CS146 Homework 5 $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

1.- $T(N) = 2T(N-1) + 1$

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$T(N-1) = T(N-1) = 2T(N-2) + 1$

substituting

$T(N) = 2(2T(N-2)+1) + 1 = 4T(N-2) + 2 + 1 = 4T(N-2) + 3$

Expanding $T(N-2)$:

$T(N-2) = 2T(N-3) + 1$

substituting:

$T(N) = 4(2T(N-3)+1) + 3 = 8T(N-3) + 4 + 3 = 8T(N-3) + 7$

so $T(N) = 2^K T(N-K) + (2^K - 1)$

because $K = N$

$T(N) = 2^N c + (2^N - 1) = \quad T(N) = 2^N - 1 = O(2^N)$

2.- $T(N) = 3T(N-1) + n$

3.- $T(N) = 9T(N/2) + n^2$

$a = 9$ $\quad f(n) = n^2$ $\quad \log_2 9 = 3.17$ $\quad \log_b a = 3.17 > K = 2$

$b = 2$ $\quad K = 2$

so CASE 1 of Master Method = $\Theta(n^{\log_2 9})$

4.- $T(N) = 100T(N/2) + n^{\log 2cn + 1}$

$a = 100$ $\quad f(n) = n^{\log 2cn + 1}$

$b = 2$ $\quad K = \log^{2cn}$

5.- $T(N) = 4T(N/2) + n^2 \log n$

$a = 4$ $\quad f(n) = n^2 \log n$ $\quad K = 2 = \log_b a = 2$ CASE 2 = $\Theta(n^2 \log n)$

$b = 2$ $\quad K = 2$

$\log_2 4 = 2$

6.- $T(N) = 5T(N/2) + n^2/\log n$ $\log_b a = \log_2 5 =$

$a = 5$ $f(n) = n^2/\log n$

$b = 2$

Problem 2.-

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yetAnotherFunc(n):
    if n>1:
        for (i=0; i<10n; i++)
            do someting;
        yetAnotherFunc(n/2);
        yetAnotherFunc(n/2);
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The loop runs 10n times, it is a $O(n)$ time complexity.
It calls itself twice with $n/2$ so:

$$T(N) = 2T\left(\frac{N}{2}\right) + O(n)$$

$a = 2$ $\log_b a = \log_2 2 = 1$ $f(n) = O(n)$ $1 = 1$

$b = 2$ $k = 1$

$\log_b a = 1 = k = 1$

CASE 2 = $\Theta(n\log n)$