Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance

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Breaking the (Benford) Law: Statistical Fraud Detection in Campaign Finance

Wendy K. Tam CHO and Brian J. GAINES

Benford's law is seeing increasing use as a diagnostic tool for isolating pockets of large datasets with irregularities that deserve closer inspection. Popular and academic accounts of campaign finance are rife with tales of corruption, but the complete dataset of transactions for federal campaigns is enormous. Performing a systematic sweep is extremely arduous; hence, these data are a natural candidate for initial screening by comparison to Benford's distributions.

KEY WORDS: Data irregularities; Data mining; FEC; First-digit distributions; Politics.

Benford's Law is a fine example of a deeply nonintuitive and intriguing mathematical result, simple enough to be described (if not fully explained) even to those without any formal training in math. The law pertains to the first digits of a collection of numbers. Most people's intuition is that, in "ordinary" large sets of numbers, each integer from 1 through 9 should appear as the leading digit with roughly equal probability. By contrast, Benford's Law reports that the digit 1 leads approximately 30% of the time and each successive digit is less common, with 9 occurring less than 5% of the time. Strikingly, this pattern holds for a diverse set of numbers that have no apparent connection to one another.

Although the law now sports Benford's name, the astronomer and mathematician Simon Newcomb was the first to note, in an 1881 *American Journal of Mathematics* article, that not all possible first digits appear with equal frequency in large sets of "natural numbers." Newcomb never proffered a theoretical explanation for this phenomena, but his observation, "the law of probability of the occurrence of numbers is such that all mantissæ of their logarithms are equally probable," (p. 40) suggests a convenient expression for the empirical distribution of first digits.

$$P(d) = \log_{10}\left(\frac{1+d}{d}\right) \quad \text{for} \quad d \in \{1, \dots, 9\}.$$

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Benford, in turn, set out empirically to test Newcomb's hypothesis. He collected data from a wide variety of datasets, including areas of rivers, population figures, addresses, American League baseball statistics, atomic weights of elements, and numbers appearing in *Reader's Digest* articles, among others (Benford 1938). In his analysis of this varied and large set of numbers (consisting of 20,229 individual numbers), there was a surprisingly good fit to the distribution of leading digits first laid out by Newcomb (1881). Thus, as so often happens, the law was named not for its discoverer, but for its first popularizer. Others followed up by confirming the (approximate) fit of still more sources of numbers, including stock market data, census statistics, some accounting data, stock market prices, and eBay bids (Hill 1995; Giles 2007; Ley 1996). About a century after Newcomb's discovery, rigorous proof of the law (and derivation of when and why it holds) were finally developed (Hill 1995).

1. FRAUD AND IRREGULARITY DETECTION BY BENFORD'S LAW

An interesting application of Benford's Law has emerged in recent decades. Whenever first digits should follow Benford's Law, it follows that deviations from the known distribution in data expected to conform signal some type of irregularity, possibly deliberate fraud. Accordingly, Benford's Law has been put to use as a simple and effective way to test for fraudulent manipulation of data, as might exist in accounts when embezzlement has occurred (Nigrini 1999; Durtschi, Hillison, and Pacini 2004). In this sense, the use of Benford's Law is in keeping with the philosophy of data mining wherein one searches large volumes of data for patterns, agnostic about any theory of the data-generating process. If a data source generally conforms to the law, random deletion would not induce a worse fit, but if entries are being falsified or there are accidental systematic omissions, violations can follow. Experimental research has shown that people do a poor job of replicating known data-generating processes, for instance by over-supplying modes or under-supplying long runs (Camerer 2003, pp. 134–138). Benford's law is widely applicable but not widely known, so it seems very unlikely that those manipulating numbers would seek to preserve fit to the Benford distribution. In that sense, it could be an unusually good diagnostic, at least until it becomes widely known.

An important caveat is that not all numbers follow the law. In accounting, for example, the first-digit distribution can become skewed when receipts include a very large number of identical transactions, reflecting sales of an especially popular item whose price is constant. In another context, election returns are unlikely to follow the law in many typical situations, where districts are

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of nearly equal size and the level of competition conspires to fix most vote totals in a limited range. Taking the top three vote getters in all 2002 U.S. House elections, almost 38% of the vote totals had 1 as the leading digit, while 4 was next most common, at about 12%. On the other hand, vote totals in pre-existing geographic units that are not artificially constructed subject to an equal-size constraint (such as counties) conform to Benford's Law much better.

Benford's Law is more "robust" than one might imagine. For instance, while not all numbers will conform to the Benford distribution, if distributions are randomly selected and random samples are taken from each of the distributions, then the frequency of digits of this combined set will converge to Benford's distribution even if the separate distributions deviate from Benford's distribution (Hill 1995, 1998).

Durtschi, Hillison, and Pacini (2004) provided guidelines on when to expect Benford compliance.

1. Numbers that result from mathematical combination of numbers (e.g., quantity × price)
2. Transaction-level data (e.g., disbursements, sales)
3. Large datasets
4. Mean is greater than median and skew is positive

On the flip side, numbers that would not follow Benford's Law have the following characteristics.

1. Numbers are assigned (e.g., check numbers, invoice numbers)
2. Numbers influenced by human thought (e.g., prices set by psychological thresholds ($1.99))
3. Accounts with a large number of firm-specific numbers (e.g., accounts set up to record $100 refunds)
4. Accounts with a built-in minimum or maximum
5. Where no transaction is recorded

These restrictions apply to many data sources, and clearly comparison to Benford's proportions is not always warranted. Although interesting applications of Benford's Law have emerged, few have ventured into the world of politics where corruption of various kinds is commonly alleged. If history is any guide, there must be myriad instances where one will be astonished by Benford's applicability in the political realm. Here, we explore data on campaign finance, a field rife with allegations of fraud, cheating, and corruption.

2. FEC FILINGS

Campaign finance regulations are nearly a century old, and the long history of ever-changing laws and scandals, large and small, suggests that incentives to slip through loopholes and twist (or ignore) restrictions are a persistent feature of competitive politics. The data describing most financial transactions undertaken by candidates seeking federal office have, in recent years, become fairly easy to access, as the Federal Election Commission (FEC) has made a practice of posting all reports to public electronic databases. A simple method of examining FEC data for signs of fraud is appealing partly because the very reason the FEC provides these data to the public is to guard against abuses of the system. By its very existence, the FEC archive enlists all interested parties in the task of monitoring the flow of money in federal elections.

In general, however, the FEC data archive would not seem to be a good prospect for data that follow Benford's Law because of numerous laws regulating political contributions. For instance, individuals have historically been limited to donating a maximum of $2,000 to federal candidates per election cycle ($1,000 designated for the primary campaign and $1,000 for the general campaign). A large proportion of all donors give the maximum amount, making the number 1 even more modal than usual in the donation records. The data, in other words, violate item 3 in the checklist above.

2.1 In-Kind Contributions and Joint Fundraising Committees

One variety of transaction that might escape this tendency to cluster is the in-kind contribution. In general, individuals are permitted to donate services to candidates without fixing a dollar amount on their efforts. Thus, volunteers can work as many hours as they please for a congressional campaign without running afoul of FEC regulations. However, under some circumstances, donors must declare a cash value for services or goods donated to a campaign. When a third party pays the bills on behalf of a campaign committee (whether the recipient is a celebrity performing for a fee, a commercial landlord collecting rent, etc.), the individual footing the bill is making an in-kind donation to the campaign. If an organization puts paid workers at the service of a campaign, the total wage bill represents an in-kind donation. All such donations are subject to the same limits as cash contributions. The key aspect of in-kind donations for present purposes is that they seem comparatively unlikely to cluster at the maximum permitted value, since they are often computed according to retail prices or pre-set wages and hours worked. In addition, the limits mentioned above apply to donations to candidate committees, but not to donations of so-called "soft" money to party committees. Candidates generally accept donations through campaign committees, however, they can also set up "Joint Fundraising" committees (JFC) that raise both regulated ("hard money") contributions and soft money simultaneously. The JFCs, in turn, redistribute the money, generally by passing on the maximum permitted amount to the candidate committee as hard money, and then channeling the balance to a party committee as soft money. Hence, the category of in-kind contributions to joint fundraising committees represents an unusual subset of the FEC domain, since neither contribution limits nor round-value focal points constrain the data strongly.

One high-profile scandal involving alleged fraud in campaign accounting involved Hillary Clinton, and revolved around in-kind donations to a JFC. In January 2005, David Rosen, the Director of Finance for Clinton's 2000 Senatorial campaign, was indicted on four counts of causing false campaign finance reports to be filed with the FEC. Prosecutors alleged that Rosen

Table 1. Committee-to-committee in-kind contributions (first digits), 1994–2004.

	Newcomb	Benford data	1994	1996	1998	2000	2002	2004
1	30.1	28.9	32.9	24.4	27.4	26.4	24.9	23.3
2	17.6	19.5	18.7	21.7	18.5	21.1	22.6	21.1
3	12.5	12.7	13.6	15.8	15.3	11.1	10.7	8.5
4	9.7	9.1	7.9	9.6	10.3	10.7	11.6	11.7
5	7.9	7.5	8.9	10.2	11.8	10.1	10.5	9.5
6	6.7	6.4	8.3	6.3	5.9	4.3	4.3	4.2
7	5.8	5.4	4.1	4.8	3.7	6.4	3.4	3.7
8	5.1	5.5	2.4	3.2	3.9	2.4	3.0	4.0
9	4.6	5.1	3.2	4.0	3.3	7.5	9.0	14.1
N		20,229	9,632	11,108	9,694	10,771	10,348	8,396
χ^2		85.1	349	507	431	4,823	1,111	2,181
V_N^*		2.9	5.7	10.1	8.1	5.5	7.8	8.7
d^*		0.024	0.052	0.081	0.061	0.071	0.097	0.131

repeatedly and knowingly under-reported in-kind contributions to New York Senate 2000, Clinton's JFC (Tonken 2004). The main incentive for such obfuscation would have been that FEC rules at the time allowed candidates to pay for fundraising events with soft money provided that the costs were no more than 40% of the total hard money raised. Thus, minimizing costs allowed the Clinton campaign to preserve precious hard money, which could be used for direct campaign advertisements in the bruising and expensive air war that lay ahead. Rosen faced up to five years in prison, but was acquitted. The defense, accepted by the jury, did not deny fraud and shoddy accounting, but blamed others, claiming that Rosen was unaware of the shenanigans (Ryan 2005).

Unfortunately, although the FEC is diligent about collecting and posting candidate reports, the data are not coded in such a way that one can easily identify in-kind donations, let alone in-kind donations to JFCs. For obscure reasons, the only in-kinds that are distinctly marked are those made from one committee (as opposed to an individual) to another committee. The reasons money is shuffled between committees are, again, somewhat arcane, and related to the fact that campaign finance regulations are: (a) ever-changing, as they seem almost without exception to produce some unintended consequences; (b) constrained in several manners by a tangled jurisprudence incorporating, for instance, first-amendment protection of political speech; and (c) created by plainly not disinterested actors, namely incumbent politicians.

3. ANALYSIS OF IN-KIND CONTRIBUTIONS

The data reported in Table 1 are the first digit relative frequencies (as percentages) for all committee-to-committee, in-kind contributions cataloged by the FEC for each of the last six election cycles. For comparison, the second column shows the values for the 20 datasets Benford discussed in his 1938 paper.

A casual perusal reveals that fit to the Newcomb-Benford theoretical distribution for the FEC data seems to have gotten worse over time. Figure 1 is a graphical depiction of the data's conformity. In each plot, the expected "Benford's law" values are plotted by the solid line. The first plot covers the elections of 1994, 1996, and 1998. The second plot pertains to contributions from the 2000, 2002, and 2004 election cycles. On average, the fit to Benford seems to have become poorer in the more recent elections. The figure makes clear, for instance, that since 2000, each election has seen an increasing surplus of leading 9s. Leading 1s, by contrast, are too few in number, and in decline over these three elections.

To make these comparisons more precise, one can compute formal test statistics. One alternative is to conduct a χ^2 goodness-of-fit test. The null hypothesis is that the data follow the Benford distribution, shown in the column labeled "Newcomb" in Table 1. The test statistic is

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i},$$

where O_i and E_i are the observed and expected frequencies for digit i, respectively. The test statistic follows a χ^2 distribution with 8 degrees of freedom, so the null hypothesis is rejected if $\chi^2 > \chi^2_{\alpha,8}$, where α is the level of significance. As the table shows, for every year we analyze, the χ^2 test produces huge values that lead one to reject the null hypothesis at any conceivable significance level (the critical value for the 0.001-level here is 26). Indeed, one can reject the null hypothesis for the very data that Benford used to demonstrate the accuracy of Newcomb's law. Of course, χ^2 tests are very sensitive to sample size, having enormous power for large N, so that even quite small differences will be statistically significant. This test appears to be too rigid to assess goodness-of-fit well, especially since the Benford proportions do not represent a true distribution that one would expect to occur in the limit (Ley 1996; Giles 2007).

A second alternative is a modified Kolmogorov-Smirnov test statistic (Kuiper 1962),

$$V_N = D_N^+ + D_N^-,$$

where

$$D_N^+ = \sup_{-\infty<x<\infty} [F_N(x) - F_0(x)],$$

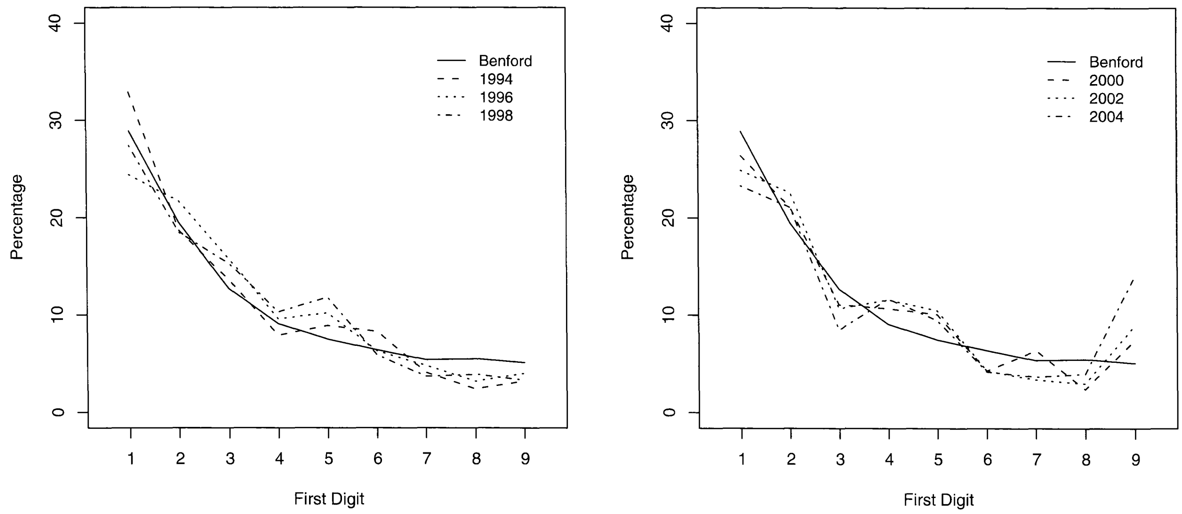

Figure 1. In-kind contributions and Benford's Law.

and

$$D_N^- = \sup_{-\infty < x < \infty} [F_0(x) - F_N(x)].$$

Giles (2007), citing Stephens (1970), favors a modified form of the V_N test statistic, V_N^*, which is independent of sample size and has critical value of 2.001 for $\alpha = 0.001$. The table shows that our values for V_N^*, like the χ^2 values, lead us to reject conformity with Benford's law for all of our FEC datasets. Again, though, we would also reject the null hypothesis for Benford's data on the basis of this test, suggesting that perhaps it is too rigid. The very naming of the "law" after Benford reflects the common understanding that he demonstrated that Newcomb's idea is widely applicable to real-world data, not the contrary. Since no one has suggested that Benford's Law holds asymptotically, a preferable statistic would be less sensitive to sample size than the χ^2 statistic. Arguably, it is also not a natural context for computing p values.

One other possible measure of fit, then, not connected to a hypothesis-testing framework and insensitive to sample size, is based on Euclidean distance from Benford's distribution in the nine-dimensional space occupied by any first-digit vector. Here, let

$$d = \sqrt{\sum_{i=1}^{9} (p_i - b_i)^2},$$

where p_i and b_i are the proportions of observations having i as the leading digit and expected by Benford's distribution, respectively. Because these vectors are compositional, we can compute the maximum possible distance, associated with a distribution where the first digit expected to occur least often (9) is the only one observed. Division by this maximum value converts the distance-from-Benford value, d, for any given empirical distribution to a score bounded by 0 and 1. The bottom row of the table shows these scores (labeled "d^*"). Again, the last two elections stand out as exhibiting somewhat worse fit than their earlier counterparts, and the Benford data provide a rough sense for what constitutes a realistic, small value.

None of these figures, of course, pinpoint why the data describing in-kind contributions for the latter elections have departed from the prior pattern of loose fit to Benford's law. This analysis merely identifies the years as anomalous and worthy of further inspection. The origin of the poor fit to Benford's law could be bad record keeping, new practices in donations that correspond to the checklist of Benford inapplicability, changes associated with the "McCain-Feingold" Bipartisan Campaign Reform Act, or increasing incidence of actual fraud or other irregularity in financial transactions.

To explore further when and how discrepancies between actual data and the theory occur, one can examine subsets of the data. Indeed, there is an especially strong rationale for re-examining the data by size of contribution. Thus far we have followed common practice by neglecting a point Benford emphasized in his 1938 article, that the Newcomb distribution is a "law for large numbers" (p. 554). Benford derived alternative distributions for numbers having only one-, two-, or three- digits, since the "limiting order" routinely described as "Benford's distribution" turns out to be a crude approximation for such small numbers.

In Table 2 we disaggregate the FEC data according to the size of the contribution, and we report Benford's theoretical distributions for one-, two-, and three- digit numbers, to which the FEC data can be compared. (For each of these distributions, we also report their standardized distance from the familiar, large-N Benford distribution in the d^* column.) It is evident, and not terribly surprising, that fit is always worse in subsets (as compared with the totals in Table 1). Aggregation plays no small part in the Benford tendency. The other main pattern is that fit is generally poor for the one-digit numbers and the four-or-more-digit numbers, and better for the intermediate categories. A striking oddity is that 2000 bears little resemblance to the other election years in regard to the smallest contributions—it has by far the best fit to the one-digit theory because of a large number of (inherently suspicious) \$1 transactions. A second interesting trait, worthy of further investigation, is that the increase in leading 9s over the last three election cycles is largely due to \$90–\$99 contributions. Although how best to generalize the distance score to acknowledge the fact that the predicted values are now a 9×4 matrix

Table 2. In-kind contributions by contribution size.

	1	2	3	4	5	6	7	8	9	N	d^*
Benford ($i < 10$)	0.393	0.258	0.133	0.082	0.053	0.036	0.024	0.015	0.007		0.140
Benford ($9 < i < 100$)	0.318	0.179	0.124	0.095	0.076	0.064	0.054	0.047	0.042		0.018
Benford ($99 < i < 1000$)	0.303	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.045		0.002
Newcomb-Benford ($i > 999$)	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046		
1994											
$1 – $ 9	0.090	0.067	0.073	0.060	0.062	0.502	0.054	0.034	0.058	536	0.535
$10 – $ 99	0.349	0.206	0.126	0.083	0.083	0.047	0.051	0.027	0.027	3,493	0.051
$100 – $ 999	0.305	0.187	0.153	0.077	0.104	0.075	0.038	0.023	0.038	4,902	0.055
$1000+	0.579	0.190	0.108	0.081	0.027	0.000	0.001	0.011	0.001	701	0.294
1996											
$1 – $ 9	0.057	0.116	0.210	0.099	0.080	0.080	0.080	0.077	0.202	352	0.389
$10 – $ 99	0.159	0.218	0.154	0.096	0.109	0.088	0.085	0.048	0.043	3,875	0.166
$100 – $ 999	0.259	0.226	0.172	0.090	0.108	0.056	0.028	0.024	0.036	5,925	0.093
$1000+	0.558	0.191	0.073	0.127	0.044	0.002	0.005	0.000	0.000	956	0.278
1998											
$1 – $ 9	0.101	0.084	0.054	0.027	0.104	0.191	0.054	0.289	0.097	298	0.437
$10 – $ 99	0.188	0.144	0.192	0.105	0.110	0.100	0.060	0.046	0.054	3,305	0.153
$100 – $ 999	0.282	0.192	0.158	0.113	0.141	0.037	0.029	0.027	0.022	5,017	0.090
$1000+	0.548	0.306	0.039	0.065	0.039	0.001	0.001	0.000	0.000	1,074	0.305
2000											
$1 – $ 9	0.427	0.036	0.056	0.021	0.053	0.167	0.062	0.058	0.120	468	0.274
$10 – $ 99	0.184	0.213	0.101	0.077	0.105	0.045	0.101	0.031	0.144	4,297	0.176
$100 – $ 999	0.249	0.203	0.142	0.154	0.117	0.040	0.047	0.021	0.027	4,855	0.100
$1000+	0.560	0.308	0.045	0.050	0.036	0.000	0.001	0.000	0.000	1,151	0.316
2002											
$1 – $ 9	0.034	0.073	0.069	0.019	0.203	0.165	0.119	0.111	0.207	261	0.466
$10 – $ 99	0.195	0.206	0.124	0.078	0.097	0.051	0.038	0.030	0.181	4,356	0.183
$100 – $ 999	0.250	0.234	0.107	0.172	0.118	0.038	0.032	0.031	0.018	4,760	0.123
$1000+	0.543	0.316	0.040	0.041	0.057	0.000	0.000	0.001	0.002	971	0.307
2004											
$1 – $ 9	0.035	0.031	0.040	0.035	0.256	0.172	0.154	0.181	0.097	227	0.495
$10 – $ 99	0.165	0.155	0.089	0.071	0.055	0.052	0.041	0.055	0.316	3,345	0.305
$100 – $ 999	0.238	0.231	0.095	0.180	0.129	0.035	0.037	0.027	0.028	3,836	0.136
$1000+	0.490	0.359	0.040	0.043	0.064	0.002	0.000	0.002	0.000	988	0.292

Note: i denotes contribution amounts in whole dollars.

rather than a 9-tuple is not self-evident, if one simply computes weighted averages of the distance for each subset, then, once again, the 2004 data seem to offer markedly worse conformity to Benford's distributions (the values are, in order, from 1994 to 2004: 0.097; 0.144; 0.146; 0.161; 0.174; 0.231). A Cox-Stuart test for trend using all 24 d^* values indicates that the last three years, when blocked by level of contribution, have seen significantly worse fit to Benford ($p < 0.001$).

4. CONCLUSION

Benford's Law is a powerful, objective, simple, and effective tool for identifying anomalies in data. It is especially valuable for large datasets on deception-prone activities. Comparison of empirical data to these theoretical distributions will not usually locate a "smoking gun," but it can be a good diagnostic for where to go sniffing for that gun. This is how Benford's Law is currently used in contexts such as tax auditing (Nigrini 1996). The ability to ferret through millions of tax returns—or campaign contributions—quickly to identify a suspicious set clearly enhances efficiency. Moreover, Benford's Law is applicable to surprisingly large and diverse classes of data. Recent applications include self-reported toxic emmissions data (de Marchi and Hamilton 2006), quality of survey data (Judge and Schechter 2006), and election returns (Mebane 2006).

Of course, first-digit laws are not universally applicable, so other tools are necessary for particular applications. Where cheating is a temptation for test administrators, runs tests and applications of simple combinatorics have proven useful (Jacob and Levitt 2003). In campaign finance, because most FEC records seem likely to violate Benford because of predictable modes caused by regulation, data mining techniques are helpful to screen for suspicious subsets. Benford's Law is only one of many tools for identifying patterns. In its favor, Benford's Law concerns a pattern in the data known at the outset, making it unusually easy to implement. Grendar, Judge, and Schechter (2007) have proposed alternative formulations for first significant digit distributions, and set forth a theoretical model wherein the Benford distribution is regarded as one member of a family of distributions of first-significant-digit occurrence rates. Their framework allows a considerable broadening of the Benford test.

One might also broaden the scope of Benford in the FEC data archive, for instance, by partitioning the in-kind data by state, recipient committee, party of the candidates, and so on. Benford also derived a nonuniform distribution for second digits, and the first- and second-digit distributions are not independent, so comparison of data to their joint distribution might prove fruitful in some contexts. In the end, all of these tests are primarily helpful in identifying cases that warrant closer inspection, and their main attraction is speed and efficiency. Organizations like the FEC, that post very large quantities of data, should consider implementing tests of this kind as a routine procedure, to ensure the quality of their data, which may be compromised by any number of recording issues unrelated to fraud.

The preliminary analysis reported above does not constitute proof of illegality, but we have identified pockets of data that merit more careful inspection. We have suggested that the most suitable subset of the FEC archive for first-digit checks is probably in-kind contribution to JFCs, a large class of transactions that, unfortunately, is not presently identifiable. We look forward to better data collection and processing practices by the FEC to facilitate more careful scrutiny of these important data.

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