# Gravitational Waves Project

Signal Processing (Prof. Dr. A. Coillet)

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**Abstract**

This project analyzes gravitational wave signals from GW190814 (black hole merger) and GW170817 (binary neutron star inspiral) by implementing numerical methods in MATLAB for time-domain, frequency-domain, and spectrogram analyses, along with power spectral density and energy estimation. The time-domain analysis revealed characteristic chirp patterns: a short, sharp signal for GW190814 and a longer, gradual signal for GW170817. Frequency-domain analysis, implemented via the Fast Fourier Transform (FFT), identified spectral peaks at 500 Hz and 1000 Hz for GW190814, and 500 Hz, 1000 Hz, and 1500 Hz for GW170817. Spectrograms were created using sliding FFTs, and confirmed the characteristic upward frequency sweeps of these events. Energy estimation, based on numerical integration of the strain squared, yielded small values due to the faint nature of gravitational wave signals at Earth. The results demonstrate the effectiveness of MATLAB-based signal processing techniques in characterizing extreme astrophysical events and align with the results published by LIGO.

## 1 Introduction

Gravitational waves, predicted by Einstein's general relativity, are ripples in spacetime caused by accelerating massive objects such as merging black holes or neutron stars. The detection of gravitational waves provides an unprecedented way to study astrophysical phenomena and test fundamental physics [3, 1, 2].

In the linearized approximation of general relativity, gravitational waves are described as small perturbations $h_{\mu\nu}$ to the flat spacetime metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

$h_{\mu\nu}$ satisfies the wave equation in vacuum, therefore, in the transverse-traceless (TT) gauge, a gravitational wave propagating in the $z$-direction takes the form [2]:

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cos(\omega t - kz) \tag{2}$$

Where $h_+$ and $h_\times$ are the two polarization modes of the wave. These modes stretch and compress spacetime in perpendicular directions transverse to the wave's propagation.

The Laser Interferometer Gravitational-Wave Observatory (LIGO) uses Michelson interferometers to detect these spacetime perturbations.

Gravitational waves induce small changes in the relative lengths of the interferometer arms, producing a measurable strain:

$$h(t) = \frac{\Delta L}{L} \tag{3}$$

Where $\Delta L$ is the change in arm length caused by the wave, and $L$ is the original arm length. For LIGO's 4-km-long arms, these changes are on the order of $10^{-21}$ [2].

Also, the energy radiated by gravitational waves was estimated using:

$$E_{\text{GW}} = \frac{c^3}{G} \int h^2(t)\, dt \tag{4}$$

where $h(t)$ is the strain, and $c = 3 \times 10^8\,\text{m/s}$ and $G = 6.674 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$ are the speed of light and gravitational constant, respectively.

The strain data $h(t)$ recorded by LIGO represents the relative change in length over time due to passing gravitational waves, combined with noise from environmental and instrumental sources. To extract meaningful signals from this data, Fourier analysis and matched filtering are applied [2]. In the present project, a Hanning window function is used as filter to improve results visualisation.

The two events analyzed in this report, GW190814 and GW170817, showcase the power of gravitational wave astronomy. GW190814, a black hole merger, produced a short-duration, high-frequency chirp signal [3]. GW170817, a neutron star merger, exhibited a longer-duration signal with lower-frequency components [1]. Both events were detected using advanced signal processing techniques and represent milestones in gravitational wave science [2].

# 2 Methodology

- For the analysis of gravitational wave strain data recorded by LIGO, it was developed the MATLAB script `"BECOGravitationalWavesLIGOProject.m"` to process the signals from two distinct events: GW190814 (a black hole merger) and GW170817 (a neutron star merger). Signals were stored in the datasets named after each event and its corresponding detector (L1 for Livingston and H1 for Hanford): **GW190814-L1-BH**, **GW190814-H1-BH**, **GW170817-L1-NS**, **GW170817-H1-NS**. The script can be found in the following GitHub repository: `https://github.com/bcastiblancoo/LIGO-Gravitational-Waves-Project/tree/main`.
- Each dataset contains strain data sampled at a frequency $F_s = 4096\,\text{Hz}$, with a corresponding sampling period $T_s = 1/F_s$.
- The analysis was performed by the following steps:
  1. **Time-Domain Analysis:** The time-domain signal was plotted to observe the temporal evolution of the strain.
  2. **Frequency-Domain Analysis:** The frequency content of the signals was analyzed using the Fast Fourier Transform (FFT), and plotting the modulus spectrum.
  3. **Spectrogram Analysis:** Then, to capture the temporal evolution of the frequency content a sliding Fourier transform was computed. A Hanning window of size $N_w = 2^{11}$ was used, with a step size of $N_w/16$. The spectrogram matrix was constructed as:

     $$\text{FFT}(t_i, f_k) = \text{FFT}\left(\text{window} \cdot h(t)\right), \tag{5}$$

     where $i$ denotes the time slice and $k$ denotes the frequency bin.

     The spectrograms were plotted in logarithmic (dB) and linear scale, focusing on the frequency range $40\,\text{Hz} \leq f \leq 250\,\text{Hz}$ to highlight the gravitational wave signal.
  4. **Power Spectral Density (PSD):** The Power Spectral Density (PSD) was computed using the periodogram method to quantify the distribution of signal power across frequencies. It identifies key frequency contributions from gravitational wave signals and noise, expressed in decibels as:

     $$P_{\text{dB}}(f) = 10 \log_{10}(P_{xx}(f)), \tag{6}$$

     Where $P_{xx}(f)$ is the power spectral density at frequency $f$.
  5. **Energy Estimation:** By approximating numerically the integral in eq. (4) as:

     $$\int h^2(t)\, dt \approx \sum h^2(t_n) \Delta t, \tag{7}$$

     With $\Delta t = 1/F_s$, where $F_s$ is the sampling frequency. The results reflect only the detected portion of the gravitational wave signal

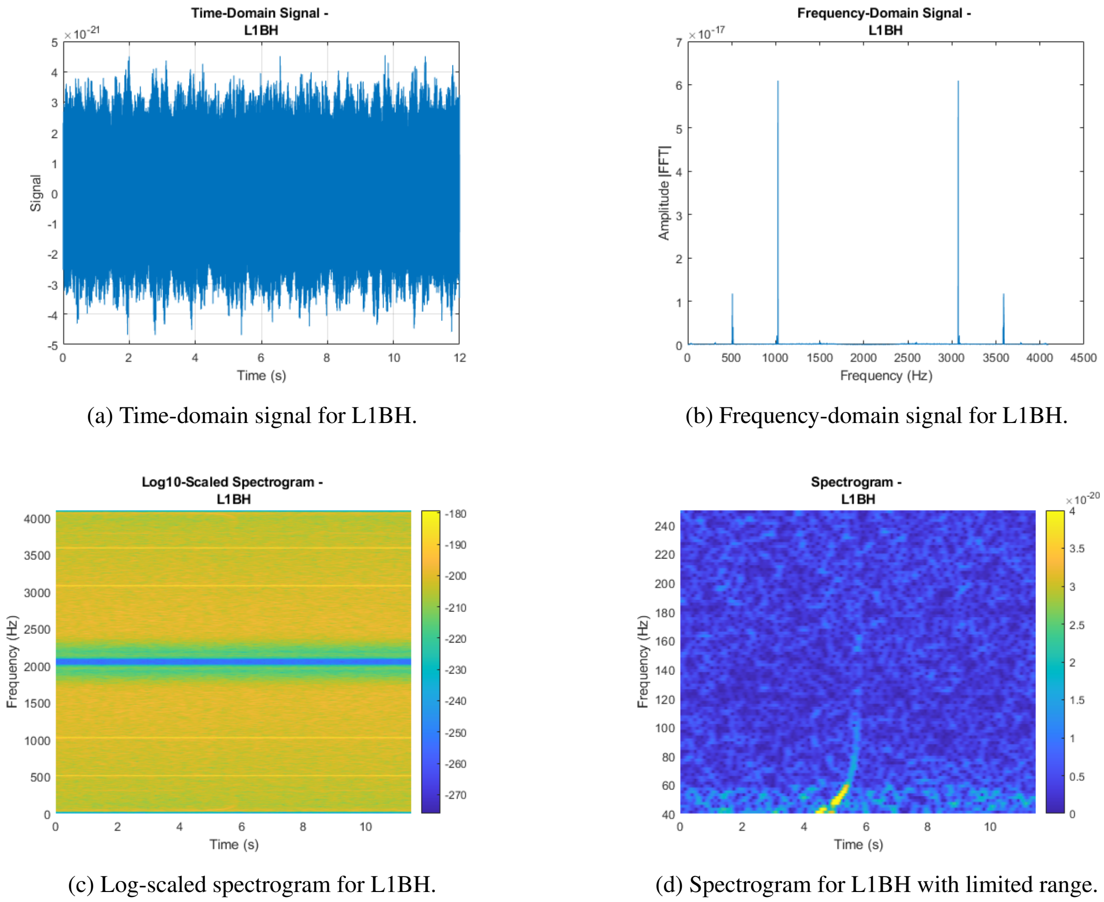
# 3 Results and Analysis

The analysis involves time-domain visualization, frequency-domain analysis, and spectrogram-based investigations for signals recorded at the Livingston (L1) and Hanford (H1) and detector, for the studied events.

## 3.1 GW190814: Black Holes Merging

Analysis for the black holes merging event (GW190814) was performed by analyzing the data set for each detector, named as L1BH and H1BH. Results are shown in figures 1 and 2.

Time-domain signal for L1BH is shown in figure 1a, and reveals a really short-duration chirp signal with a rapid increase in amplitude, which is an expected pattern from gravitational waves. From frequency-domain analysis shown in 1b is clear that the greatest contributions to modulus spectrum are around 500 and 1000 Hz frequencies (which are repeated later in 3600 and 3100 Hz approximately). However, no considerable contributions are visible in the expected range 50-250 Hz. This behaviour can be seen better by applying an sliding Fourier transform, i.e., an spectrogram, to analyze contributions to frequency spectrum in time evolution. The resulting spectrogram for L1BH is depicted in dB scale in figure 1c, where more yellow lines correspond to greater contributions to modulus spectrum as in previous one, but contributions from range 50-250 Hz are not visible yet. In figure 1d the spectrogram in linear scale is shown, and by setting the limits of strain amplitude between 0 and $10^{-24}$ a clear upward frequency sweep is seen in yellow, considerable in the range 40-100 Hz, lasting for about 1 to 2 seconds. This characteristic chirp is a hallmark of black hole mergers and it is in concordance with results published in [3].




(a) Time-domain signal for L1BH.

(b) Frequency-domain signal for L1BH.

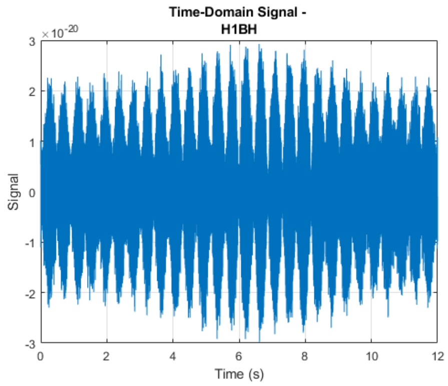
(c) Log-scaled spectrogram for L1BH.

(d) Spectrogram for L1BH with limited range.

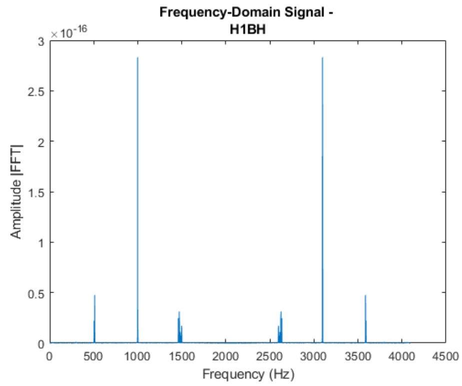
Figure 1: Analysis of the L1BH gravitational wave signal in various domains.

Figure 2 replicates the above analysis for signal collected in Hanford detector (H1BH). Time-domain signal is not as chirp as in L1BH but reveals a shor-duration as well. Frequency-domain graph agrees with what was found for L1BH, with main contributions around 500 and 1000 Hz, which is also emphasize in the logarithmic spectrogram in 2c. Linear spectrogram for H1BH is shown in 2d, where a distinct increase in frequency, represented by yellow hues, is observed around 50 Hz.

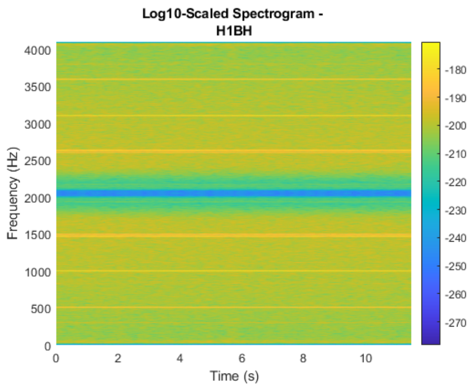
Therefore, signals recorded at Hanford (H1) and Livingston (L1) for the GW190814 event show minor variations in amplitude and phase due to differences in detector sensitivity and orientation. Despite these small discrepancies, the overall structure of the chirp is consistent across detectors. Thus, the frequency spectra from both detectors are consistent, demonstrating the coherence of the detected signal across the LIGO network.



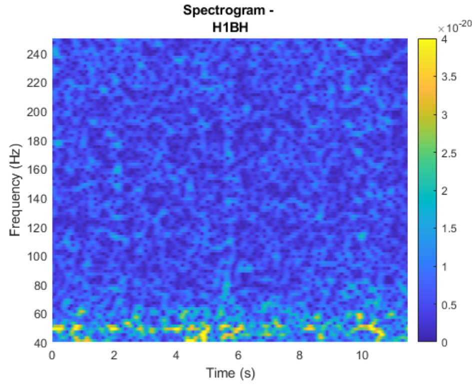

(a) Time-domain signal for H1BH.




(b) Frequency-domain signal for H1BH.




(c) Log-scaled spectrogram for H1BH.




(d) Spectrogram for H1BH with limited range.

Figure 2: Analysis of the H1BH gravitational wave signal in various domains.

The PSD analysis for GW190814, shown in figure 3, emphasizes dominant power at 500 Hz and 1000 Hz. These frequencies match the modulus spectrum results and reflect the physical characteristics of the sources.

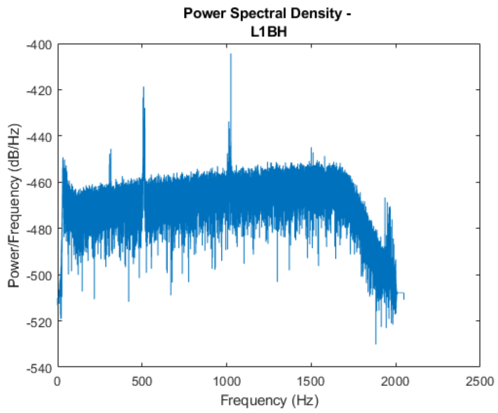
The estimated energy values were computed for each detector:

- L1: $E_{\text{GW}} \approx 1.711 \times 10^{-5}\,\text{J}$
- H: $E_{\text{GW}} 5.329 \times 10^{-4}\,\text{J}$

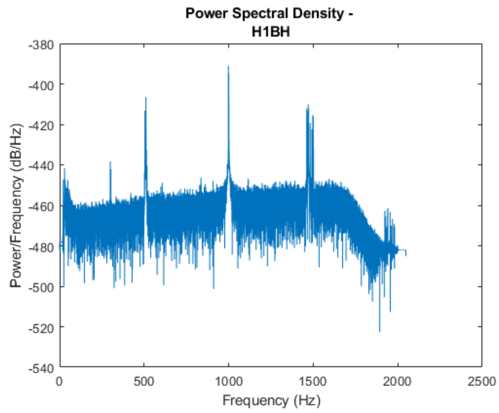
These small values are consistent with the faint nature of gravitational wave signals by the time they reach Earth. The detector sensitivity, finite observation time, and large distances to the sources contribute to the discrepancy from the total astrophysical energy emitted.

### 3.2 GW170817: Binary neutron stars inspirals

By following the same analysis approach performed previously, datasets from the GW170817 (binary neutron stars inspirals) corresponding to both detectors were analyzed, named L1NS and H1NS. Results are shown in figures 4 and 5.



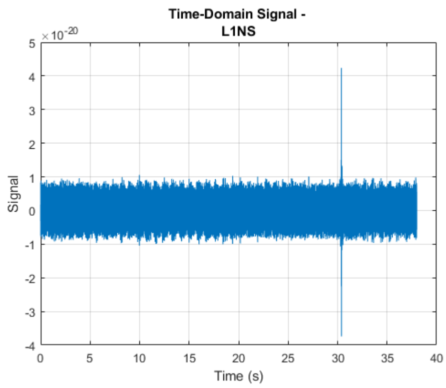

(a) Power Spectral Density - L1BH



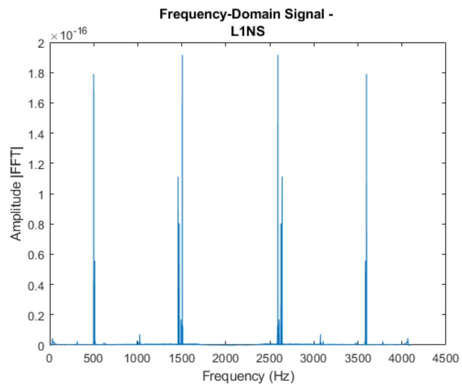

(b) Power Spectral Density - H1BH

Figure 3: Power Spectral Density for GW190814 (Black Hole Merger).

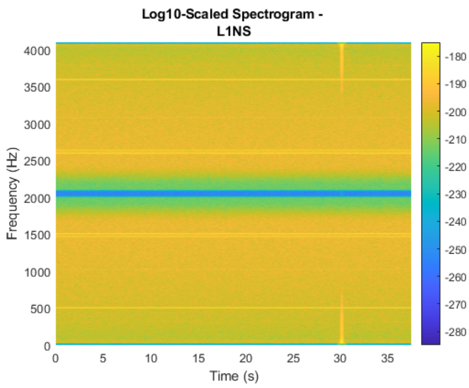
For the L1 detector, main contributions are for frequencies around 500 and 1500 Hz (see figure 4b), depicting a great peak in amplitude signal slightly after 30 seconds in time-domain graph 4a, which also can be spotted in spectrograms shown in 4c and 4d. This spectrogram shows a gradual upward frequency sweep, consistent with the slower inspiral phase of neutron stars. The frequency evolution terminates near 120 Hz, reflecting the merger, where an abrupt contribution from all frequencies is measured. The logarithmic spectrogram highlights lower-power components over a broad frequency range.



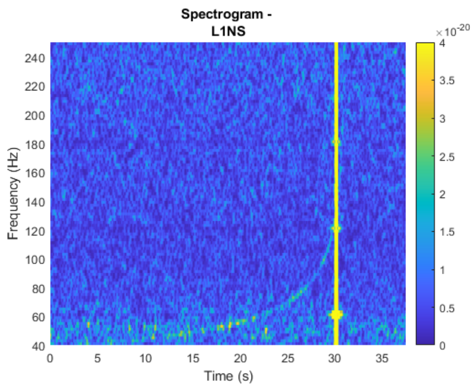

(a) Time-domain signal for L1NS.




(b) Frequency-domain signal for L1NS.




(c) Log-scaled spectrogram for L1NS.




(d) Spectrogram for L1NS with limited range.

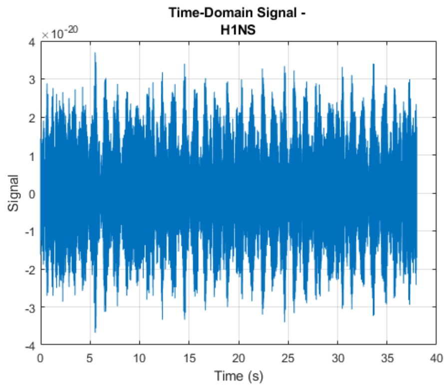
Figure 4: Analysis of the L1NS gravitational wave signal in various domains.

On the other hand, for H1 detector main contributions are for frequencies around 1000 and 1500 Hz (see figure 5b), and from spectrogram in 5d, contributions in the range 40-120 Hz are spotted.

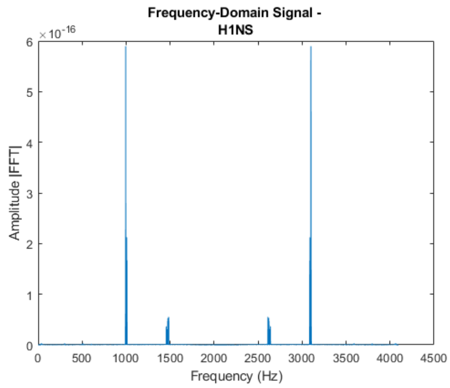
Time-frequency patterns observed in the spectrograms from H1 and L1 align well, confirming the coherence of the detected signals across the LIGO network.

Furthermore, notice that GW170817 exhibits a longer-duration signal with a lower amplitude compared to GW190814, making significant changes in the spectrogram. This is because of the nature of what was observed, in this case (GW170817), binary neutron stars inspirals.

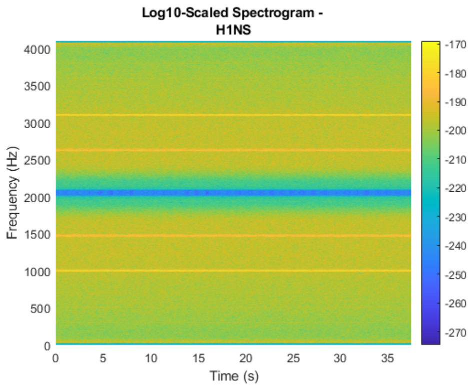
The time-domain analysis for GW170817 exhibits a longer-duration signal with a lower amplitude compared to GW190814. This behavior is consistent with neutron star mergers, where the inspiral phase is more gradual due to the lower masses of the stars. The chirp-like structure of the signal reflects the increasing amplitude as the neutron stars spiral closer together before merging. Signals from Hanford (H1) and Livingston (L1) are similar, with minor differences in amplitude and phase.



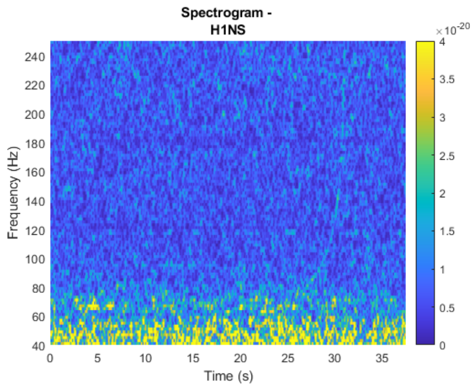

(a) Time-domain signal for H1NS.




(b) Frequency-domain signal for H1NS.




(c) Log-scaled spectrogram for H1NS.




(d) Spectrogram for H1NS with limited range.

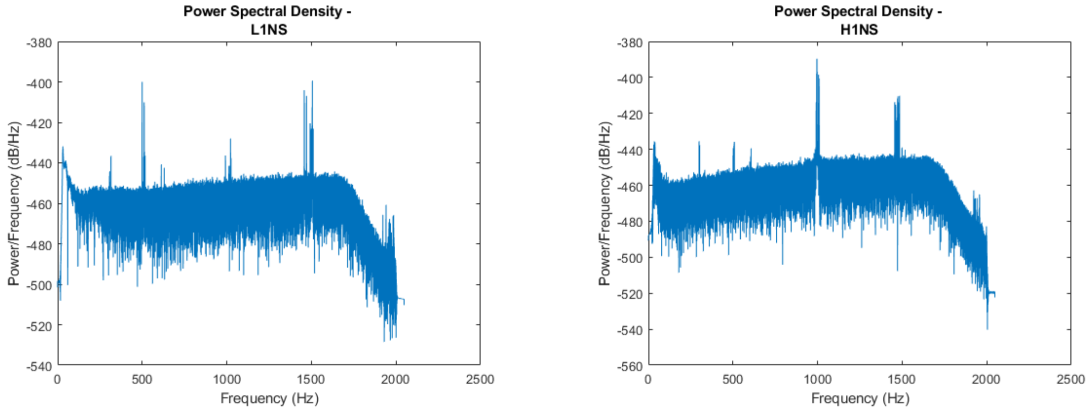
Figure 5: Analysis of the H1NS gravitational wave signal in various domains.

The PSD analysis for GW170817, shown in figure 6, reveals that the Livingston detector (L1) highlighted peaks at 500 Hz and 1500 Hz, while the Hanford detector (H1) showed contributions at 1000 Hz and 1500 Hz, with harmonics near 3100 Hz and 3600 Hz. These frequency values agrees with the modulus spectrum results and reveals the physical characteristics of the sources.

The estimation of the energy were computed for each detector:

- L1: $E_{\text{GW}} \approx 1.547 \times 10^{-4}$ J.
- H1: $E_{\text{GW}} \approx 1.832 \times 10^{-3}$ J.

These smalls values agrees with the faint nature of gravitational wave signals, as well as close magnitude order with the ones for the GW190814 event.




(a) Power Spectral Density - L1NS

(b) Power Spectral Density - H1NS

Figure 6: Power Spectral Density for GW170817 (Binary Neutron Star Inspiral).

# 4 Conclusions

This project analyzed gravitational wave signals from GW190814 (black hole merger) and GW170817 (binary neutron star inspiral) using time-domain, frequency-domain, and spectrogram analyses, complemented by energy estimations. The findings provide detailed insights into the physical nature of these events and the effectiveness of signal processing techniques.

In the time-domain, the characteristic chirp patterns of gravitational waves were clearly observed. GW190814 exhibited a sharp, short-duration signal consistent with the rapid merger of black holes. In contrast, GW170817 displayed a smoother, longer-duration signal, reflecting the gradual inspiral phase of neutron star systems.

On the other hand, frequency-domain analysis revealed key spectral features consistent with the dynamics of the events. For GW190814, significant contributions to the modulus spectra were observed at 500 Hz and 1000 Hz, with harmonics at 3100 Hz and 3600 Hz. GW170817, on the other hand, showed peaks at 500 Hz and 1500 Hz in the Livingston (L1) detector and at 1000 Hz and 1500 Hz in the Hanford (H1) detector.

Also, spectrogram analysis provided a time-frequency representation of the gravitational wave signals, revealing clear upward frequency sweeps. GW190814 demonstrated a rapid increase in frequency over a short duration, characteristic of black hole mergers. GW170817 exhibited a slower, more gradual frequency sweep over a longer period, consistent with the inspiral phase of neutron stars.

Finally, the energy estimation yielded small values for the radiated gravitational wave energy: $1.711 \times 10^{-5}$ J (L1) and $5.329 \times 10^{-4}$ J (H1) for GW190814, and $1.547 \times 10^{-4}$ J (L1) and $1.832 \times 10^{-3}$ J (H1) for GW170817. These values reflect the limited fraction of energy detectable on Earth due to the faint nature of gravitational wave signals.

Overall, this project demonstrates the power of time-frequency signal processing techniques as FFT and Hanning window in identifying and analyzing gravitational wave events. The results align with the ones reported in scientific literature, showing the importance of advanced data analysis in deepening our understanding of extreme cosmic phenomena.

# References

[1] B. P. Abbott, R. Abbott, T. D. Abbott, et al. "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral". In: *Physical Review Letters* 119.16 (2017), p. 161101. DOI: 10.1103/PhysRevLett.119.161101. URL: https://arxiv.org/abs/1710.05832.

[2] B. P. Abbott, R. Abbott, T. D. Abbott, et al. "Review of the Advanced LIGO Gravitational Wave Observatories Leading to Observing Run Four". In: *Classical and Quantum Gravity* 40.6 (2023), p. 065006. DOI: 10.1088/1361-6382/acba1b. URL: https://iopscience.iop.org/article/10.1088/1361-6382/acba1b.

[3] R. Abbott, T. D. Abbott, S. Abraham, et al. "GW190814: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 Solar Mass Compact Object". In: *The Astrophysical Journal Letters* 896.2 (2020), p. L44. DOI: 10.3847/2041-8213/ab960f. URL: https://iopscience.iop.org/article/10.3847/2041-8213/ab960f.