## 2         A         C      -0.1154701
## 3         B         C      -0.7395740
```