Teaching Portfolio

Brandon Caruso

New York State Initial Certificate,
Mathematics 7-12
Background in Computer Science

Hello.
I’m Mr. Caruso.

My name is Brandon Caruso and I’m a STEM educator. I am a Chautauqua County native, looking forward to inspiring the next generation of students in our community. I hope to engage them in the creative and exciting possibilities found in both Mathematics and Computer Science. I have a B.S. in Software Engineering from SUNY at Oswego and a Master’s of Human-Computer Interaction and Design from the University of Washington. I’m currently working on a B.S. in Mathematics/Adolescence Education with Initial NYS Certification from SUNY at Fredonia. I have taught CS and Math to students (K-12) in a variety of settings, from classroom to MakerSpace, in-person and virtually. Before entering the world of education, I was the Guest Experience Designer for the National Comedy Center in Jamestown, NY, where I designed and developed interactive museum experiences. I interned at the NASA Jet Propulsion Laboratory and the Walt Disney World Resort. My passion extends to the arts, where I play the trumpet for local theatres and music ensembles. I look forward to opening the eyes of my students, showing them their unique abilities to support them in crafting their own story.

Designer of Learning Experiences

Statement of Teaching Philosophy

Guide the discovery of one's own capabilities

My mathematics and computer science classroom will not only be a place of content discovery, but a place where *all* students see firsthand their own capabilities to engage in material that they may be told is only approachable for a small subset of the academic elite. I hope that my students see they have the abilities to persevere, problem-solve, and create with the mathematical and computer science skills they are learning in my classroom. I want them to leave my classroom understanding they have the capabilities to obtain any skill, develop their own passions, and utilize them to create, solve problems, and promote change in areas that they are personally passionate about. My students will realize that their individual strength relies on supporting and collaborating with others to leverage all of our collective abilities to tackle future challenges and problems.

Prepare to engage in the digital and physical world

My classroom centers around developing and exposing students to the skills necessary to engage in their communities now and in the future, both physical and digital. I believe it is essential that we prepare students with not only the technical skills necessary to engage productively in our communities, but also prepare them to be exceptional and critical digital citizens. I am not creating cohorts of future mathematicians and computer scientists. I am engaging students to see the relevance mathematics, computer science, and technology have on their daily lives and relationships, regardless of future profession or calling.

Interactive, Creative, Collaborative

It might be obvious at this point that my classroom will center around these three pillars: interactive, creative, and collaborative. My philosophy and practice are centered around the idea that, as an educator, I design *experiences* for my students to learn and expose themselves to new ideas and skills. In these experiences, they build on prior knowledge, exchange ideas and learn from peers and mentors, construct new knowledge, and build an intrinsic curiosity to explore the next question or solve the next problem. Learning isn't necessarily straightforward, it is iterative, messy, and we must embrace failure and capitalize on our mistakes. My classroom will be a community where these ideas are embraced and championed.

Brandon Caruso
New York State Initial Certificate,
Mathematics 7-12
Background in Computer Science

40 N Ralph Ave., Falconer, NY 14733
716.397.7515 | brandonjcaruso1@gmail.com

Table of Contents

Brandon Caruso

40 N Ralph Ave., Falconer, NY 14733
716.397.7515 | brandonjcaruso1@gmail.com
Portfolio | brandoncaruso.com/teachingportfolio

Certification

Candidate for **New York State Initial Certificate, Mathematics 7-12**
Passing Score on EAS, ATS-W, Math CST Certification Exams as of 3/2022

Education

Bachelor of Science in Mathematics/Adolescence Education, expected May 2022
State University of New York at Fredonia, Spring 2021 – present | GPA: 4.0

Master of Human-Computer Interaction & Design, August 2017
University of Washington, Fall 2016 – Summer 2017 | GPA: 3.96

Bachelor of Science in Software Engineering, August 2015
State University of New York at Oswego, Fall 2013 – Summer 2015 | GPA: 4.0 Summa Cum Laude

Associate in Science in Individual Studies, May 2013
Jamestown Community College, Fall 2012 – Spring 2013 | GPA: 4.0

Honors

SUNY Fredonia Dean's List, Spring 2021, Fall 2021
Earl G. Mathewson Scholarship, 2021
Oebele G. VanDyk Outstanding Computer Science Senior award, 2015
Oswego President's List, all semesters Fall 2013 - Summer 2015
JCC Dean's List, Fall 2020, Fall 2012, Spring 2013

Student Teaching

Math 7, Fredonia Middle School, present

- Instructed Advanced Math 7 in probability, statistics, and systems of equations
- Assisted in preparation of Math 7 students for the NYS assessments

Algebra 1, Westfield Academy & Central School, 1/2022-3/2022

- Taught three units in 9th Grade Algebra I, and led Algebra I Lab and 8th Grade AIS
- Developed a custom unit on graphing functions that included assessment tools
- Developed lab activities to enhance class content and increase relevance, incorporated technology thoughtfully to improve student engagement
- Adapted and augmented curriculum to meet needs and interests of students
- Managed classroom routines and behaviors effectively, maximizing instructional time
- Communicated with parents regularly and advocated to showcase student work

Teaching Experience

Field Experience, Applied Geometry, Jamestown High School, Fall 2021

- Developed and led instruction, in a diverse classroom, that focused on coordinate geometry and compass and straight edge constructions
- Led morning warm-ups that reviewed and focused on Algebra I concepts
- Assisted classroom teacher by supporting instruction and helping individual students

Substitute Teacher, Falconer Central School (Middle/High School), present

Teaching Assistant, Jamestown YMCA/Jamestown Public School, Summer 2021

- Worked 6th, 7th, and 8th graders, some English Language Learners, for 7 weeks
- Co-taught 4 hours of daily ELA and Math instruction with NYS certified teachers
- Developed engaging enrichment activities that strengthened student math thinking skills, computer science skills, and developed a variety of STEAM activities
- Monitor growth over the summer through pre and post standardized assessments

Field Experience, Middle School Math Tutor, Dunkirk Middle School, 2/2021 – 5/2021

- Developed engaging lessons for a virtual environment, considering student's weaknesses and strengths in Math and Science
- Monitor growth through multiple types of assessments

Educational Outreach and Staff Training, National Comedy Center, 2/2018 – 8/2020

- Developed and led educational programming for students (9-12, BOCES, & University) on the technology and design behind the interactive museum exhibits

Program Assistant (K-6), Prendergast Library Makerspace, 11/2017 – 2/2018

- Introduced children to 3D printing and modeling, robotics, and programming
- Developed and consulted on plans for the future of the Makerspace with library staff

Guest Teacher (6th Grade), Falconer Public School, 1/2018

- Developed, planned, and co-taught a multi-week CS lesson to novice programmers
- Monitored student progress and provided appropriate support tailored to student's abilities

Summer Recreation Counselor (K-12), Village of Falconer**,** Summer 2013, 2014

- Planned and coordinated various programs, including STEM activities for children
- Ensured health, safety, and well-being of all children

Other Relevant Experience

Design Researcher, Seattle Children's Hospital, 2/2017 – 8/2017

- Developed and performed co-design workshops and design research activities with pediatric dialysis patients (age 3-13)

Previous Work Experience

Web and Exhibit Design Consultant, National Comedy Center, 8/2020 - present

Guest Experience Researcher and Developer, National Comedy Center, 2/2018 – 8/2020

- Developed and created new interactive and immersive museum experiences
- Facilitated soft-opening and initial beta-testing of interactive exhibits
- Maintained and supported exhibit technology
- Coordinated and executed accessibility services and led accessibility initiatives
- Performed exhibit design, graphic production, interactive development, media production, and exhibit fabrication and installation
- Launched, designed, and developed custom features for the online video platform, *National Comedy Center Anywhere*

Cast Member, Walt Disney World Resort, 1/2016 – 8/2016

Software Engineering Intern, NASA Jet Propulsion Laboratory, 6/2015 – 8/2015

Conference Participation

Mission: Rocky Rover, Presenter, *Make it,Take it!*, 71st Annual AMTNYS Conference, 2021

Professional Development

Teaching Digital Citizenship Training, Common Sense Education, 2021

Professional Memberships

Association of Mathematics Teachers of New York State (AMTNYS), present
Science Teachers Association of New York State (STANYS), present

Volunteer/Community Service

Website Design, Our Lady of Loreto Roman Catholic Church,

Graphic/Media Design, The Lucille Ball Little Theatre of Jamestown,

Jamestown Municipal Band, Jamestown Area Community Orchestra

References

Mrs. Connie Riedesel
Mathematics Teacher 7-12
Westfield Academy & Central School
203 East Main Street
Westfield, NY 14787

o. (716) 326-2151
m. (716) 581-0199
criedesel@westfieldcsd.org

Dr. Keary Howard
Professor (Coordinator Mathematics Education)
Department of Mathematical Sciences
SUNY Fredonia
208 Fenton Hall
Fredonia, NY 14063

o. (716) 673-3873
m. (716) 294-0846
keary.howard@fredonia.edu

Ms. Mary Plumb
STEAM Coordinator K-12
Falconer Central School
2 East Avenue, N.
Falconer, NY 14733

m. (716) 720-0070
mplumb@falconerschools.org

Mrs. Michelle Garvin
Mathematics Teacher 7-12
Jamestown High School
350 East 2nd St
Jamestown, NY 14701

o. (716) 483-3470
michelle.l.garvin@jpsny.org

Dr. Julia Wilson
Associate Professor Mathematics (Chair)
Department of Mathematical Sciences
SUNY Fredonia
208 Fenton Hall
Fredonia, NY 14063

o. (716) 673-3865
julia.wilson@fredonia.edu

Ms. Laura LaPlaca, PhD
Director of Archives & Research
National Comedy Center
203 West Second Street
Jamestown, NY 14701

o. (716) 484-2222 x 305
llaplaca@comedycenter.org

Letters of Recommendation

Westfield Academy & Central School District

Committed to Excellence

Westfield Academy and Central School
203 E Main St
Westfield NY 14787
(716)326-2151

March 18, 2022

To Whom It May Concern:

My name is Connie Riedesel, and I am a mathematics teacher at Westfield Academy and Central School. I have had the privilege of working with Brandon Caruso for 7 weeks while he student taught with me. Brandon is a very professional, well qualified candidate and will make a great teacher.

In my classroom, Brandon planned and implemented lessons aligned to the NYS standards. While working in my Algebra 1 classes, he taught three chapters, one of which he wrote himself. He also taught my Algebra 1 labs and an 8th grade AIS. With a background in computer science, he has an amazing ability to integrate technology to supplement the material and engage students. His strategies included direct instruction, cooperative groups, discovery, and games.

Brandon's biggest success was in his creation of many interactive labs and activities that supplemented and reinforced the classroom material. He had students create parabolas using a rope, a line and a point. Students made playing cards of angle relationships. Students were able to explore parabolas through a Desmos activity that he modified, looking at how parabolas moved horizontally and vertically, as well as the effect that amplitude had on the parabola. He had students design roller coasters using piecewise functions. He wrote a short unit he called "Getting' Funky with Functions" that explored transformations with quadratics, absolute value, square root, piecewise, cubic and cube root functions. He wrote the notes, homework (he called them "strength training"), review, labs and test for this unit. This unit is more professionally written than anything I have seen online or through a publishing company.

Brandon brings a lot of energy to the classroom, while still keeping a controlled and very organized classroom. He is able to recognize the challenging students and takes steps to prevent disruptions through eye contact, physical presence, and reminders about behavior expectations. Brandon worked well with students, often having at least five students a day staying for help (and many days more). He is able to explain things in more than one way and is patient with the at-risk learners. He is very professional in his dealings with students, staff, administration and parents. Brandon communicates very well with students, staff and parents. Very knowledgeable and highly organized, he is able to make connections between the content and the real world. His appearance, attitude, and behavior exuded professionalism.

Brandon is a highly qualified candidate who will make an excellent addition to any school. I have had many student teachers and he rises to the top as one of the most promising with natural ability. I highly recommend Brandon Caruso for any position that he seeks within your district.

Sincerely,

Connie Riedesel

Connie Riedesel
criedesel@westfieldcsd.org
(716) 326-2151 ext. 329
Mathematics Teacher
Westfield Academy and Central School

FREDONIA
STATE UNIVERSITY OF NEW YORK

March 23, 2022

Dear Department Chair and/or Principal:

I am writing you to offer my analysis of Brandon Caruso's qualifications as he begins what I believe will be an extraordinary career in mathematics education. My responsibilities at the university include the supervision of prospective secondary teachers in mathematics, the teaching of undergraduate mathematics courses, and the direction of Masters studies in mathematics education. Over the last two years, I have worked closely with Brandon and look forward to helping him reach his potential as he enters the profession.

Brandon is a gifted teacher. He is fearless and innovative in the classroom. Although I was aware of these attributes early on in my Introduction to Contemporary Mathematics Education Course (MAED 105/106) and Literacy/Technology Course (MAED 276), they became manifest during his student teaching field placements. His adventurous spirit continues to shape his lesson plan development and implementation. These are readily available in his burgeoning teaching portfolio, which I encourage you to explore. Brandon is also gifted in mathematics as is evident by both his GPA and accumulation of a variety of honors and awards.

It is what Brandon has accomplished in mathematics education outside of the rigors of the collegiate classroom that is most impressive to me. His academic talents are well documented. It is what he has done without notice and accolade that must be recognized: These include (in no particular order):

- Presenting integrated STEM lessons at the 2021 Association of Mathematics Teachers of New York State (AMTNYS) Conference.
- Headlining station activities at Eden Middle School's Virtual STEM Night
- Chasing trophies at SUNY Fredonia's 2021 Pi Day with his high school student at Westfield Academy and Central Schools
- Completing a Master's degree in Information Technology prior to tackling mathematics and education coursework at SUNY Fredonia.

In short, I believe that Brandon will serve as superb mathematics educator/mentor for many generations of students. He couples a depth of content knowledge with a genuine willingness to help others – a perfect combination for a young teacher. Simply put, I would trust him with the mathematical education of my own children – an accolade I reserve for my very best students. Please feel free to contact me at any time to discuss Brandon's potential and his fit for your district.

Sincerely,

Keary J. Howard, Ph.D.
Professor of Mathematics Education
208 Fenton Hall SUNY Fredonia
Department of Mathematical Sciences,
Fredonia, New York 14063
Phone: (716) 673-3873 e-mail: keary.howard@fredonia.edu

March 30, 2022

Dear Search Committee:

I heartily recommend that you invite Brandon Caruso to join your team of educators. Brandon's collaborative outlook, energy for innovation, and spirit of inclusivity will be significant assets to your shared work together.

As Brandon's supervisor and collaborator during his tenure at the National Comedy Center, I witnessed his unique inclination to approach every goal as, simultaneously, an artist, an engineer, and a humanist. His multifaceted approach to projects big and small was a testament to the power of bridging divides between the arts and sciences, and to the potential inherent in welcoming ideas, perspectives, and connections from all corners. Put bluntly: Brandon's defining characteristic, and the quality I valued most, was his organic and insuppressible enthusiasm for new ideas. His passion is fostering a sense of curiosity and joyful discovery in others—and that passion pushed everyone he worked with to evolve their creativity.

In his capacity as the National Comedy Center's Guest Experience Researcher and Designer, Brandon expanded his own job description (to our entire team's delight) to include the development of educational materials and experiences for K-12 student groups, age-appropriate interpretive materials for children visiting the museum with their families, and specialized tours and talks for advanced students studying information technology, exhibit media, and interactive design. He was an instrumental part of the curatorial team, offering creative solutions to the challenges inherent in communicating dense, complex, and nuanced information in efficient and engaging ways. More abstractly, Brandon contributed immensely to the team's shared enthusiasm for our work, always bringing compelling questions and an investment in the subject matter (even when outside his core areas of expertise) to the table.

I was grateful for Brandon's recognition of the importance of frequent and honest assessment, which ensured that our team was always keyed into questions of effectiveness and impact. His priorities were clear: delivering quality content that reached our visitors—from young students to seasoned lifelong learners—in ways that were memorable, enriching, and in line with our mission. Above all, he consistently measured success in terms of whether and how we were welcoming and speaking to the museum's diverse constituencies of users—most prominently including our cohort of local and regional K-12 students.

Indeed, Brandon's dual focuses on creating accessible interpretive tools and reaching out to serve the Chautauqua County community was energizing and inspiring. Brandon's seemingly innate ability to connect substantive educational material to everyday lived experience foregrounds his encompassing belief in the personal, communal, and societal impacts of engaged, enthusiastic learning. I am heartened at the thought that his future students will have the benefit of a teacher (and mentor) who is keyed into their individual strengths and excited to celebrate their differences.

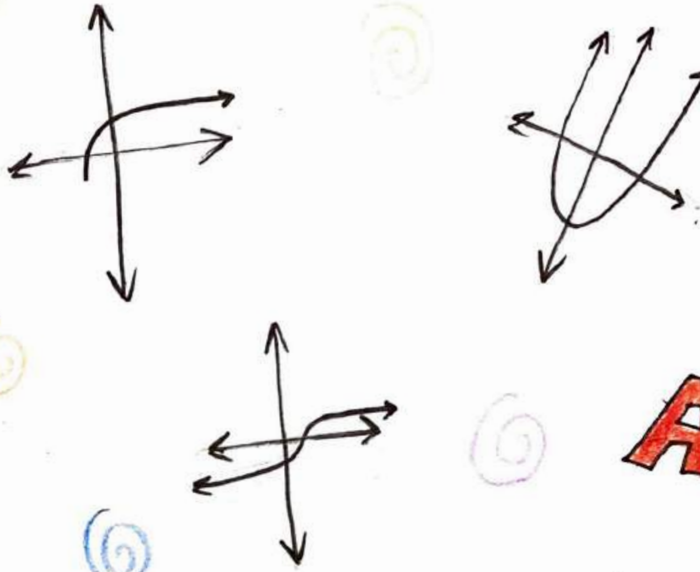
With best wishes for the continued growth and success of your students,

Laura LaPlaca, PhD
National Comedy Center
Director, Carl Reiner Department of Archives & Preservation
(716) 484-2222 x305
llaplaca@comedycenter.org
203 West Second Street, Jamestown, New York, 14701

Thank you for teaching me math in the time that you were here. Your way of teaching was new and interesting. It was very different from what I'm used to, but I still enjoyed it. You are very good at explaining things when I have a question. In the future, I believe you will become a very fun, great teacher. Keep working hard and soon enough you'll have a classroom of your own!

THANK YOU!

-Alina



A+

GETTIN' FUNKY WITH FUNCTIONS!

$y=a(x-h)^2+k$

Desmos

Observation Rubrics

College of Education | FREDONIA STATE UNIVERSITY OF NEW YORK

Student Teaching Formative Observation Form

1 = Ineffective 2 = Developing 3 = Effective (*Minimum Performance Target*) 4 = Highly Effective N = Not Observed

Student Teacher: Brandon Caruso	Date of Observation: 2/3/21	Lesson Topic: Fun with Factoring
School District and Building: Westfield Academy and Central School	Grade/Subject: Algebra I	Cooperating Teacher: Connie Riedesel
Observation: Observation #1	Quarter/Situation: Q3 Spring 2022	Clinical Field Supervisor: Dr. Keary Howard

DOMAIN 1: PLANNING AND PREPARATION

Component	Rating	Component	Rating
A. Demonstrating knowledge of content and pedagogy	3	D. Demonstrating knowledge of resources	4
B. Demonstrating knowledge of students	3	E. Designing coherent instruction	4
C. Selecting instructional outcomes	3	F. Assessing student learning	3

DOMAIN 2: THE CLASSROOM ENVIRONMENT

Component	Rating	Component	Rating
A. Designing an environment of respect and rapport	4	D. Managing student behavior	3
B. Establishing a culture for learning	3	E. Organizing physical space	3
C. Managing classroom procedures	3		

DOMAIN 3: INSTRUCTION

Component	Rating	Component	Rating
A. Communicating with students	3	D. Using assessment in instruction	3
B. Using questioning and discussion techniques	4	E. Demonstrating flexibility and responsiveness	3
C. Engaging students in learning	3		

DOMAIN 4: PROFESSIONAL RESPONSIBILITIES

Component	Rating	Component	Rating
A. Reflecting on teaching in terms of accuracy and use in further teaching	3	D. Participating in a professional community	N
B. Maintaining accurate records	3	E. Developing and growing professionally	3
C. Communicating with families	N	F. Demonstrating professionalism	4

COMMENTS: Planning, Instruction, Assessment of Student Understanding
Brandon: How cool is it to 'see' you in-person with middle schoolers teaching a subject you enjoy and pursuing your passion. It's a testament to your hard work and dedication. In general, you are comfortable and at ease with your students and have a solid grasp of all of the elements of direct instruction. Kudos to your emphasis on mathematical vocabulary given the challenging topic of factoring methods/techniques. Great use of alternative guided practice strategies in your Factoring Lead Challenge. In a blocked classroom with almost 90 minutes of dedicated time you need to provide a variety of instructional methods – well done here. Thanks for providing a time limit for a number of station activities/guided practice problems and holding them accountable for accurate solutions. It's a veteran strategy that keeps you on task and the class engaged. As we head to mid February, be prepared to take on a complete role as a teacher. Grading, lesson planning, unit assessments, communication with parents, etc. are all likely to be on your plate. Embrace these challenges and be sure to enjoy the experience – it will fly by with little time to reflect. BTW – great use of technology to produce questions/solutions for the solutions to your Guided Notes. It made things move quickly and efficiently. It's another example of the power of preparation. Prep time = smooth math classes.

FINAL COMMENTS	
General Commendations	Focus Areas
• Great start as expected. • The student as teacher method (leading a question in your case) is powerful in more ways than you know. You'll use it throughout your career.	• Continue to make connections with individual students. • The weather and the drive will get better – think spring!

REMINDERS	
Next Observation Date	Requirements
Week of 2/7-2/11	Remember to upload required OFE lessons in the appropriate folder. Submit GoReact formative evaluation video #2 following our in-person observation.

College of Education | FREDONIA STATE UNIVERSITY OF NEW YORK

Student Teaching Formative Observation Form

1 = Ineffective   2 = Developing   3 = Effective (*Minimum Performance Target*)   4 = Highly Effective   N = Not Observed

Student Teacher: Brandon Caruso	Date of Observation: 2/8/22	Lesson Topic: Polynomial Puzzles and More
School District and Building: Westfield Academy and Central School	Grade/Subject: Algebra I	Cooperating Teacher: Connie Riedesel
Observation: Observation #2	Quarter/Situation: Q3 Spring 2022	Clinical Field Supervisor: Dr. Keary Howard

DOMAIN 1: PLANNING AND PREPARATION

Component	Rating	Component	Rating
A. Demonstrating knowledge of content and pedagogy	3	D. Demonstrating knowledge of resources	4
B. Demonstrating knowledge of students	3	E. Designing coherent instruction	4
C. Selecting instructional outcomes	3	F. Assessing student learning	3

DOMAIN 2: THE CLASSROOM ENVIRONMENT

Component	Rating	Component	Rating
A. Designing an environment of respect and rapport	4	D. Managing student behavior	3
B. Establishing a culture for learning	3	E. Organizing physical space	3
C. Managing classroom procedures	3		

DOMAIN 3: INSTRUCTION

Component	Rating	Component	Rating
A. Communicating with students	3	D. Using assessment in instruction	4
B. Using questioning and discussion techniques	4	E. Demonstrating flexibility and responsiveness	3
C. Engaging students in learning	3		

DOMAIN 4: PROFESSIONAL RESPONSIBILITIES

Component	Rating	Component	Rating
A. Reflecting on teaching in terms of accuracy and use in further teaching	3	D. Participating in a professional community	N
B. Maintaining accurate records	3	E. Developing and growing professionally	4
C. Communicating with families	N	F. Demonstrating professionalism	4

COMMENTS: Planning, Instruction, Assessment of Student Understanding
Brandon: I enjoyed spending some time with Ms. Riedesal discussing your first few weeks in the classroom. She has been thoroughly impressed. In particular, your curricular creativity (Algebra lab activities, G10 graphing unit, etc.) rivals anything that is commercially available. At some point, we should discuss a grant collaboration to look at curriculum development that could compete with Weiler's e math instruction. Specifically, here is what Ms. R. mentioned during our mid term roundtable: Brandon is doing well. He engages well with the students, calling on all students every day. He starts labs with headscratchers, designed to get students thinking about math and allows for multiple correct responses. He has students share their answers. He is doing exceptionally well with classroom management-- normally an area of weakness for student teachers. The content that he writes is better than any textbook I've seen. He plans great activities that tie into the material we are covering in class. He selects pairings that enhance learning while breaking up some unhealthy cliques and dependencies. He reflects on his lessons and adjusts. He seeks out and receives feedback very well. He is by far the best student teacher that I've had. Rock on young man…

FINAL COMMENTS	
General Commendations	Focus Areas
• I appreciate your continued emphasis on differentiated instructional methods. Group work, direct instruction, paired problem solving, etc. all make appearances in your classroom.	• Love your use of 'Headscratchers.' What should (or is) the incentive for a student to solve one? • Speaking of incentives – see me for Pi Day plans that your Algebra gang can participate in with t-shirts and trophies that no amount of money can buy.

REMINDERS	
Next Observation Date	Requirements
Digital mid term check in with Ms. R. next week.	Submit GoReact formative evaluation video #2 following our in-person observation Continue to update OFE Student Teaching Folders.

College of Education | FREDONIA STATE UNIVERSITY OF NEW YORK

Student Teaching Formative Observation Form

1 = Ineffective 2 = Developing 3 = Effective (*Minimum Performance Target*) 4 = Highly Effective N = Not Observed

Student Teacher: Brandon Caruso	Date of Observation: 3/3/22	Lesson Topic: Quadratic Quandaries
School District and Building: Westfield Academy and Central School	Grade/Subject: Algebra I	Cooperating Teacher: Connie Riedesel
Observation: Observation #3	Quarter/Situation: Q3 Spring 2022	Clinical Field Supervisor: Dr. Keary Howard

DOMAIN 1: PLANNING AND PREPARATION

Component	Rating	Component	Rating
A. Demonstrating knowledge of content and pedagogy	4	D. Demonstrating knowledge of resources	4
B. Demonstrating knowledge of students	3	E. Designing coherent instruction	4
C. Selecting instructional outcomes	3	F. Assessing student learning	3

DOMAIN 2: THE CLASSROOM ENVIRONMENT

Component	Rating	Component	Rating
A. Designing an environment of respect and rapport	4	D. Managing student behavior	3
B. Establishing a culture for learning	4	E. Organizing physical space	3
C. Managing classroom procedures	3		

DOMAIN 3: INSTRUCTION

Component	Rating	Component	Rating
A. Communicating with students	3	D. Using assessment in instruction	3
B. Using questioning and discussion techniques	4	E. Demonstrating flexibility and responsiveness	3
C. Engaging students in learning	3		

DOMAIN 4: PROFESSIONAL RESPONSIBILITIES

Component	Rating	Component	Rating
A. Reflecting on teaching in terms of accuracy and use in further teaching	3	D. Participating in a professional community	N
B. Maintaining accurate records	3	E. Developing and growing professionally	4
C. Communicating with families	N	F. Demonstrating professionalism	4

DEPARTMENT OF MATHEMATICAL SCIENCES
280 Central Ave. 223 Fenton Hall Fredonia, NY 14063 T 716.673.3243 www.fredonia edu

COMMENTS: Planning, Instruction, Assessment of Student Understanding
Brandon: It makes your Math Ed. Professor proud to see both forms of the quadratic on your board. Couple it with your emphasis on completing the square and I'm confident that the instruction your WACS students are receiving is some of the best in the country. No lie (a purposeful pun in today's quadratic project 'Two Truths and a Lie.' From Ms. Riedesel: Brandon communicates well. He posts frequently on Classroom, sends newsletters to the parents sharing what students have been studying as well as sample student work, he has worked with administration and the technology department to loop the students' graphical art and piecewise roller coasters on the tv in the lobby as well as the school web page. Brandon is very professional in all that he does. He has exceptional classroom management. He is very knowledgeable about math and its many connections to other fields. He is extremely well organized, plans engaging activities and is a fantastic teacher. He also has shown up at several athletic contests to see his students in another light and has established strong relationships with his students. By far, Brandon is the best student teacher that I have had (I have had at least a dozen). I wish we had an opening for him-- I would hire him immediately.

FINAL COMMENTS	
General Commendations	Focus Areas
• Superb performance in your initial placement. • Thanks for taking the time to visit your Q4 placement over the winter break!	• Continue to display your boundless enthusiasm for teaching mathematics as you move to Fredonia Middle School.

REMINDERS	
Next Observation Date	Requirements
Digital end-of-placement reflection with Ms. Riedesel week of 3/7. Q4 meet-and-greet week of 3/21.	Upload your final Q3 video and complete your self-reflection requirements.

Sample Classroom Syllabus

NextGen Regents

GEOMETRY

10th Grade & 9th Grade A ***Syllabus***

2021-22 School Year

Room 314

Instructor

Mr. Brandon Caruso

bcaruso@fredonia.edu

716.555.5555 x314

Mathematics is beautiful. *Don't believe me?* On our journey through Geometry, we will encounter this beauty on a daily basis. We finally dive deep into a topic we have been exploring since elementary school and build upon our previous knowledge to grow deeper understanding. This course prepares you with foundational skills that will be utilized throughout the rest of your mathematics education, from Algebra II/Trigonometry onward. This course aligns with the NYS Next Generation Mathematics Learning Standards and will prepare students to complete the NYS Geometry Regents Exam.

Topic Overview

- Congruence, Proof, and Constructions
- Transformations and Rigid Motion
- Similarity, Proof, and Trigonometry
- Extending to Three Dimensions
- Connecting Algebra and Geometry
- Circles With and Without Coordinates

Goals

- Visualize and use spatial reasoning to analyze properties of geometric objects
- Apply transformations, symmetry, and coordinate geometry to analyze real-life problems and situations
- Identify and justify geometric relationships, formally and informally

A Day In the Life of a Geometry Student

We do math on our feet, in our seats, and on the streets. Geometry is hands-on, creative, visual, and collaborative. Therefore, every day we will take advantage of these characteristics. Outside of daily classroom discussions and discovery labs, don't be surprised if our class is in the band room, on the football field, or in nature. Geometry will bring us to all these places.

Check-in	Discover	Discuss	Reflect

Classroom Community Expectations

We learn best when we learn together and from each other. We are a community of unique individuals with different abilities and experiences. This means, we expect everyone to **be in class every day to contribute** and we expect everyone to **respect themselves and others.** You are responsible for your own success, but we are a community that is here to support you.

Let's Unplug a Bit! Cellphones will *never* be needed during class. These will remain in lockers or they will be removed by the instructor for the class period.

Let's Talk

Instructor Availability

Work Sessions

Tue, Wed, Thu

2:30 - 3:30 pm

Office Hours

A/B Days - Period 3

C/D Days - Period 4

Materials

3-Ring Binder

Lined Filler Paper

School One-to-One Device

Provided

Construction Tools

Calculators

Geogebra (Online)

Notes & Readings

No Course Textbook

Online Classroom

All course materials and resources will be found on our online classroom.

Work submissions will be submitted online, unless otherwise specified.

Link:
classroom.google.com

We Put in the Work To Grow! Grading and Assessment

Discovery Sets **15%**

A collection of problems, activities, or readings that prepare students for in-class activities. These are graded for completion and collected for growth feedback. *Due at the start of each class*

***You Pick* Problem Sets** **30%**

A collection of problems of varying difficulty. Students choose 2 questions at each level to complete. These are graded for correctness and for feedback. Corrections are required and will receive half-credit back. Assigned *Bi-weekly.*

NYS Geometry Regents **20%**

There is no local final. Passing the regents exam is required to pass the course. *June 21st, 9:15 am*

Strength Check-Ins **20%**

These assessments look to find where we can grow. Corrections are required and will receive half-credit back. *In-class throughout school year, at least 4.*

Deep Dive Projects **10%**

We put what we have learned to work. Students will immerse themselves in fun and engaging problems. They will collaborate and present mathematical work to their peers. *At least 3.*

Engagement & Perseverance Log **5%**

Attend and be active in class and after select classes, we reflect on how we have grown and what we achieved during class.

A: 90-100% **B: 80-89%** **C: 70-79%** **D: 60-69%** **F: 0-59%**

Late Work All work shall be completed by the due date and must be turned in prior to the *Strength Check-In* for that topic. 10% will be deducted from any late assignment and late Discovery Sets will receive zero points. Parents should email the teacher about any circumstances that may result in late work.

School Year Highlights and Traditions

The Math Art Gala **10.1.2021**

Students create geometric masterpieces by hand or digitally. *Families welcome.*

Mathmas **12.22.2021**

Students give the gift of problem solving and present solutions to fun math problems to their peers.

Pi Day **3.14.2022**

Celebrate π ! Fun activities throughout the day in the Media Center.

STEAM Night 2022 **5.4.2022**

Students create engaging math based activities to share with Elementary Students.

Hello. I'm Mr. Caruso.

My name is Brandon Caruso. I am a Chautauqua County native and I hold a B.S. in Mathematics/Adolescence Education from SUNY at Fredonia. I also hold a B.S. in Software Engineering from SUNY at Oswego and a Master's of Human-Computer Interaction and Design from the University of Washington. I have taught Math and CS to students (K-12) in a variety of settings, from the classroom to the MakerSpace. I also play the trumpet for local theatres and music ensembles.

Dear, Parents. *You know your child best.* Please reach out, through email, with any best ways to work with your child and any concerns or questions throughout the year. We will collaborate to make sure your child succeeds. Also, please help me defeat the stigma around mathematics, use positive language at home when discussing math. Instead of signing this, please keep this document and help your student complete their first *Discovery Set*. There are a few questions they will need your help to complete.

Lesson Plan Exemplar

Lesson Title: Going Digital | Binary Numbers - Binary Bracelets

Teacher: Brandon Caruso
Grade Level: 8
Math/Science Content Area: Exponents, Number Systems of Different Bases, Data Encoding
Unit of Study: Exponents & Scientific Notation
Time: 15 min. Flipped Lesson Video and 80 min. Class Period

Central Focus for the Learning Segment

Students will engage and extend their knowledge of exponents by applying that mathematical knowledge to computer science concepts and ideas. Students will explore different number systems and data representations, both familiar (decimal) and new (binary). Students will learn how to convert between decimal and binary representations and extend that to encoding data like text. Through a hands-on in-class activity, students will encode a message in binary using bead on a bracelet. Lastly, students will draw connections between the fundamental building blocks of digital electronics and binary representation. They will be introduced to transistors and some of the hardware that make up our digital electronics.

Content Standard(s):

NY-8.EE.1 - Know and apply the properties of integer exponents to generate equivalent numerical expressions.
NY-8.EE.3 - Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
CSTA.2.DA.07* - Represent data using multiple encoding schemes.
CSTA.3A.DA.09* - Translate between different bit representations of real-world phenomena, such as characters, numbers, and images.

Learning Objectives - Students will:

1. Represent decimal and binary numbers as sums of single digits multiplied by powers of 10 or 2.
2. Convert decimal numbers to binary numbers and binary numbers to decimal numbers.
3. Encode a character string, associated with decimal numbers, to a binary string.
4. Identify a fundamental building block of digital electronics, transistors, and their relation to binary.

Instructional Resources and Materials

Lecture Video, Guided Notes, Binary Bracelet Procedure Document, Variety of Colored Beads, Elastic String, Scissors

Language Function Students Will Develop. Additional Language Demands and Language Supports:

exponent, power, base, number system, digit, decimal, binary, encode, decode, conversion

Guided notes provide a structured way to document and introduce students to these terms. For words like **binary**, students are shown how to draw the meaning from the word itself. Students use this language throughout the lesson.

Differentiation and Planned Universal Supports:

A variety of representations are included throughout the lesson. The content of the lesson is provided in video form so students can watch and reviewing at their own pace. Guided notes are provided to help students document important information, organize, and retain ideas from the lesson. In the lesson, conversion of binary numbers is introduced, first, visually with dot cards, then connected with the underlying mathematical notation, and then a written algorithm. Finally, a

*Computer Science Teachers Association (2017). CSTA K-12 Computer Science Standards, Revised 2017. Retrieved from http://www.csteachers.org/standards.

hands-on activity provides an opportunity to explore binary representation in a tangible way by creating a bracelet. Students that have difficulty with threading beads or physically making the bracelet can use a partner to build their bracelet. Message length and the table conversion activity can be shorten for students that may require modifications.

Instructional Strategies, Learning Tasks, and Protocol

A **flipped lesson** is used so students learn the content prior to class through a video lesson and guided notes. The teacher use modeling throughout the video to show students how to convert between systems. The video introduces:

Lights Please - The teacher links a visual, turning on and off a light, to the complex capabilities of our modern electronics. The teacher makes the organic connection to show that binary is used by these devices.

Familiar Decimal Numbers - The teacher presents decimal numbers, using a positional representation, as a familiar foundation to help students then apply similar ideas to binary numbers.

Binary Dot Cards - These cards provide a visual approach as the first way to illustrate the conversion from decimal to binary. These cards decompose decimal numbers to powers of two and then to binary digits. This then lead into the more formal algorithm for converting decimal to binary notation.

Binary Bracelet Activity - A creative hands-on activity that applies what students have learned about decimal and binary numbers systems and explores how data, like text, can be encoded and represented using binary numbers.

Student Assessments and Content Assessed

Informal Assessment:
Students present the teacher with the secret access code and completed guided notes as an entry ticket. Students will be given opportunities to respond during the warm-up and review activity at the start of the lesson. Students will put their initials next to the numbers they converted in the warm-up activity. The teacher will use this to judge address any misconceptions prior to starting the in-class activity. During the bracelet activity, students will be asked to fill-in the procedure document and the teacher, roaming the room, will probe and ask students questions during the activity.

Formal Assessment:
At the conclusion of the lesson, students will be asked to complete a written assessment that asks students to encode a string of characters to binary (simulating the bead activity), convert between decimal and binary numbers, and demonstrate understanding of transistors. Students will also be to challenge their knowledge with non-scored "Dig Deeper" questions.

Evaluation Criteria:

Students should be prepared to participate in the in-class activity by completing their guided notes prior to class. Students will also be expected to engage in the in-class activity and discussion. Mastery of the lesson will be determined by completing the bracelet activity and scoring 85% or higher on the formal assessment.

Relevant Theories and/or Research Best Practices:

Constructivism is heavily used in this lesson as students learn through experience and by engaging with knowledge in a non-passive form. The teacher serves as a mentor in the exploration and acquisition of knowledge. The lesson utilizes **project based learning** to engages students through collaborative and hands-on activities to improve retention and attitudes toward learning. Lastly, **guided notes** provide support for students to help them organize, synthesis, associate, and retain the content of the lesson.

Lesson Timeline: This is a Flipped Lesson. There are two parts to this lesson.

Pre-Lesson Video - Learning the Content (15 mins)

Hook (3 mins)

The teacher shows a flashlight turning on and off, illustrating a two state system. They then discuss that these simple states can be used to represent data in digital electronics and introduces binary numbers. Guided Notes are completed by teacher along with students.

Review Decimal Numbers (4 mins)

The teacher then reviews the decimal numeral system, completing the guided notes, and showing students how a sum of the product of digits and powers of ten can be used to represent decimal numbers ($12 = 1 \times 10^1 + 2 \times 10^0$). The teacher completes two examples of representing numbers in this form.

Explore Binary (7 mins)

The teacher first shows converting from binary to decimal. Then by using binary dot cards, the teacher starts to illustrate the algorithm to convert from decimal to binary. The algorithm is then official introduced, by the teacher, through completing two examples. Again, completing guided notes throughout.

Close (1 mins)

The teacher reviews the covered material, asks students to convert the number 7 to binary as the secret access code and to complete the **Give it a Try** questions for entry into class.

In-Class Activity - Using the Content (80 mins)

Entry Activity and Review (20 mins) - This could be replaced with the pre-lesson for a non-flipped lesson. Students show the teacher the secret access code and guided notes. On the whiteboard, students find one decimal and one binary number to convert to the other representation. When completed, students discuss the takeaways from the pre-lesson and guided notes with a partner. The teacher reviews the algorithm for converting between binary and decimal by selecting two examples on the board and demonstrating the conversion. Students help recall the algorithm.

Binary Bracelet Activity - Lead Into Activity (2 mins)

The teacher asks students how might letters be encoded in the computer? They then show that by associating a letter with a decimal number, we can then encode these letters using binary numbers.

Prepare for Binary Bracelet Activity (5 mins)

The teacher, wearing their binary bracelet, introduces the activity and goes over the procedure document with students. Providing time for students to ask questions.

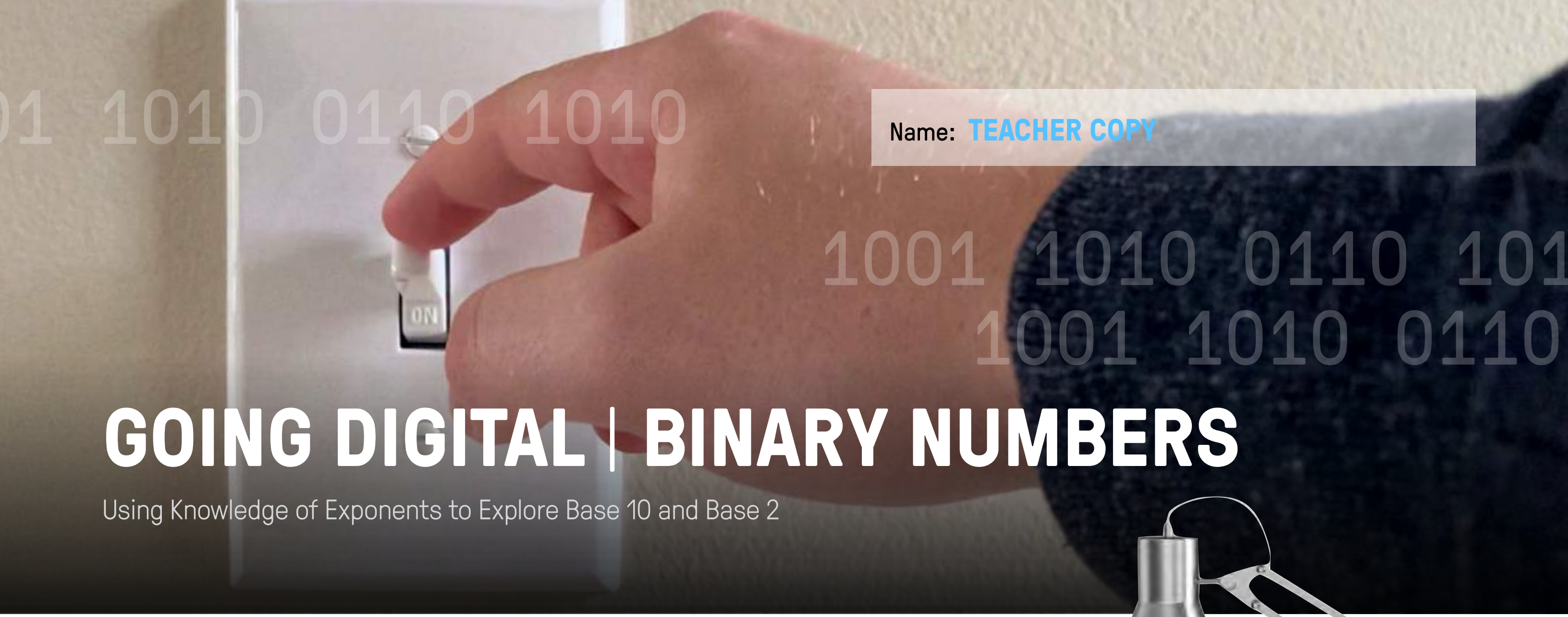
Create Binary Bracelets (30 mins)

Students follow the procedure document and work with a partner to complete aspects of the document. Students create their own bracelet, converting text to the binary representation. The teacher floats among students and provides feedback and answers questions. Supplies are available at a central location in the room.

Decode Each-others Bracelets (5 mins)

When students are done, they need to find another student, different than their partner, and decode each others binary bracelets.

Show Off What You Know (20 mins) - Individually, students will be give a written ticket out the door to complete and to “show off what they know.” The students will also show the teacher that they completed the activity by showing their finished bracelet when they submit the assessment.



Name: TEACHER COPY

GOING DIGITAL | BINARY NUMBERS

Using Knowledge of Exponents to Explore Base 10 and Base 2

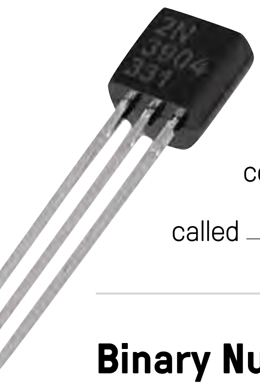
Lights Please!

We have all turned a lamp on and off using a switch. It might be hard to believe, but these two simple states, on and off, can be used to:

represent information, the letter 'A' or number '36',

transmit information, a text message or TikTok video,

make decisions or calculate, turn on your alarm or control an autonomous vehicle.



This type of two state system is the fundamental building block of how our digital electronics work today. Many of the components of our digital electronics, like processors, (the control units of a computer), and memory, use small fundamental semiconductor switches called transistors that can be either on or off.

Binary Number System

In the binary number system there are only two digits called bits, coming from "**bi"**nary dig"**its"**, they are:

0 1

Take a Step Back

The prefix "bi" in the word **binary** give us a hint that this system only has two digits, "bi-" means two.

Great! Our digital electronics are good at working with two states, so binary is the perfect representation!

Decimal Numeral System - Now This Looks Familiar

As a child, we learn to count in the decimal numeral system, where there are ten digits:

Digits of the Decimal Numeral System:

0	1	2	3	4	5	6	7	8	9

So, how do we get the number 132 from these ten digits?

We must position the digits. For example, we refer to the digit in the ones place, the tens place, the hundredths place, and so on. In fact, we are multiplying each digit by a power of 10 and adding them together.

1	3	2
10^{2}	10^{1}	10^{0}
100	10	1
hundredths	tens	ones

Take a Step Back
Notice how **10** is in the base of the exponent? The decimal number system is sometime referred to as **base 10**.

Check this out! Here is how we get the decimal number **132**:

$$1 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$$

$$1 \times 100 + 3 \times 10 + 2 \times 1$$

$$100 + 30 + 2$$

$$132$$

Give it a try! Write these decimal numbers as a sum of the digits 0-9 multiplied by powers of 10:

56
$$5 \times 10^1 + 6 \times 10^0$$

1,234
$$1 \times 10^3 + 2 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$$

202
$$2 \times 10^2 + 0 \times 10^1 + 2 \times 10^0$$

Back to Binary - What if we only have 2 digits?

Similar to the decimal system, we must ___position___ the bits ___0 and 1___. These are the only two digits in the binary number system. **However**, instead of multiplying by powers of 10 , we multiply **0** or **1** by a ___power of 2___ and add them together. Binary is also sometimes called ___base 2___.

What is the decimal value of the binary number 10110?

1	**0**	**1**	**1**	**0**
2 ^{4}	**2** ^{3}	**2** ^{2}	**2** ^{1}	**2** ^{0}
16	8	4	2	1

Here is how we get the decimal value of the binary number 10110:

$$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 1$$

$$16 + 0 + 4 + 2 + 0$$

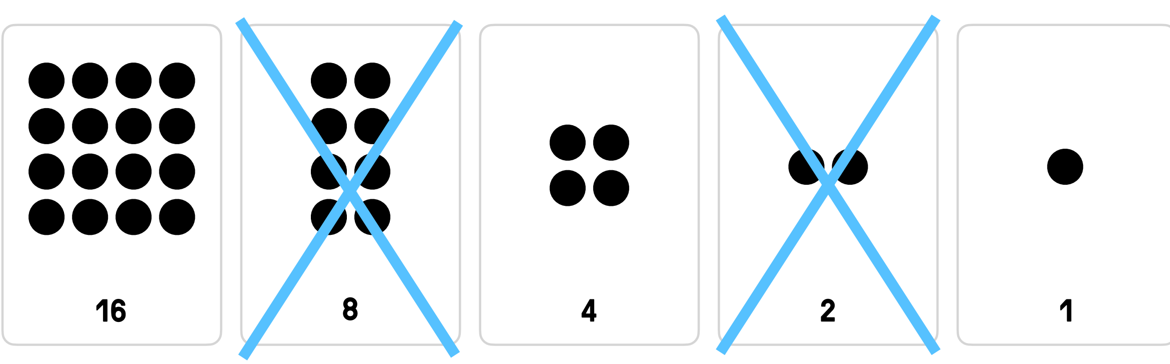
$$22$$

POWERS OF 2

2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
1	2	4	8	16	32	64	128	256	512	1024

STARTING WITH BINARY CARDS - Converting 21 from decimal to binary

Each card represents a binary digit or bit with their corresponding power of 2 dots on them. We decide which bits we want to keep or turn on **(1)** and which we want to flip over or turned off **(0)**, so the dots visible equal **21.**



10101

Our Decimal Number To Binary Number Conversion Algorithm

We have an **algorithm**, a series of step, we can follow to convert from decimal to binary:

1) Place a **1** in the position of the largest power of 2 that is less than or equal to the decimal number, then subtract that power of two from the decimal number.

2) Using the resulting decimal number, go to the next power of two, if the power of two is less than or equal to the decimal number place a **1** in the position and, again, subtract that power of two from the decimal number. If it is not less than or equal to the decimal number place a **0**, and move to the next power of two.

3) Repeat step (2) until the remaining decimal number is either a 1 or a 0 in the 2^0 position.

Let's Practice! Convert **78** from decimal to binary.

Our largest power of two that is less than or equal to 78 is 64 **.**

$78 - 64 = 14$, place a 1 on 64
32 and 16 are both too large, place a 0.
$14 - 8 = 6$, place a 1 on 8
$6 - 4 = 2$, place a 1 on 4
$2 - 2 = 0$, place a 1 on 2
0, that a binary number place it on the end.

1	0	0	1	1	1	0
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

Give it a try! Convert these decimal numbers to binary numbers and the binary numbers to decimal numbers:

56
111000 32+16+8 = 56

35
100011 32+2+1 = 35

101001
41 32+8+1 = 41

100111
39 32+4+2+1 = 39

Secret Access Code

1011

Binary Bracelet | In-Class Activity

We can represent letters in binary by first associating them with a decimal number. If we have a decimal number, we can then convert that to a binary number for storage or sending to other digital devices. In class, we will be making **Binary Bracelets** that have a secret message; pick your initials, maybe make a friendship bracelet, or maybe a short secret word (no more than 4 letters). First, we need an encoding chart that has the characters, A-Z, their associated with decimal numbers, and then their binary representation. Others we have no clue how to decode our finished bracelet.

Since we have 27 distinct characters (we included SPACE), how many bits are needed to represent all the characters?

$32 = 2^5$, 5 bits

A-Z and SPACE - CHARACTER ENCODING TABLE

Work with a partner and fill in the binary representation for each of the decimal numbers and their associated letter:

Character	Decimal	Binary	Character	Decimal	Binary
SPACE	0	00000	N	14	01110
A	1	00001	O	15	01111
B	2	00010	P	16	10000
C	3	00011	Q	17	10001
D	4	00100	R	18	10010
E	5	00101	S	19	10011
F	6	00110	T	20	10100
G	7	00111	U	21	10101
H	8	01000	V	22	10110
I	9	01001	W	23	10111
J	10	01010	X	24	11000
K	11	01011	Y	25	11001
L	12	01100	Z	26	11010
M	13	01101			

Making Your Bracelet Student Messages will Vary

Materials: Scissors, Three Different Colored Beads, Elastic String

1) Select Your Message You Want to Encode:

MY MESSAGE
LOL

2) Select the color beads you want to represent your bits:

0	1
AQUA	BLUE

Using the chart, What is your message converted to decimal numbers?

12 15 12

Using the chart, What is your message converted to binary numbers?

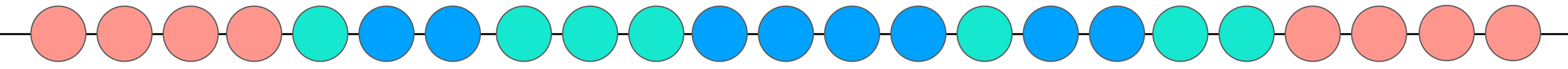
01100 01111 01100

3) Cut a piece of elastic string (longer that you think you will need) and thread the beads in the correct order to match the binary representation of your message.

4) Pick a third bead color and fill in the remainder of the bracelet so that is fits your wrist. Add equal number of beads before your binary message and after your binary message. Have a friend help tie the bracelet to your wrist (not too tight) and cut off the extra elastic.

5) Find another student, different from your partner, and decode their bracelet.

Here's my secret message bracelet for **LOL**:

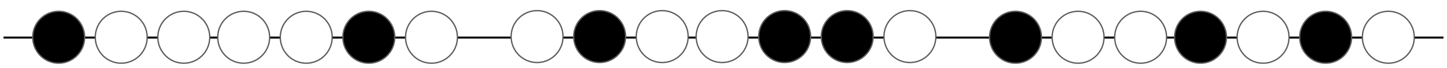


What color did I choose to represent 0, how about 1?

We made some amazing binary bracelets in class! Mr. Caruso would like to make a friendship bracelet for him and his friend. He wants it to say, **B & J**, on it! Fill out this empty bracelet as blueprint to give to Mr. Caruso so he can make the friendship bracelet. Fill in the circles for **1 =** ● and leave the circles empty for **0 =** ○.
Use the ASCII code table to the right. ASCII is a 7-bit code per character and we will ignore spaces.

ASCII Character	Decimal Number
B	66
J	74
&	38

Encode: **B & J**



SHOW YOUR WORK

$$B : 66 = 1 \times 2^6 + 1 \times 2^1 = 64 + 2 = 66$$

1	0	0	0	0	1	0
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

$$J : 74 = 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^1 = 64 + 8 + 2$$

1	0	0	1	0	1	0
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

$$\& : 38 = 1 \times 2^5 + 1 \times 2^2 + 1 \times 2^1 = 32 + 4 + 2 = 38$$

0	1	0	0	1	1	0
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

Convert the following from decimal (base 10) to binary (base 2):

234 [Students should show work] 11101010	17 [Students should show work] 10001	132 [Students should show work] 10000100

Convert the following from binary (base 2) to decimal (base 10):

111101 [Students should show work] 61	1010 [Students should show work] 10	110001 [Students should show work] 49

An iPhone 11 A13 processor chip has 8.5 billion of these fundamental semiconductor devices. What did we call these fundamental switches that are one of the building blocks of our digital electronics? **Transistor**

DIG DEEPER - Try Your Best!

What are the minimum number of bits you need to represent, the decimal number, **2056** in binary?

12 bits, 100000001000

There are 8 bits in 1 byte. What is the range of decimal numbers you can represent in 2 bytes?

00000000 00000000 to 11111111 111111111 , in decimal would be 0 to 65,535

Lesson Title: "Tree"-gonometry? Introduction to Right Triangle Trigonometry

Grade Level: 10th Grade
Mathematical Content Area: Right Triangle Trigonometry - Sine, Cosine, and Tangent
Unit of Study: Geometry

Central Focus for the Learning Segment (Lesson Introduction Video: https://youtu.be/rDzbmP3eXnQ):
Through the lens of a forester measuring the height of a majestic *Sequoiadendron giganteum,* students are introduced to the trigonometric ratios and functions found using right triangles. Students examine the question, *"How can we find lengths of sides of a right triangle with only knowing an angle and one other side length?"* Students review right triangles, examine the relationship between angle and side length measures, define the basic trigonometric functions, and apply this knowledge to solve an applied problem calculating the height of a giant sequoia.

Content Standard(s): NYS Math Common Core and/or Performance Indicators
GEO-G.SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of sine, cosine and tangent ratios for acute angles.
GEO-G.SRT.8. Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems.

Learning Objectives associated with the content standards:
Students will define the sine, cosine and tangent ratios for acute angles based on side lengths and a specified angle.
Students will use sine, cosine, tangent to solve right triangles in applied problems given an angle and side length.

Instructional Resources and Materials to engage students in learning**:** Students Guided Notes, TI-84 Calculator

Language Function Students Will Develop. Additional Language Demands and Language Supports:
ratio, relationship, function, sine, cosine, tangent, trigonometry, right triangle, side length, angle measure, hypotenuse, opposite, adjacent, inclinometer, theta, alpha

Differentiation and Planned Universal Supports: Depending on students needs, guided notes can be provided completed and made available digitally for any student. This allows them to utilize any assistive technology necessary to read and view the notes. A captioned video recording of completing the guided notes will be made available to also student to reference later. Alternate translations of the notes can be made available to ELL students.

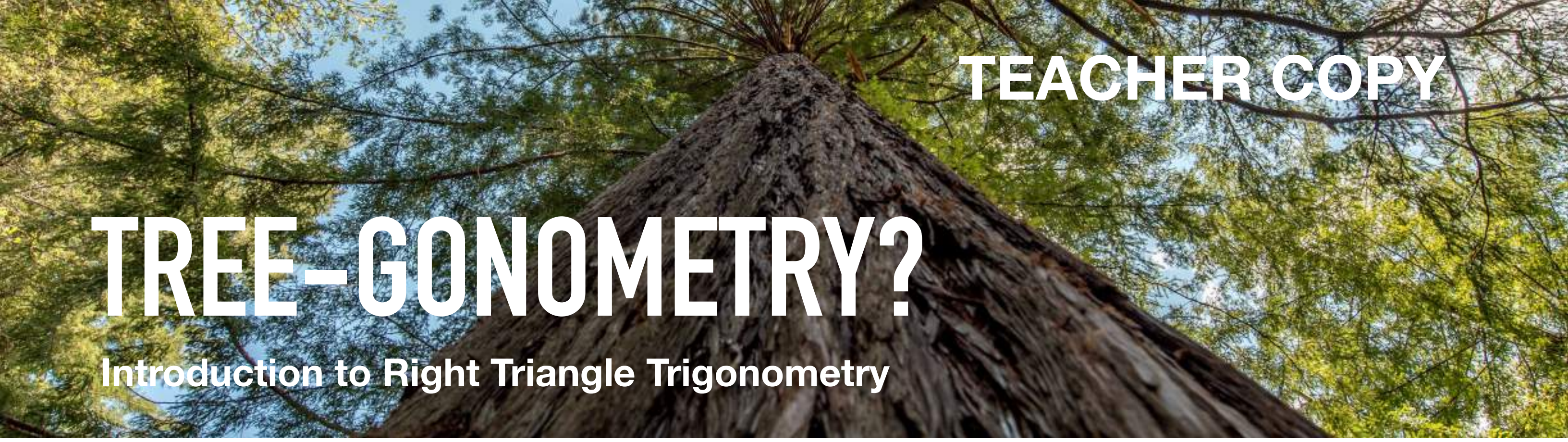
Instructional Strategies, Learning Tasks, and Protocol that support diverse student needs: This lesson utilized guided notes where students fill in along with the teacher. Embedded in the notes are practice problems that have students identify the trigonometric ratios based on sides and an angle and identify missing side lengths. Visual representations are explored along side the mathematical representations. Students are provided with a mnemonic to help recall the various relationships.

Student Assessments and Content Assessed
Informal Assessment: While completing the notes, the teacher will question students to review different trigonometric ratios. The teacher will gradually remove support and ask students how to do the guided practice.
Formal Assessment: Teacher will grade in-note independent practice and the final applied practice problem.

Evaluation Criteria: Students will correctly identify 90% of the side ratios given an angle and trig. function, determine the lengths of the unknown sides with 90% accuracy, and correctly complete the applied problem.

Relevant Theories & Research Best Practices: Polya Problem Solving & Posner's Research in Conceptual Change

Lesson Timeline:
Pre-Assessment (5 min): Students will find a side of a triangle given an angle and second side length.
Introduction (10 min): Teacher introduces the forestry context and problem. They define trigonometry and review right triangles. They define the different trigonometric functions, derive tangent using sine and cosine, and provide a mnemonic to help recall the various relationships.
Guided Practice (2 min): Teacher models how to find the side length trig. ratios for angle θ (pg. 3).
Independent Practice (2 min): Students complete the side length ratio for angle α (pg. 3).
Guided Practice (5 min): Models finding the side lengths for the second triangle (pg. 4).
Independent Practice (10 min): Students find the side lengths for the first and last triangles (pg. 4).
Closure (1 min): Teacher reviews the different trigonometric functions and the possible application of these functions.
Post-Assessment (5 min): Students will complete an applied problem (pg. 4).



TEACHER COPY

TREE-GONOMETRY?

Introduction to Right Triangle Trigonometry

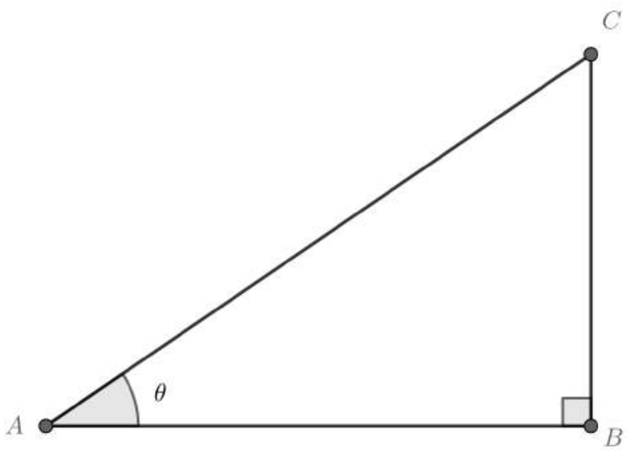
The giant sequoia, *Sequoiadendron giganteum,* is the world's largest tree and largest living thing by volume. With a height of 164–279 feet or more, a diameter of 20–30 feet, and an estimated lifespan of 1800–2700 years, the giant sequoia is among the tallest, widest and longest-lived of all organisms on Earth.

How can we make such height measurements without climbing these trees with a long tap measure? Well, we can actually use our knowledge of triangles to measure the height of tall structures while staying safely on the ground.

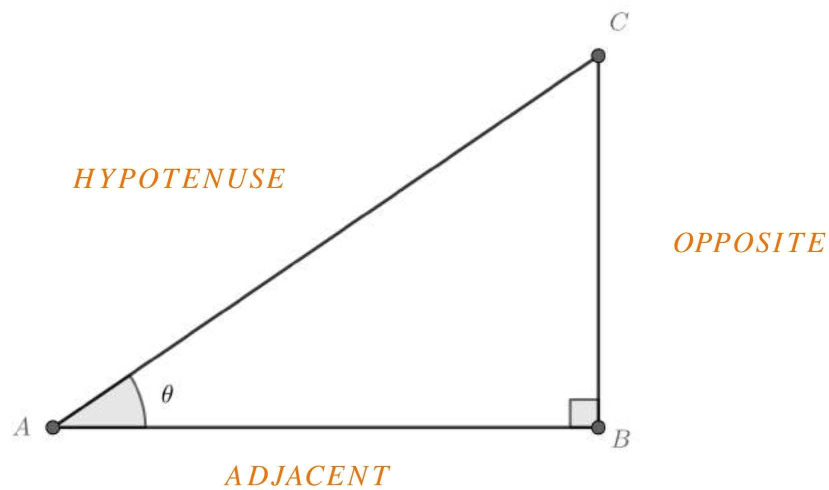
REALLY TRIANGLES? SAY HELLO TO TRIGONOMETRY

Trigonometry is a branch of mathematics that is all about triangles. In fact, "Trigonon" and "metron" are Greek for **"triangle"** and **"measure"**. So, **Trigonometry** focuses on the relationships between the **side lengths** and the **angle measures** in a triangle.

Special relationships abound when we focus on **right triangles**, for instance, $\triangle ABC$, with an angle θ at A and a $90°$ angle at B.



RATIOS OF SIDES – SINE, COSINE, AND TANGENT

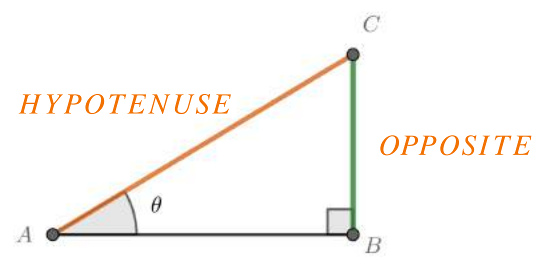


All of our trigonometric functions relate an angle to a ratio of corresponding side lengths.

SINE

Our first relationship, and most foundational, is called **sine**. The $\sin\theta$, "sine of theta", is a function of θ that is equal to the ratio of the *length* of the side opposite θ and the *length* hypotenuse.

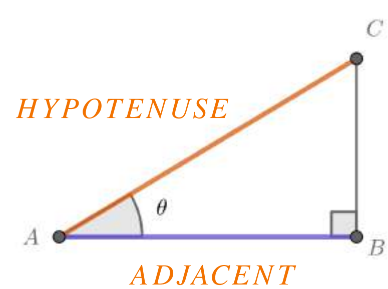
$$\sin\theta = \frac{OPPOSITE\ SIDE}{HYPOTENUSE} = \frac{BC}{AC}$$



COSINE

Our second relationship is called **cosine.** The $\cos\theta$, "cosine of theta", is a function of θ that is equal to the ratio of the *length* of the side adjacent (next to) θ and the *length* hypotenuse.

$$\cos\theta = \frac{ADJACENT\ SIDE}{HYPOTENUSE} = \frac{AB}{AC}$$

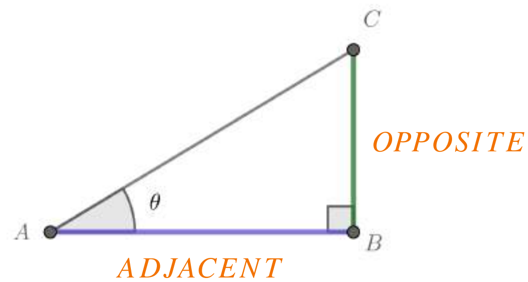


SOUND FAMILIAR? The co-sine is actually the sine of the complement of θ. *Think About!*

TANGENT

Our third relationship is called **tangent**. The $\tan\theta$, "tangent of theta", is a function of θ that is equal to the ratio of the *length* of the side opposite (next to) θ and the *length* of the side adjacent θ.

$$\tan\theta = \frac{OPPOSITE\ SIDE}{ADJACENT\ SIDE} = \frac{BC}{AB}$$



LOOK FAMILIAR? Tangent, $\tan\theta$, is actually the ratio of the $\sin\theta$ and $\cos\theta$. Check it out…

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{OPPOSITE\ SIDE}{HYPOTENUSE}}{\frac{ADJACENT\ SIDE}{HYPOTENUSE}} = \frac{OPPOSITE\ SIDE}{\cancel{HYPOTENUSE}} \cdot \frac{\cancel{HYPOTENUSE}}{ADJACENT\ SIDE} = \frac{OPPOSITE\ SIDE}{ADJACENT\ SIDE} = \tan\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

There are other trigonometric functions, but these three are the most foundational.

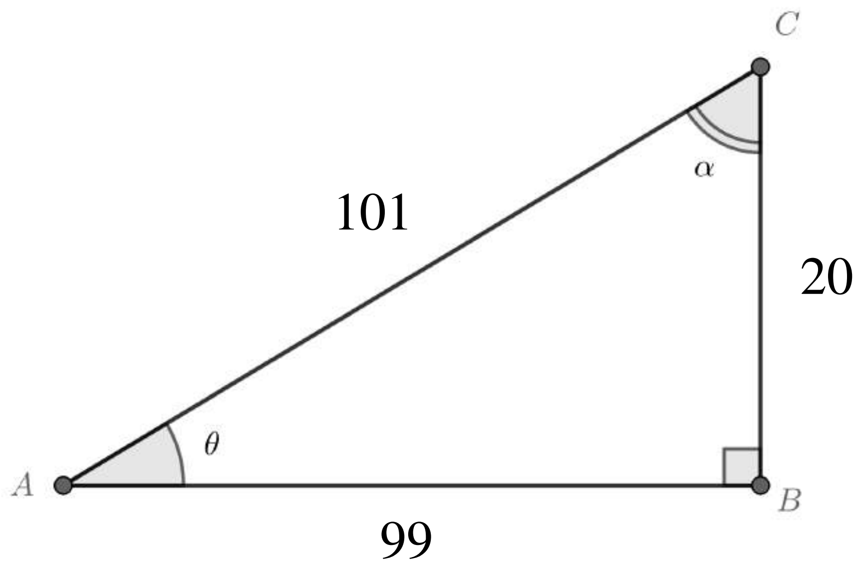
HOW CAN I REMEMBER ALL THIS?

This can seem daunting to remember at first. However, remember all these ratios stem from our right triangles. There is a mnemonic that can be used to remember these ratios, **SOH CAH TOA.**

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cosine} = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{Tangent} = \frac{\text{Opposite}}{\text{Adjacent}}$$

GETTIN' "TRIG"GY WITH IT

First, let's calculate all the different ratios for θ and α, given the side lengths of $\triangle ABC$.



$\sin\theta = \frac{20}{101}$

$\tan\alpha = \frac{99}{20}$

$\cos\alpha = \frac{20}{101}$

$\tan\theta = \frac{20}{99}$

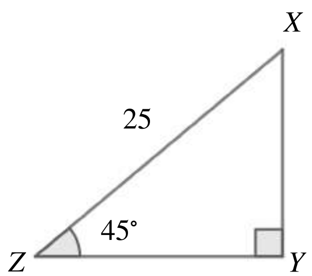
$\sin\alpha = \frac{99}{101}$

$\cos\theta = \frac{99}{101}$

THE POWER OF SINE, COSINE, & TANGENT

If we are given a measure of an angle and one side length, we can find all the other side lengths of a right triangle. Let calculate the missing side lengths for each of the following triangles:

CALCULATOR WARNING! Since we are using **degree** measure for our angles, make sure your calculator is in **Degree** mode. The other mode is **Radian** mode, we will talk about that soon enough!

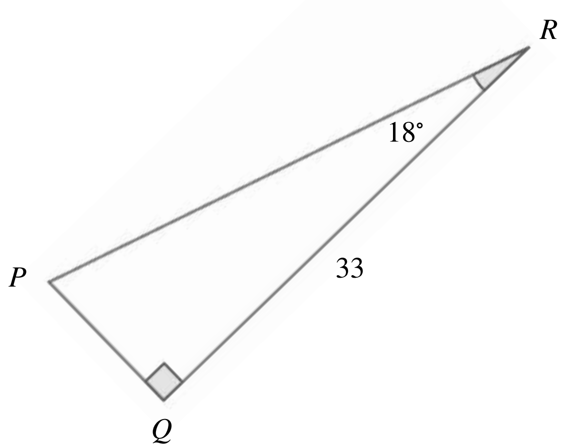


$$\sin(45^\circ) = \frac{XY}{25}$$

$$XY = \sin(45^\circ) \cdot 25 \approx 17.68$$

$$\cos(45^\circ) = \frac{ZY}{25}$$

$$ZY = \cos(45^\circ) \cdot 25 \approx 17.68$$

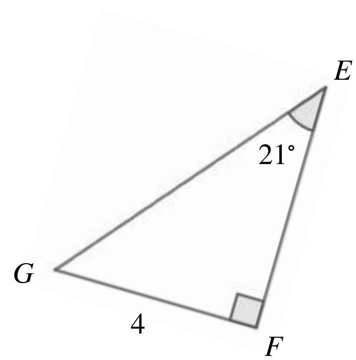


$$\cos(18^\circ) = \frac{33}{PR}$$

$$PR = \frac{33}{\cos(18^\circ)} \approx 34.7$$

$$\tan(18^\circ) = \frac{PQ}{33}$$

$$PQ = \tan(18^\circ) \cdot 33 \approx 10.72$$



$$\sin(21^\circ) = \frac{4}{EG}$$

$$EG = \frac{4}{\sin(21^\circ)} \approx 11.16$$

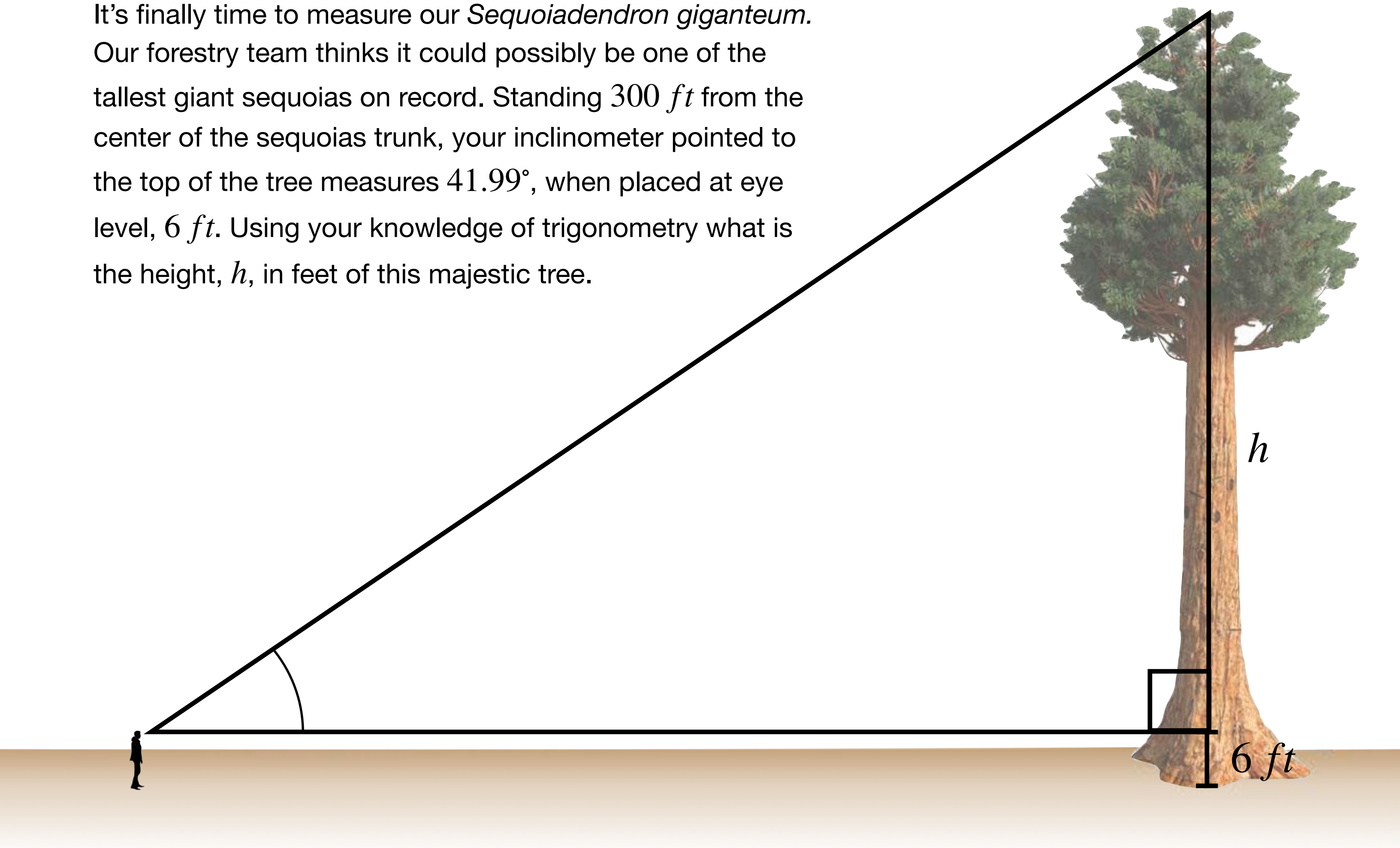
$$\tan(21^\circ) = \frac{4}{FE}$$

$$FE = \frac{4}{\tan(21^\circ)} \approx 10.42$$

DON'T TRUST THESE WEIRD FUNCTIONS? We know the Pythagorean Theorem holds for any right triangle. Check to see if your side lengths still satisfy the Pythagorean Theorem.

I'M A LUMBERJACK AND I'M OKAY!

It's finally time to measure our *Sequoiadendron giganteum.* Our forestry team thinks it could possibly be one of the tallest giant sequoias on record. Standing $300\ ft$ from the center of the sequoias trunk, your inclinometer pointed to the top of the tree measures $41.99°$, when placed at eye level, $6\ ft$. Using your knowledge of trigonometry what is the height, h, in feet of this majestic tree.



$$\tan(41.99°) = \frac{h}{300\ ft}$$

$$300\ ft \cdot \tan(41.99°) = h$$

$$h = 270\ ft$$

$$Total\ \ Height = 270\ ft + 6\ ft = 276\ ft$$

Projects/Labs

AMTNYS 2021 | Make It Take It
Brandon Caruso
bcaruso@fredonia.edu
Dominique Martello
dmartello@fredonia.edu

MISSION: **ROCKY ROVER**

Introduction (Link to Video: MissionBrief_MissionRockyRover.mp4):
In this simulated planetary rover mission, students collaborate to design, implement, and analyze data from Rocky - The Rover's first mission on Planet Z. Students will utilize math, earth science, computer science, and writing skills to navigate the landing zone, collect and analyze data, make inferences about Planet Z's past, plan new objectives, and communicate findings to others to complete the mission. This activity is well-suited for 10th Grade students enrolled in Geometry and Earth Science, though could possibly be extended to 9th and 11th graders. This should take 80 minutes.

NYS Next Generation Mathematics, Science, ELA Standards:

Math Standards:

GEO-G.SRT.8 Use sine, cosine, tangent, the Pythagorean Theorem and properties of special right triangles to solve right triangles in applied problems.

GEO-G.MG.3. Apply geometric methods to solve design problems.

Science Standards:

HS-ESS2-5. Plan and conduct an investigation of the properties of water and its effects on Earth materials and surface processes.

HS-ESS1-6. Apply scientific reasoning and evidence from ancient Earth materials, meteorites, and other planetary surfaces to construct an account of Earth's formation and early history.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

HS-ETS1-3. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.

ELA Standards:

9-10SL4: Present claims, findings, and supporting evidence clearly, concisely, and logically; organization, development, substance, and style are appropriate to task, purpose, and audience.

Objectives:

At the conclusion of this activity, students will be able to:

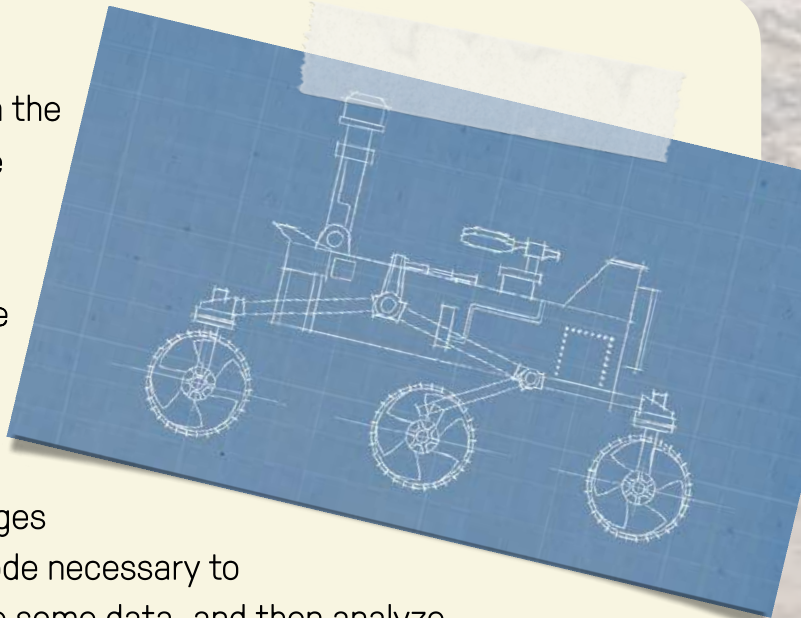
- Use trigonometric ratios and functions, the distance formula, and properties of right triangles to solve problems using a coordinate grid.
- Read topographic maps and make informed decisions about navigational pathways.
- Analyze geologic features in the environment to make inferences on past geologic processes.
- Design solutions to real-world problem by breaking them down into smaller parts and evaluate those solutions.
- Communicate scientific findings to others in a logical, engaging, and accurate way.

Materials: Teacher and Student Copies of Mission File

Instructional Protocol:

Break students up into small groups of two to four. Give each team member a copy of the Mission File. Students will turn in one final completed Mission File as a team. It may be helpful to assign roles to students in the group. Students can take lead at different portions of the activity, though all students should be contributing to all aspects of the mission file. Suggested roles might be; mathematician, geologist, programmer, and scientific writer/social media manager. When each group is done, reflect as a class, have teams share their code, social media post, and findings as a group and have students perform a code review to make sure each solution works.

MISSION BRIEF



Success! Rocky, our newest rover, has landed on the surface of Planet Z. This is the first time we have landed a rover on this planet. Our science teams want to look at geological characteristics of the unknown planet and find some potential evidence of water. These teams have identified three locations of interest that they would like Rocky to explore at its first landing zone. They would like to capture some geological samples and images of the environment. Our goal is to develop the code necessary to navigate the rover to different locations, capture some data, and then analyze the data and images we get back from Rocky. **Was there ever water on Planet Z?**

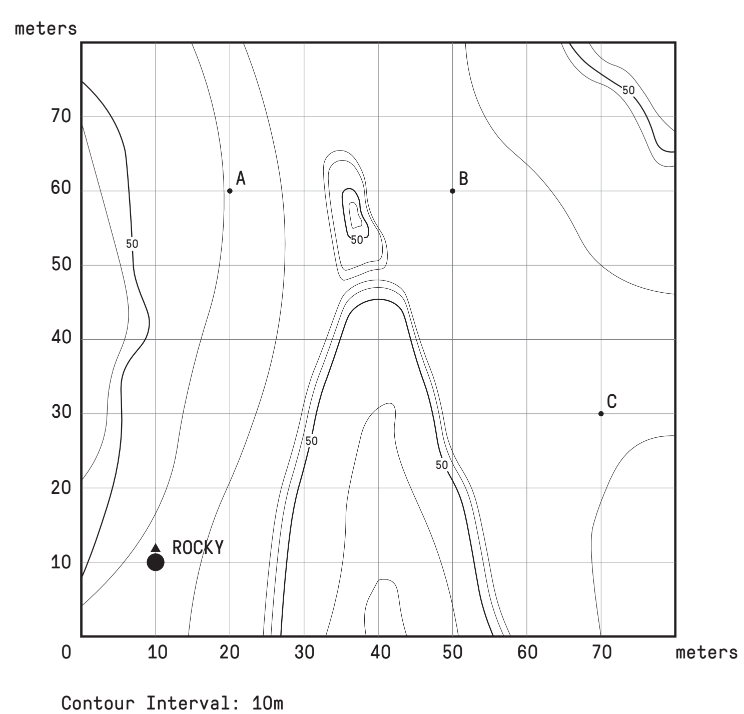
The Landing Zone

Using height data collected by MAPO, our satellite orbiting Planet Z, we have been able to construct a topographical map of the landing zone. We have also overlaid a standard Cartesian coordinate grid to help us safely traverse the landing zone. The scale is in meters. A large map is included later in the mission file.

What topographical features do you notice in the landing zone? Are there any locations we want Rocky to avoid?

ANSWERS WILL VARY

Students should mention there are steep slopes surrounding Rocky. There is no direct route between A and B. The area between Rocky and point A, as well as, points B and C are relatively flat and safe to navigate. The area above points A and B is also safe to navigate.



OBJECTIVE 1

Develop the code to navigate the landing zone. Rocky must go to locations **A**, **B,** and **C** and run a routine at each location to capture some data. Rocky does **not** need to return to starting location.

Specification

Rocky can only travel 140m for this objective before having to recharge. Rocky's navigation system comes with two commands that we can use to maneuver the rover on the surface.

```
turn( deg )        move( m )
```

`turn()` is a function that lets us rotate Rocky a certain number of **degrees clockwise**. That number is a parameter we place in between the parentheses of the `turn()` function. `move()` is a function that lets us move Rocky **forward** a certain number of **meters**. That number is also a parameter you place in between the parentheses of the `move()` function. Round the numbers for your parameters to the hundredths place.

Important! The `move()` function has Rocky return to the initial heading and reorient. Therefore, Rocky will point in the positive `y`-direction after the rover moves forward the specified distance. Consider this when determining the angle measure you have to turn Rocky.

Rocky's initial location and heading is indicate with the large dot and triangle on the map. Please avoid dangerous terrain with steep slopes and travel in straight lines.

Once you get to locations **A**, **B**, and **C**, run the following routines to capture data for the team to analyze.

A - `captureEnvironmentImages()`
B - `captureEnvironmentImages()`
C - `captureChemicalAnalysis()`

Lastly, our pre-planning team came up with an initial navigation plan but it was rejected by our testing team. Take a look at the plan and see where it can be improved. **We can do better than this!**

The next few pages will help you plan your code.

PRE-PLAN DOC: A rejected navigation plan developed by our initial pre-planning team.
LANDING ZONE: A larger map of the landing zone.
PLANNING DOC: A place to do your math work and planning.
MISSION CODE: A place to write your code to move to the different locations and capture the data.

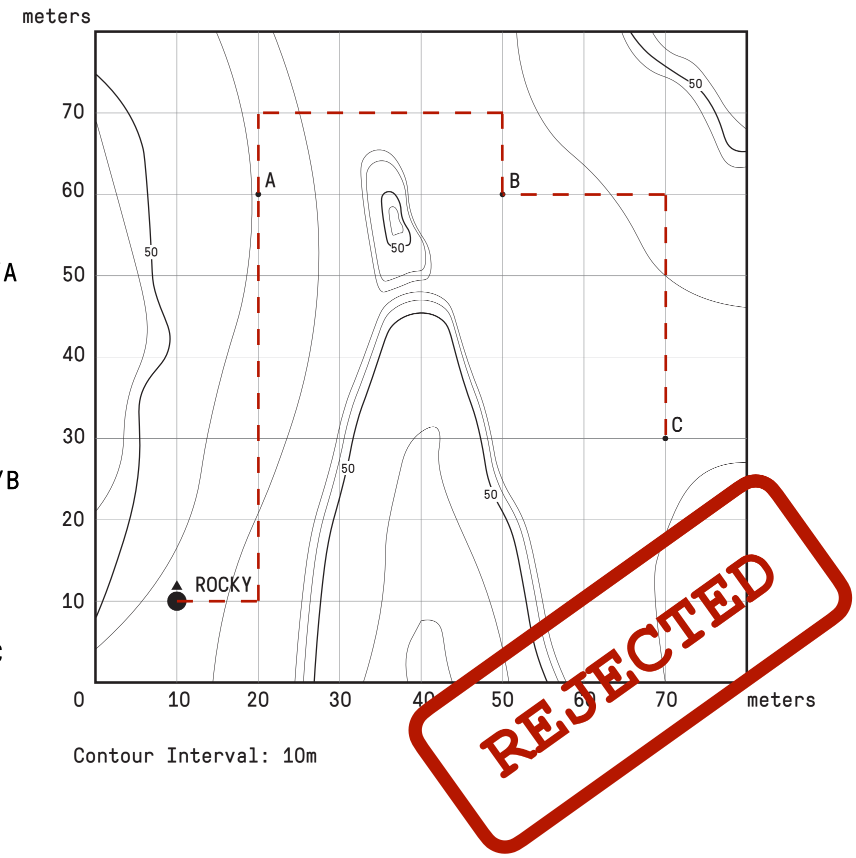
MISSION: ROCKY ROVER
TEACHER VERSION

PRE-PLAN DOC

MISSION CODE:

```
//REMINDER:
//Rocky REORIENTS after moving
//forward in move()

turn(90)
move(10)
move(50)
captureEnvironmentImages() //A
move(10)
turn(90)
move(30)
turn(180)
move(10)
captureEnvironmentImages() //B
turn(90)
move(20)
turn(180)
move(30)
captureChemicalAnalysis() //C
```



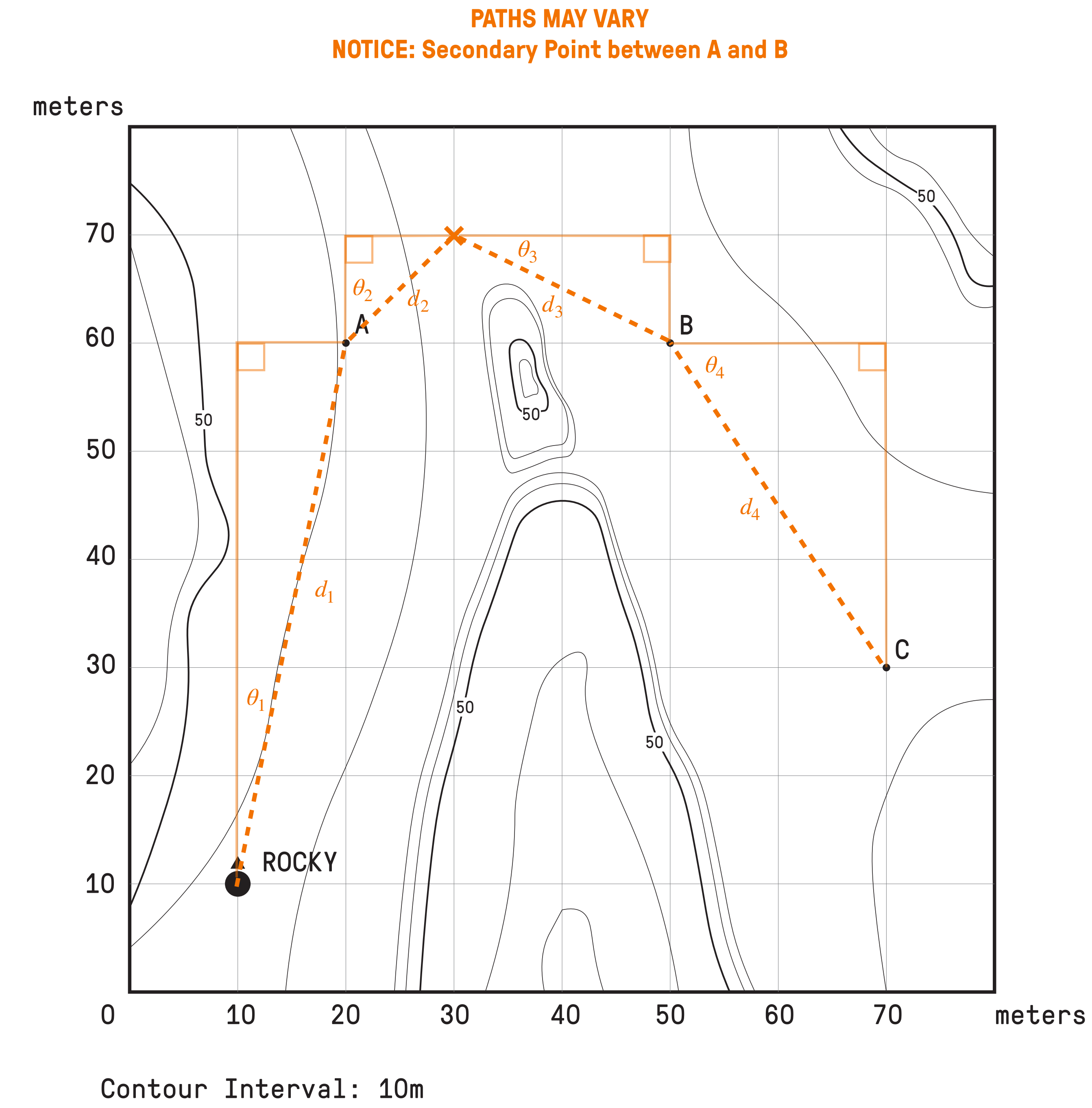
Why was this plan rejected?

This path's total distances is 160m, which exceeds the total distance allowed for the mission.

Student's might also mention diagonal distance is more efficient, could be less lines of code, etc, but they must include the failure to meet the constraint.

LANDING ZONE

It might help to draw Rocky's path on the map.



MISSION: ROCKY ROVER
TEACHER VERSION

PLANNING DOC

INITIAL LOCATION TO A

INITIAL LOCATION : $(10,10)$
A : $(20,60)$

`turn(11.31)`

`move(50.99)`

Distance:

$$d_1 = \sqrt{(20-10)^2 + (60-10)^2} \approx 50.99\ m$$

Angle:

$$\theta_1 = \tan^{-1}\left(\frac{10}{50}\right) \approx 11.31^\circ$$

A TO B

MAY VARY BASED ON PATH

A : $(20,60)$
X : $(30,70)$

`turn(45)`

`move(14.14)`

Distance:

$$d_2 = \sqrt{(30-20)^2 + (70-60)^2} \approx 14.14\ m$$

Angle:

$$\theta_2 = \tan^{-1}\left(\frac{10}{10}\right) = 45^\circ$$

X : $(30,70)$
B : $(50,60)$

`turn(116.57)`

`move(22.36)`

Distance:

$$d_3 = \sqrt{(50-30)^2 + (60-70)^2} \approx 22.36\ m$$

Angle:

$$\theta_3 = \tan^{-1}\left(\frac{10}{20}\right) \approx 26.57^\circ$$

ROCKY REORIENTS
Add 90° for turn()

B TO C

B : $(50,60)$
C : $(70,30)$

`turn(146.31)`

`move(36.06)`

Distance:

$$d_4 = \sqrt{(70-50)^2 + (60-30)^2} \approx 36.06\ m$$

Angle:

$$\theta_4 = \tan^{-1}\left(\frac{30}{20}\right) \approx 56.31^\circ$$

ROCKY REORIENTS
Add 90° for turn()

TOTAL DISTANCE TRAVELED $d_{total} = 123.55\ m$ **MUST BE LESS THAN 140m**

MISSION: ROCKY ROVER
TEACHER VERSION

MISSION CODE

Write your code to move to the different locations and capture the data below (one function per line and all lines might not be used.). CODE WILL VARY, BUT MUST RESULT IN PLANNED PATH

```
turn(11.31)
move(50.99)
captureEnvironmentImages()
turn(45)
move(14.14)
turn(116.57)
move(22.36)
captureEnvironmentImages()
turn(146.31)
move(36.06)
captureChemicalAnalysis()
```

OBJECTIVE 2

Excellent work! The routine you developed worked great and Rocky sent back some amazing data. Let's analyze some of the images and chemical analysis data we got back. The science team is extremely excited with what we can determine about the past of Planet Z.

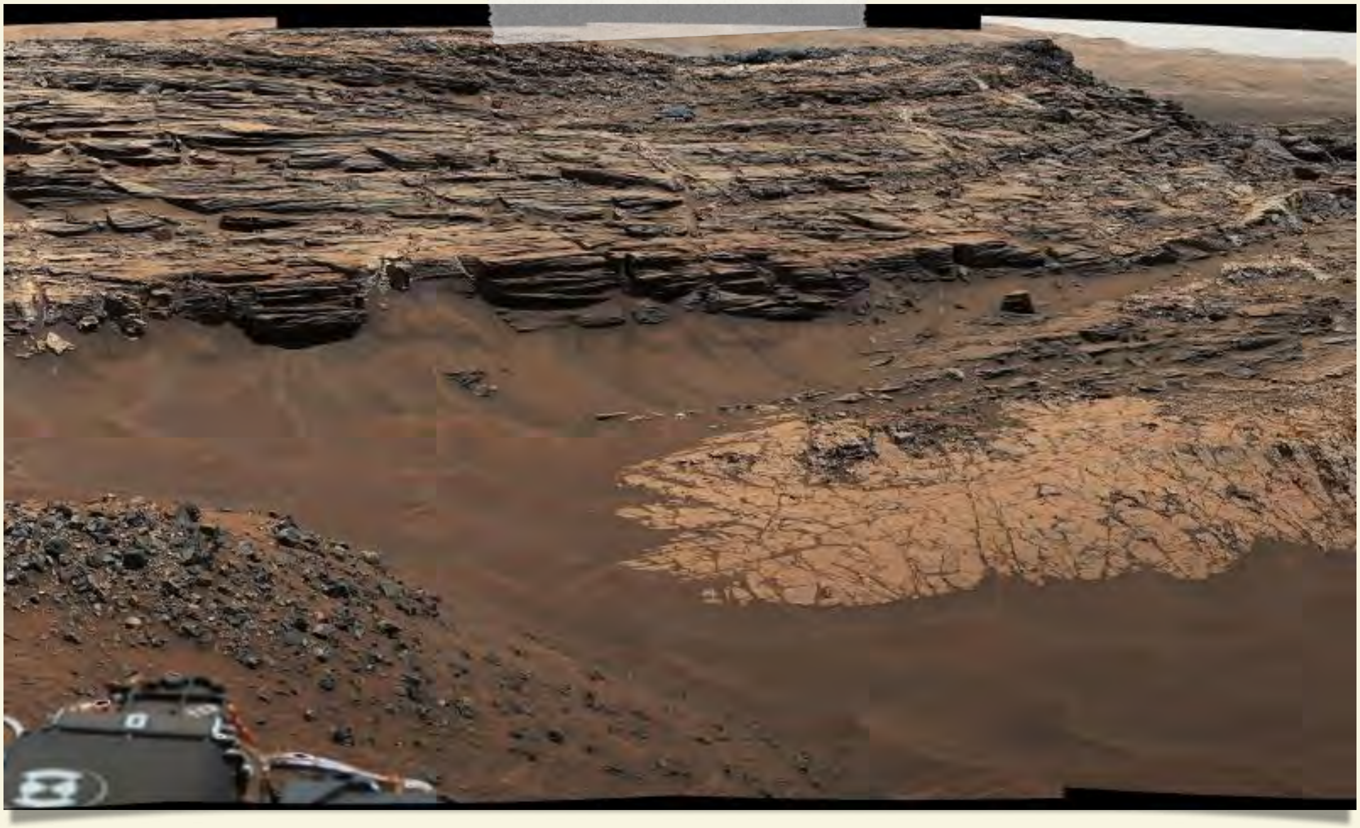
LOCATION A:

At **Location A** we got back the above image. **Is this rock sedimentary, igneous, or metamorphic? How can you tell? What does that tell you about this region of Planet Z?**

This rock is igneous. You can tell this rock is igneous because of its dark color and fine texture. It lacks the foliation of metamorphic rocks and it lacks the rounded grains and bedding of sedimentary rocks. This region of Planet Z had volcanic activity some time in its past.

OBJECTIVE 2

LOCATION B:

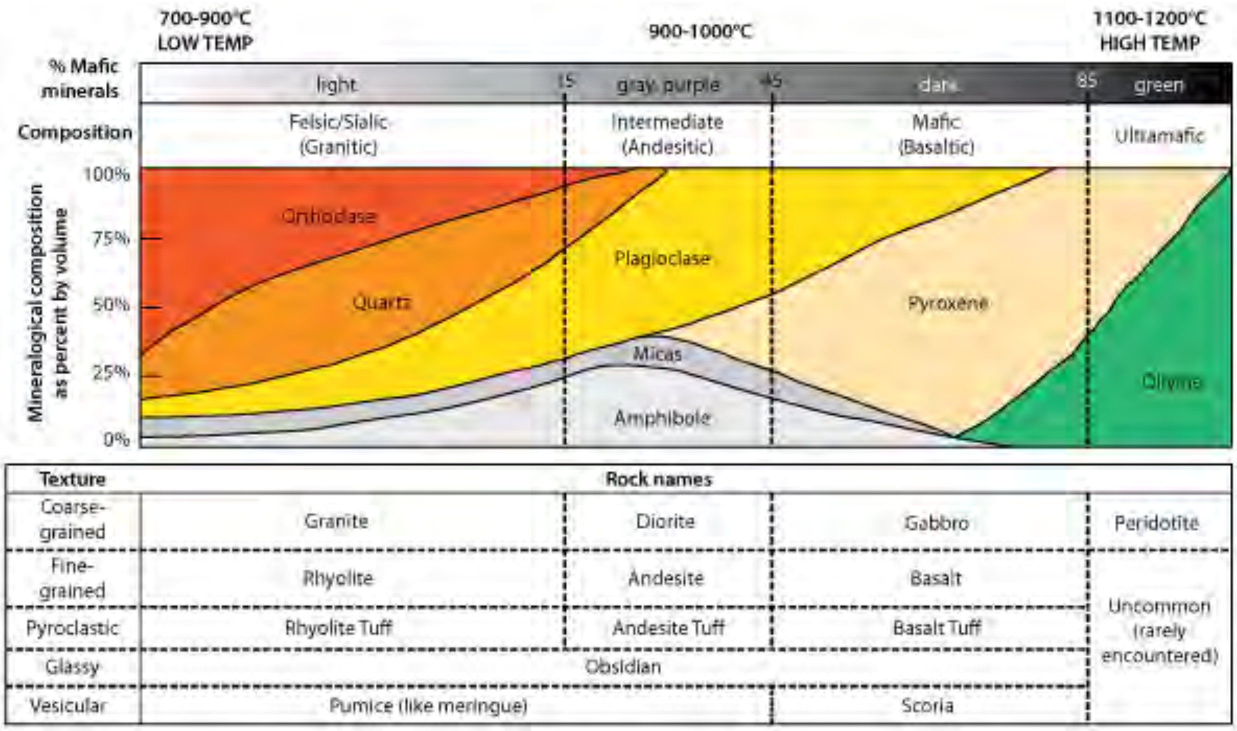


At **Location B** we got back the above image. Take note of the sedimentary structures of the rock face. **What do you see? What kind of rock do you think is shown? What does that tell you about the depositional environment?**

There are layers in the rock. The rock is shale/mudstone. The depositional environment must have been a calm body of water.

OBJECTIVE 2

LOCATION C:



At **Location C** we performed a chemical analysis of one of the rocks found on Planet Z. We found that the rock contains `~60% Pyroxene`, `~25% Olivine`, and `~15% Plagioclase`. **If the rock is fine grained, what is the name of the rock?** Refer to the chart above.

Basalt

DEBRIEF

NEXT STEPS

Provide two new mission objectives. What questions remain? Are there any other locations in the landing zone that you would like to explore? Provide specific points and also explain why.

ANSWERS WILL VARY

The students should mention something about the planet's water.
Ex) If there was water on this planet in the past, where is it today?

Students might also mention locating volcanic activity.

We are looking for students to select objectives that build on what was found in activity.

SOCIAL MEDIA POST

Summarize your findings into a short social media post to share with the public. What interesting science did you uncover? What can this tell us about Planet Z? What is next for the mission? Include an intriguing first line and the image you would like to include.

ANSWERS WILL VARY

Students should craft a casual, concise, engaging, yet scientifically accurate post. They should provide scientific reasons for selecting the image that accompanies the post.

LEGO Master Builders

Exploring LEGO Set Price Estimation Through Linear Regression

You and your partner have just been given a chance to interview for a LEGO Designer job. You have been tasked with creating a new LEGO set. There's one new requirement, you must include predicted price in your proposal. LEGO will not give you the formula because it's confidential and for employees only, but that doesn't stop you! By using data from LEGO.com and your math knowledge, determine the price of any set you could possibly imagine. **The title, Master Builder, is on the line!**

Image (right): Nathan Sawaya, Yellow, Lego Sculpture, https://www.brickartist.com

Let's Get Started

1. Collect a diverse sample of LEGO set data. Using the LEGO website, www.lego.com, collect a sample of **15** different LEGO sets of different prices (in **US $**) and the number of pieces for that set. Create or use a table to store your data. Take a look at sets in the **City** and **Creator Expert Line** themes.

2. Generate a scatter plot and a line of best fit, using your graphing calculator. Include your scatter plot and line of best fit graphed together and the linear regression results you received using your calculator.

3. What is the equation for your line of best fit? Use variable and function names that make sense. x and y, what do those have to do with LEGO?

4. That's nice. What does it mean? Explain the relationship between number of pieces and price. Explain what the **slope** and **y-intercept** mean using everyday language, not technical math descriptions.

5. Put your equation to work. Let's make a predication.

Nathan Sawaya is a contemporary artist that uses LEGO bricks to make jaw dropping and inspiring LEGO sculptures. One of his creations is called **T-Rex,** a 20 ft LEGO dinosaur that uses 80,000 LEGO bricks. **If LEGO were to sell this sculpture as a set, how much would it cost using your equation?**

Image and Data Source: https://www.dailyartmagazine.com/nathan-sawayas-lego/

Let's create!

6. Together with your partner, make a small LEGO creation **(no more that 60 pieces)**. Once you are done, you will create the artwork for the front of your LEGO set box. **On the artwork include:** a cool name for your set, a picture of your set, the number of pieces, and the price you found using your equation. Include your calculations for price and the max possible price your set could be given the piece limit, on the back. Discuss any limitations with this equation or other observations you might have about this linear model.

DIG DEEPER Does LEGO sell individual pieces? Is your price per piece reasonable? What are some pieces you can actually buy at your price per piece? Why aren't the pieces all the same price like your model implies?

Image (right): LEGO, Corner Garage, https://www.lego.com/en-us/product/corner-garage-10264

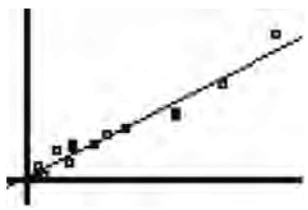
LEGO Master Builders **Teacher Solution Guide**

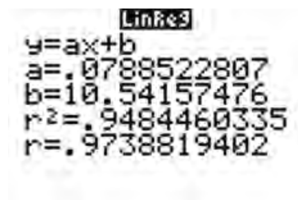
Exploring LEGO Set Price Estimation Through Linear Regression

Num. of Pieces (x)	Price $ (y)
118	$9.99
212	$39.99
244	$29.99
328	$19.99
533	$79.99
756	$49.99
790	$99.99
806	$89.99
1167	$99.99
1334	$119.99
1686	$139.99
2480	$169.99
2504	$179.99
3231	$249.99
4124	$379.99

1.Table of Values. Students select a diverse sample, have pieces as the independent variable (x) and price in US $ as the dependent variable (y). Sample table/data provided.

2. Graph and Line of Best Fit. Students provide screenshot of their scatter plot fully in view, they include line of best fine graph and read out for the LinReg command. Graph is linear. If not, they use the appropriate line of best fit. Sample:





3. What is the equation for your line of best fit? Students use descriptive variable names or provide a key for letter variables and the equation matches the line of best fit. Sample:

$$\text{price} = .08(\text{pieces}) + 10.54$$

4. Explain Relationship Students use everyday language to describe relationship (positive linear), slope (price per piece), and y-intercept (base price w/ no pieces). Sample: As the number of pieces in a LEGO set increases, so does its price. In fact, for every piece added to a set the price increase by ~ 8 cents. Additionally, according to the y-intercept, you can still charge $10.54 for an empty set.

5. Make some predications. Students show substituting in 80,000 pieces in their formula and calculate the price. Sample: $\text{price} = .08(80{,}000) + 10.54 \quad \text{price} = \$6{,}410.54$

6. Make some predications. Students make a creation no more the 60 pieces, their artwork includes set name, picture, piece count, price. On back, students calculate their cost using the formula and number of pieces. Students calculate max price using their formula and 60 pieces. Students discuss limitations of equation and data. Students show creativity and quality in artwork. Sample:

$$\text{price}_{\text{set}} = .08(45) + 10.54,\ \text{price}_{\text{set}} = \$14.14 \qquad \text{price}_{\max} = .08(60) + 10.54,\ \text{price}_{\max} = \$15.34$$

It looks like it might over estimates price for smaller piece counts and under estimates price for larger piece counts. Looking at the date, price is rounded to a nice retail price indicating pricing may be more categorical.

DIG DEEPER Students identify some bricks they can buy for their price, provide reasonable explanation for the differences in prices, and explain why their price is reasonable. Sample: Yes, individual pieces can be bought for $0.05 to $0.86, with a majority in the lower price range. Our price of $0.08 would seem to be reasonable. You can buy a round flat tile and a LEGO gold ingot for $0.08. Price might vary because of material used or mold complexity needed to create the piece.

“I Scream, You Scream, We All Scream for Ice Cream”

Exploring Ice Cream Production in the United States Through Sinusoidal Regression

Americans are crazy about ice cream. Whether at Boxcar Barney’s, The Shack, Big Dipper, Big Tree, Bemus Point Market, Frosty Treat, or Farmer’s Daughter, you know that on a nice summer day in Chautauqua County people flock to treat their taste buds to a cool, creamy, and sweet treat! To keep up with demand, New York State companies like Fieldbrook Foods or Perry’s Ice Cream, and others around the US, are working year-round to produce this delicious dessert, but how does this production change from month to month? Well, you’re about to find out.

Let’s Get Started

1. Collect ice cream production data from 2020. Using this dataset form United State Department of Agriculture’s (USDA) website:

https://www.ers.usda.gov/webdocs/DataFiles/48685/Dairyglance.xlsx?v=8214.7

Open the file using Excel or similar software package. Collect the production of “Frozen Products” for each month of the year in 2020 and organize the data into a table.

2. Using your graphing calculator, generate a scatter plot and a regression curve for this data. Include your scatter plot and curve of best fit graphed together and the sinusoidal regression results you received using your calculator.

3. Sinu-what? Wait a second! Why aren’t we using a quadratic equation to model ice cream production? It looks close, right?

4. What is the equation for your curve of best fit? Use variable and function names that make sense. x and y, what do those have to do with ice cream?

5. Hungry Yet? First, what does it mean? Explain the relationship between month and ice cream production in millions of gallons. Explain what the frequency tell us, how about the amplitude and vertical shift? When is the rate of production increasing the fastest? How can you tell?

6. Keep'em Honest The International Dairy Foods Association said the following, "[In the US,] Most ice cream is made March through July. July is the busiest production month for ice cream makers." Can your model be used to justify this claim? Source: https://www.idfa.org/news-views/media-kits/ice-cream/ice-cream-sales-trends

Photo by Anthony Fomin on Unsplash

7. Put your equation to work. Sunday, July 19, 2020 was National Ice Cream Day. While Americans around the country consumed some frozen dairy goodness, using your equation, how much ice cream was produced in the US on July 19, 2020?

THINK A BIT

8. Perry's Ice Cream executives have contacted you after finding out you have done some modeling of ice cream production in the US. They want to start producing ice cream in Australia for a purely Australian market. They ask you to give them some insights into what production might look like in Australia, compared to production in the US. They are confident Australians will demand ice cream the same as Americans, they just need to tap into the market. Develop a sketch of a production graph and short description for the Perry's Team.

WHERE DID THAT COME FROM? We have all said it, **"I Scream, You Scream, We All Scream for Ice Cream."** Where did this common ice cream idiom come from?

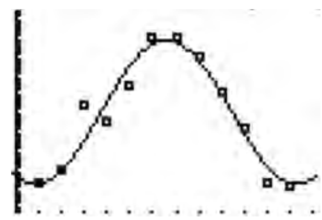
“I Scream, You Scream, We All Scream for Ice Cream” Teacher Solution Guide

Exploring Ice Cream Production in the United States Through Sinusoidal Regression

1. **Table of Values.** Students include all data from 2020, they have month (1-12) as the independent variable (x) and ice cream in millions of gallons as the dependent variable (y). Sample table/data provided.

Month (x)	1	2	3	4	5	6	7	8	9	10	11	12
Production, mill. gall. (y)	85.6	88.8	104.9	101.3	110.4	122.6	122.4	117.2	108.3	99.5	85.3	84.3

2. **Graph and Curve of Best Fit.** Students provide screenshot of their scatter plot fully in view. They include curve of best fit graph and read out for the SinReg command. Graph is sinusoidal. Sample:



3. **Sinusoidal Explanation:** Students explain that the data shows a cyclic production pattern, making a trigonometric model better than a quadratic model.

4. **What is the equation for your line of best fit?** Students use descriptive variable names or provide a key for letter variables and the equation matches the curve of best fit. Sample: $\text{production} = 18.42 \cdot \sin(.55(\text{month}) - 2.05) + 103.42$

5. **Explain Relationship** Students use everyday language to describe the relationship as cyclic, production increases in spring and early summer months and decreases in winter. From the frequency and period, the production cycles around 12 months, and amplitude and vertical shift show max and min production. Rate of production is most around March/April. They discuss idea of derivative but don’t need to explicitly state it. Sample:

$\text{period} = 2\pi/.55 = 11.42 \text{ months} \quad \max/\min = 103.42 \pm 18.42 \text{ mill. gal.}$

Ice Cream production roughly follows the cycle of the seasons, where production is lowest in winter months (December, January) and highest in spring and summer months (May, July). The period of 11.42 months, suggests this is roughly yearly. The max production, in mid-June, is around 121.8 mil. gal. and the minimum production, in December, is around 85 mil. gal.. Late March, early April seems to be the months where the production is increasing the fastest, because the curve is the steepest at this point.

6. **Keep’em Honest** Students should confirm that their model would reasonably justify this claim, but state that their model does show, however, the production seemed to peak just slightly earlier in mid-June of 2020.

7. **Put your equation to work.** Students convert the date 7/19 into a decimal and then substitute into their equation. Sample:

Convert 7/19: $7 + 19/31 = 7.61$ Substitute: $\text{production} = 18.42 \cdot \sin(.55(7.61) - 2.05) + 103.42 = 119 \text{ mil. gal.}$

8. **THINK A BIT** Students observe that their data might show a relationship between production and seasonal change. Students will then consider the seasonal patterns in Australia to be opposite those in the United States. Their report would suggest an estimated production cycle that is flipped across the median line. They draw a reasonable graph.

9. **WHERE DID THAT COME FROM?** Students cite the late 1920s novelty song “Ice Cream (I Scream, You Scream, We All Scream for Ice Cream)” with words and music by Howard Johnson, Billy Moll, and Robert A. King

Math & Science Night Activity

GET DOWN & BOOGIE

Choreograph the next dance craze

Students group up (3-4 students) to simplify expressions or solve single-step and two-step equations of various levels to acquire points to unlock dance moves. They have 4 minutes to complete as many as they can. You can find example problems on the next page. Students then have 1 minute to use their points to select their dance moves.

Dance Moves

Clap	1 point	**The Robot**	5 points
Jump	1 point	**The Floss**	5 points
Slide	1 point	**Whip & Nae Nae**	5 points
Twist	1 point	**Moonwalk**	5 points
Kick	1 point	**Freestyle**	10 points

It's time to choreograph...

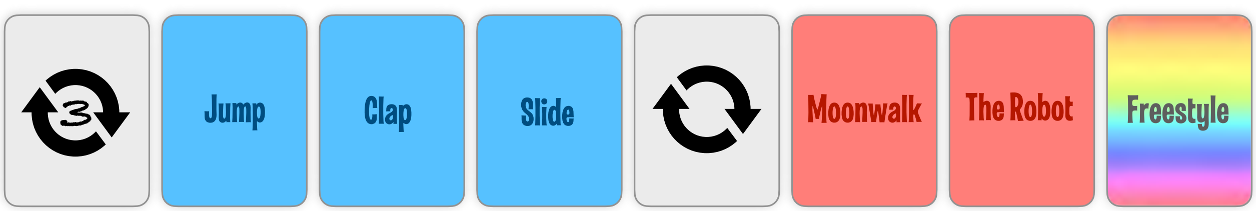
Before choreographing, students must select a dancer who cannot help create the dance (bonus points if Mom or Dad jump in and strut their stuff). Within 4 minutes, students will tape their dance move cards onto a choreography sheet to create a sequence. Once they have used up all their cards the dance is done.

Let's get loopy

Using what's called a loop in computer programming, students can repeat a sequence of dance moves a specific number of times. To indicate a loop on their choreography sheet, they can use loop cards at the start and end of the sequence, indicating on the first card the number of times they will loop those dance moves.

Time to Dance

Teams give the choreography sheet to their designated dancer and they will have 1 minute to practice. The choreographers cannot correct the dancer. Every dancer will perform their dance at the end of the session to a Top 40 hit. The judges decide the best dance at the end of the night for the coveted Boogie Trophy.



Example Dance Sequence

Earn Points for Moves

Simplify the following expressions or solve the following equations to earn points to earn moves for your dance.

Level 1 1 Point Each

$3(4) + 3$	$(7 \div 7) + 4$	$x + 1 = 5$
$9 \times 63 \times 0$	$4x = 12$	$3 + 9 \div 3$

Level 2 5 Points Each

$5(3 + z) = 15$	$10(9 \times 10)$	$4x + 2 = 10$
$2p + 1 = 15$	$2x + x$	$3y - 1 = 8$

Level 3 10 Points Each

3^2	$2z - \frac{1}{2} = \frac{1}{2}$	$9(3(4 \div 2) + 4)$
$9 + 3 \times 6 - 4$	96×10^5	$7(m + 1) = 56$

Answers are left to right

Level 1: $15, 5, x = 4, 0, x = 3, 6$

Level 2: $z = 0, 900, x = 2, p = 7, 3x, y = 3$

Level 3: $9, z = \frac{1}{2}, 90, 23, 9{,}600{,}000, m = 7$

MYSTERY PIXEL ART

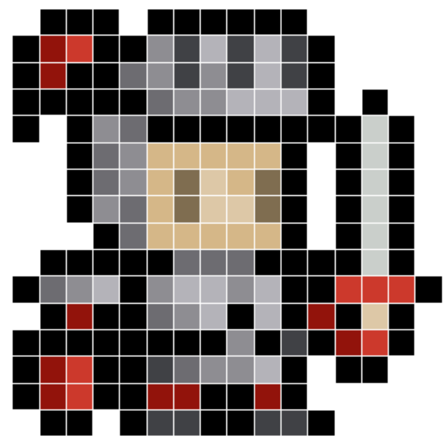
Brandon Caruso
bcaruso@fredonia.edu

Alyssa Wright
wrig3786@fredonia.edu

Ashley Mills
ammills@fredonia.edu

OVERVIEW

A collection of mystery pixelated masterpieces are on their way to Eden Central School. It will be up to you and your team to decode these pieces of mystery artwork.



WHAT WILL YOU RECEIVE?

- A Four Quadrant Cartesian Grid (16 x 16)
- A Set of Secret Decoding Cards
- 8 Crayons - Brown, Red, Orange, Yellow, Green, Blue, Purple, Black

CHALLENGE | LET'S GET PIXELATED

To begin, you and your team will watch the short introduction video. Then, from the set of decoding cards, each member of your team will receive at least one secret decoding card. Your team must complete all the decoding cards to unveil the complete art piece. Your team has 15 minutes to decode the mystery pixel art. Once the team has completed decoding the artwork, go off grid and make it your own.

DECODING CARD | WHAT WILL BE ON A DECODING CARD?

There will be a color key that matches a number to a colored crayon. You will then see a collection of coordinates and secret expressions. You will evaluate the secret expressions to tell you the number of the color that you need to plot at a given coordinate (i.e. $(2, 3)$). There may be more than one coordinate associated with an expression. These expressions can evaluate to both positive and negative integers.

FINAL SUBMISSION | WE NEED PROOF

After you have finished decoding and adding your own touch, send us a short uncut video of you and your team working on decoding the artwork, plotting the pixels, and, finally, showcasing your finished pixelated masterpiece.

LET'S GET DECODING

MYSTERY PIXEL ART

It will be up to you and your team to decode a pixelated masterpiece. To get ready for the challenge, here is a practice decoding set for some random pixels. This is not a piece of art, it's just practice. Evaluate the expression to get the number of the color of the pixel that should be plotted at the given x and y coordinates.

COLOR KEY							
	1	0	-5	3	-3	4	10

Evaluate each expression to identify the color pixel at the given position,(x, y):

1. $(3, -4)$
   $8 \times 2 \div 4$

2. $(1, 2), (4, 4)$
   $9(2) \div 6$

3. $(-3, 3)$
   $(5 + 6) \div (2 + 9$

4. $(-2, -4)$
   $-3 \times 3 + 6$

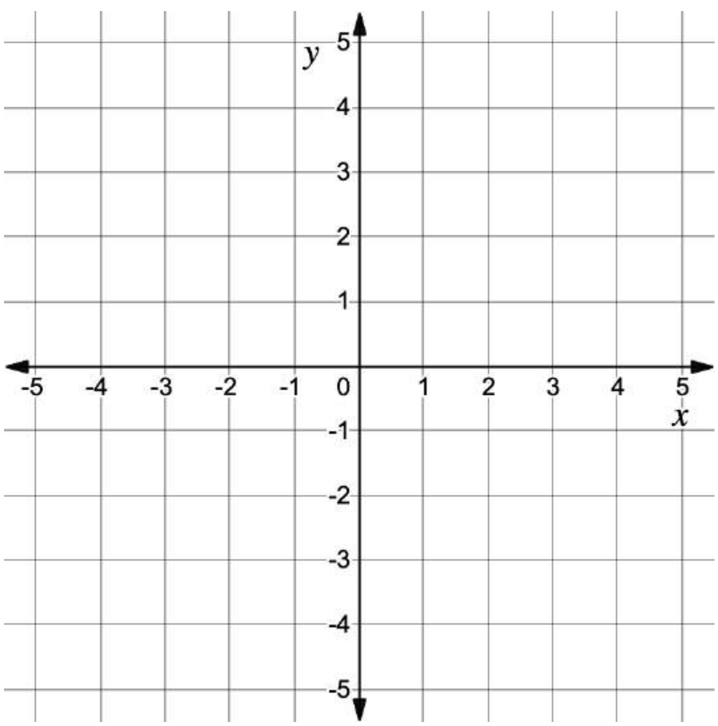
5. $(0, 0)$
   $4 \times 4 - \frac{18}{3}$

6. $(-1, -2)$
   $25 \div (-5)$

7. $(2, 3), (-5, 4)$
   $36 \times 2 - 77$

8. $(3, 2)$
   $(347/580) \times 0$

Using crayons, plot your pixels (or plot little squares) on this Cartesian grid:



Answers: (1) 4 (2) 3 (3) 1 (4) -3 (5) 10 (6) -5 (7) -5 (8) 0
Hey, You! No Cheating!

Teaching with Technology

AMTNYS 2021 | Make It Take It
Brandon Caruso
bcaruso@fredonia.edu

All Systems Solved

Exploring Basic Systems of Equations

Introduction (Link to Introduction Video: AllSystemsSolved_Introduction.mp4):
Through this WebQuest, students will explore solving basic systems of linear equations of two variables. Students will find the solution, if one exists, for two linear equations using a variety of methods, graphically and algebraically (substitution and elimination). Students will identify what systems of linear equations are, when they can be used, review linear equations, and explore what methods are available to solve these systems of equations. Students will explore this topic using a variety of online resources, activities, and graphing tools. They will complete their exploration through an application problem related to US track athlete Dave Wottle's 1972 Summer Olympics 800m dash comeback. This exploration is recommended for Grade 8 students and can serve as a review for Algebra I students.

NYS Next Generation Mathematics Standards
NY-8.EE.8 Analyze and solve pairs of simultaneous linear equations.
NY-8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Recognize when the system has one solution, no solution, or infinitely many solutions.
NY-8.EE.8.b Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection.
NY-8.EE.8.c Solve real-world and mathematical problems involving systems of two linear equations in two variables with integer coefficients.
AI-A.REI.6a Solve systems of linear equations in two variables both algebraically and graphically.

Objectives
At the end of the lesson student should be able to say,
- I can describe, identify, and create a system of linear equations.
- I can solve a system of linear equations using graphically methods.
- I can solve a system of linear equations algebraically, using both substitution and elimination.
- I can apply system of linear equations to real world problems.

Materials
All Systems Solved - Student Copy Print Out
Tablet with Camera or Computer with access to the Internet

Instructional Protocol
Have students work with a partner to complete the WebQuest. Give each student a printed copy of the *All Systems Solved* WebQuest worksheet. The quest is divided into stops. Have students answer the questions for each stop, using the information found at each of the QR Codes. Prior to starting the activity, walkthrough with your students on how to use the tablet's camera application to capture a QR Code and open the corresponding URL. Students will find information pertaining to the activity at the various URLs. If students do not have access to a device with a camera, they can enter URLs into a web browser. Provide the links on a class website for easy access or provide an interactive PDF. During the activity, the instructor can provide assistance and clarification, but should mostly be monitoring group progress and collaboration. Have students discuss together as a group at the end of the activity, what they learned during the activity, what questions remain, and what method they prefer to use to solve these systems of equations.

All Systems Solved

How do we solve basic systems of equations? Why would we want to solve systems of equations?

Stop 1: A system of what?

What is a system of equations? Write an example.

A system of equations is a set of two or more equations that share variables. In a system, we have at least as many equations as variables. We solve these systems to find any common solution.

$$2x + 3y = 5$$
$$x - y = 1$$

Scan Me!

https://www.mathsisfun.com/definitions/system-of-equations.html

List a few situations where we can use systems of equations.

Any of the following (or related topics):

Rate, Distance and Time
Economic Problems
Mixtures

Scan Me!

https://sciencing.com/10-can-used-everyday-life-8710568.html

Stop 2: Hey, that looks familiar

Let's take a step back and look at linear equations. What is a linear equation?

A linear equation is an equation of a line, with a slope and a y-intercept. There are only simple variables in linear equations.

Scan Me!

https://www.mathsisfun.com/algebra/systems-linear-equations.html

Write this linear equation, $y = 4x + .6$ in as many different ways as you can think of.

$y - 4x = .6$

$y = 2(2x - .3)$

$y - .6 = 4x$

$y - 4x - .6 = 0$

Generate a couple of linear equations that can make up your own system of linear equations. Why do these make up a system?

Answer will vary

$2x + 3y = 3$
$2x + y = -1$

Students should have at least two linear equations that use the same variables. Students should align equations so that variables are in the same column. If not provide a comment to encourage students to do this in the future. (Don't mark incorrect.)

These two equations share they same variables. We have at least as many equations as variables.

Stop 3: Seeing is Believing

When considering systems of equations, we can actually identify the solution(s) (or absents of a solution(s)) visually. We do this by inspecting the graph of the equations in our system. Remember our goal is to find a solution that works for all the equations in the system. These solutions are where these equations intersect!

What is the solution for this system of equations, using the graph?

Scan Me!

https://www.desmos.com/calculator/lpf9lc7wxr

$x = 8$ $y = 2$

or

$(8,2)$

Time to play… is there a solution?

Identify the solution, if it exists.

A

Scan Me!

https://www.desmos.com/calculator/2k7duosbyb

$x = 10$ $y = 5$

or $(10,5)$

B

Scan Me!

https://www.desmos.com/calculator/ldzmeibnny

Same Equation
Infinitely Many Solutions

C

Scan Me!

https://www.desmos.com/calculator/pdhcgzmxzr

No Solutions
Parallel Lines

Stop 4: Getting Hands-On

Seeing is believing, but can we just trust our eyes? There are different methods we can use to solve systems of equations using many of the math skills we already know.

Substitution

Solve this system of equations using the substitution methods.

Scan Me!

https://www.mathplanet.com/education/algebra-1/systems-of-linear-equations-and-inequalities/the-substitution-method-for-solving-linear-systems

$$2x - 9y = 14$$

$$x + 6y = 7$$

Solve for one of the variables.

Approach will vary. Accept any mathematical equivalent response, that utilizes the substitution method.

$$x = 7 - 6y$$

Substitute that into the second equation to find the value of the second variable.

$$\begin{aligned} 2(7 - 6y) - 9y &= 14 \\ 14 - 12y - 9y &= 14 \\ -21y &= 0 \\ y &= 0 \end{aligned}$$

Finally, substitute that value into the first equation to solve for the value of the final variable.

$$\begin{aligned} x + 6(0) &= 7 \\ x &= 7 \end{aligned}$$

Elimination

Solve this system of equations using the elimination method.

$$10x - 9y = 24$$

$$y - x = -2$$

Scan Me!

https://www.mathplanet.com/education/algebra-1/systems-of-linear-equations-and-inequalities/the-elimination-method-for-solving-linear-systems

Align all like terms and equal signs in the same columns.

Approach will vary. Accept any mathematical equivalent response, that utilizes the elimination method.

$$\begin{aligned} 10x - 9y &= 24 \\ -x \quad + y &= -2 \end{aligned}$$

If you can't eliminate one of the variable using addition or subtraction, find a constant to multiple one of the equations by so that we can eliminate one of the variables.

$$\begin{aligned} 10x - 9y &= 24 \\ 9(-x \quad + y &= -2) \end{aligned}$$

Solve for one of the variables.

$$\begin{aligned} 10x - 9y &= 24 \\ + \ -9x + 9y &= -18 \\ \hline x \qquad &= 6 \end{aligned}$$

Finally, substitute that value into either of the original equations to solve for the value of the final variable.

$$\begin{aligned} y - x &= -2 \\ y - 6 &= -2 \\ y &= 4 \end{aligned}$$

Stop 5: A Mind Blowing Comeback

There is arguably nothing better than an unforgettable sports comeback. Check out the following video and then comeback here…

Scan Me!

https://www.youtube.com/watch?v=67tPlnNMKSc

It's time for the state championships and Scott and Samir have been training all year for the 800 meter dash. Through their training, Scott and Samir know the following about their running ability.

Scott can run 375 meters in 1 min, $d = 375t$.
Samir can run 500 meters in 1 min, $d = 500t$.
Where d is in meters and t is in minutes.

In their practice run, the starter's pistol fires and Samir notices his shoe is untied. Scott has a head start, while Samir spends 30 seconds, $\frac{1}{2}$ min, tying his shoes. Therefore, his distance function has changed, $d = 500(t - \frac{1}{2})$. Even with this stumbling block, Samir still wins the race!

Using our knowledge of systems of equations, at what time and at what distance will Samir catch up to Scott? ***Approach will vary. Accept any mathematical equivalent response.***

$$d = 500(t - \tfrac{1}{2}) = 500t - 250$$

$$\begin{aligned} d - 500t &= -250 \\ -d - 375t &= 0 \\ \hline -125t &= -250 \\ t &= 2 \end{aligned}$$

$$d = 375t$$
$$d = 375(2)$$
$$d = 750$$

SOLUTION: $t = 2 \text{ mins}$, $d = 750 \text{ m}$

Unit Development with Assessment Measures

“Poly” want a “nomial”? | Unit Plan

Brandon Caruso
Dr. Howard and Dr. Jabot
MAED and SCED 419/417

Introduction:

This unit is designed to introduce students to polynomials. The unit starts by teasing the fact that projectile motion can be modeled with polynomials. Students then being by learning the language used to describe polynomials. They use this language to describe polynomials to other students in Polynomial Speed Dating. They explore how polynomials are similar to numbers, by learning how to add, subtract and multiply polynomials. They are introduced to the repeated distribution and the area model for multiplication. The last lesson of the unit looks into factoring. It presents methods for factoring using the greatest common factors, perfect squares, and strategies for trinomials, revisiting the area model. Students end the unit with an application activity that uses polynomials as abstractions for the dimensions of a house.

NYS Next Generation Standards:

AI-A.SSE.1.a Write the standard form of a given polynomial and identify the terms, coefficients, degree, leading coefficient, and constant term.

AI-A.SSE.2 Recognize and use the structure of an expression to identify ways to rewrite it.

AI-A.APR.1 Add, subtract, and multiply polynomials and recognize that the result of the operation is also a polynomial. This forms a system analogous to the integers.

Objectives:

At the conclusion of this unit, students will be able to:

- Use mathematics language to describe and communicate polynomials to others.
- Write polynomials in simplest standard form with 80% accuracy.
- Find the sum and difference of polynomials with 80% accuracy.
- Multiply different combinations of monomials, binomials, and trinomials with 80% accuracy.
- Use the Area Model or repeated distribution to multiple polynomials.
- Factor polynomials, up to trinomials, into an equivalent product with 80% accuracy.
- Apply polynomials as an abstraction of numbers in a real-world geometry application.

Materials: Teacher and Student Copies of Guided Notes, Teacher and Student Copies of Pre/Post Assessments, Index Cards, Whiteboards

Instructional Protocol:

Each lesson is composed of guided notes with embedded practice. Complete the notes with students and fade support as you complete each practice section.

Pre-assessment: Prior to starting the unit, have students complete the *Headscratcher* quiz.

Lesson 1 (Terms, Terms, Terms): This lesson introduces all the necessary language surrounding polynomials; terms, coefficients, degree, leading coefficient, and constant term. We end the lesson with Polynomial Speed Dating. Use the inside-outside circle strategy to make partnering easy and efficient. You will need Index Cards and Whiteboards for this activity.

Lesson 2 (Numbers in Disguise): This lesson starts by introducing combining like terms. Students then learn how to add and subtract polynomials. To introduce multiplication, first, review the distributive property and multiplying exponents. Then start with monomials and work to other products of polynomials, using repeated distribution and the area model.

Lesson 3 (From Polynomials to Products): Start by having students guess how to factor polynomials, then introduce different strategies; factoring greatest common factors, factor perfect squares, and factoring trinomials, or a combination of all these strategies.

Project (Polynomial Floor Plans): In small groups or pairs, students will use polynomials as abstractions for the dimensions of a house. Using the floor plan, students will factor, add, subtract, and multiply polynomials. They will also evaluate polynomials given a value for the unknown, in an applied problem.

Post-assessment: To end the unit, have students complete the *Show Off What You Know* exam.

TEACHER COPY

TERMS, TERMS, TERMS

Learning the lingo around Polynomials

Have you every played catch or tossed a ball in the air? Have you ever watched the launch of a rocket or a cannon ball? Maybe you're interested in starting a start-up and want to find out your profit? These seem like totally different situation, but they have at least one thing in common. In mathematics, we often represent these situations using expressions called **polynomials**. This is a huge idea in algebra. We may have encountered mathematical expressions like $x + 2$ or $2y + 5$ in the past. These mathematical expressions were polynomials. Let's get to know the characteristics of polynomials.

A SpaceX rocket launching from Cape Canaveral in Florida. Notice, the trajectory of the rockets launch. This arc can be represented in math using a polynomial.

POLYNOMIALS

$$3x^2 + 2x - 4$$

This expression is a **polynomial**. List somethings you notice about this expression?

three terms, + and -, a variable x, there is an exponent, etc.

Just like with words like tri-angle or bi-cycle, the meaning of the word polynomial can be found by breaking apart the word itself, poly-nomial.

"poly" - **multiple**

"nominal" - **term**

A polynomial is **an expression with multiple terms.**

WHAT'S A TERM?

$$\underline{3x^2} + \underline{2xy} - \underline{4}$$

A **term** is either a single number or variable, or numbers and variables **multiplied** together. Terms are separated by addition (+) and subtraction (–) signs.

Number of Terms	Prefix	Name	Example
1	"mono-"	monomial	$2x^3$ or 3
2	"bi-"	binomial	$9k + 4$
3	"tri-"	trinomial	$3p + 5y - 3$
4 or more	"poly-"	polynomial	$3x^3 - 2x + y + 8$

COUNT THE TERMS?

Count the number of terms and identify if it's a monomial, binomial, trinomial, or polynomial.

$x^5 - 4$ **2, binomial**

$2zx$ **1, monomial**

$p^2 + 5p - 2$ **3, trinomial**

$x^2 + y + 2x + 3y + 1$ **5, polynomial**

ANATOMY OF A TERM

Degree

$$\underline{10}x^{2}$$

Coefficient Variable

variable	**is a symbol for a number we don't know yet**
coefficient	**is a number used to multiply a variable**
degree	**the exponent or power of the variable**

TALK THE TALK When referring to terms in a polynomial often call terms by the variable, for example:

$3x^3$ - "the cubed term" $9x^2$ - "the x-squared term" $10xy$ - "the xy term"

LIKE TERMS

Just like grouping similar things, like apples with apples and oranges with oranges, we can combine terms that are alike to simplify our polynomials. However, we can only combine "like terms". **"Like terms"** are terms that have the same variables with the same powers. Their coefficients do not need to match.

$9x^2$	$3x^3$
$7x^2$	$3x^2$
LIKE TERMS	UNLIKE TERMS

FIND THEIR MATCH Circle the like terms below.

$3x^2 \quad x^2$ $\qquad$ $9xy \quad -3xy$ $\qquad$ $7x^2 \quad 7x$ $\qquad$ $2x^2y \quad -yx^2$

THE ANATOMY OF A POLYNOMIAL

Degree

Leading Coefficient — $3x^2 + 2xy - 9$

Leading Term $\qquad$ Coefficient $\qquad$ Constant Term

leading coefficient	**the number in front of the variable with the highest power, in the example, 3**
degree	**the highest power of any of the terms, the example is a second-degree trinomial**
constant term	**a term that contains *only* a number, in the example, 9**
leading term	**the term with the highest power in the expression, in the example, $3x^2$**

Polynomials can have constants, variables and positive integer exponents (0,1,2,3,…). They never has division by a variable, negative exponents, or variables as exponents.

PICK THE POLYNOMIALS

Circle the expressions below that are polynomials.

$4x^2 - 8x + 1$ $\quad$ $9x^{-2} + 2x - \dfrac{1}{x}$ $\quad$ $2^x + 3^x + 4^x$ $\quad$ $2x^2 + 5x^3 - x + 8$

WHAT'S THE DEGREE?

The **degree** of a polynomial the highest degree of any of the terms. Write the degree of the following polynomials.

$2x^2 + 5x^3 - x + 8$	$x + 2$	$x^3 + 2 + y^9 - 9$	$3x^4 + 5x - 2x^5$
third degree	**first degree**	**ninth degree**	**fifth degree**

STANDARD FORM

We often place polynomials in their standard form. The standard form of a polynomial is written with the the powers on the variables always in **descending** order from left to right.

$3x^6 - 2x^3 + 5x + 8$	$8 + 3x^6 + 5x - 2x^3$
STANDARD FORM	NOT STANDARD FORM

Notice, these two expressions are equivalent! They are the same polynomial. However, one is in standard form and the other is not.

GIVE IT A SHOT

Write the following polynomials in standard form.

$3x^2 + 5x^3 + 7 - 8x$	$9x^4 + 2x - x^2 + 1$	$3 - 2x - 5x^2$
$5x^3 + 3x^2 - 8x + 7$	$9x^4 - x^2 + 2x + 1$	$-5x^2 - 2x + 3$

POLYNOMIAL SPEED DATING

Will you make a match?

We learned a lot of mathematical language. We often just think math is all about numbers, but language is so important in mathematics. It is important because it helps use communicate and understand the mathematical ideas we have in our head or on our papers. Let's put your language to the test.

> **You Need:**
> Index Card (or Piece of Paper)
> Whiteboard, Marker, and Eraser

On your index card, write down three different polynomials. Include some variety by using different numbers of terms, different degrees, and include some different coefficients.

When the round starts describe your polynomial in words to your partner. You can't just tell your partner your expression. Use the language we have developed today in class to accurately describe your polynomial.

Have your partner, write down the polynomial you are describing on their whiteboard. If it is a match, you both get a point. If not, work with your partner to see what you could have said to ensure a match next time. Each round will be 30 seconds. Have the other partner share one polynomial, then change partners.

PRACTICE

Given the following clues, write the polynomial?

I'm a trinomial.
My degree is 5.
The coefficient of the x-squared term is 3.
The constant term is -5.

$$x^5 + 3x^2 - 5$$

TEACHER COPY

NUMBERS IN DISGUISE

Adding, Subtracting, and Multiplying Polynomials

Polynomials may seem different at first, but they are actually **abstract representations of the numbers** that we see every day. They are like numbers in disguise. If we think of polynomials in this way, they behave like these numbers as well. In fact, we can add, subtract, and multiply polynomials!

COMBINING LIKE TERMS

"Like terms" are terms that have the same variables with the same powers. Their coefficients do not need to match. When we have "like terms" we can add and subtract them, by adding or subtracting their coefficients, depending on the desired operation. For example,

$$3x^2 + 4x^2 = 7x^2$$
$$7xy - 2xy = 5xy$$

Notice, these are equivalent expressions. Therefore, we can write polynomials in simplest standard form by first combining like terms and then writing the polynomial.

GIVE IT A SHOT

Combined like terms and write the following polynomials in simplest standard form.

$$3x^2 + 4x - 5 + 3x - x^2$$

$$3x^2 - x^2 + 4x + 3x - 5$$
$$2x^2 + 7x - 5$$

$$3 - 5x^3 + 8xy - 4 + xy$$

$$-5x^3 + 8xy + xy - 4 + 3$$
$$-5x^3 + 9xy - 1$$

ADDING POLYNOMIALS

Since polynomials are abstract representations of numbers, we can add polynomials together. We add them by simply combining the "like terms" in each of the polynomials.

$$(3x^2 - x + 3) + (4x^2 + 2x - 5)$$
$$3x^2 + 4x^2 - x + 2x + 3 - 5$$
$$7x^2 + x - 2$$

GIVE IT A SHOT Find the sum of the polynomials in simplest standard form.

$$(6x^2 - 2x + 8) + (3x^2 + 7x - 2)$$
$$6x^2 + 3x^2 - 2x + 7x + 8 - 2$$
$$9x^2 + 5x + 6$$

$$(x^3 + 4x^2 - 8x + 3) + (x^3 - x + 1)$$
$$x^3 + x^3 + 4x^2 - 8x - x + 3 + 1$$
$$2x^3 + 4x^2 - 9x + 4$$

SUBTRACTING POLYNOMIALS

We can also subtract polynomials. Being careful about the order, we subtract the "like terms" in each of the polynomials.

$$(x^2 + x - 1) - (-4x^2 + 2x - 5)$$
$$(x^2 - (-4x^2)) + (x - 2x) - (1 - (-5))$$
$$5x^2 - x + 4$$

GIVE IT A SHOT Find the difference of the polynomials in simplest standard form.

$$(5x^2 + 3x - 1) - (3x^2 - 6x + 4)$$
$$5x^2 + 3x - 1 - 3x^2 + 6x - 4$$
$$5x^2 - 3x^2 + 3x + 6x - 1 - 4$$
$$2x^2 + 9x - 5$$

Subtract $3x^2 + 2x + 4$ from $4x^2 + 6x - 3$.

$$(4x^2 + 6x - 3) - (3x^2 + 2x + 4)$$
$$4x^2 + 6x - 3 - 3x^2 - 2x - 4$$
$$4x^2 - 3x^2 + 6x - 2x - 3 - 4$$
$$x^2 + 4x - 7$$

MULTIPLYING POLYNOMIALS

We can also multiply polynomials. Let's start off by multiplying monomials.

BLAST FROM THE PAST When we multiply exponents with like bases, we add the powers.

$$x^a \cdot x^b = x^{a+b}$$

MULTIPLYING MONOMIALS

When we multiply monomials, we multiply the coefficients and also multiply any exponents with similar bases.

$$5x^3 \cdot 2x^2$$

$$5 \cdot 2 \cdot x^3 \cdot x^2$$

$$10 \cdot x^{3+2}$$

$$10x^5$$

$$-3x \cdot -8x$$

$$-3 \cdot -8 \cdot x \cdot x$$

$$24x^2$$

$$\frac{1}{2}x^2y^5 \cdot \frac{3}{4}x^9y$$

$$\frac{1}{2} \cdot \frac{3}{4} \cdot x^2 \cdot x^9 \cdot y^5 \cdot y$$

$$\frac{3}{8} \cdot x^{2+9} \cdot y^{5+1}$$

$$\frac{3}{8}x^{11}y^6$$

BLAST FROM THE PAST The Distributive Property,

$$a(b+c) = a \cdot b + a \cdot c$$

REPEATED DISTRIBUTIVE PROPERTY

When we multiply polynomials, we utilize repeated use of the distributive property.

$$(x-3)(x^2-3x+2)$$

$$(x-3)(x^2-3x+2)$$

$$(x(x^2) - x(3x) + x(2)) + ((-3)(x^2) + (-3)(-3x) + (-3)(2))$$

$$x^3 - 3x^2 + 2x - 3x^2 + 9x - 6$$

$$x^3 - 6x^2 + 11x - 6$$

PHEW, THAT IS A LOT! | USING AN AREA MODEL

Things can get rather complex when using the repeated distributive property method. We can use an area model to keep track of everything. Let's take a look at an example.

$$(x-3)(x^2-3x+2)$$

1) **Draw a grid, using the number of terms in the first factor as the number of rows and number of terms in the second factor as the number of columns.**

2) **Write the terms in the first factor down the side, one term for each row. Then, write the terms in the second factor along the top, one term for each column. Include the sign!**

	x^2	$-3x$	$+2$
x			
-3			

3) **For each cell, Multiple the terms for that row and column.**

	x^2	$-3x$	$+2$
x	$x \cdot x^2 = x^3$	$-3x^2$	$+2x$
-3	$-3x^2$	$+9x$	-6

4) **Add all the terms in each cell together and combine like terms.**

$$x^3 - 3x^2 - 3x^2 + 9x + 2x - 6$$
$$x^3 - 6x^2 + 11x - 6$$

LET'S PRACTICE

Using either method, write each of the following products in standard polynomial form. Please verify your answer.

$(x + 2)(x - 6)$

$x^2 - 4x - 12$

$(3x - 2)(5x - 1)$

$15x^2 - 3x - 10x + 2$

$15x^2 - 13x + 2$

$(2x + 7)(x + 3)$

$2x^2 + 6x + 7x + 21$

$2x^2 + 13x + 21$

$(2x + 5)^2$

$4x^2 + 10x + 10x + 25$

$4x^2 + 20x + 25$

$(x + 2)(x^2 + 4x + 3)$

$x^3 + 4x^2 + 3x + 2x^2 + 8x + 6$

$x^3 + 6x^2 + 11x + 6$

$(x - 4)(x + 3)(x - 5)$

$(x - 4)(x^2 - 5x + 3x - 15)$

$(x - 4)(x^2 - 2x - 15)$

$x^3 - 2x^2 - 15x - 4x^2 + 8x + 60$

$x^3 - 6x^2 - 7x + 60$

TEACHER COPY

FROM POLYNOMIAL TO PRODUCT

Factoring Polynomials

In the past, when you heard the word **factor** you might of thought about greatest common factor of two numbers or an algebraic expression that is part of product. In this case, a factor is a thing or things. But factor can also be a mathematical action. If we want to to rewrite an algebraic expression as a product, we can **factor** that original expression.

To **factor** is to rewrite an algebraic expression as an **equivalent product.**

FROM PRODUCT TO POLYNOMIAL

In the last lesson, we learned how to multiply polynomials.

$$(x-10)(x-4) \underset{?}{\rightleftarrows} x^2-14x+40$$

$$x(x^2-2x+3) \underset{?}{\rightleftarrows} x^3-2x^2+3x$$

HOW DO WE REVERSE IT? Let's take this as a puzzle. What factors do we need to multiple to get our original polynomials? Factor the following polynomial. Remember, the result is an **equivalent** product. Multiple the resulting factors to double check you get the original polynomial.

$$x^2-3x+2$$

$$(x\ \underline{-}\ \underline{1})(x\ \underline{-}\ \underline{2})$$

TALK WITH A FRIEND Were there any strategies you tried to figure out what the factors would be?

FACTORING | FROM POLYNOMIAL TO PRODUCT

Depending on the polynomial, there are different approaches to factoring. We will cover a few techniques.

THE GREATEST COMMON FACTOR

One way to factor a polynomial, is to “factor out” the greatest common factor and reverse the distributive property. The **greatest common factor** for a polynomial is the largest monomial (coefficient and/or variable) that is a factor of each term of the polynomial. Let’s look at an example

$$30x^3 + 5x^2 - 25x$$

1) **Look at the coefficients. Is there a number you can divide every term by?**

   Yes, 5. Greatest Common Factor of 30, 5, and 25 is 5.

2) **Look at the variables. Can you factor out the same variable from every term?**

   Yes, x.

3) **These two things make up the largest monomial we can factor out. Divide every term by $5x$.**

$$\frac{30x^3}{5x} + \frac{5x^2}{5x} - \frac{25x}{5x} = 6x^2 + x - 5$$

4) **We can now write our final product. Write in final factored form.**

$$5x(6x^2 + x - 5)$$

PERFECT SQUARES

$$x^2 + 8x + 16$$

Notice, the lead term (x^2) and constant term $(4^2 = 16)$ are perfect squares. Additionally, the middle term is two times the product of the numbers that are squared, $2(4 \cdot x) = 8x$. Make sure to consider negatives, as well. This is a special case, we can now factor this as,

$$(x + 4)^2$$

VERIFY Check our work by expanding $(x + 4)^2$. It should equal $x^2 + 8x + 16$, does it?

TRINOMIALS | WITH A LEADING ONE

Let's say we have a trinomial with a leading coefficient of one. Like this:

$$x^2 - 7x + 10$$

1) **Look at the constant term. What are its pairs of factors?**

$$1,10 \qquad 5,2$$

2) **Which of these pairs sum to the coefficient for the x-term?**
   Don't forget they can be negative!

$$-5 - 2 = -7$$

3) **Write in final factored form.**

$$(x - 5)(x - 2)$$

LET'S PRACTICE

Using any of the method, factor each of the following polynomials.

$$6x^3 - 8x^2 + 2x$$

GCF of Coefficients: 2

Variable to factor: x

Largest Monomial Factor: $2x$

$$2x(3x^2 - 4x + 1)$$

$$x^2 - 12x + 36$$

Perfect Square!

$$(x - 6)^2$$

$$x^2 - 8x + 15$$

Factors of 15: $1,15 \;\; 3,5$

$$-8 = -5 - 3$$

$$(x - 5)(x - 3)$$

$$x^2 - 12x + 20$$

Factors of 20: $1,20 \;\; 4,5 \;\; 2,10$

$$-12 = -10 - 2$$

$$(x - 10)(x - 2)$$

RETURN OF THE AREA MODEL

We used an area model to multiply polynomials. We can also use it to factor trinomials, when our trinomial doesn't have a leading coefficient of 1 or -1, we can utilize the area model to help us factor these trinomials. Let's look at an example:

$$3x^2 - 2x - 8$$

1) **Draw a 2 x 2 grid. Place the leading term in the top-left cell and the constant term in the bottom right cell.**

$3x^2$	
	-8

2) **For the remaining cells, we are looking for two x-terms whose coefficients multiply to the product of the leading coefficient (3) and the constant term (-8). They must also add to the coefficient of our middle term.**

$$3 \cdot -8 = -24$$

Factors of 24: $12{,}2 \quad 6{,}4 \quad 1{,}24 \quad 3{,}8$

Which add to -2?

$$-6 + 4 = -2$$

$3x^2$	$4x$
$-6x$	-8

3) **Now we need to find what to place at the start of the row. This is the greatest common factor of the terms in that row. For the terms at the top of the columns, divide each term in row by that factor.**

GCF of $3x^2$ and $4x$ is x.

$$\frac{3x^2}{x} = 3x \qquad \frac{4x}{x} = 4$$

GCF of $-6x$ and -8 is -2.

$$\frac{-6x}{-2} = 3x \qquad \frac{-8}{-2} = 4$$

	$3x$	$+4$
x	$3x^2$	$4x$
-2	$-6x$	-8

4) **The terms on the side of the box form one factor and the terms on the top form our second factor.**

$$(3x + 4)(x - 2)$$

VERIFY Check our work by expanding $(3x + 4)(x - 2)$. It should equal $3x^2 - 2x - 8$, does it?

LET'S PRACTICE

Using any of the methods we learned, factor each of the following trinomials. Please verify your answer.

$x^2 + 8x - 9$

Factors of 9: $1{,}9 \quad 3{,}3$

$8 = 9 - 1$

$(x - 1)(x + 9)$

$x^2 - 3x - 18$

Factors of 18: $1{,}18 \quad 2{,}9 \quad 6{,}3$

$-3 = 3 - 6$

$(x - 6)(x + 3)$

$7x^2 + 11x - 6$

$7 \cdot -6 = -42$

Factors of 42: $1{,}42 \quad 7{,}6 \quad 2{,}21 \quad 3{,}14$

Which add to 11**?**

$11 = 14 - 3$

	x	$+2$
$7x$	$7x^2$	$14x$
-3	$-3x$	-6

$(7x - 3)(x + 2)$

$11x^2 - 10x - 1$

$11 \cdot -1 = -11$

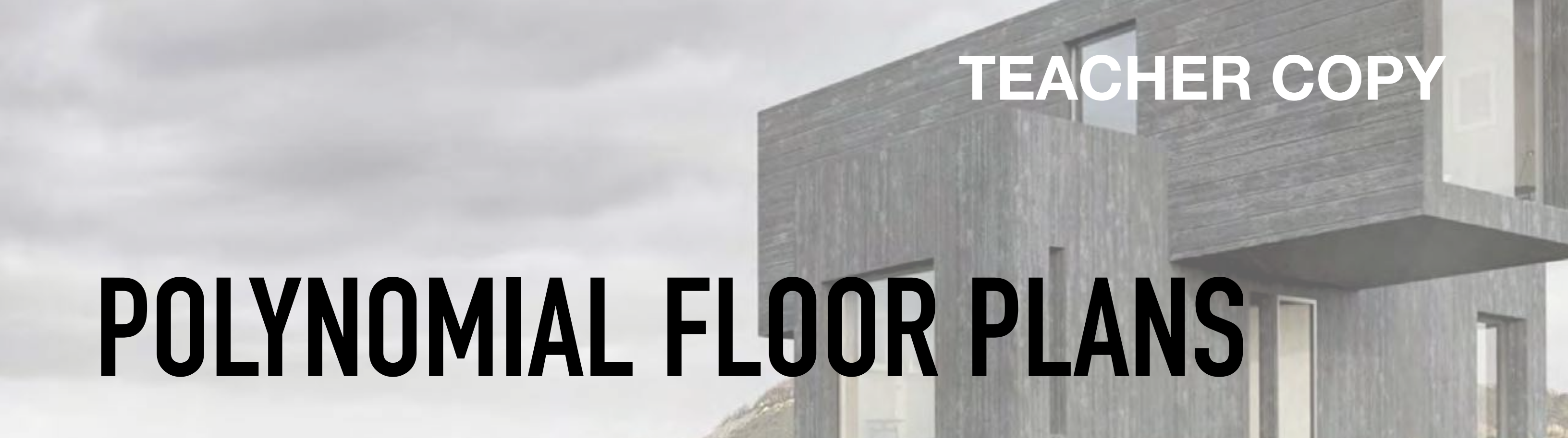
Factors of 11: $1{,}11$

Which add to -10**?**

$-11 + 1 = -10$

	x	-1
$11x$	$11x^2$	$-11x$
$+1$	$+1$	-1

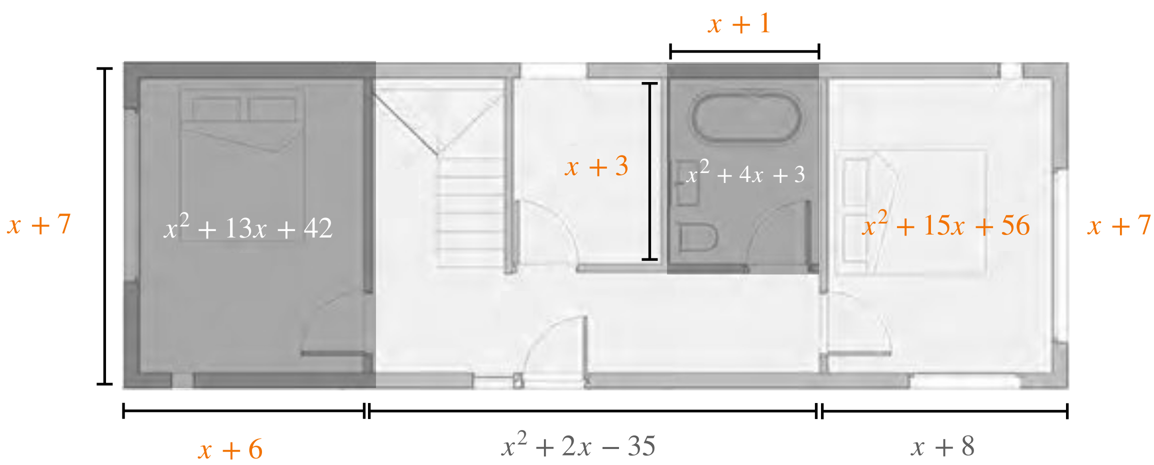
$(11x + 1)(x - 1)$



TEACHER COPY

POLYNOMIAL FLOOR PLANS

KOTO is an architecture firm that designs modular architecture. They design cabins and homes that are prefabricated, modern, and sustainable. One of their home designs is the YKSI HOUSE. Here is the blueprint of the first floor:



Notice, the dimensions and areas of the rooms are given as polynomials. We will figure out the actual dimensions later. For now, using your knowledge of polynomials, answer the following questions from your client. Give your responses as polynomials.

What's the width and length of the left bedroom?

$$x^2 + 13x + 42 = (x+7)(x+6)$$

Width: $(x+6)$ **Length:** $(x+7)$

What's the width and length of the bathroom?

$$x^2 + 4x + 3 = (x+3)(x+1)$$

Width: $(x+1)$ **Length:** $(x+3)$

What's the total length of the first floor?

Length: $(x+6)+(x^2+2x-35)+(x+8)=x^2+4x-21$

What's the square footage (area) of the first floor?

Area: $(x+7)(x^2+4x-21)=x^3+11x^2+7x-147$

Your client mentioned only needing one bedroom. What's the area of the right bedroom? What would the new length of the house be and the new total square footage if you removed the right bedroom?

Area of Bedroom: $(x+7)(x+8)=x^2+15x+56$

Length: $x^2+4x-21-(x+8)=x^2+3x-29$

Area Without: $(x+7)(x^2+3x-29)=x^3+10x^2+8x-203$

We know that $x=7ft$. What are the actual dimensions of the house with only the left bedroom?

	Length	Width	Area
Bathroom	10 ft	8 ft	80 sq ft
Bedroom	14 ft	13 ft	182 sq ft
First Floor	41 ft	14 ft	574 sq ft

DIG DEEPER If is costs $225 a square foot to build this house, how much would the client save on building the first floor from removing the right bedroom from the first floor?

Area With: $(7)^3 + 11(7)^2 + 7(7) - 147 = 784 \text{ sq ft}.$

Cost With: $\$225 \text{ per sq ft.} \cdot 784 \text{ sq ft}. = \$176{,}400$

Cost Without: $\$225 \text{ per sq ft.} \cdot 574 \text{ sq ft}. = \$129{,}150$

Savings: $\$176{,}400 - \$129{,}150 = \$47{,}250$

Accept mathematically equivalent solutions.

HEAD SCRATCHER

Name: KEY

Give every question a shot! You may not know how to do a problem and that is okay!

1. Simplify the following expressions.

$2(x+4)$

$2x+8$

$5(6-x+5)$

$-5x+55$

$3x(x+2)$

$3x^2+6x$

$x^5 \cdot x^6$

x^{11}

$2x \cdot x$

$2x^2$

$3x^2y \cdot y^4$

$3x^2y^5$

2. Find each of the following sums and differences. Write your answer in simplest standard form.

$(3x-2)-(2x+4)$

$x-6$

$4x^2+6x-3-3x^2+2x+4$

x^2+8x+1

4. Write each of the following products in standard polynomial form.

$(x + 5)(x + 3)$

$x^2 + 8x + 15$

$(2x - 3)(5x + 1)$

$10x^2 - 13x - 3$

5. Factor the following polynomials.

$x^2 - 8x + 12$

$(x - 2)(x - 6)$

$2x^2 + 5x - 33$

$(2x + 11)(x - 3)$

SHOW OFF WHAT YOU KNOW

Name: KEY

Polynomials

1. For the following expressions, if it is a polynomial, write it in standard form. Then name the number of terms, the leading coefficient, the degree, and the constant term. If it is not a polynomial, indicate why.

$-2x + 3x^9 + 3$

$3x^9 - 2x + 3$

Terms: Trinomial, 3
L.C.: 3
Degree: 9
Constant: 3

$\dfrac{2 + 3x + 4}{x + 1}$

Not Polynomial

$4x^2 + xy - 9x^4 + 2$

$-9x^4 + 4x^2 + xy + 2$

Terms: 4
L.C.: -9
Degree: 4
Constant: 2

$3^x + 3x + 4$

Not Polynomial

$4x^2 + y + 5z^3 - 3$

$5z^3 + 4x^2 + y - 3$

Terms: 4
L.C.: 3
Degree: 3
Constant:-3

$3x^{-3} + 3x^2 + 4x$

Not Polynomial

2. Find each of the following sums and differences. Write your answer in simplest standard form.

$6x^2 + 2x - 3 - x^2 + 4x - 1$

$5x^2 + 6x - 4$

$6x^2 + 2x - 3 - (x^2 + 4x - 1)$

$5x^2 - 2x - 2$

3. Write each of the following products in standard polynomial form. Please verify your answers.

$(x - 10)(x - 4)$

$x^2 - 14x + 40$

$(x + 3)^2$

$x^2 + 6x + 9$

$(2x + 3)(5x + 8)$

$10x^2 + 31x + 24$

$(x + 3)(x - 2)(x - 8)$

$x^3 + 7x^2 - 14x + 48$

4. Write $x(x^2 - 2x - 3)$ **as a polynomial.**

$x^3 - 2x^2 - 3x$

5. Rewrite each of the following trinomials in completely factored form. Please verify your answers.

$x^2 + 3x - 40$

$(x + 8)(x - 5)$

$5x^2 - 21x + 4$

$(5x - 1)(x - 4)$

$2x^2 + 13x + 21$

$(2x + 7)(x + 3)$

$x^2 + 8x + 16$

$(x + 4)^2$

6. Which of the following products is equivalent to the trinomial $x^2 - 5x - 24$ **?**

(a) $(x - 12)(x + 2)$

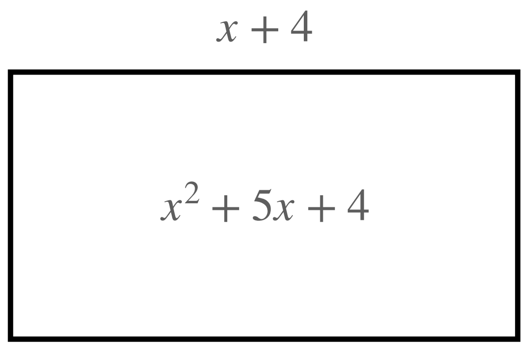
(b) $(x + 12)(x - 2)$

(c) $(x - 8)(x + 3)$

(d) $(x + 8)(x - 3)$

C

7. A rectangle has dimensions as shown below in terms of an unknown variable, x.



(a) **Find a binomial expression for the width of the rectangle in terms of x. Justify your answer based on the expressions for the rectangle's length and area.**

(b) **If the width of the rectangle is 21 inches, what is the length and the area? Use appropriate units and explain how you found your answer.**

a) $(x + 1)$

Justify: $(x + 4)(x + 1) = x^2 + 5x + 4$

b)

$x + 1 = 21 \qquad x = 20$

Length: $(x + 4) = 20 + 4 = 24$

Area:

$21 \cdot 24 = 504\ sq\ in$

$x^2 + 5x + 4 = (20^2 + 5(20) + 4) = 504\ sq\ in$

GETTIN' FUNKY WITH FUNCTIONS

Exploring Absolute Value, Square & Cube Roots, and Piecewise Functions

Say hello to my function, f …

$$f(x) = 2x^2 + 3x + 4$$

A ***function*** is like a machine, it takes an ***input***, does something with it, and the relates it to an ***output***.

$$\underset{\text{input}}{x} \rightarrow \boxed{\begin{matrix} f \\ 2x^2 + 3x + 4 \end{matrix}} \rightarrow \underset{\text{output}}{f(x)}$$

ANATOMY OF A FUNCTION – FUNCTION NOTATION

$$f(x) = 2x^2 + 3x + 4$$

Function Name   Input   Output

In the past, we have worked with relationships in terms of a dependent and independent variable. For example, the **linear equation**, $y = 4x + 5$. In this relationship, our x is our input and our y is our output. We could rewrite $f(x)$ as $y = 2x^2 + 3x + 4$.

DOMAIN AND RANGE

When we are given a function, say $g(x)$, how can we describe its possible inputs and outputs?

The set of all possible inputs that work or can be plugged in to our function is called **the domain**. This set are the x values where the functions is **defined**.

We call the set of all possible outputs of our function **the range**. This set is the value of $f(x)$ given all possible x values. In a relationship $y = 2x + 4$, the range is the set of all possible y values.

SPECIFY DOMAIN & RANGE USING INEQUALITIES, INTERVAL NOTATION, OR SET-BUILDER NOTATION

We can use a variety of different notations to describe the domain and range of a function. Let's get familiar with inequalities, interval notation, and set-builder notation, looking just at the number line.

	Inequality Notation	Set-Builder Notation	Interval Notation
			
			
			
			
			
			

COMPOUND INEQUALITIES AND CONJUNCTIONS

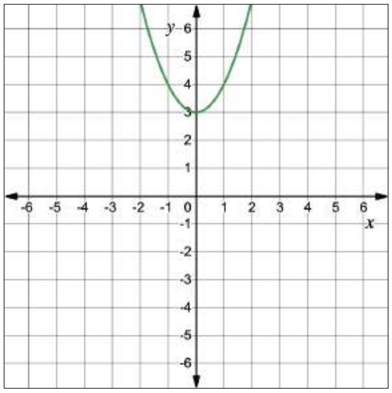
A **compound inequality** includes two inequalities in one statement. For example,

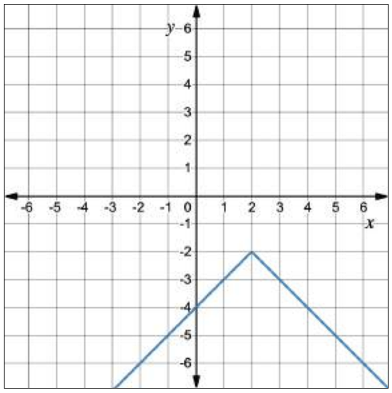
$$-3 < x \leq 6$$

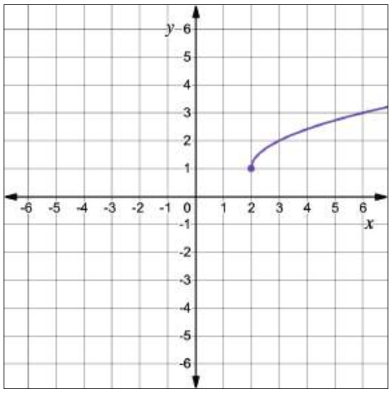
These can be rewritten as a **conjunction** of two inequalities,

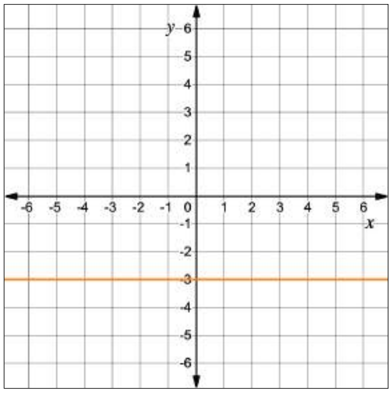
WHAT'S THE DOMAIN AND RANGE?

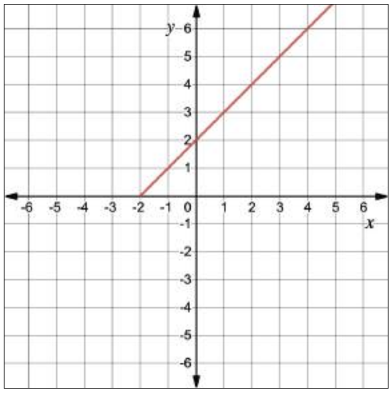
For each of the following graphs, describe the domain and range of the function.

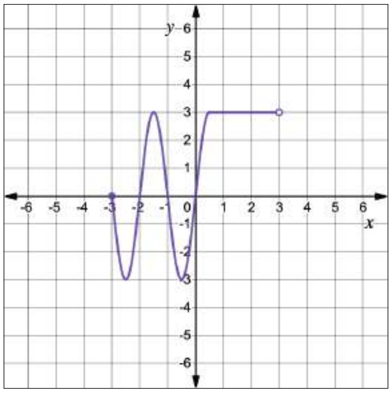




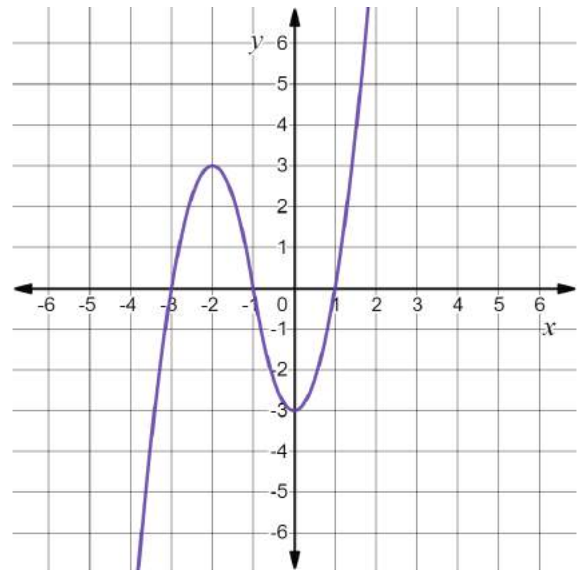








INCREASING AND DECREASING



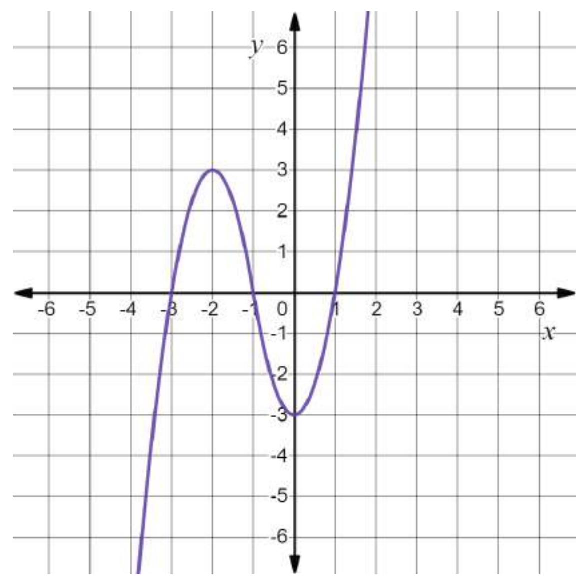
When we look at the graphs of our functions, we can identify intervals over which the function is changing in specific ways.

We say that a function is **increasing** on an interval if the function values increase as the input values increase within that interval.

Similarly, a function is **decreasing** on an interval if the function values decrease as the input values increase over that interval.

DIG DEEPER Over what intervals is $g(x)$ increasing and decreasing?

MAXIMUM AND MINIMUM



Some times functions are increase or decreasing across their entire domain, like linear functions. Other functions **increase** or **decrease** over the course of their domian.

A value of the input of function where it changes from increasing to decreasing is called a **local maximum**.

A value of the input of function where it changes from decreasing to increasing is called a **local minimum**.

Collectively these are called **local extrema**, extreme values.

DIG DEEPER Name the local extrema for $g(x)$.

ABSOLUTE VALUE

Getting Funky with Functions

Let's take a look at a function that uses **absolute value**.

$$f(x) = |x|$$

The ***absolute value*** of a number refers to how far the number is from zero. It is notated using vertical bars around the value. For example,

$|-4| = 4$ $\quad |8| = 8$ $\quad |-3.51| =$ $\quad |2.45| =$ $\quad |0| =$

Notice, the absolute value turns the number inside the vertical bars into its non-negative value.

ABSOLUTE VALUE STANDARD FORM

$$f(x) = a|x - h| + k$$

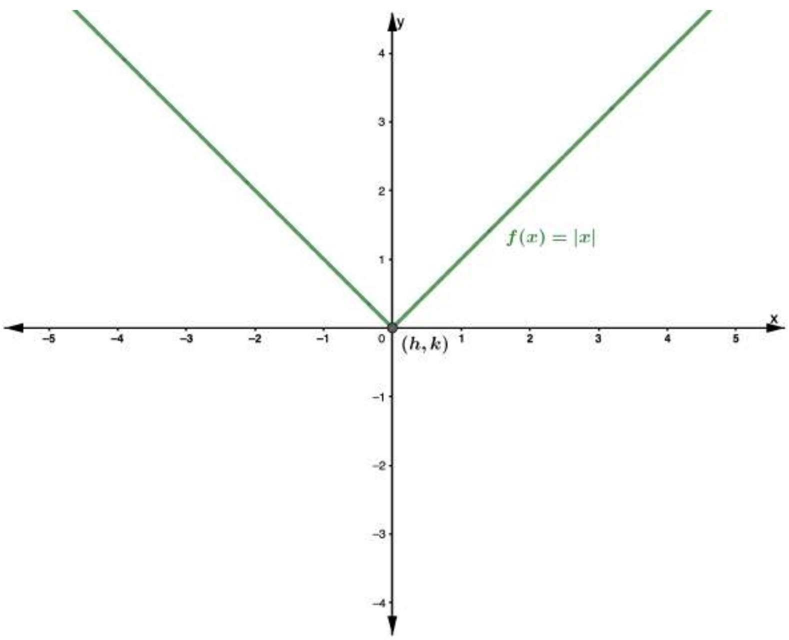
What does h do?
moves right (–) or left (+)

What does k do?
moves up (+) or down (–).

What does a do?
graph stretches vertically, slope, and whether the graph opens up or down.

The vertex is the point (h, k).

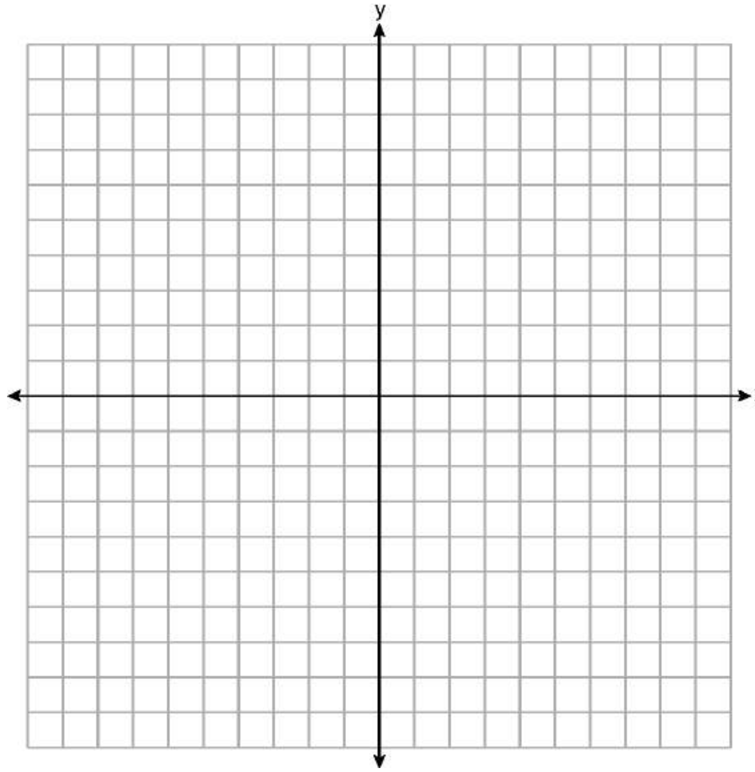
PLAY ON DESMOS: https://www.desmos.com/calculator/xz9bgasurw



LET'S GRAPH AN ABSOLUTE VALUE FUNCTION

Graph the following function $a(x) = |x + 3| - 2$.

x	$a(x) = \vert x + 3 \vert - 2$	$a(x)$
-6		
-4		
-2		
0		

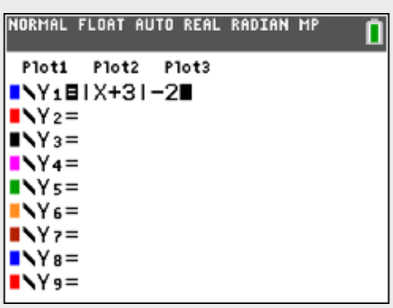


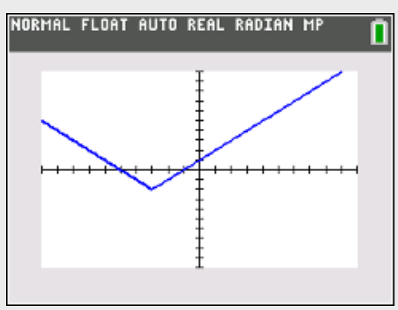
DIG DEEPER What's the vertex, the domain, and range of $a(x)$?

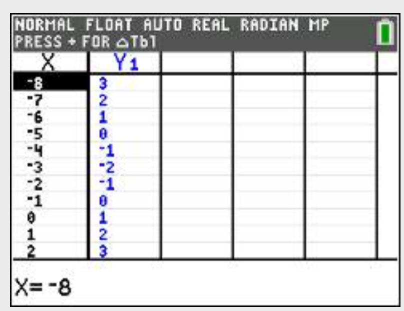
"TI" TIP | GRAPHING USING YOUR CALCULATOR

We can use our graphing calculators to help us graph these functions.

1. Make sure the equation is in standard form. (i.e. $f(x) = |x + 3| - 2$ or $y = |x + 3| - 2$)



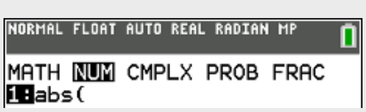




2. Press y= and input the equation into the calculator where it says Y1. Press enter.

3. To view your graph, press graph.

4. To view a table of values, press 2nd graph. This shows the points to graph the function.

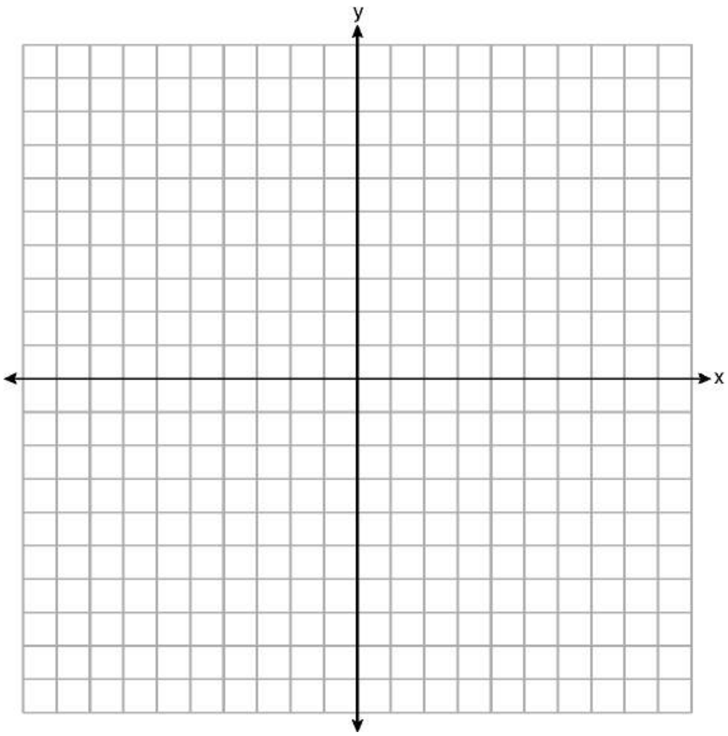


How to get absolute value? Press math → NUM → 1:abs(

BECOME A PRO

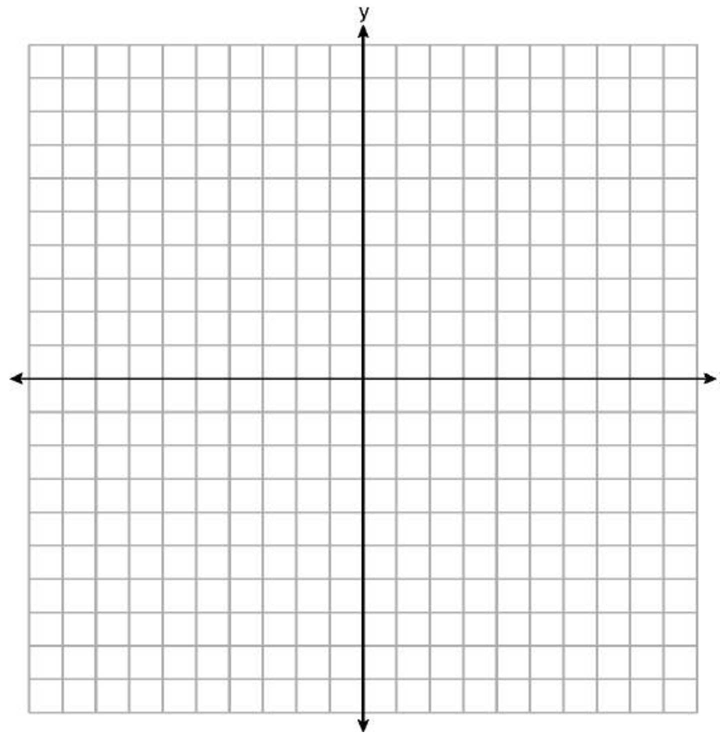
Graph each of the functions and state their vertex, domain, and range.

$z(x) = |x - 2| - 5$



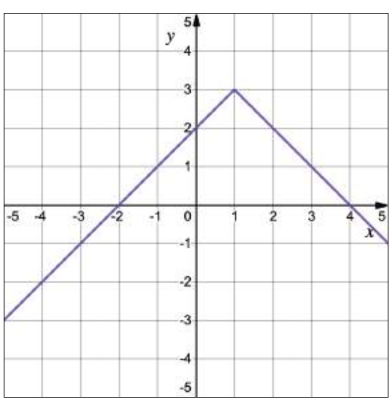
Vertex: **Domain:** **Range:**

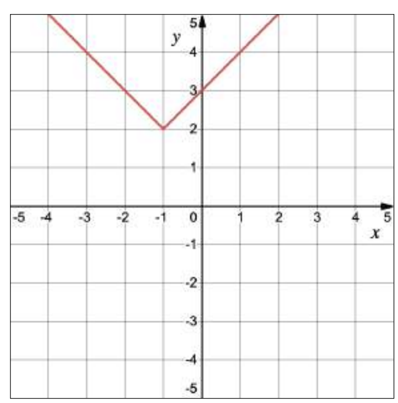
$y = |x + 3|$

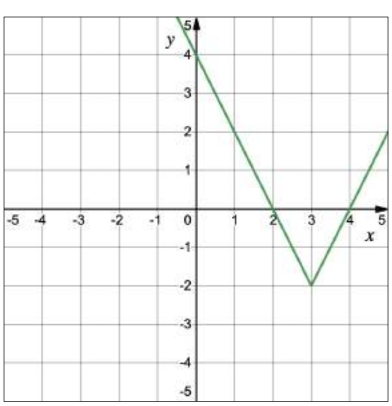


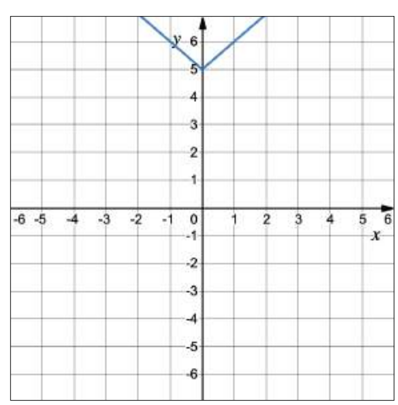
Vertex: **Domain:** **Range:**

Describe the transformation from $y = |x|$ to the graphs below. Write the function for each graph.









SQUARE & CUBE ROOTS

Getting Funky with Functions

Let's take a look functions that are defined using the **square root**.

$$f(x) = \sqrt{x}$$

The **square root** of a number refers to a number that when multiplied by itself gives the number. It is the inverse operation to squaring a number. For example,

$4 \cdot 4 = 4^2 = 16$, therefore, the square root of 16 is $\sqrt{16} = 4$

$\sqrt{9} =$ $\qquad\qquad$ $\sqrt{121} =$ $\qquad\qquad$ $\sqrt{81} =$ $\qquad\qquad$ $\sqrt{4} =$

SQUARE ROOT STANDARD FORM

$$f(x) = a\sqrt{x-h} + k$$

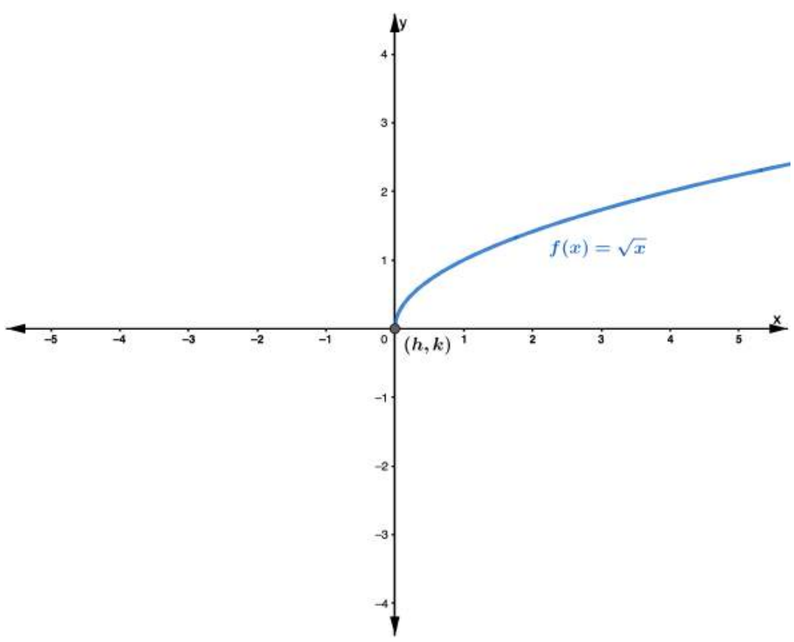
What does h do?
moves right (–) or left (+)

What does k do?
moves up (+) or down (–).

What does a do?
graph stretches vertically, and reflects across x-axis when negative

The endpoint is the point (h, k).

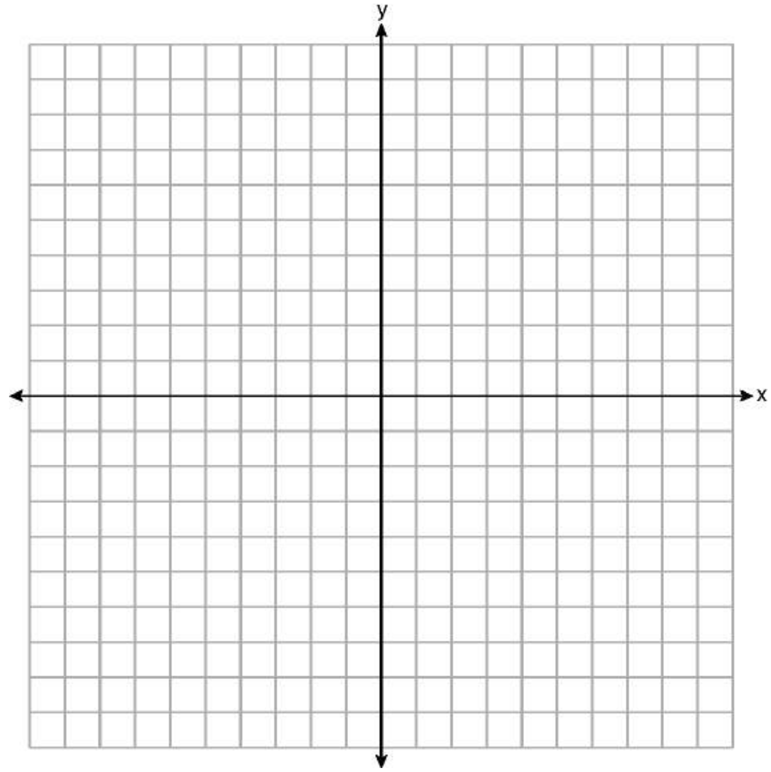
PLAY ON DESMOS: https://www.desmos.com/calculator/bmlp5gnd22



LET'S GRAPH A SQUARE ROOT FUNCTION

Graph the following function $z(x) = \sqrt{x} + 1$.

x	$z(x) = \sqrt{x} + 1$	$z(x)$
0		
4		
9		
16		

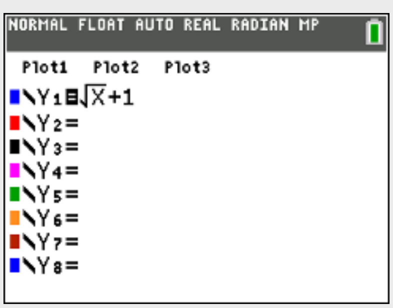


DIG DEEPER What's the domain and range of $z(x)$?

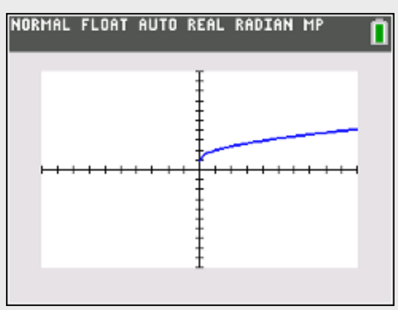
"TI" TIP | GRAPHING USING YOUR CALCULATOR

We can use our graphing calculators to help us graph these functions.

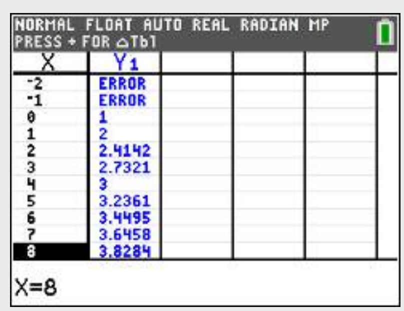
1. Make sure the equation is in standard form. (i.e. $f(x) = \sqrt{x} + 1$ or $y = \sqrt{x} + 1$)



2. Press y= and input the equation into the calculator where it says Y1. Press enter.



3. To view your graph, press graph.



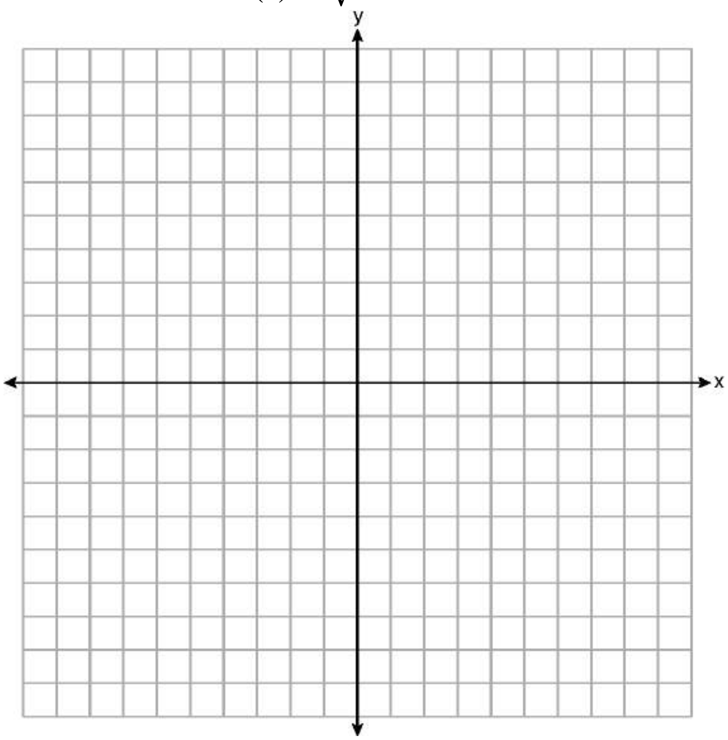
4. To view a table of values, press 2nd graph. This shows the points to graph the function. Notice, there is an **ERROR** for undefined points.

How to get square root? Press 2nd x^2

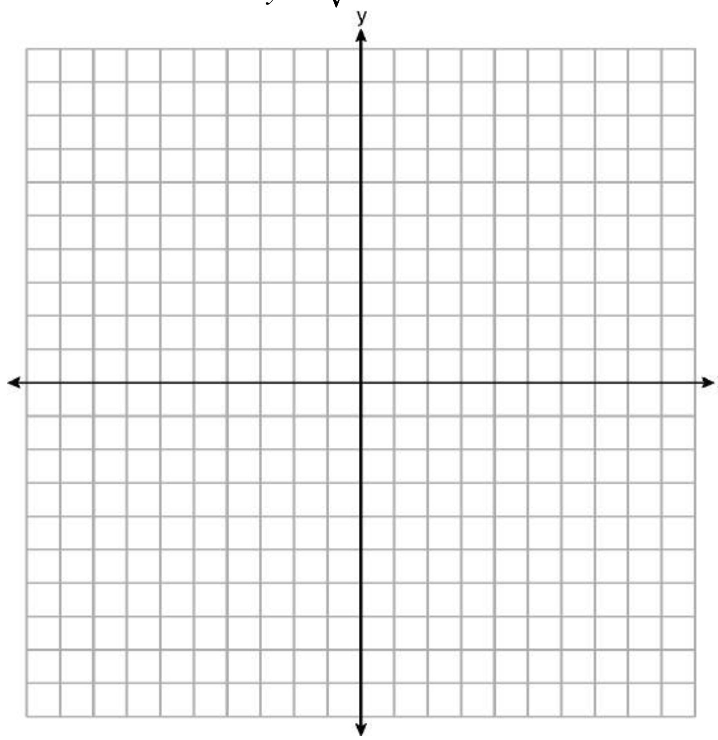
BECOME A PRO

Graph each of the functions and state their domain and range.

$s(x) = \sqrt{x + 2} - 4$



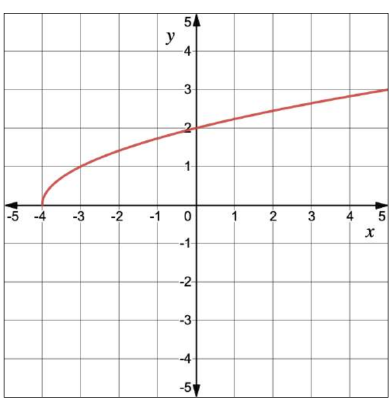
$y = \sqrt{x} + 3$

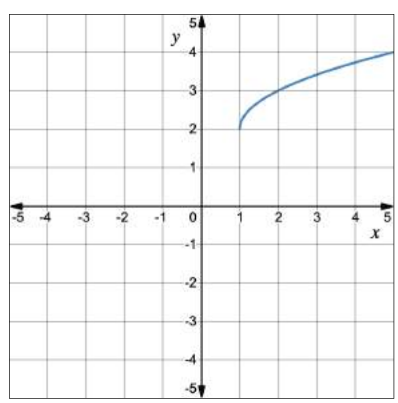


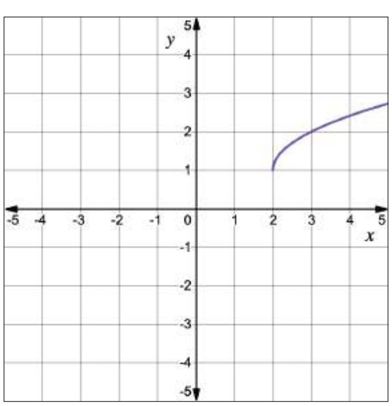
Domain: **Range:**

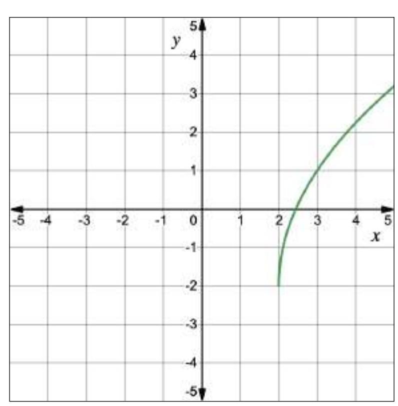
Domain: **Range:**

Write the function for each graph. State their domain and range.









WHAT ABOUT CUBE ROOT FUNCTIONS?

Let's take a look at functions that use **cube roots**.

$$f(x) = \sqrt[3]{x}$$

Similar to the square root, the cube root of a number refers to a number that when multiplied by itself **three times** gives the number. It is the inverse operation to cubing a number or raising a number to **the third power**. For example,

$2 \cdot 2 \cdot 2 = 2^3 = 8$, therefore, the cube root of 8 is $\sqrt[3]{8} = 2$.

$\sqrt[3]{27} =$ $\quad$ $\sqrt[3]{64} =$ $\quad$ $\sqrt[3]{1} =$ $\quad$ $\sqrt[3]{-8} =$

Notice, the cube root is defined for negative numbers.

CUBE ROOT STANDARD FORM

$$f(x) = a\sqrt[3]{x - h} + k$$

What does h do?

moves right (–) or left (+)

What does k do?

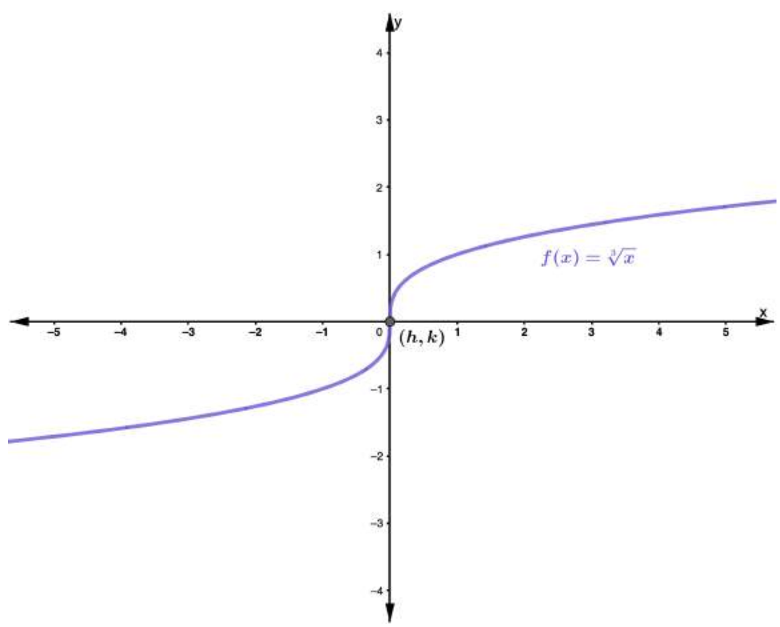
moves up (+) or down (–).

What does a do?

graph stretches vertically, and reflects across x-axis when negative

The point of inflection is (h, k).

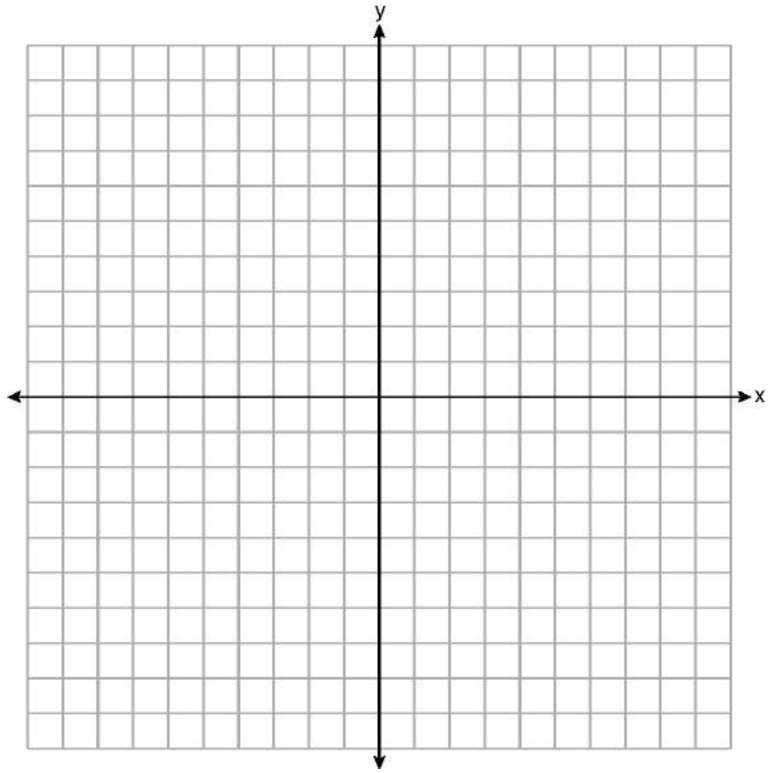
PLAY ON DESMOS: https://www.desmos.com/calculator/sfarhddobk



LET'S GRAPH A CUBE ROOT FUNCTION

Graph the following function $f(x) = \sqrt[3]{x} + 3$.

x	$f(x) = \sqrt[3]{x} + 3$	$f(x)$
-8		
-1		
0		
1		
8		

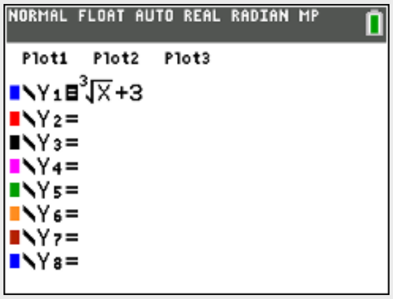


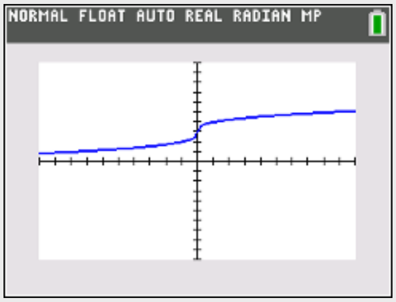
DIG DEEPER What's the domain and range of $f(x)$? What's the point of inflection?

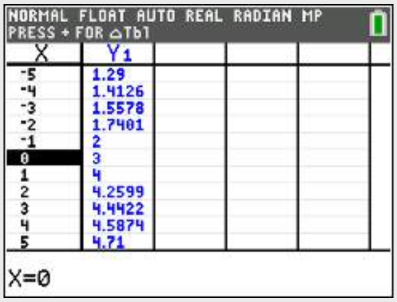
"TI" TIP | GRAPHING USING YOUR CALCULATOR

We can use our graphing calculators to help us graph these functions.

1. Make sure the equation is in standard form. (i.e. $f(x) = \sqrt[3]{x} + 3$ or $y = \sqrt[3]{x} + 3$)







2. Press y= and input the equation into the calculator where it says Y1. Press enter.

3. To view your graph, press graph.

4. To view a table of values, press 2nd graph. This shows the points to graph the function.

How to input cube root? Press math 4, 4: ³√(

How to input any nth root? Press math 5, 5: ˣ√, enter a value for your root.

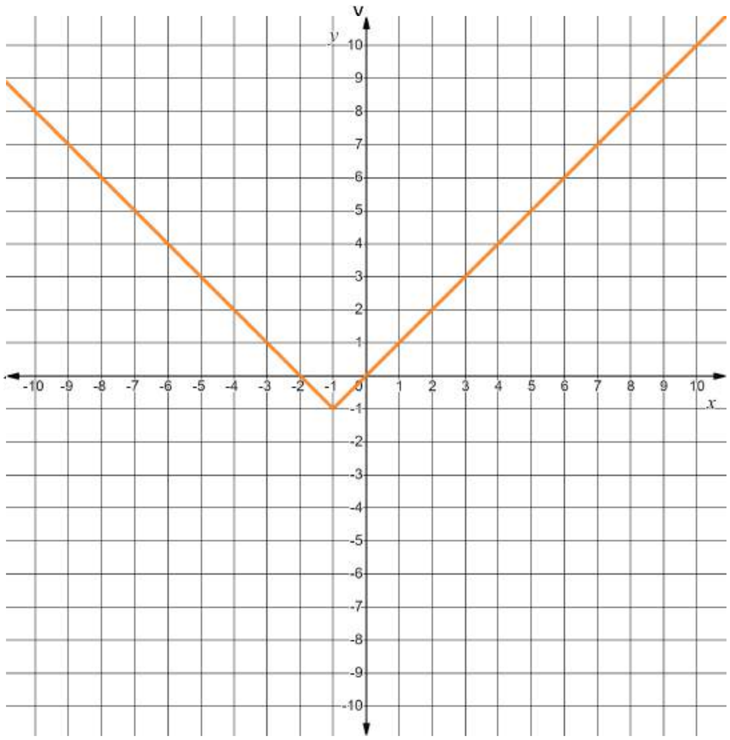
STRENGTH TRAINING 1

Name: SOLUTIONS

Funky with Functions | Absolute Value and Roots

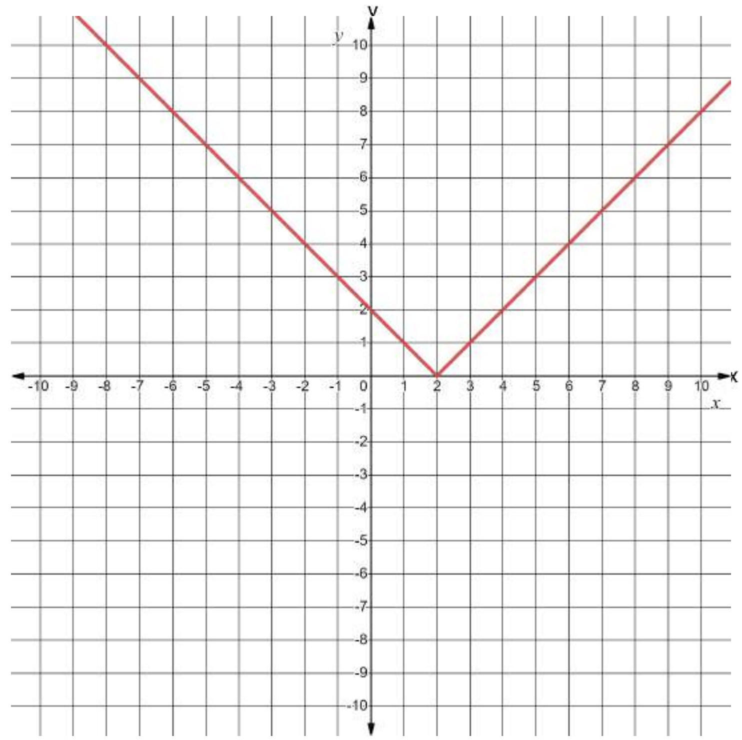
Graph each of the functions and state the vertex, domain, and range.

$y = |x+1| - 1$



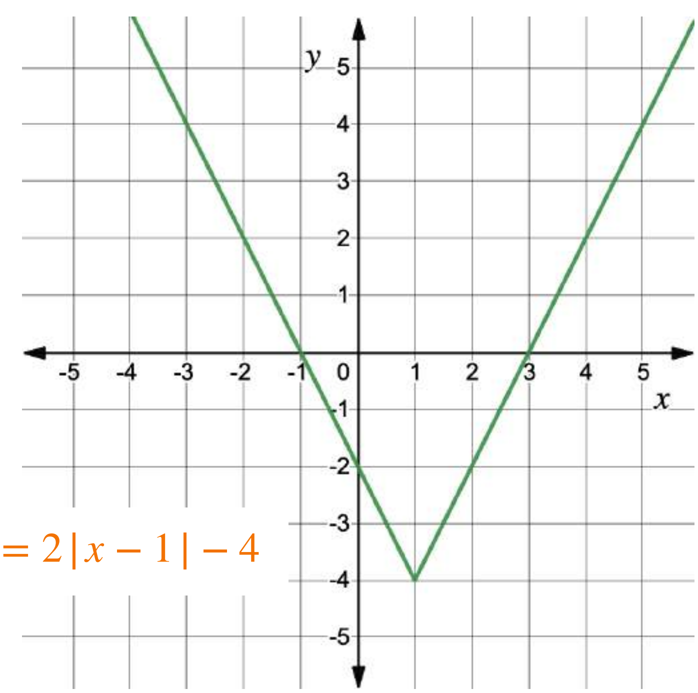
Vertex: $(-1, -1)$ **Domain:** $\{x \mid x \in \mathbb{R}\}$ **Range:** $\{y \mid y \geq -1\}$

$y = |x-2|$



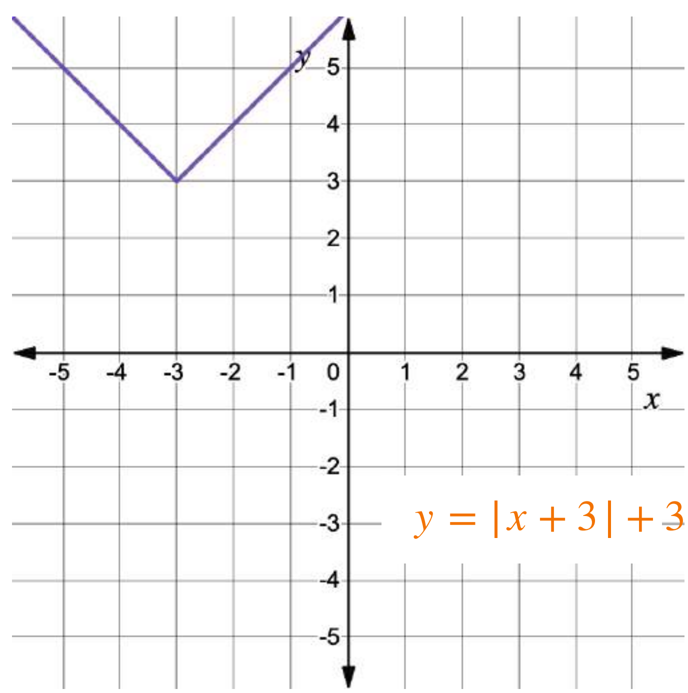
Vertex: $(2,0)$ **Domain:** $\{x \mid x \in \mathbb{R}\}$ **Range:** $\{y \mid y \geq 0\}$

Write the function represented in the graph and state the vertex, domain, and range.



$y = 2|x-1| - 4$

Vertex: $(1, -4)$ **Domain:** $\{x \mid x \in \mathbb{R}\}$ **Range:** $\{y \mid y \geq -4\}$

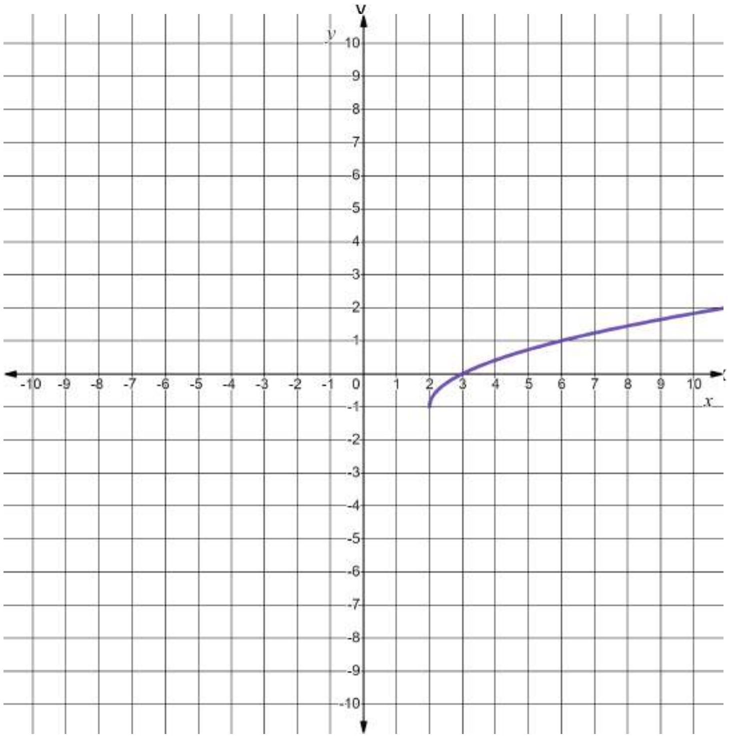


$y = |x+3| + 3$

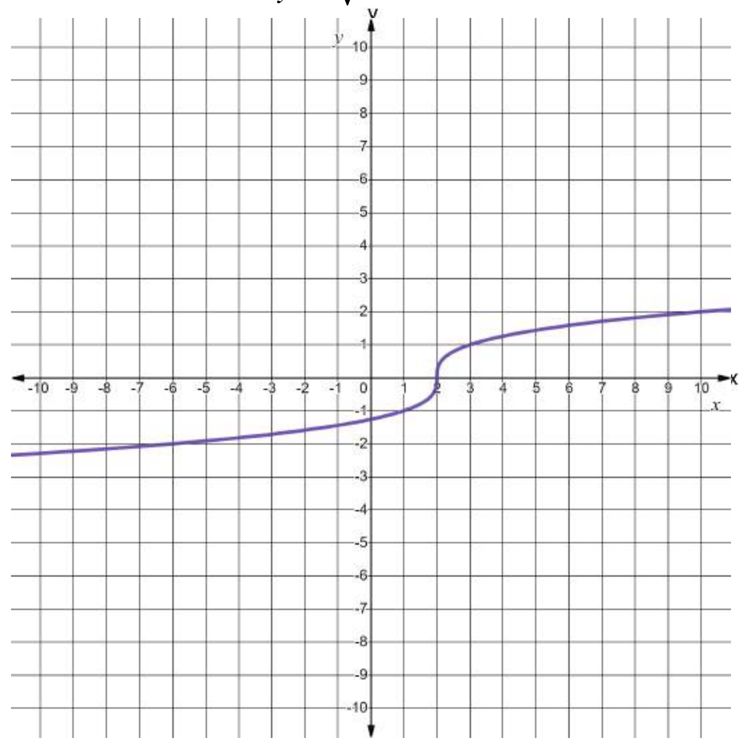
Vertex: $(-3,3)$ **Domain:** $\{x \mid x \in \mathbb{R}\}$ **Range:** $\{y \mid y \geq 3\}$

Graph each of the functions and state the domain and range.

$$y = \sqrt{x-2} - 1$$



$$y = \sqrt[3]{x-2}$$

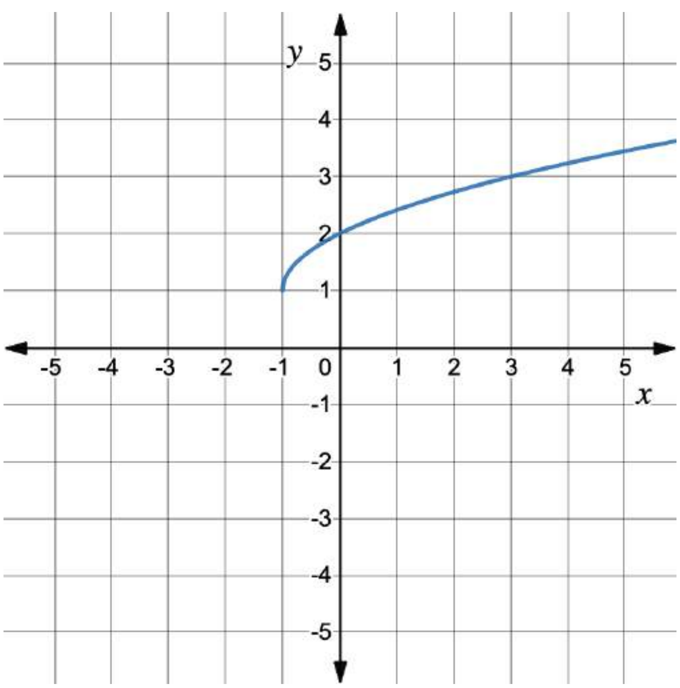


Domain: $\{x \mid x \geq 2\}$ **Range:** $\{y \mid y \geq -1\}$

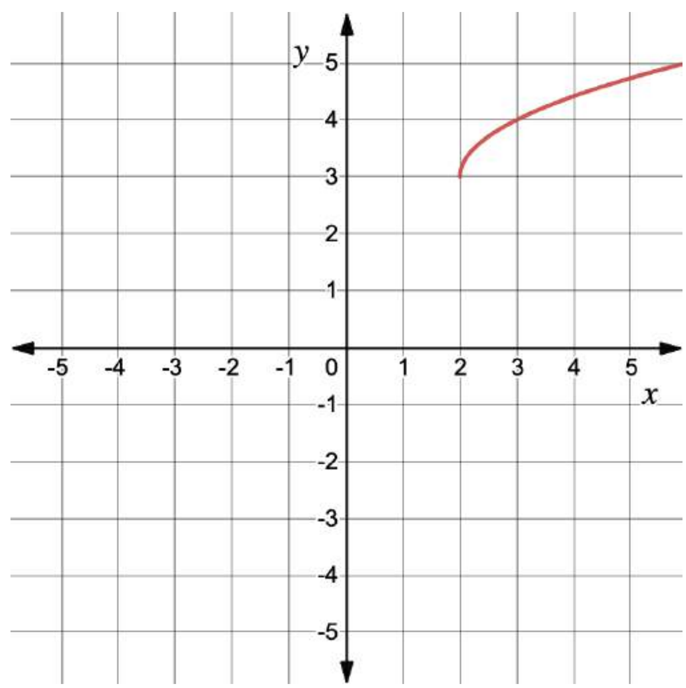
Domain: $\{x \mid x \in \mathbb{R}\}$ **Range:** $\{y \mid y \in \mathbb{R}\}$

Write the function represented in the graph and state the domain and range.

$$y = \sqrt{x+1} + 1$$



$$y = \sqrt{x-2} + 3$$



Domain: $\{x \mid x \geq -1\}$ **Range:** $\{y \mid y \geq 1\}$

Domain: $\{x \mid x \geq 2\}$ **Range:** $\{y \mid y \geq 3\}$

PIECEWISE FUNCTIONS

Getting Funky with Functions

We can craft functions that behave differently depending on the input of the function. These functions defined in pieces are call **piecewise function**.

$$f(x) = \begin{cases} 2 & if\ x \geq 5 \\ -2x & if\ -2 \leq x \leq 3 \\ 2 - x^2 & if\ x < -2 \end{cases}$$

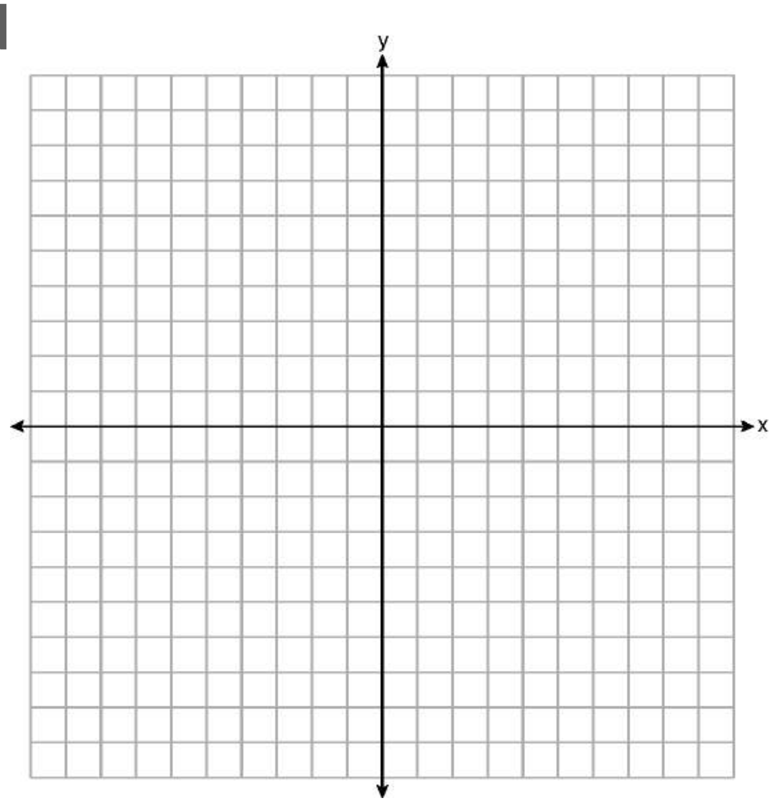
Notice, this function is made up of three pieces. For example, if $x = 7$, which is an $x \geq 5$, $f(7) = 2$. What is the value of $f(x)$ when,

$f(5) =$ $\qquad$ $f(-4) =$ $\qquad$ $f(4) =$ $\qquad$ $f(-2) =$

LET'S GRAPH A PIECEWISE FUNCTION

Graph $f(x) = \begin{cases} x - 1 & if\ x \leq 3 \\ 2 & if\ x > 3 \end{cases}$.

x	$f(x)$
-5	
0	
3	
6	
10	



"TI" TIP | GRAPHING USING YOUR CALCULATOR

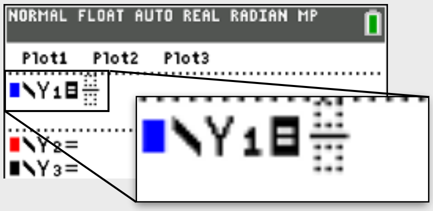
We can use our graphing calculators to help us graph these functions.

1. Make sure the equations are in standard form. Press y=.

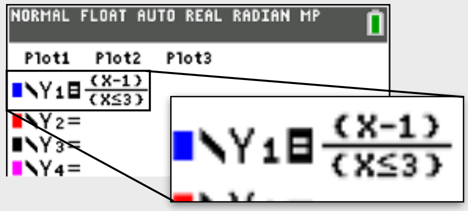
We can input piecewise functions a few different ways. Let's graph the function: $f(x) = \begin{cases} x - 1 & if\ x \leq 3 \\ x^2 & if\ x > 3 \end{cases}$

Method 1: Input each function using fractions, where the numerator is the function and the denominator is the inequality.

1. Input each piece of the function separately. Start by inputing the first equation in Y_1.



2. Press alpha x,T,θ,n to get a fraction.

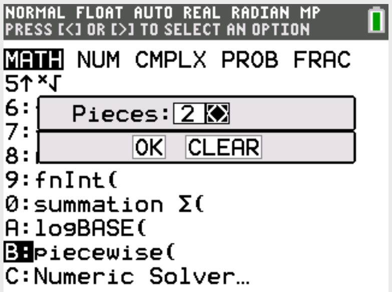


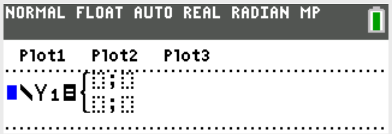
3. Input the equation in the numerator and the inequality in the denominator. **Enclose both in parentheses.** Press enter.

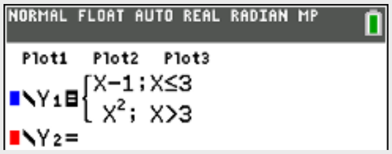
4. Input the remaining pieces.

Method 2: Use the piecewise function, by specifying the number of pieces.

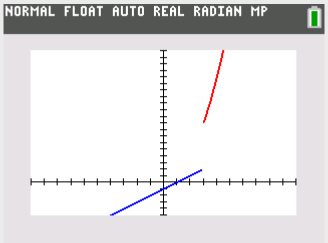
1. Input the equation where it says Y_1.
2. Press math alpha apps → B:piecewise(.
3. Input the number of pieces in your function.
4. Press enter twice.
5. Input the function, followed by the inequality.

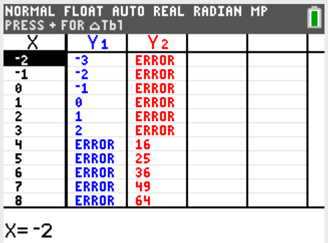






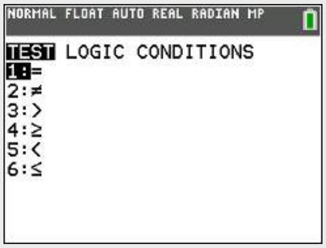
Once you have completed either method:





2. To view your graph, press graph.

3. To view a table of values, press 2nd graph. This shows the points to graph the function. Use this to determine open and closed endpoints.

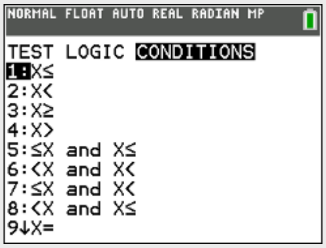


How to input inequalities?

Press 2nd math, select desired inequality.

How to input compound inequalities as conjunctions?

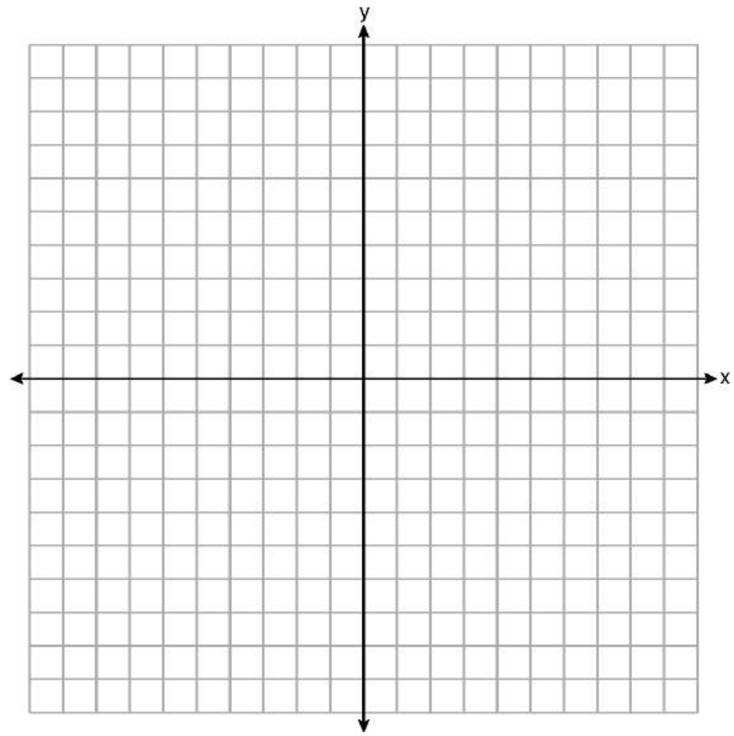
Press 2nd math CONDITIONS, select desired conjunction.



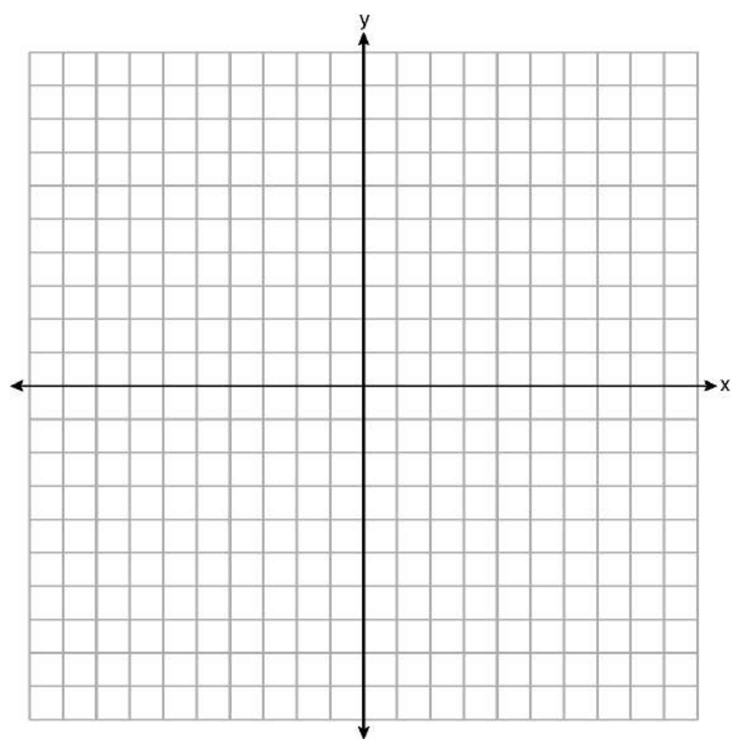
BECOME A PRO

Graph each of the functions

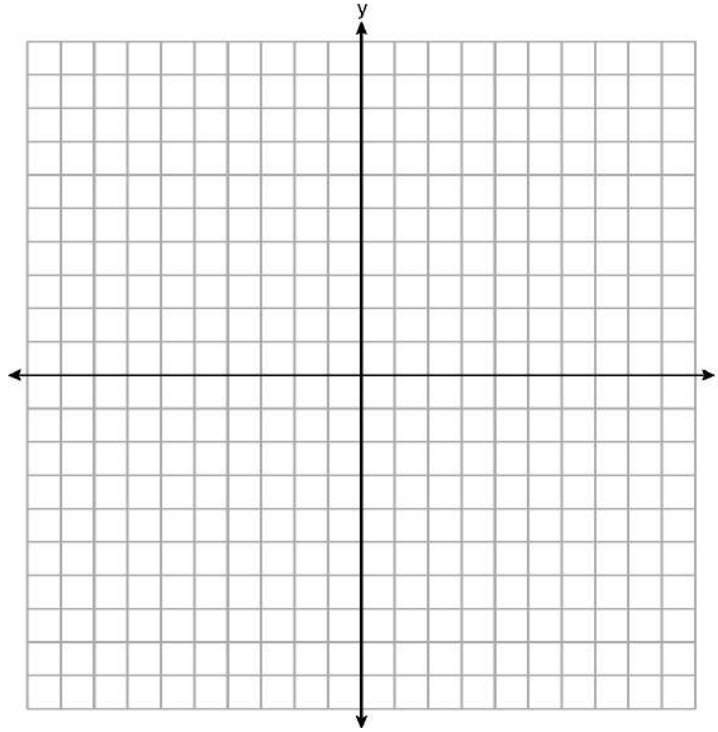
$$f(x) = \begin{cases} 2 & if\ x \geq 5 \\ -2x & if\ -2 \leq x \leq 3 \\ 2 - x^2 & if\ x < -2 \end{cases}$$



$$m(x) = \begin{cases} 2 + x & if\ x < -4 \\ -x & if\ -4 \leq x \leq 2 \\ \frac{x}{3} & if\ x > 2 \end{cases}$$

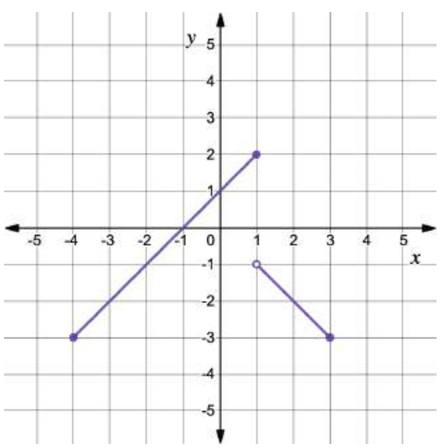


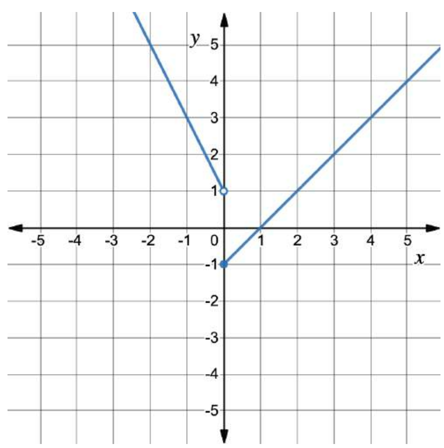
$$f(x) = \begin{cases} x + 5 & if\ x < -2 \\ x^2 + 2x + 3 & if\ \ -x \geq -2 \end{cases}$$

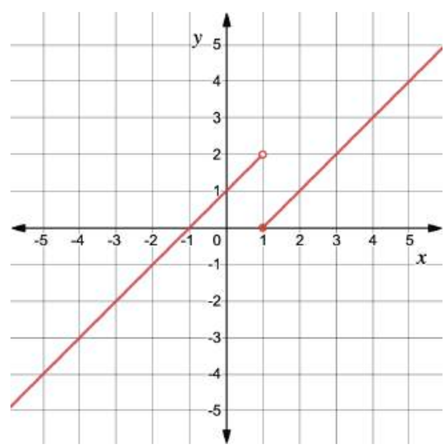


WRITE THE EQUATION

Write the equation represented in the following graphs:







Find the value of $r(x)$ when $x = -5$ and $r(x) = \begin{cases} x+2 & if\ x < 2 \\ 2x+1 & if\ x \geq 2 \end{cases}$

Find the value of $p(x)$ when $x = 4$ and $p(x) = \begin{cases} x+4 & if\ x < 1 \\ 2 & if\ 1 \leq x \leq 4 \\ x-5 & if\ x > 4 \end{cases}$

STRENGTH TRAINING 2

Name: SOLUTIONS

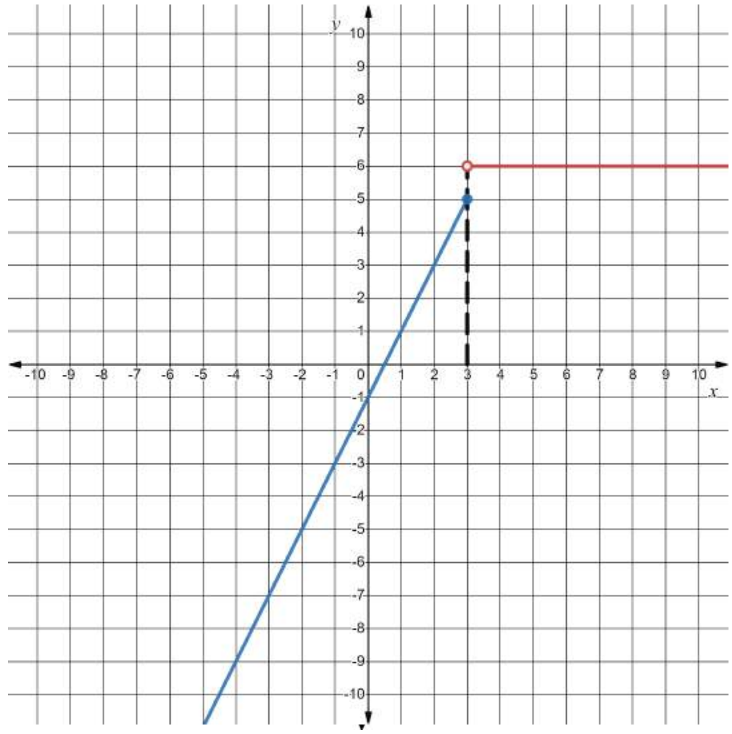
Funky with Functions | Piecewise

Graph each of the functions. Show a table of values.

$$f(x) = \begin{cases} 2x - 1 & if\ x \leq 3 \\ 6 & if\ x > 3 \end{cases}$$

x	$2x - 1$
-3	-7
0	-1
2	3
3	5

x	6
3	undef.
4	6
5	6
6	6

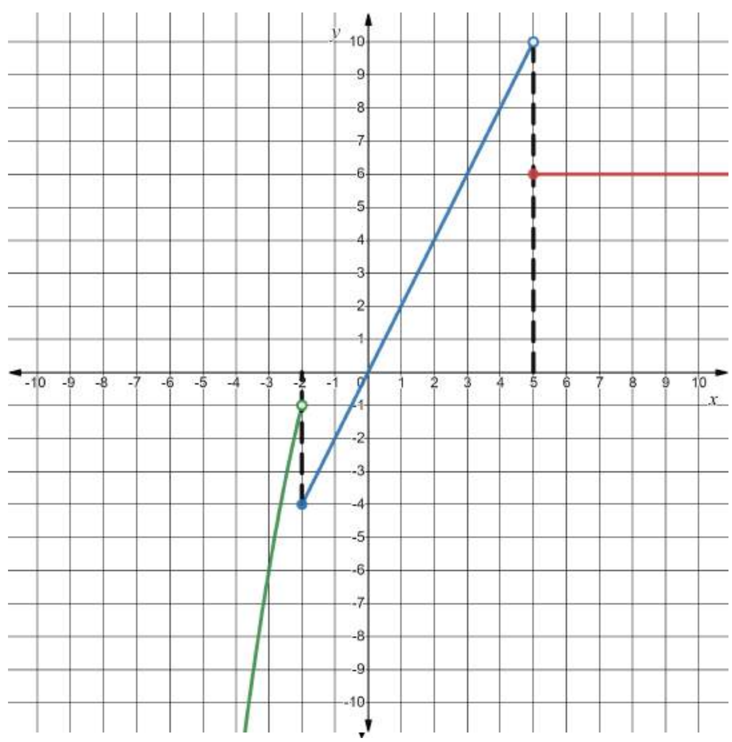


$$m(x) = \begin{cases} 6 & if\ x \geq 5 \\ 2x & if\ -2 \leq x < 5 \\ 3 - x^2 & if\ x < -2 \end{cases}$$

x	$2x$
-2	-7
0	-1
2	3
3	5

x	6
5	6
6	6
7	6

x	$3 - x^2$
-2	-1
-3	-6
-4	-13



Let $f(x) = \begin{cases} -3x & if\ x \geq 6 \\ 2x & if\ \ 0 \leq x < 6 \\ x^2 & if\ x < 0 \end{cases}$, evaluate the following:

$f(6) = -3(6) = -18$

$f(0) = 2(0) = 0$

$f(-1) = (-1)^2 = 1$

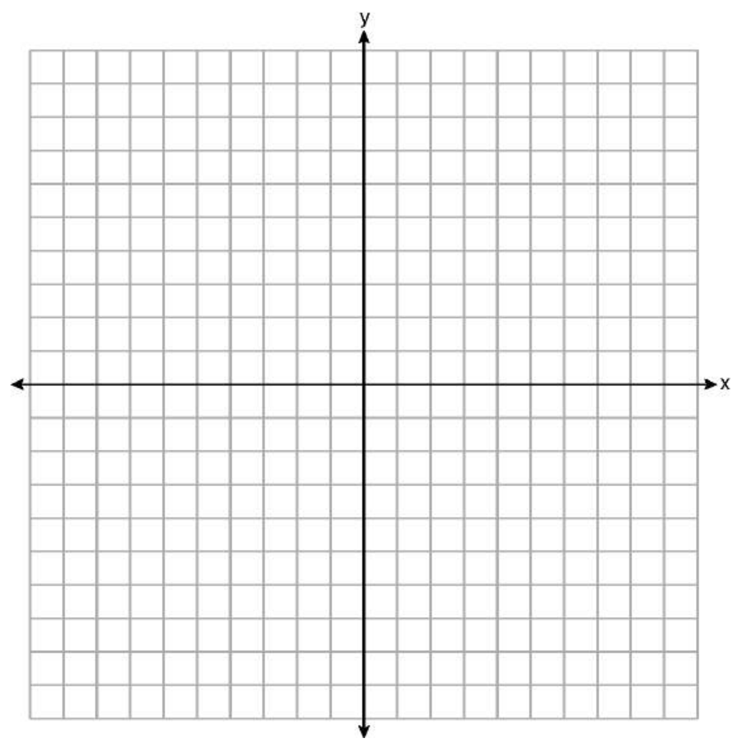
$f(3) = 2(3) = 6$

$f(7) = -3(7) = -21$

$f(x)$, when $x = 1$

$f(1) = 2(1) = 2$

Create your own piecewise function that has ***three*** different pieces. Write the definition and graph your function.



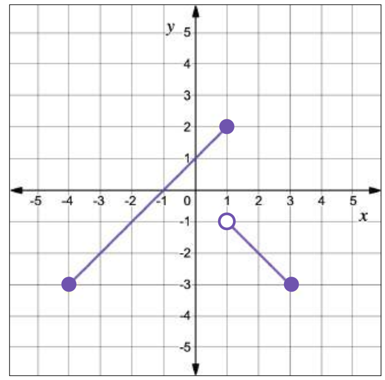
STRENGTH CHECK-IN

Name: SOLUTIONS

Funky with Functions | Review

1. Which of following describes the graph shown?

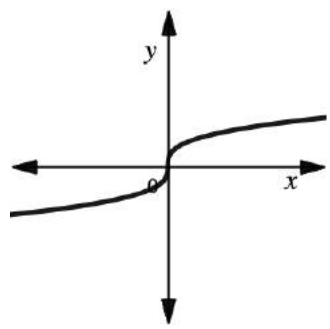
    1. $f(x) = \begin{cases} x & if\ x < 1 \\ -x & if\ 1 < x \leq 3 \end{cases}$
    2. $f(x) = \begin{cases} x & if\ -4 \leq x \leq 1 \\ x^2 & if\ x > 1 \end{cases}$
    3. $f(x) = \begin{cases} x+1 & if\ x \leq -4 \\ -x & if\ x > 1 \end{cases}$
    4. $f(x) = \begin{cases} x+1 & if\ -4 \leq x \leq 1 \\ -x & if\ 1 < x \leq 3 \end{cases}$



4 ________

2. Which of following describes the graph shown?

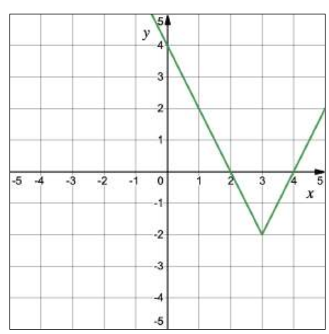
    1. $f(x) = x^2$
    2. $f(x) = x^3$
    3. $f(x) = \sqrt[3]{x}$
    4. $f(x) = |x|$



3 ________

3. What is the domain and range of the function shown?

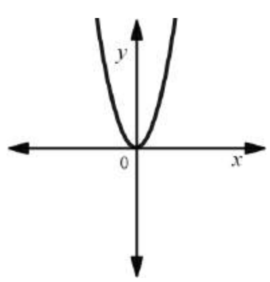
    1. $\{x \mid x \in \mathbb{R}\}, \{y \mid y \geq -2\}$
    2. $\{x \mid x \geq -2\}, \{y \mid y \geq -2\}$
    3. $\{x \mid x \in -2\}, \{y \mid y \geq \mathbb{R}\}$
    4. $\{x \mid x \geq -2\}, \{y \mid y \geq -2\}$



1 ________

4. The function $f(x) = x^2$

    1. increases then decreases.
    2. always increases.
    3. always decreases.
    4. decreases then increases.



4 ________

5. What is the domain of the function $f(x) = \sqrt{x+5} - 1$?
    1. $(-5, \infty)$
    2. $[1, \infty)$
    3. $(-\infty, \infty)$
    4. $[-5, \infty)$

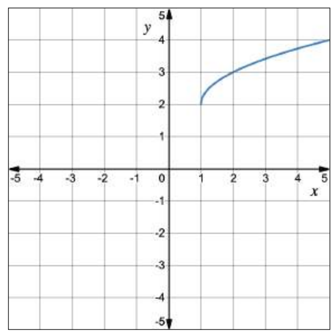
4

6. Let $z(x) = \begin{cases} x^2 - 5 & if\ x \leq -3 \\ 3x + 4 & if\ x > -3 \end{cases}$, what is $z(-3)$?
    1. 4
    2. -5
    3. -3
    4. 6

$z(-3) = (-3)^2 - 5 = 4$

1

7. Which of following describes the graph shown?
    1. $f(x) = \sqrt{x} + 2$
    2. $f(x) = \sqrt{x-1} + 2$
    3. $f(x) = \sqrt{x+2} - 1$
    4. $f(x) = \sqrt{x-1}$



2

8. What is the range of $f(x) = |x+5| - 4$
    1. $\{x \mid x \geq 5\}$
    2. $\{y \mid y \in \mathbb{R}\}$
    3. $\{x \mid x \in \mathbb{R}\}$
    4. $\{y \mid y \geq -4\}$

4

9. State the domain and range of $y = \sqrt{x+4} - 1$

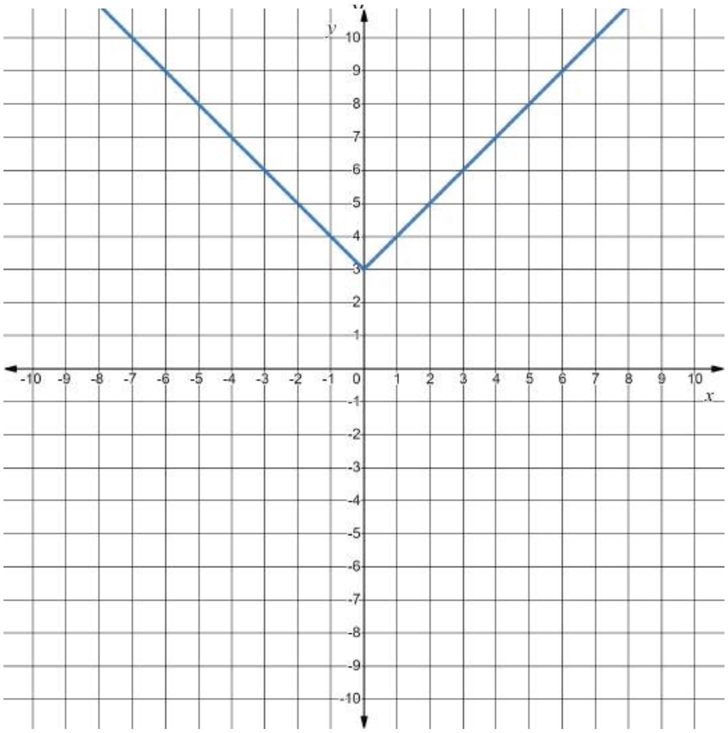
$\{x \mid x \geq -4\}, \{y \mid y \geq -1\}$

10. State the domain and range of $y = \sqrt{x} + 2$

$\{x \mid x \geq 0\}, \{y \mid y \geq 2\}$

11. Graph the following functions and state the domain and range.

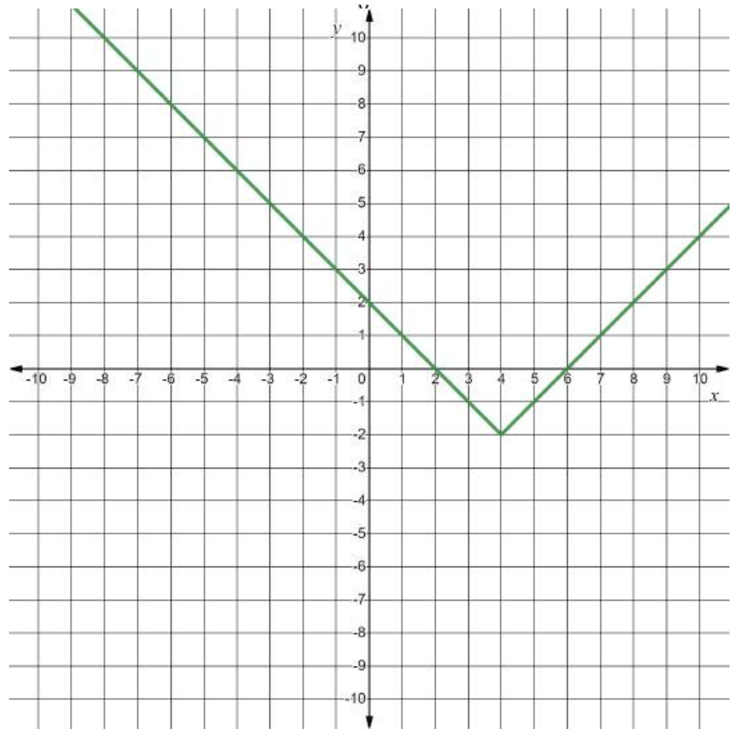
$y = |x| + 3$



Domain: **Range:**

$\{x \mid x \in \mathbb{R}\}, \{y \mid y \geq 3\}$

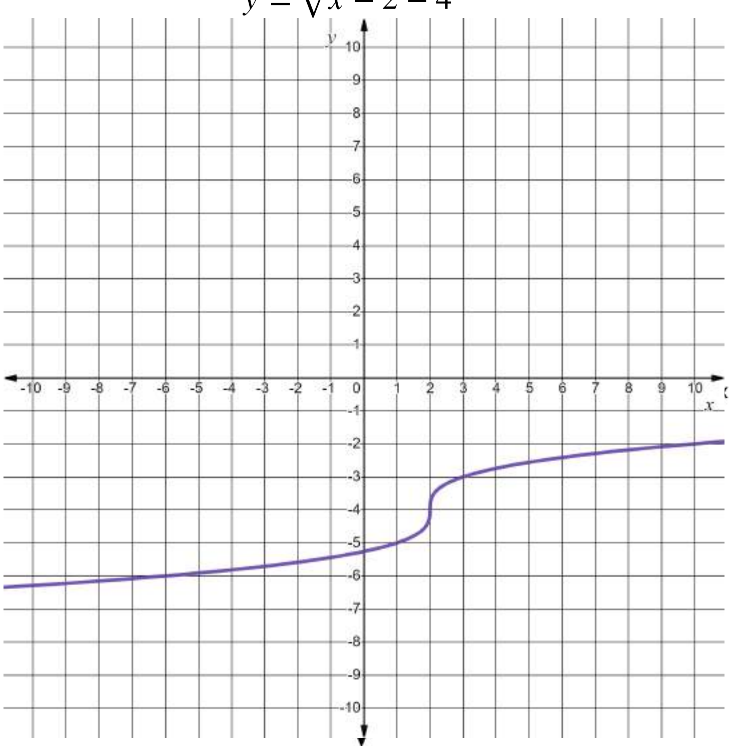
$y = |x - 4| - 2$



Domain: **Range:**

$\{x \mid x \in \mathbb{R}\}, \{y \mid y \geq -2\}$

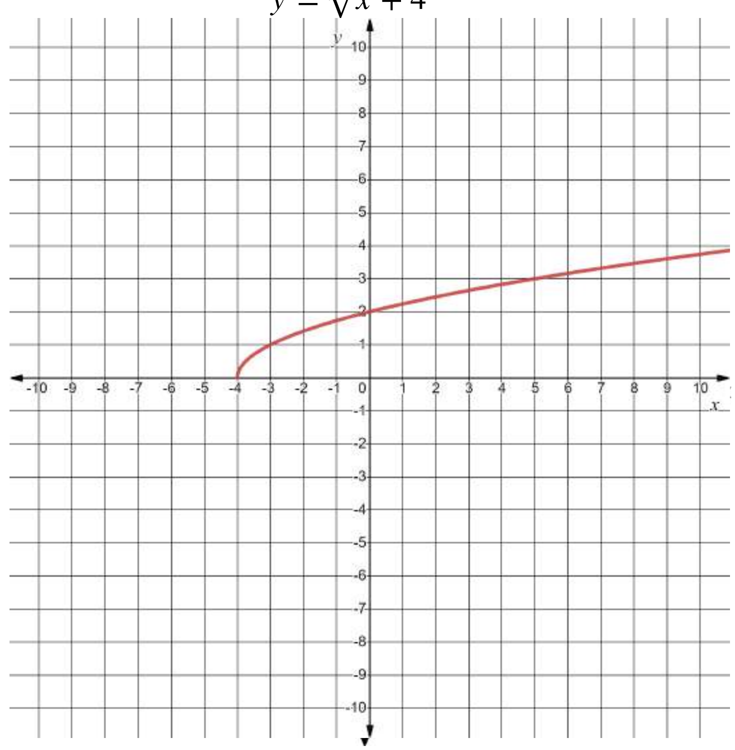
$y = \sqrt[3]{x - 2} - 4$



Domain: **Range:**

$\{x \mid x \in \mathbb{R}\}, \{y \mid y \in \mathbb{R}\}$

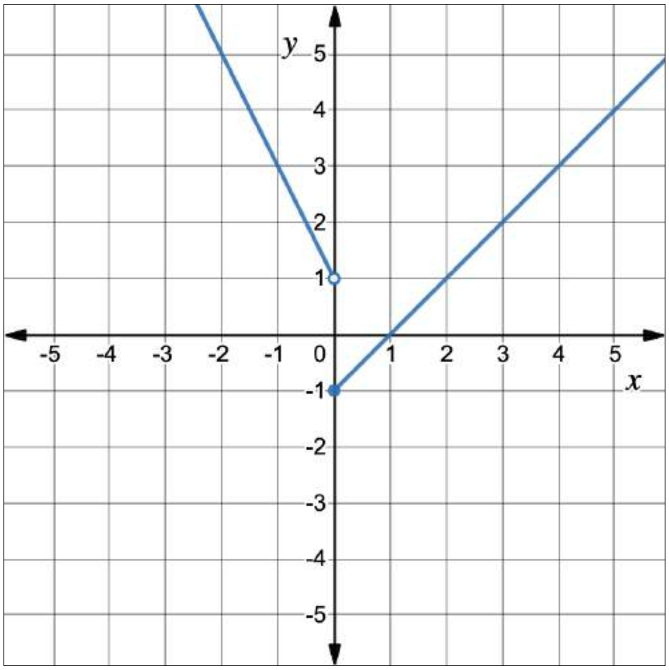
$y = \sqrt{x + 4}$

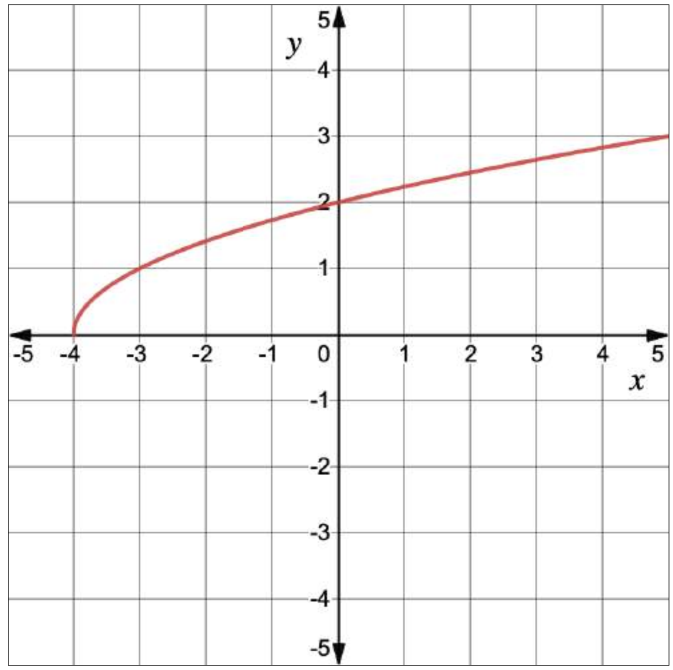


Domain: **Range:**

$\{x \mid x \geq -4\}, \{y \mid y \geq 0\}$

12. Write the function represented in the following graphs:



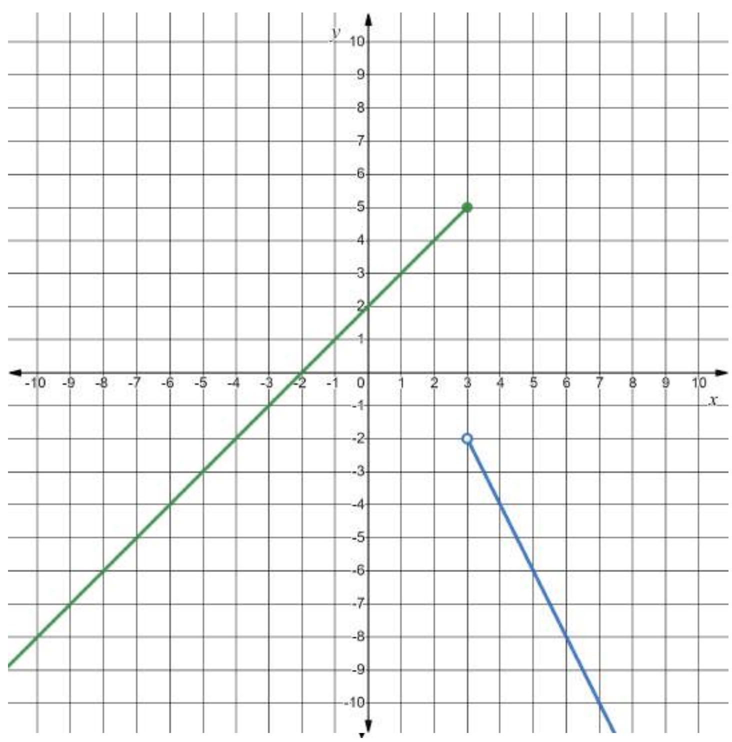


$$f(x) = \begin{cases} -x+1 & if\ x<0 \\ x-1 & if\ x \geq 0 \end{cases}$$

$$y = \sqrt{x+4}$$

13. Graph the following function on the axes provided.

$$m(x) = \begin{cases} x+2 & if\ x \leq 3 \\ -2x+4 & if\ x > 3 \end{cases}$$

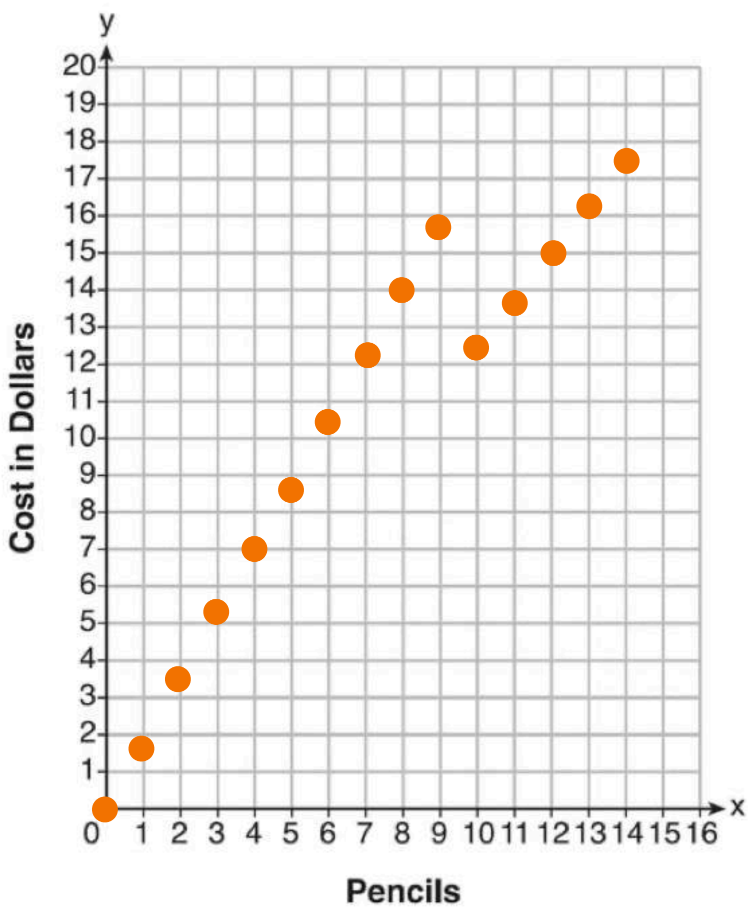


14. At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is \$1.75. If a customer purchases 10 or more pencils, the cost of each pencil is \$1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a ***whole number.***

$$c(x) = \begin{cases} 1.75x & if\ 0 \le x \le 9 \\ 1.25x & if\ x \ge 10 \end{cases}$$

Create a graph of c on the axes provided.

A customer brings 8 pencils to the cashier. The cashier suggests that the total cost to purchase 10 pencils would be less expensive. State whether the cashier is correct or incorrect. Justify.



The cashier is correct. First, by looking at the graph the value of the function at $x = 10$ is less than the fable of the function at $x = 8$. Further,

$$c(8) = 1.75(8) = 14$$

$$c(10) = 1.25(10) = 12.50$$

Notice, $c(8) > c(10)$. The cashier is correct.

15. Let $f(x) = \begin{cases} 6 & if\ x \ge 5 \\ 2x & if\ -2 \le x < 5 \\ 3 - x^2 & if\ x < -2 \end{cases}$, evaluate the following:

$f(-2) = 2(-2) = -4$

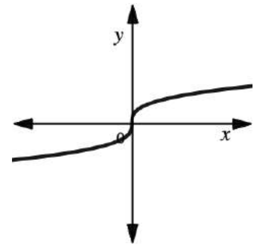
$f(5) = 6$

$f(0) = 2(0) = 0$

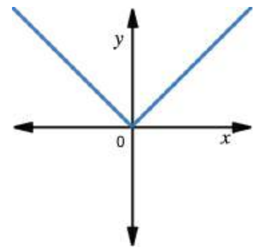
$f(-3) = 3 - (-3)^2 = 3 - 9 = -6$

16. Match the equation on the left with the corresponding graph on the right.

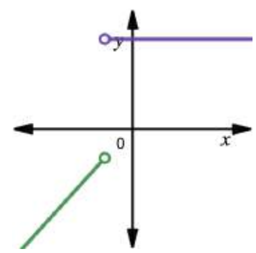
$y = \sqrt[3]{x}$



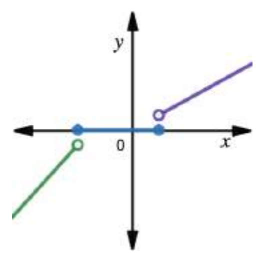
$y = \sqrt{x}$



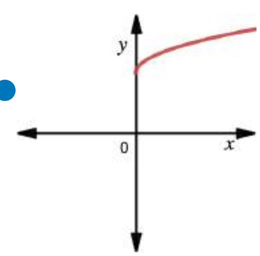
$$c(x) = \begin{cases} x & if\ x < -2 \\ 6 & if\ x > -2 \end{cases}$$



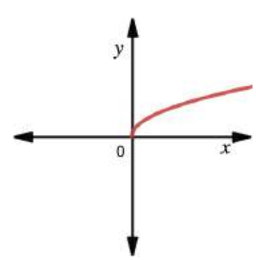
$y = |x|$



$y = \sqrt{x} + 4$



$$f(x) = \begin{cases} x + 3 & if\ x < -4 \\ 0 & if\ -4 \le x \le 2 \\ \frac{1}{2}x & if\ x > 2 \end{cases}$$



FUNCTION FINDER

Find the graph - Exploring Absolute Value

Function mayhem has ensued! Mr. Caruso has lost it and has posted graphs of functions around the room. Each function has a corresponding letter. These letters form a secret message. Below there is a collection of equations and tables of values. Find the corresponding graph and place the letter on the blank with the corresponding number. You can use your graphing calculator.

1. $y = \frac{1}{4}|x|$

2. $y = -\frac{1}{4}|x|$

3. $y = |x+4|$

4. $y = |x+2|$

5. $y = |x| + 2$

6. $y = |x| + 4$

7. $y = |x| - 2$

8. $y = |x+3| + 3$

9. $y = |x+3| - 3$

10. $y = |x-3| + 3$

11. $y = |x-3| - 3$

12. $y = |x-2|$

13.

x	y
-2	2
-1	1
0	0
1	1
2	2

14.

x	y
-2	-2
-1	-1
0	0
1	-1
2	-2

15.

x	y
-2	6
-1	3
0	0
1	3
2	6

16.

x	y
-2	-6
-1	-3
0	0
1	-3
2	-6

WHAT'S THE SECRET MESSAGE?

13	7	12	5	4

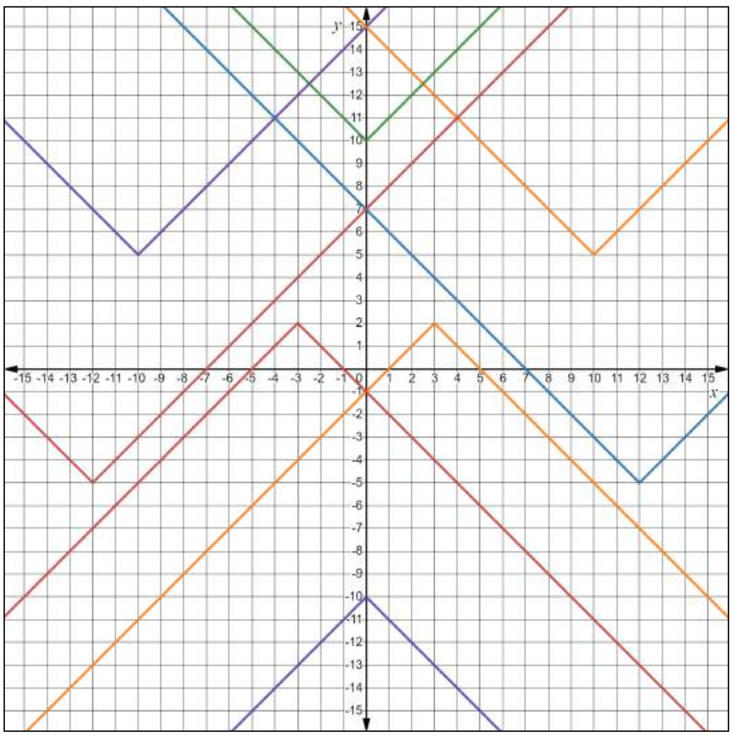
14	16	15	1

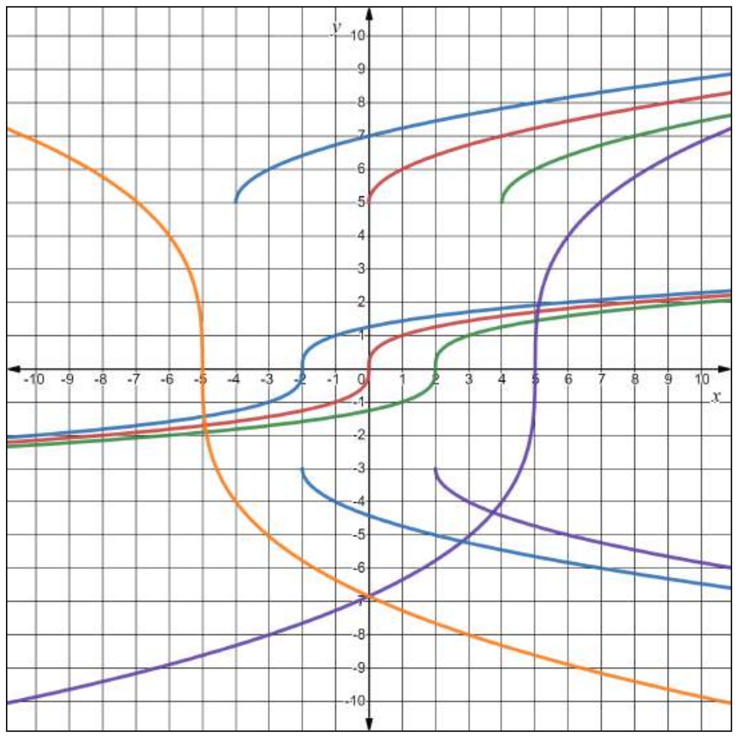
2	9	10	8	6	3	11

COUNTERFEIT GRAPHICAL ART

Produce a Graphical Masterpiece - Can you spot the fake?

Counterfeit art, or art forgery, is a serious issue for art collectors and museum. It can spark awe and anger. In this activity, you have been tasked with forging the artwork of famed mathematical artist, ***Pascal the Painter***.





Use Desmos to help forge the artwork. Submit an image of each counterfeit art piece to the Google Classroom. We shouldn't be able to spot the fake.

GET CREATIVE

Using functions that use either absolute value, square roots, or cube roots, make your own graphical masterpiece. It must be comprised of at least 6 different functions. Share an image of your masterpiece to the Counterfeit Art Gallery JamBoard.

PIECEWISE COASTERS

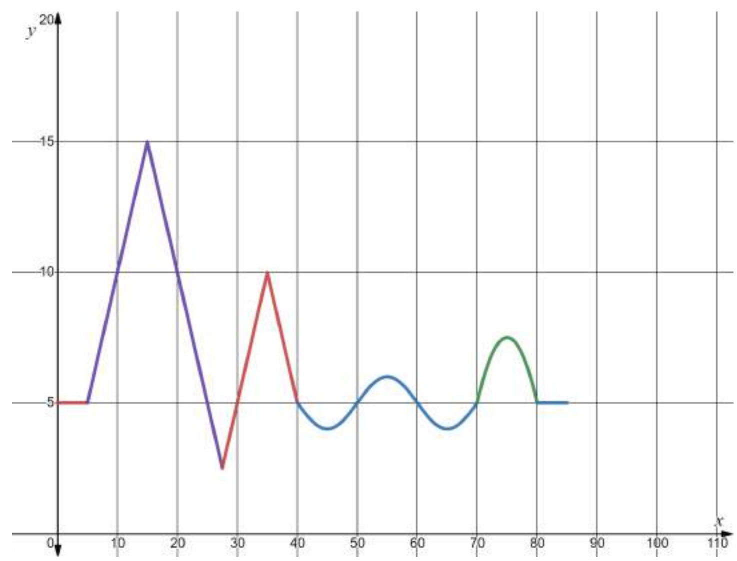
Create Some Crazy Coasters

A staple of growing up in Chautauqua County is concurring *The Little Dipper* roller coaster at Midway State Park. With its max height of a whopping 10 feet, this steel family coaster is a classic.

Families are seeking coasters with more thrill, while still being family friendly. It's your turn to design the next Midway State Park coaster classic.

Using a piecewise function, define a function that describes the height of the coaster as a function of time, x. This will describe the layout of the coaster. Here is an example:

$$c(x) = \begin{cases} 5 & if \ \ 0 \leq x \leq 5 \\ -|x-15|+15 & if \ \ 5 < x \leq 27.5 \\ -|x-35|+10 & if \ \ 27.5 < x \leq 40 \\ \cos(\frac{1}{10}\pi x - 11)+5 & if \ \ 40 < x \leq 70 \\ -\frac{1}{10}(x-70)(x-80)+5 & if \ \ 70 < x \leq 80 \\ 5 & if \ \ 80 < x \leq 85 \end{cases}$$



Use Desmos to graph your piecewise coaster function. When you are done share an image of your coaster to the Coaster Gallery JamBoard.

REQUIREMENTS

- Coaster must start and end at the same height. The stations at the start and end of the coaster must be at least 5 units long.
- No loops, it must be a function.
- Must utilize at least 6 different functions (including the stations)

Complete the coaster spec. sheet and turn in for your lab

PIECEWISE COASTER SPEC. SHEET

COASTER NAME:

Designer Names:

Write the formal definition for your coaster function:

On what intervals is the coaster increasing and decreasing?

What are the local extrema of your coaster?

Would this be a feasible layout? Explain why.

SHOW OFF WHAT YOU KNOW

Name: SOLUTIONS

Gettin' Funky with Functions | Exam

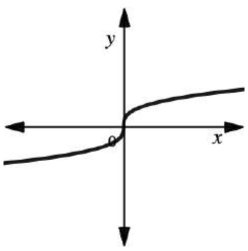
Part 1: Multiple Choice | Show your work. Partial credit may be awarded. **[2 pts. each]**

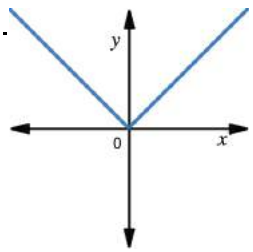
1. What is the range of $f(x) = |x-3| + 2$?

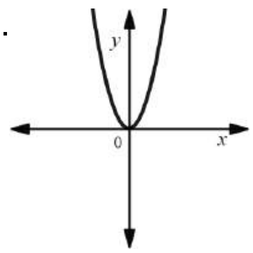
   1. $\{y \mid y \geq 2\}$ 2. $\{x \mid x \in \mathbb{R}\}$ 3. $\{y \mid y \in \mathbb{R}\}$ 4. $\{x \mid x \geq 3\}$

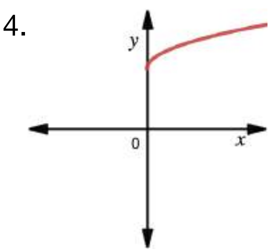
   1

2. Which of following best represents a function involving the absolute value of a variable?

   1. 

   2. 

   3. 

   4. 

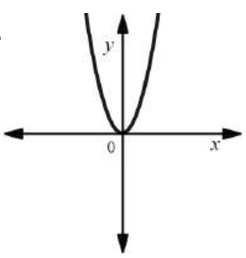
   2

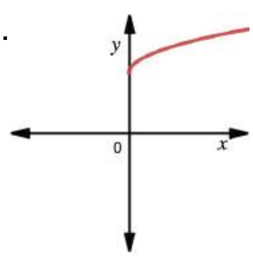
3. What is the domain of $f(x) = \sqrt{x-2} + 3$?

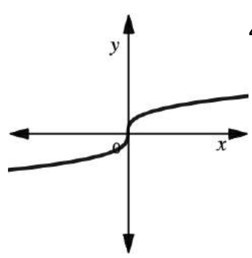
   1. $(2,\infty)$ 2. $[3,\infty)$ 3. $(-\infty,\infty)$ 4. $[2,\infty)$

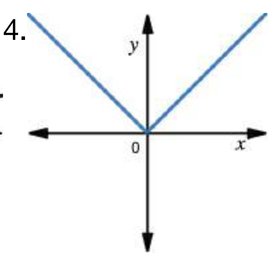
   4

4. Which of following best represents a function involving a square root of a variable?

   1. 

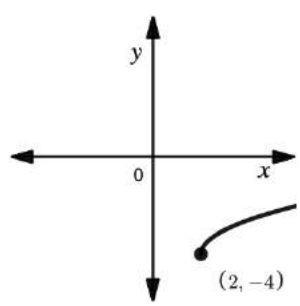
   2. 

   3. 

   4. 

   2

5. What is the domain and range of the function shown?
   1. $\{x \mid x > 2\}; \{y \mid y > -4\}$
   2. $\{x \mid x \geq -4\}; \{y \mid y \geq 2\}$
   3. $\{x \mid x \geq 2\}; \{y \mid y \geq -4\}$
   4. $\{x \mid x > -4\}; \{y \mid y > 2\}$

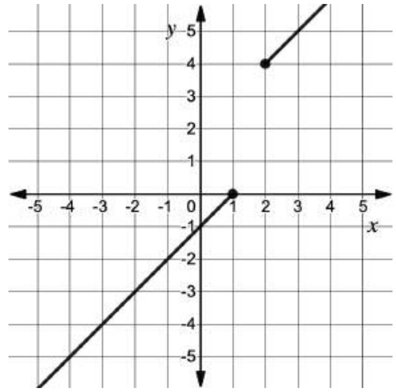
   3

6. The function $f(x) = x^3$
   1. increases then decreases.
   2. always increases.
   3. always decreases.
   4. decreases then increases.

   2

7. Which of the following describes the graph shown?

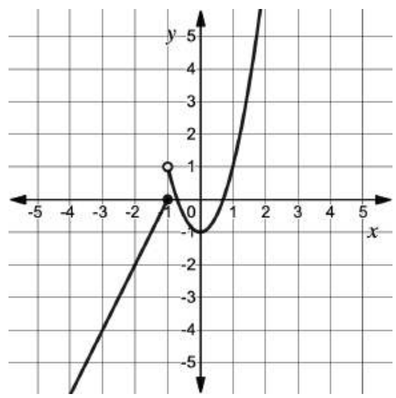
1. $f(x) = \begin{cases} 2x & if\ x \geq 2 \\ -x & if\ x \leq 1 \end{cases}$
2. $f(x) = \begin{cases} x+2 & if\ x \geq 2 \\ x-1 & if\ x \leq 1 \end{cases}$
3. $f(x) = \begin{cases} 2 & if\ x \geq 2 \\ -1 & if\ x \leq 1 \end{cases}$
4. $f(x) = \begin{cases} 2 & if\ x \geq 2 \\ 1 & if\ x \leq -1 \end{cases}$



2

8. Which of the following describes the graph shown?

1. $f(x) = \begin{cases} \frac{1}{2}x+2 & if\ x \geq -1 \\ \frac{1}{2}x^2-1 & if\ x \leq 1 \end{cases}$
2. $f(x) = \begin{cases} 2x+2 & if\ x \leq -1 \\ 2x^2-1 & if\ x > -1 \end{cases}$
3. $f(x) = \begin{cases} \frac{1}{2}x+2 & if\ x \leq -1 \\ \frac{1}{2}x^2-1 & if\ x > -1 \end{cases}$
4. $f(x) = \begin{cases} 2x+2 & if\ x < -1 \\ 2x^2-1 & if\ x \geq -1 \end{cases}$



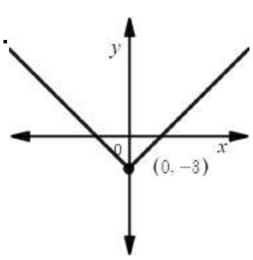
2

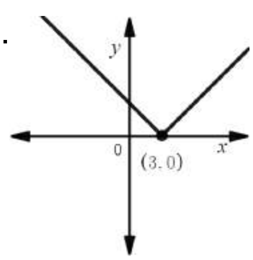
9. Let $f(x) = \begin{cases} x^2+1 & if\ x \leq -1 \\ 2x-3 & if\ x > -1 \end{cases}$, what is $f(-2)$?

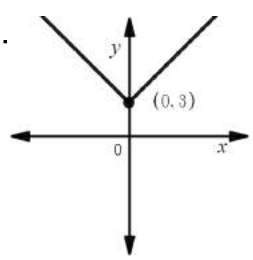
1. 5 2. -5 3. -3 4. -7

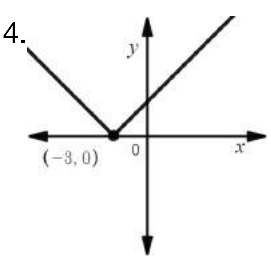
1

10. Which of the following is the graph for $y = |x| - 3$?

1. 

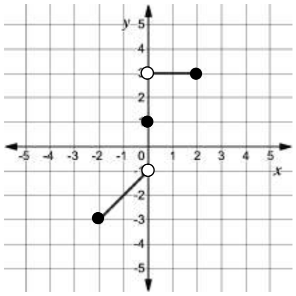
2. 

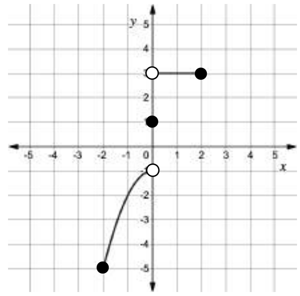
3. 

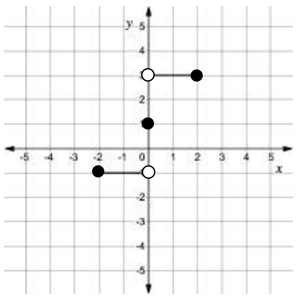
4. 

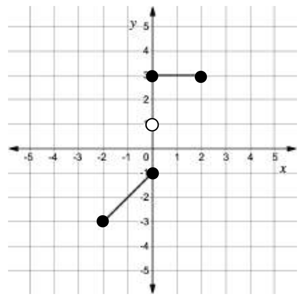
1

11. Which one of the following graphs is the graph of $f(x) = \begin{cases} x-1 & if\ \ -2 \le x < 0 \\ 1 & if\ \ x = 0 \\ 3 & if\ \ 0 < x \le 2 \end{cases}$?

1. 

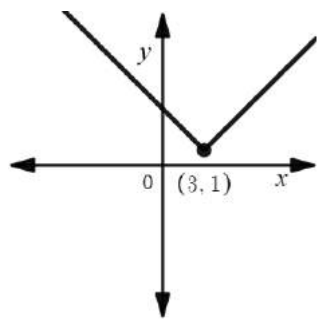
2. 

3. 

4. 

1

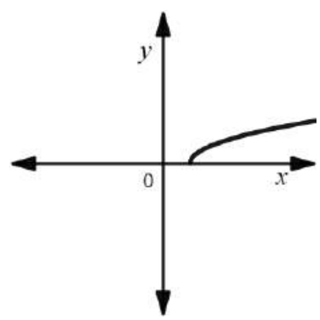
12. Which of the following describes the graph shown?



1. $y = |x+3| - 1$
2. $y = |x-3| + 1$
3. $y = (x-3)^2 + 1$
4. $y = |x| - 3$

2

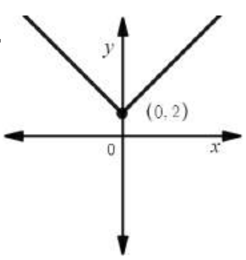
13. Which of the following best describes the graph shown?

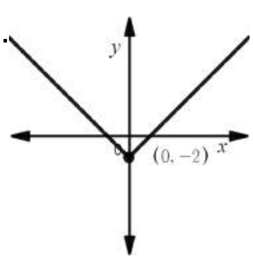


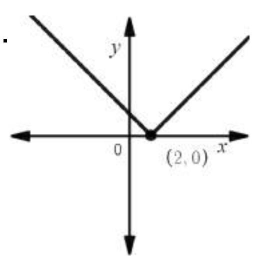
1. $y = \sqrt{2-x}$
2. $y = -\sqrt{2-x}$
3. $y = \sqrt{x-2}$
4. $y = -\sqrt{x-2}$

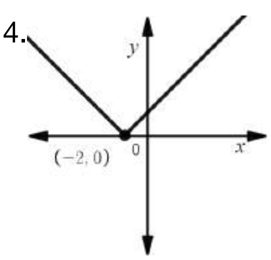
3

14. Which of the following is a graph of $y = |x+2|$?

1. 

2. 

3. 

4. 

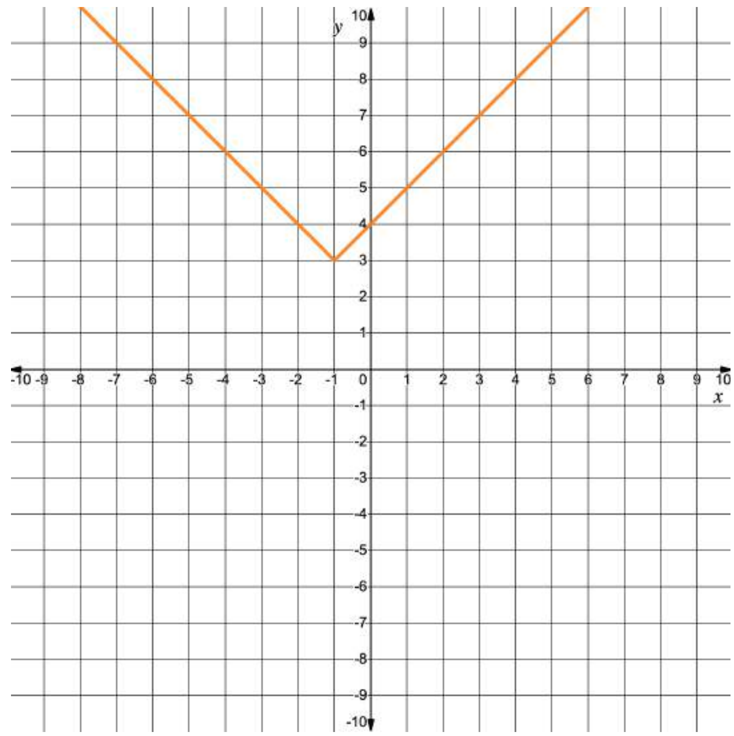
4

Part 2: Graphing | Show your work. Partial credit may be awarded. **[10 pts. each]**

Graph $f(x) = |x + 1| + 3$ on the provided axes and state the **domain** and **range**.

$\{x \mid x \in \mathbb{R}\}$

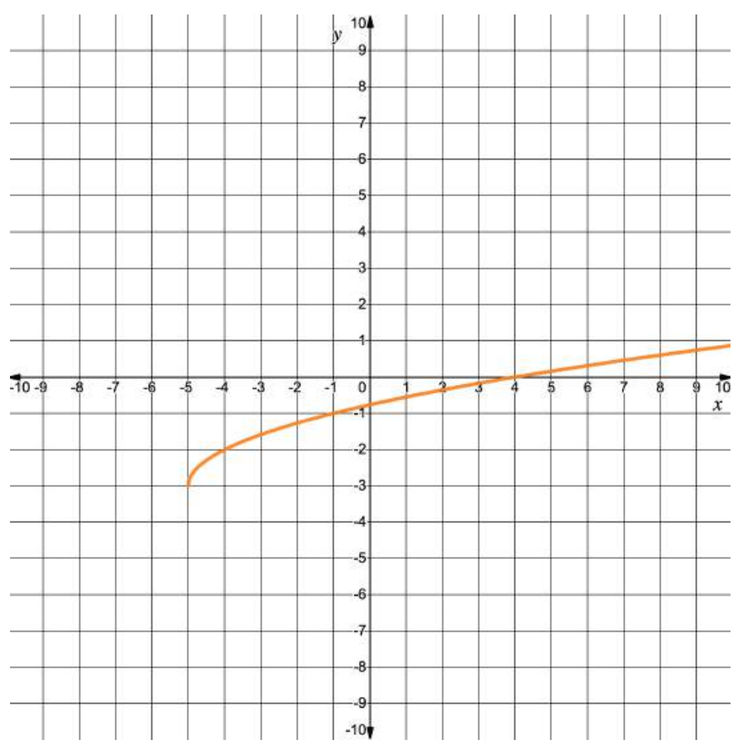
$\{y \mid y \geq 3\}$



Graph $f(x) = \sqrt{x + 5} - 3$ on the provided axes and state the **domain** and **range**.

$\{x \mid x \geq -5\}$

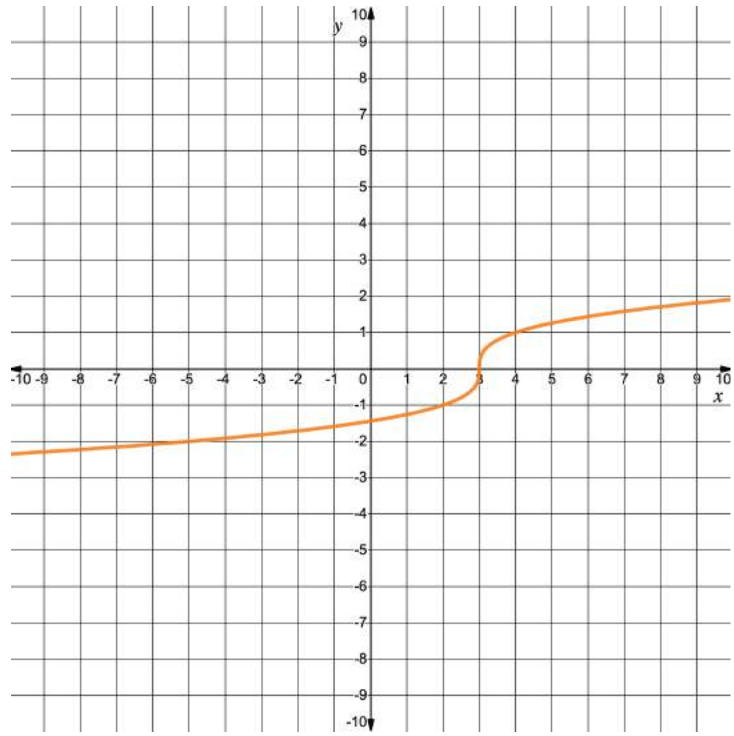
$\{y \mid y \geq -3\}$



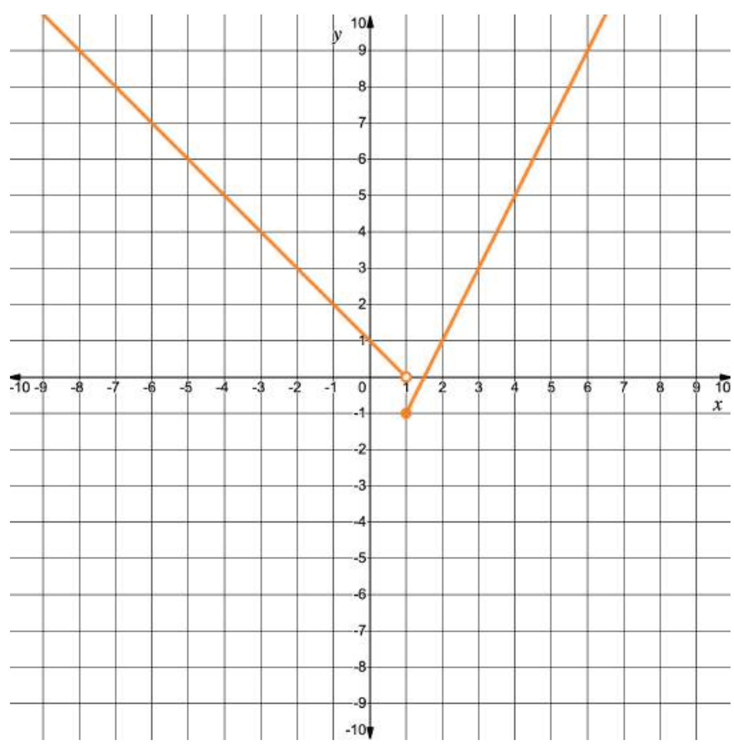
Graph $f(x) = \sqrt[3]{x - 3}$ on the provided axes and state the **domain** and **point of inflection.**

$\{x \mid x \in \mathbb{R}\}$

(3,0)



Graph $f(x) = \begin{cases} 2x - 3 & if\ x \geq 1 \\ -x + 1 & if\ x < 1 \end{cases}$ on the provided axes.



Part 3A: Open Ending | Show your work. Partial credit may be awarded. **[2 pts. each]**

Let $f(x) = \begin{cases} x^3 - 2 & if\ x \leq 0 \\ x^2 & if\ \ 0 < x \leq 1 \\ 2x - 1 & if\ x > 1 \end{cases}$, find the following values of $f(x)$:

$f(1) =$ $1^2 = 1$

$f(0) =$ $(0)^3 - 2 = -2$

$f(-1) =$ $(-1)^3 - 2 = -3$

$f(\frac{1}{2}) =$ $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$f(-\frac{1}{2}) =$ $\left(-\frac{1}{2}\right)^3 - 2 = -2\frac{1}{8}$

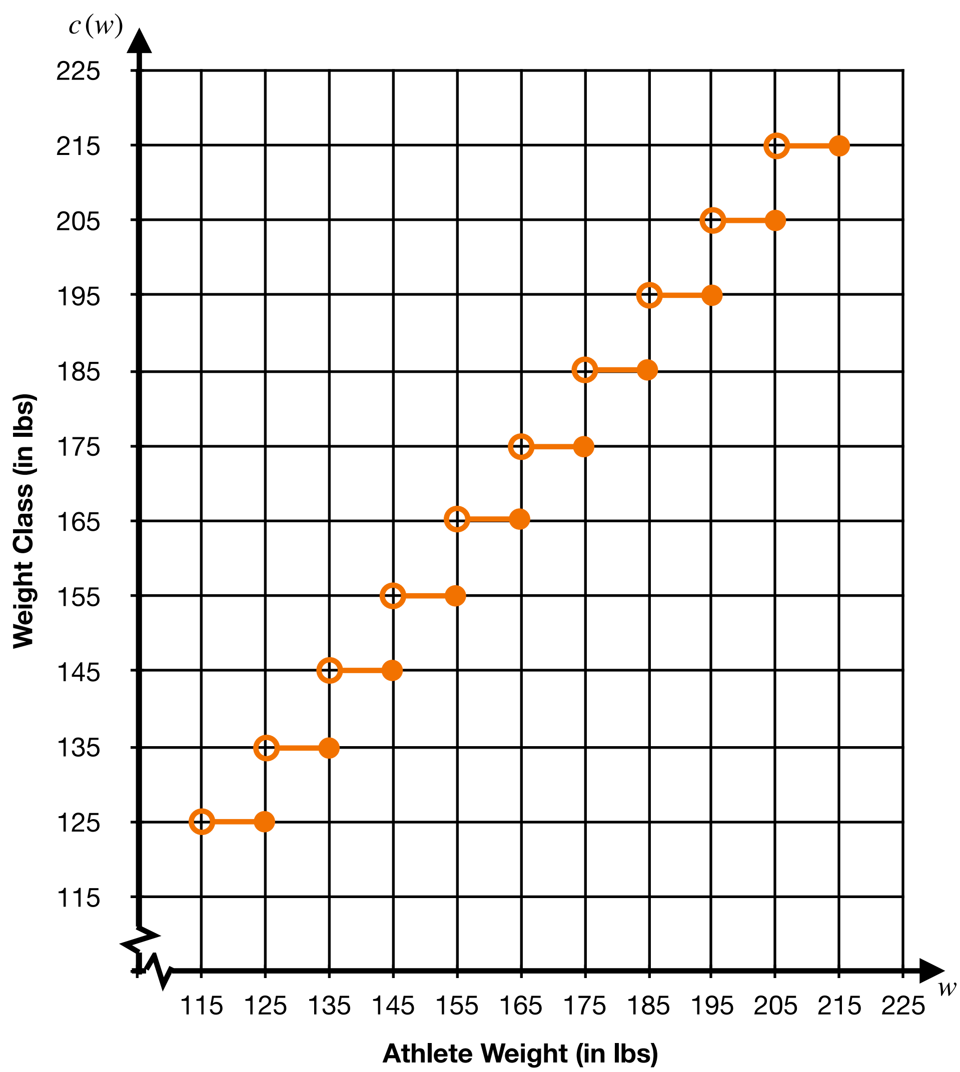
$f(2) =$ $2(2) - 1 = 3$

Part 3B: Open Ending | Show your work. Partial credit may be awarded. **[20 pts. Total]**

The Westfield Wolverine Wrestling Team is off to the first annual *Caruso Classic* wrestling tournament. Mr. Caruso knows little about wrestling, but he was able to define a piecewise function to help him sort the athletes into the appropriate weight classes. His function $c(w)$ is define below, where w is the wrestler's weight in lbs., and $c(w)$ is the the wrestler's wrestling class, which is also specified as a weight, for example "155 lbs. weight class".

Graph the weight class function, $c(w)$, on the provided axes.

$$c(w) = \begin{cases} 125 & if\ \ 115 < w \leq 125 \\ 135 & if\ \ 125 < w \leq 135 \\ 145 & if\ \ 135 < w \leq 145 \\ 155 & if\ \ 145 < w \leq 155 \\ 165 & if\ \ 155 < w \leq 165 \\ 175 & if\ \ 165 < w \leq 175 \\ 185 & if\ \ 175 < w \leq 185 \\ 195 & if\ \ 185 < w \leq 195 \\ 205 & if\ \ 195 < w \leq 205 \\ 215 & if\ \ 205 < w \leq 215 \end{cases}$$



Fill in the following table with the correct weight classes for each Athlete, using $c(w)$:

	Weight (in lbs)	**Weight Class** (in lbs)
Mason	172.0	**175**
Jack	145.0	**145**
Marty	205.4	**215**
Lance	126.0	**135**
Bryce	137.8	**145**

Extra-Curricular Qualifications and Skills

Let’s put on a show!

Since learning to play the trumpet in 4th Grade, music has always been a central part of my life. In fact, a young Brandon once wanted to become a High School music teacher. Since then, I have been actively engaged in the local theatre, playing trumpet for or performing in over 20 different productions in Chautauqua County, both community theatre and local high school productions. I’m eager to lead a hand and support my students in any way possible, from pit orchestra to backstage sound and lighting. I look forward to helping support the arts in any school I’m a part of. Whether it is assisting with a marching band, the school musical or play, or perhaps visual communication, web/graphic design club.

Get Techie with it!

Technology is my second love. I have worked with kids from K to 12 and beyond, introducing them to 3D printing, computer programming, design, and more. Our students interact with technology almost every minute of their lives, it's time they jump in to understand what makes this stuff work. Introducing our students to computer science and the social issues that surround widespread technology is critical. I look forward to developing a computer science curriculum in any school I’m a part of. From after-school coding club to elective coursework in CS, I can’t wait to open the eyes of my students to what they can create and the possibilities that abound with technology.

Brandon Caruso

New York State Initial Certificate,
Mathematics 7-12
Background in Computer Science

40 N Ralph Ave., Falconer, NY 14733
716.397.7515 | brandonjcaruso1@gmail.com