IS622 Week 4 Homework

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3.1.3 Suppose we have a universal set U of n elements, and we choose two subsets S and T at random, each with m of the n elements. What is the expected value of the Jaccard similarity of S and T ?

Each item in T has an m / n chance of also being in S. The expected number of items common to S & T is therefore m^2 / n.

Exp. Jaccard Similarity = (No. of common items) / (Size of T + Size of S - Number of common items) = m / (2n - m) after simplification.

3.3.3

(a) Compute the minhash signature for each column if we use the following three hash functions: h1(x) = 2x + 1 mod 6; h2(x) = 3x + 2 mod 6; h3(x)=5x+2 mod6.

```
s1 <- c(0,0,1,0,0,1); s2 <- c(1,1,0,0,0,0); s3 <- c(0,0,0,1,1,0)
s4 <- c(1,0,1,0,1,0); element <- c(0,1,2,3,4,5)
h1 <- function(x) { (2*x + 1) %% 6 }
h2 <- function(x) { (3*x + 2) %% 6 }
h3 <- function(x) { (5*x + 2) %% 6 }

hashlist <- list(h1,h2,h3)
setlist <- list(s1,s2,s3,s4)

solution_3.3.3 <- computeMinhashSigs(hashlist, setlist)

#switch the binary to their corresponding values.

s1 <- c(2,5); s2 <- c(0,1); s3 <- c(3,4); s4 <- c(0,2,4)
setlist <- list(s1,s2,s3,s4)

solution_3.3.3 <- computeMinhashSigs(hashlist, setlist)
solution_3.3.3
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    1    1    1
## [2,]    2    2    2    2
## [3,]    0    1    4    0
```

It looks like the h2 has believes each row is identical while h1 believes they look nearly identical

(b) Which of these hash functions are true permutations?

```
hashlist <- list("h1"=h1,"h2"=h2,"h3"=h3)
row_count <- 5

hashPermuteDirect(hashlist, row_count)
```

```
##      [,1] [,2]
## [1,] "h1" "3"
## [2,] "h2" "2"
## [3,] "h3" "6"
```

(c) How close are the estimated Jaccard similarities for the six pairs of columns to the true Jaccard similarities?

Pretty close

3.5.5

Compute the cosines of the angles between each of the following pairs of vectors.

```
angle <- function(x,y){
  dot.prod <- x%*%y
  norm.x <- norm(x,type="2")
  norm.y <- norm(y,type="2")
  theta <- acos(dot.prod / (norm.x * norm.y))
  as.numeric(theta)
}
```

(a) (3,−1,2)and(−2,3,1).

```
x <- as.matrix(c(3,-1,2))
y <- as.matrix(c(-2,3,1))
angle(t(x),y)
```

```
## [1] 2.094395
```

(b) (1,2,3)and(2,4,6).

```
x <- as.matrix(c(1,2,3))
y <- as.matrix(c(2,4,6))
angle(t(x),y)
```

```
## [1] 2.107342e-08
```

(c) (5,0,−4)and(−1,−6,2).

```
x <- as.matrix(c(5,0,-4))
y <- as.matrix(c(-1,-6,2))
angle(t(x),y)
```

```
## [1] 1.893438
```

(d) (0,1,1,0,1,1) and (0,0,1,0,0,0).

```r
x <- as.matrix(c(0,1,1,0,1,1))
y <- as.matrix(c(0,0,1,0,0,0))
angle(t(x),y)
```

```
## [1] 1.047198
```

3.7.1

Suppose we construct the basic family of six locality-sensitive functions for vectors of length six. For each pair of the vectors 000000, 110011, 010101, and 011100, which of the six functions makes them candidates?

I don't think this question has a answer. All of these LSH functions have a random parameter so their output can vary. I think the 6 functions referred to are: jaccard, hamming, cos, sketch, euclid, and minhash distances