The Effective Use of Benford's Law to Assist in Detecting Fraud in Accounting Data

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The Effective Use of Benford's Law to Assist in Detecting Fraud in Accounting Data

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Benford's law has been promoted as providing the auditor with a tool that is simple and effective for the detection of fraud. The purpose of this paper is to assist auditors in the most effective use of digital analysis based on Benford's law. The law is based on a peculiar observation that certain digits appear more frequently than others in data sets. For example, in certain data sets, it has been observed that more than 30% of numbers begin with the digit one. After discussing the background of the law and development of its use in auditing, we show where digital analysis based on Benford's law can most effectively be used and where auditors should exercise caution. Specifically, we identify data sets which can be expected to follow Benford's distribution, discuss the power of statistical tests, types of frauds that would be detected and not be detected by such analysis, the potential problems that arise when an account contains too few observations, as well as issues related to base rate of fraud. An actual example is provided demonstrating where Benford's law proved successful in identifying fraud in a population of accounting data.

INTRODUCTION

In the past half-century, more than 150 articles have been published about Benford's law, a quirky law based on the number of times a particular digit occurs in a particular position in numbers (Nigrini 1999). In the past 10 years a subset of these articles have promoted the use of this law (the study of digits or digital analysis) as a simple, effective way for auditors to not only identify operational discrepancies, but to uncover fraud in accounting numbers. Recent audit failures and the issuance of Statement on Auditing Standard No. 99, *Consideration of Fraud in a Financial Statement Audit* (AICPA 2002) have set the profession in search of analytical tools and audit methods to detect fraud. Specifically, SAS No. 99 (paragraph 28) reiterates SAS No. 56 in requiring auditors to employ analytical procedures during the planning phase of the audit with the objective to identify the existence of

unusual transactions, events and trends. At the same time, SAS No. 99 cautions that relying on analytical procedures, which are traditionally done on highly aggregated data, can provide only broad indications of fraud. The purpose of this paper is to assist auditors in the most effective use of digital analysis based on Benford's law. When used properly, digital analysis conducted on transaction level data, rather than aggregated data, can assist auditors by identifying specific accounts in which fraud might reside so that they can then analyze the data in more depth.

Specifically, we provide guidance to auditors to help them distinguish between those circumstances in which digital analysis might be useful in detecting fraud and those circumstances where digital analysis cannot detect fraud. Further, we provide guidance on how to interpret the results of such tests so auditors can better assess the amount of reliance they should place on digital analysis as a way to detect fraud. Coderre (2000) states, "Auditors should use discretion when applying this method … as it is not meant for all data analysis situations." Our paper attempts to provide auditors with facts needed to exercise that discretion.

We begin by providing an overview of Benford's law, and how it applies to accounting and in particular fraud detection. We next detail the types of accounting numbers that might be expected to conform to a Benford distribution and which accounting numbers may not, thus highlighting situations where digital analysis is most useful. We next discuss which digital analysis tests should be conducted and how the results are interpreted. The following section discusses the limitations of digital analysis for detecting certain types of fraud. We provide some insight into the overall effectiveness of digital analysis conditioned on the underlying fraud rate. We include an example of digital analysis using data from an actual entity and show the results of a normal account versus an account that was later determined to contain fraud. In summary, we conclude that if used appropriately, digital analysis can increase an auditor's ability to detect fraud.

ORIGIN OF BENFORD'S LAW

In 1881, Simon Newcomb, an astronomer and mathematician, published the first known article describing what has become known as Benford's law in the *American Journal of Mathematics.* He observed that library copies of books of logarithms were considerably more worn in the beginning pages which dealt with low digits and progressively less worn on the pages dealing with higher digits. He inferred from this pattern that fellow scientists used those tables to look up numbers which started with the numeral one more often than

those starting with two, three, and so on. The obvious conclusion was that more numbers exist which begin with the numeral one than with larger numbers. Newcomb calculated that the probability that a number has any particular non-zero first digit is:

$$P(d) = Log_{10}(1+1/d) \tag{1}$$

    Where: d is a number 1, 2 …9, and
           P is the probability

Using his formula, the probability that the first digit of a number is one is about 30 percent while the probability the first digit a nine is only 4.6 percent. Table 1 shows the expected frequencies for all digits 0 through 9 in each of the first four places in any number.

Table 1
Expected Frequencies Based on Benford's Law

Digit	1st place	2nd place	3rd place	4th place
0		.11968	.10178	.10018
1	.30103	.11389	.10138	.10014
2	.17609	.19882	.10097	.10010
3	.12494	.10433	.10057	.10006
4	.09691	.10031	.10018	.10002
5	.07918	.09668	.09979	.09998
6	.06695	.09337	.09940	.09994
7	.05799	.09035	.09902	.09990
8	.05115	.08757	.09864	.09986
9	.04576	.08500	.09827	.09982

Source: Nigrini, 1996.

Formulas for expected digital frequencies:
For first digit of the number:

       Probability $(D_1 = d_1) = \log(1+(1/d_1))$; $d_1 = (1,2, 3…9)$

For second digit of the number:

$$Probability\ (D_2 = d_2) = \sum_{d_1=1}^{9} \log(1+(1/d_1 d_2)); \ d_2 = (1, 2, 3 ... 0)$$

For two digit combinations:

       Probability $(D_1 D_2 = d_1 d_2) = \log(1+(1/d_1 d_2))$
       Probability $(D_2 = d_2 \mid D_1 = d_1) = \log(1+(1/d_1 d_2))/\log(1+(1/d_1))$

Where

       D_1 represents the first digit of a number,

       D_2 represents the second digit of a number, etc.

Newcomb provided no theoretical explanation for the phenomena he described and his arti-
cle went virtually unnoticed. Then, almost 50 years later, apparently independent from
Newcomb's original article, Frank Benford, a physicist, also noticed that the first few pages
of his logarithm books were more worn than the last few. He came to the same conclusion
Newcomb had arrived at years prior; that people more often looked up numbers that began
with low digits rather than high ones. He also posited that there were more numbers that
began with the lower digits. He, however, attempted to test his hypothesis by collecting and
analyzing data. Benford collected more than 20,000 observations from such diverse data
sets as areas of rivers, atomic weights of elements, and numbers appearing in *Reader's
Digest* articles (Benford 1938). Diaconis and Freedman (1979) offer convincing evidence
that Benford manipulated round-off errors to obtain a better fit to a logarithmic law, but even
the non-manipulated data are a remarkably good fit (Hill 1995). Benford found that num-
bers consistently fell into a pattern with low digits occurring more frequently in the first
position than larger digits. The mathematical tenet defining the frequency of digits became
known as Benford's law.

For almost 90 years mathematicians and statisticians offered various explanations for this
phenomenon. Raimi's 1976 article describes some of the less rigorous explanations that
range from Goudsmit and Furry's (1944) thesis that the phenomena being the result of "the
way we write numbers," to Furlan's (1948) theory that Benford's law reflects a profound
"harmonic" truth of nature. It wasn't until 1995 that Hill, a mathematician, provided a proof
for Benford's law as well as demonstrating how it applied to stock market data, census sta-
tistics, and certain accounting data (Hill 1995). He noted that Benford's distribution, like
the normal distribution, is an empirically observable phenomenon. Hill's proof relies on the
fact that the numbers in sets that conform to the Benford distribution are second generation
distributions, that is, combinations of other distributions. If distributions are selected at ran-
dom and random samples are taken from each of these distributions, then the significant-
digit frequencies of the combined samplings will converge to Benford's distribution, even
though the individual distributions may not closely follow the law (Hill 1995; Hill 1998).
The key is in the combining of numbers from different sources. In other words, combining
unrelated numbers gives a distribution of distributions, a law of true randomness that is uni-
versal (Hesman 1999).

Boyle (1994) shows that data sets follow Benford's law when the elements result from ran-
dom variables taken from divergent sources that have been multiplied, divided, or raised to
integer powers. This helps explain why certain sets of accounting numbers often appear to
closely follow a Benford distribution. Accounting numbers are often the result of a mathe-

matical process. A simple example might be an account receivable which is a number of items sold (which comes from one distribution) multiplied by the price per item (coming from another distribution). Another example would be the cost of goods sold which is a mathematical combination of several numbers, each of which comes from a different source.

Although the mathematical proof is beyond the needs of our discussion, intuitively the law is not difficult to understand. Consider the market value of a firm. If it is $1,000,000, it will have to double in size before the first digit is a "2," in other words it needs to grow 100 percent.[1] For the first digit to be a "3," it only needs to grow 50 percent. To be a "4" the firm must only grow 33 percent and so on. Therefore, in many distributions of financial data, which measure the size of anything from a purchase order to stock market returns, the first digit one is much further from two than eight is from nine. Thus, the observed finding is that for these distributions, smaller values of the first significant digits are much more likely than larger values.[2]

BENFORD'S LAW APPLIED TO AUDITING AND ACCOUNTING

Auditors have long applied various forms of digital analysis when performing analytical procedures. For example, auditors often analyze payment amounts to test for duplicate payments. They also search for missing check or invoice numbers. Benford's law as applied to auditing is simply a more complex form of digital analysis.[3] It looks at an entire account to determine if the numbers fall into the expected distribution.

Although Varian (1972), an economist, suggests that Benford's law can be used as a test of the honesty or validity of purportedly random scientific data in a social science context, it wasn't picked up by accountants until the late 1980s. At that time, two studies relied on digital analysis to detect earnings manipulation. Carslaw (1988) found that earnings numbers from New Zealand firms did not conform to the expected distribution. Rather, the numbers contained more zeros in the second digit position than expected and fewer nines, thus implying that when a firm had earnings such as $1,900,000, they rounded up to $2,000,000. Although Carslaw used the Benford distribution as his expectation, he referred to it as "Feller's Proof." Thomas (1989) discovered a similar pattern in the earnings of U.S. firms.

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[1] The growth rate from $1,000,000 to $2,000,000 is determined as ($2,000,000-$1,000,000)/$1,000,000 = 100 percent.

[2] A similar description, in British pounds, is found in Carr (2000).

[3] An excellent operational description of the application of Benford's law for auditing is found in Drake and Nigrini (2000).

Nigrini appears to be the first researcher to apply Benford's law extensively to accounting numbers with the goal to detect fraud. According to an article published in Canadian Business (1995), Nigrini first became interested in the work on earnings manipulation by Carslaw and Thomas then separately came across Benford's work and wed the two ideas together for his dissertation. His dissertation used digital analysis to help identify tax evaders (Nigrini 1996). More recently, papers have been published which detail practical applications of digital analysis such as descriptions of how an auditor performs tests on sets of accounting numbers, how an auditor uses digital analysis computer programs, and case studies for training students (Nigrini and Mittermaier 1997).[4]

The academic literature is somewhat cautious in making claims about the effectiveness of procedures based on Benford's law to detect fraud. In particular, such work cautions that a data set which, when tested, does not conform to Benford's law, may show only operating inefficiencies or flaws in systems rather than fraud (Etteridge and Srivastava 1999). Our paper expands on those studies to discuss why certain data sets are appropriate for digital analysis and others are not. We explain why some types of fraud cannot be identified by digital analysis. We show how tests of the results of digital analysis can be interpreted as well as why care must be taken in the interpretation. All this will inform auditors' discretion as they apply digital analysis to a particular work environment.

WHEN TO USE DIGITAL ANALYSIS

When an auditor chooses to use digital analysis in an attempt to detect fraud, several issues should be considered. First, on which types of accounts might Benford analysis be expected to be effective? While most accounting-related data sets conform to a Benford distribution, there are some exceptions. And since digital analysis is only effective when applied to conforming sets, auditors must consider whether a particular data set should be expected to fall into a Benford distribution prior to conducting digital analysis. Second, what tests should be run and how should the results of those tests be interpreted? Since there are high costs associated with false positives (identifying a fraud condition when none is present) as well as false negatives (failing to identify a fraud condition when one exists), one must consider the level of significance, or threshold beyond which accounts are deemed contaminated and selected for further investigation. Third, when is digital analysis ineffective? In

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[4] York (2000) provides directions for conducting a Benford analysis in Excel. Nigrini (1999) demonstrates how a routine in Audit Command Language (ACL) can be used to perform digital analysis. Lanza (2000) describes DATAS, a statistical analysis tool, that performs Benford digital analysis. Etteridge and Srivastava (1999) provide three assignments and a completed Excel program for students to use to understand digital analysis.

other words, are there categories of fraud that cannot be signaled using digital analysis? Finally, how much assistance can auditors expect to receive from Benford's law in their ability to identify suspect accounts for further investigation? Each of these issues is addressed in the subsequent sections.

Choosing Appropriate Data Sets for Analysis

Most accounting-related data can be expected to conform to a Benford distribution, and thus will be appropriate candidates for digital analysis (Hill 1995). Such is the case because typical accounts consist of transactions that result from combining numbers. For example, accounts receivable is the number of items purchased multiplied by the price per item. Similarly, accounts payable and most revenue and expense accounts are expected to conform. Account size, meaning the number of entries or transactions, also matters. In general, results from Benford analysis are more reliable if the entire account is analyzed rather than sampling the account. This is because the larger the number of transactions or items in the data set, the more accurate the analysis.

Benford analysis will reveal various underlying peculiarities in an account. Therefore, not all accounts labeled as "non-conforming" will be fraudulent. For example, we ran digital analysis on various accounts of a large medical center. In this analysis, the laboratory expenses were flagged as not conforming to a Benford distribution when there was no reason a priori to believe it would not. Further investigation revealed that certain authorized, but repetitive transactions caused the account to fail the statistical tests. Specifically, there were numerous purchases for $11.40, which turned out to be the cost of liquid nitrogen ordered by dermatologists, as well as numerous purchases for $34.95, which were costs for cases of bottled water. Once these entries were removed, the data set conformed to the expected distribution.

Some populations of accounting-related data do not conform to a Benford distribution. For example, assigned numbers, such as check numbers, purchase order numbers, or numbers that are influenced by human thought, such as product or service prices, or ATM withdrawals, do not follow Benford's law (Nigrini and Mittermaier 1997). Assigned numbers should follow a uniform distribution rather than a Benford distribution. Prices are often set to fall below psychological barriers, for example $1.99 has been shown to be perceived as much lower than $2.00, thus prices tend to cluster below psychological barriers. ATM withdrawals are often in pre-assigned, even amounts. Other accounts which might not follow a Benford distribution will be firm specific. For example, in the medical center the "patient refund" account did not conform because most refunds involved co-payments which were

often pre-assigned amounts and applied to large numbers of patients.[5] Other examples of accounts which would not be expected to conform to a Benford distribution would be those that have a built-in maximum or minimum value. For example, a list of assets that must achieve a certain materiality level before being recorded would have a built-in minimum and therefore would not likely conform.

In addition to an auditor's judgment in determining which populations fit a Benford distribution, there exist some tests that reveal whether or not Benford's law applies to a particular data set. Wallace (2002) suggests that if the mean of a particular set of numbers is larger than the median and the skewness value is positive, the data set likely follows a Benford distribution. It follows that the larger the ratio of the mean divided by the median, the more closely the set will follow Benford's law. This is true since observations from a Benford distribution have a predominance of small values. The difficulty in relying only on such tests as a screening process, before applying digital analysis, is that if an account contains sufficient bogus observations it could fail the tests; thus, digital analysis would not be applied when, in fact, it should. Table 2 summarizes when it is appropriate to use Benford analysis, and when to use caution.

Table 2
When Benford Analysis Is or Is Not Likely Useful

When Benford Analysis Is Likely Useful	Examples
Sets of numbers that result from mathematical combination of numbers - Result comes from two distributions	Accounts receivable (number sold * price), Accounts payable (number bought * price)
Transaction-level data - No need to sample	Disbursements, sales, expenses
On large data sets - The more observations, the better	Full year's transactions
Accounts that appear to conform - When the mean of a set of numbers is greater than the median and the skewness is positive	Most sets of accounting numbers
When Benford Analysis Is Not Likely Useful	**Examples**
Data set is comprised of assigned numbers	Check numbers, invoice numbers, zip codes
Numbers that are influenced by human thought	Prices set at psychological thresholds ($1.99), ATM withdrawals
Accounts with a large number of firm-specific numbers	An account specifically set up to record $100 refunds
Accounts with a built in minimum or maximum	Set of assets that must meet a threshold to be recorded
Where no transaction is recorded	Thefts, kickbacks, contract rigging

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[5] An analysis of patient refunds from a large medical center showed a higher number of entries which began with the digit 2 and a lower than expected number of entries which began with the digits 6-9.

Interpreting Results of the Statistical Tests

Two underlying concepts should be considered when deciding how effective digital analysis based on Benford's law might be. First, the effectiveness of digital analysis declines as the level of contaminated entries drops, and not all accounts which contain fraud contain a large number of fraudulent transactions. Second, in many instances accounts identified as non-conforming do not contain fraud. An example of this was given previously in the case of the operating supplies for the medical center. These facts are particularly important when considering the usefulness of statistical tests.

As in any statistical test, digital analysis compares the actual number of items observed to the expected and calculates the deviation. For example, in a Benford distribution, the expected proportion of numbers which contain the digit one in the first position is 30.103 percent. The actual proportion observed will most likely deviate from this expected amount due to random variation. While no data set can be expected to conform precisely, at what point is the deviation considered large enough to be a significant indication of fraud?

The expected distribution of digit frequency, based on Benford's law, is a logarithmic distribution that appears visually like a Chi-square distribution. Such a distribution deviates significantly from a normal or uniform distribution. The standard deviation for each digit's expected proportion is:

$$s_i = [p_i^* \, (1 - p_i)/n]^{1/2} \qquad (2)$$

Where: s_i is the standard deviation of each digit, 1 through 9;

p_i is the expected proportion of a particular digit based on Benford's law; and

n is the number of observations in the data set (particular account).

A z-statistic can be used to determine whether a particular digit's proportion from a set of data is suspect. In other words, does a digit appear more or less frequently in a particular position than a Benford distribution would predict? The z-statistic is calculated as follows (Nigrini 1996):

$$z = (|(p_o - p_e| - 1/(2n))/s_i \qquad (3)$$

Where: p_o is the observed proportion in the data set;

p_e is the expected proportion based on Benford's law;

s_i is the standard deviation for a particular digit; and

n is the number of observations (the term 1/(2n) is a continuity correction factor and is used only when it is smaller than the absolute value term).

A z-statistic of 1.96 would indicate a p-value of .05 (95 percent confidence) while a z-statistic of 1.64 would suggest a p-value of .10 (90 percent confidence). For the proportion of observations to be significantly different from that expected, the deviation must be in the tail of the distribution. Thus arise two concerns, one intuitive and one statistical. First, intuitively, if there are only a few fraudulent transactions, a significant difference will not be triggered even if the total dollar amount is large. Second, statistically, if the account being tested has a large number of transactions, it will take a smaller proportion of inconsistent numbers to trigger a significant difference from expected than it would take if the account had fewer observations. This is why many prepackaged programs which include a Benford's law-based analytical test urge auditors to test the entire account rather than taking a sample from the account.

To understand the second point, consider two accounts, one contains 10,000 transactions while the second contains only 1,000 transactions. If all transactions within the 10,000-transaction account are used, a minimum difference of 75 transactions is required before a z-statistic would signal that the account is deviant. This translates into a proportion of .75 percent of the total account. By contrast, in the 1000-entry account, there would need to be 23 fraudulent entries (or a proportion of 2.3 percent deviant entries) before the same z-statistic flagged it as possibly fraudulent. If a 200-entry sample was drawn, it would require six deviant entries or 3 percent before it would be seen as statistically different than expected.[6]

Such a result occurs because the size of the variance of the sample proportion is dependent on the number of observations in the data set being tested. If lower confidence levels are used to detect the presence of fraud, more false positives will be signaled with an accompanying higher cost of investigation. For example, in an account which contains 10,000 entries, if the confidence level is set to 80 percent (a z-statistic of 1.28) only 58 deviant transactions would be needed to signal the possibility of fraud. In general, there is a tradeoff. The more discriminatory the test, the less likely that fraud will be detected when it is present, and the less discriminatory the test, the more likely the test will return false positives, indicating fraud when none is present.

An extension of the z-test, which tests only one digit at a time, is a chi-square test. The chi-square test combines the results of testing each digit's expected frequency with actual fre-

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[6] We solved for N, (the number of deviant entries required) using the following formula: $(N-np) / (npq)^{1/2} = $ z-statistic. Where n = sample size, p = expected proportion of the first digit, and q = (1 - p). Z-statistic used was 1.64 for a 90 percent confidence level which put 5 percent in each tail of the distribution.

quency into one test statistic that indicates the probability of finding the result. If the chi-square test rejects the hypothesis that the probability of all digits conform to a Benford distribution, then one knows the entire account warrants further examination. In general, the chi-square test will be less discriminatory than the individual z-test results but will result in fewer false positives.

Limitations Based on the Type of Fraud

Benford's analysis tests for fraudulent transactions based on whether digits appear in certain places in numbers in the expected proportion. Therefore, a significant deviation from expectations occurs under only two conditions: the person perpetrating the fraud has either added observations or has removed observations but on a basis that would not conform to a Benford distribution. Each action would result in an observable deviation from expectations, provided the number relative to the sample was large enough for statistical detection. Therefore, when a fraud is such that transactions are never recorded (i.e., off-the-books fraud), as in the case of bribes, kickbacks or asset thefts, digital analysis cannot be expected to detect the absence of transactions. This deficiency is noted by the ACL for Windows Workbook, "It is very difficult to discover any clues from records that are unexpectedly absent." (ACL 2001, p. 221).

In addition, other types of fraud exist that cannot be detected by Benford analysis because the data sets under examination are not appropriate for such analysis. For example, duplicate addresses or bank accounts cannot be detected, yet two employees with similar addresses might signal ghost employees or an employee's address which is also a vendor's address might signal a shell company. Other examples include duplicate purchase orders or invoice numbers that could signal duplicate payments fraud or shell companies. Further, Benford analysis will not detect such frauds as contract rigging, defective deliveries, or defective shipments.

The question arises as to what additional tests might complement Benford analysis. We would suggest rather than using Benford's law as the primary tool around which other analytical tools are chosen, that auditors should consider it another tool to be added to the arsenal they already employ. Such arsenal includes personal observations of assets, outside verification, keen awareness of corporate culture, an awareness of the examined firm's performance relative to others in the industry and a healthy skepticism toward management explanations of deviations in their records.

Base Rates and Conditional Probabilities

The value of Benford's law is in its use as a signaling device to identify accounts more likely to involve fraud, thus improving on the random selection process auditors generally employ when assessing the validity of a firm's reported numbers. An auditor who decides to rely on the results of digital analysis to detect fraud is making a decision in the presence of two kinds of uncertainty. First, it is unknown exactly how accurate digital analysis will be with real data. Second, it is unknown what the base probability of fraud is in real data. To accurately determine the effectiveness of Benford's law, it would be essential to compare the empirical distribution of accounts known to contain fraud with accounts known to be fraud free. Unfortunately, that data is very difficult to obtain as most firms do not want to make public their particular accounts even when no fraud has occurred. Therefore, we are left to speculate as to the accuracy. To do this, we rely on Bayes' theorem that represents probability under uncertainty as:

$$P(F|S) = \frac{P(S|F)*P(F)}{P(F)*P(S|F)+P(NF)*P(S|NF)} = \frac{P(S|F)*P(F)}{P(S)} \tag{4}$$

Where: F is fraud present;
                     NF is no fraud present;
                     S is the signal of fraud; and
                     P is the probability.

Thus, the probability of fraud existing, given the signal of fraud from digital analysis based on Benford's law, is the probability a signal will be given if fraud exists multiplied by the probability of fraud (the base rate) divided by the probability of a signal of fraud. The probability of a fraud signal, P(S), is the percent of times the test correctly identifies fraud plus the percent of times the test incorrectly signals fraud. Thus, the usefulness of Benford's law for fraud detection can be summarized as accurate fraud signals divided by total fraud signals.

As noted above, the probability or base rate of fraud, P(F), is a necessary consideration in evaluating the usefulness of a Benford analysis. Although the Association of Certified Fraud Examiners in their 1996 Report to the Nation on Occupational Fraud and Abuse report certain summary statistics, the real base fraud rate is unknown. This is due to several reasons: (1) firms are often reluctant to report that they have been victimized; (2) auditors and law enforcement know only about the frauds that have been detected or reported; and (3) no one knows the extent of undetected fraud. Further, the base rate of fraud is likely environment specific, with certain environments being more conducive to fraud (Bologna and Lindquist 1995).

It is likely that base rates of fraud in specific populations of transactions are fairly small. As an example, assume that the base rate is 3 percent and Benford analysis correctly identifies accounts which contain fraud 75 percent of the time.[7] In this case the probability of finding a fraud would be calculated as follows:

$$P\left(F\mid S\right) = \frac{P\left(S\mid F\right)*P(F)}{P(F)*P\left(S\mid F\right)+P(NF)*P\left(S\mid NF\right)} = \frac{.75*.03}{(.03*.75)+(.97*.25)} = .085 \qquad (5)$$

The conditional probability is .085 or approximately 9 percent, meaning there would be a 9 percent chance of discovery.[8] This may be judged as a significant improvement over an unassisted random sampling that would be successful 3 percent of the time. It should be noted that there has been no widespread testing of digital analysis, nor is there any way to assess its true success rate, since an auditor would seldom know of the frauds the analysis failed to detect. The above analysis does, however, provide insights into how to evaluate the effectiveness of the procedure given certain assumptions.

AN EXAMPLE OF DIGITAL ANALYSIS

We conducted digital analysis on two accounts of a large medical center in the Western United States.[9] Figure 1 on the following page, shows the distribution for the first digits of check amounts written for the office supplies account. While digits two and seven appeared to be significantly different than expected, the overall deviation falls within the conforming range.[10] However, subsequent analysis was conducted on the two non-conforming items. The analysis indicated that the variation was due to legitimate payments and did not represent a fraud.

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[7] It is not to be expected that the procedure will always find accounts with fraud since sometimes it will signal that an account is deviant when there is no fraud and other times will not signal fraud present because there are too few fraudulent entries.

[8] The numerator shows that 75 percent of the time the test will successfully signal fraud given that fraud is in 3 percent of the transaction sets. The denominator is the base rate (3 percent) multiplied by the success rate (75 percent) added to the expectation that some accounts will be signaled (25 percent) when no fraud exists (97 percent). The correct signals divided by the total signals indicating fraud provides a measure of the usefulness of the test in this case.

[9] For the analysis shown here, we used the free download software found at http://www.nigrini.com/, under DATAS software for windows.

[10] A simple test of the conformity of 9 digits given in Drake and Nigrini (2000) is called the Mean Average Deviation (The sum of the absolute values of the difference between the actual and expected percentages divided by nine (the number of digits)). This test proved to be within suggested limits.

Figure 1
Office Supplies Disbursement Check Amounts

FIRST DIGIT DISTRIBUTION
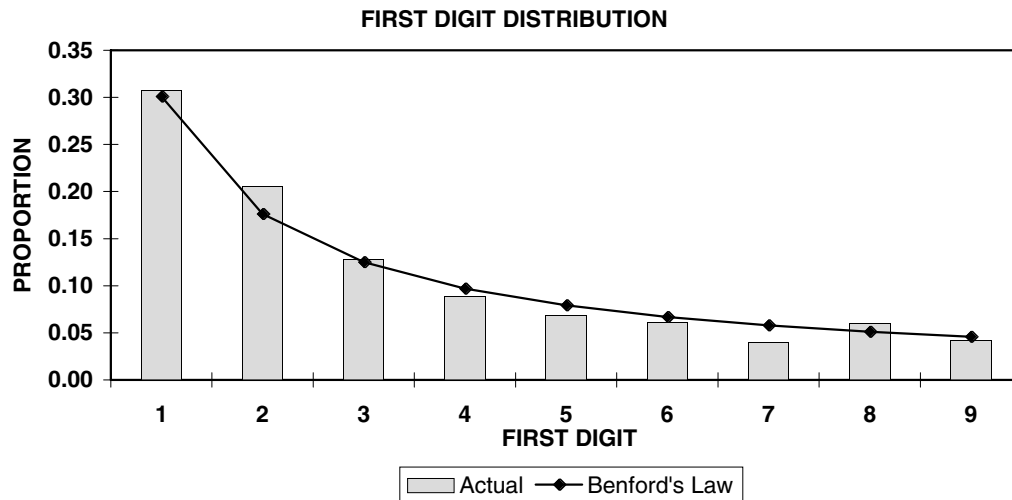
In the second Benford analysis, the insurance refund account reflected a distribution of the first digits shown in Figure 2. All digits, except the digit 2, were significantly different than expected by the Benford distribution.

Figure 2
Insurance Refund Check Amounts
FIRST DIGIT DISTRIBUTION
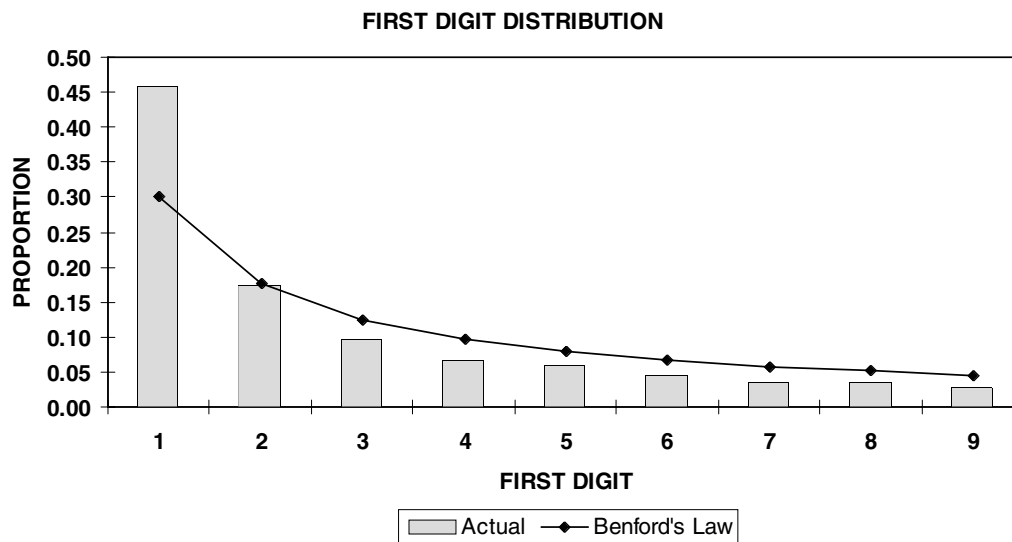
When the details of the account were inspected, it was apparent that many more refund checks of just over $1,000 had been written than in the previous period. In fact, most of the previous period's refund checks were in amounts of less than $100.00. When queried, the financial officer of the medical center responded that she had decided to accumulate refunds for large insurers in an attempt to write fewer checks.

A subsequent detailed examination of the account, however, uncovered that the financial officer had created bogus shell insurance companies in her own name and was writing large refund checks to those shell companies. While the investigation into the fraud is ongoing, it appears that approximately $80,000 had been diverted to the shell insurance companies. In this instance, digital analysis was useful in identifying a suspect account. However, it required looking beyond the easy explanation to find the fraud.

CONCLUSION

We conclude that Benford's analysis, when used correctly, is a useful tool for identifying suspect accounts for further analysis. Because of its usefulness, digital analysis tools based on Benford's law are now being included in many popular software packages (e.g., ACL and CaseWare 2002) and are being touted in the popular press. CaseWare 2002 says of this new application it "...can identify possible errors, potential fraud or other irregularities." The goal of this paper has been to help auditors more appropriately apply Benford's law-based analysis to increase their ability to detect fraud. SAS No. 99 instructs auditors to use analytical tests in the planning stages of their audit. Benford analysis is a particularly useful analytical tool because it does not use aggregated data, rather it is conducted on specific accounts using all the data available. It can be very useful in identifying specific accounts for further analysis and investigation.

Because the potential cost of undetected fraud is high, an auditor using this technique must take care not to overstate the reliability of such tests. While such tests have many advantages, certain limitations must also be considered. Specifically, (1) care must be exercised in interpreting the statistical results of the test, (2) Benford analysis should only be applied to accounts which conform to the Benford distribution, and (3) the auditor must be cognizant of the fact that certain types of frauds will not be found with this analysis.

While Benford analysis by itself might not be a "surefire" way to catch fraud, it can be a useful tool to help identify some accounts for further testing and therefore should assist auditors in their quest to detect fraud in financial statements.

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