21st IFAC World Congress 2020 – July 2020

Least Conservative Linearized Constraint Formulation for Real-Time Motion Generation

Bárbara Barros Carlos[1], Tommaso Sartor[2], Andrea Zanelli[3], Moritz Diehl[3], and Giuseppe Oriolo[1]

[1]DIAG, Sapienza University of Rome, [2]MECO Research Team, KU Leuven, [3]SYSCOP, University of Freiburg

Dipartimento di Ingegneria Informatica Automatica e Gestionale Antonio Ruberti

Sapienza Università di Roma

KU Leuven

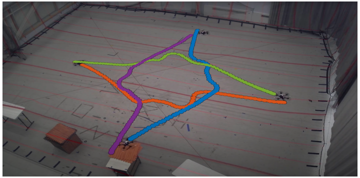
Uni Freiburg

outline

- motivation
- proposed approach
- least conservative real-time planner
- motion generation benchmark
- conclusions and future work

motivation

- autonomous robot navigation in dynamic environments is still a challenging task
- current optimization-based strategies for real-time motion generation often lead to:
  - solutions that are **computationally demanding** $\rightarrow$ application in systems with **tight runtime requirements** is not possible
  - optimal trajectories that are **over-conservative**
- how to obtain a solution for real-time motion generation for systems with tight runtime requirements that is **reliable** and guarantees **less conservative optimal trajectories**?



proposed approach

- central idea
  - novel approach for real-time motion generation for robotic systems that provides trajectories that are **closer** to the imposed lower bound on the distance from obstacles
- here
  - exploit the algorithmic ideas of the **real-time iteration (RTI) scheme** to formulate a **"least"** conservative linearized collision avoidance constraint
  - generated trajectories are **less** conservative for planners based on **Newton-type method** than for those based on a **fully converged NMPC method**
  - overcome the numerical difficulties associated with the (local) feasibility of the optimization problem

proposed approach

RTI scheme in acados [Verschueren 2019]

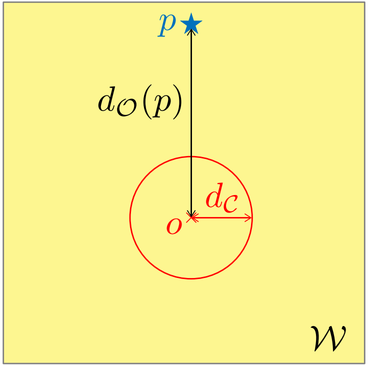
$$
\begin{aligned}
\min_{\substack{\xi_0,\ldots,\xi_N,\\ u_0,\ldots,u_{N-1},\\ s_0,\ldots,s_N}} \quad & \frac{1}{2}\sum_{i=0}^{N-1} \begin{bmatrix}\xi_i\\ u_i\end{bmatrix}^T \overbrace{\begin{bmatrix}Q_i & S_i\\ S_i & R_i\end{bmatrix}}^{H_i} \begin{bmatrix}\xi_i\\ u_i\end{bmatrix} + \begin{bmatrix}q_i\\ r_i\end{bmatrix}^T \begin{bmatrix}\xi_i\\ u_i\end{bmatrix} \\
& + \frac{1}{2}(\xi_N^T Q_N \xi_N) + q_N^T \xi_N \\
& + \frac{1}{2}\sum_{i=0}^{N}(s_i^T P_i s_i) + p_i^T s_i \\
\text{s.t.} \quad & \xi_0 - \bar{\xi}_0 = 0, \\
& \xi_{i+1} - A_i\xi_i - B_i u_i - c_i = 0, \quad i = 0,\ldots,N-1, \\
& s_i + G_i^{\xi}\xi_i + G_i^{u} u_i \leq 0, \quad i = 0,\ldots,N-1, \\
& s_N + G_N^{\xi}\xi_N \leq 0, \\
& 0 \leq s_i, \quad i = 0,\ldots,N.
\end{aligned}
$$

- an efficient **Newton-type scheme** for the approximate online solution of optimization problems [Diehl 2005]
- only **a single linearization and dense quadratic program (QP) solve** are carried out per sampling instant

least conservative real-time planner

collision avoidance constraint

- assumptions
  - workspace $\mathcal{W} \in \mathbb{R}^n$
  - obstacles are point-mass models $o \in \mathbb{R}^n$
  - single-body robot $p \in \mathcal{W}$
  - clearance $d_{\mathcal{C}} \in \mathbb{R}^+$
- definitions
  - $d_{\mathcal{O}}(p) := \|p - o\|_2$
  - $c_1(p) := \|p - o\|_2 - d$
  - $c_2(p) := \|p - o\|_2^2 - d^2$
  - $H_1(\bar{p}) := \{p : c_1(\bar{p}) + \nabla_p c_1(\bar{p})^T (p - \bar{p}) \geq 0\}$
  - $H_2(\bar{p}) := \{p : c_2(\bar{p}) + \nabla_p c_2(\bar{p})^T (p - \bar{p}) \geq 0\}$
  - $S := \{p : c_1(p) \geq 0\}$



least conservative real-time planner

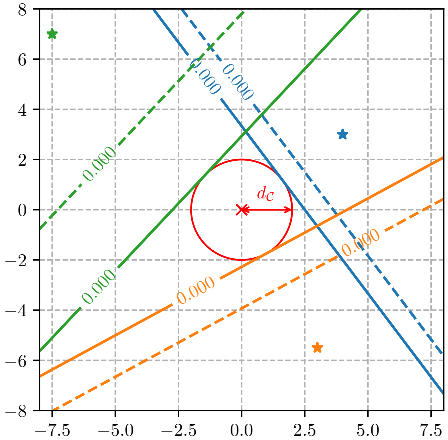
collision avoidance constraint

optimization algorithm behind the planner at hand:

- based on **Newton-type method**, e.g. `acados`
  - squared Euclidean norm $\rightarrow$ local trajectories that are **over-conservative**
  - nonsquared Euclidean norm $\rightarrow$ local trajectories that are **“least” conservative**
- based on a **fully converged NMPC approach**, e.g. `IPOPT` [Wächter 2006]
  - **in general**, no constraint formulation has a distinct advantage over the other
  - **feasible sets** are the same for both formulations

least conservative real-time planner

collision avoidance constraint



Proposition 2 – Least Conservative Linearized (LCL) constraint formulation

Consider $H_1(\bar{p})$, $H_2(\bar{p})$, $c_1(p) \geq 0$, $c_2(p) \geq 0$ and S. Then $H_1(p) \subset S$ and $H_2(p) \subset S$. Moreover, we have that $H_2(p) \subseteq H_1(p)$.

Proof.

- feasible half-spaces $H_1(p)$ and $H_2(p)$ are subsets of the true feasible set S
- the affine outer function in $c_1(p)$ is identical to its linearization, while the outer strongly convex function in $c_2(p)$, when linearized, leads to an underestimator

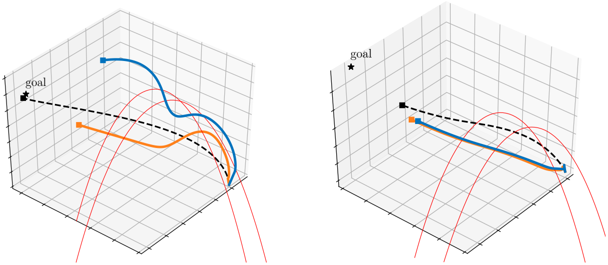
least conservative real-time planner

constraints violation, feasibility and soft constraints

- no guarantees of collision-free trajectories in-between shooting nodes $\rightarrow$ **constraint violation**
- the predicted trajectory between time points may be seen **penetrating** the obstacle
  - we rely on a reasonable choice for the size of the discretization time step
- hard constraints may render the optimization problem **infeasible**
  - we introduce **slack variables** associated with the soft constraints, and heavily penalize them
  - we consider a ℓ_1**–norm** penalization term to obtain an exact penalty function

motion generation benchmark

scenario



- a quadrotor must travel from a hover configuration to another while avoiding balls thrown at it
- balls perform a **ballistic** motion, thrown at an **unknown** time instant
- **nonlinear** model of the quadrotor [Luis 2016]
- controls: rotational speeds of the propellers, constrained as $\mathcal{U} := \{u \in \mathbb{R}^{n_u} : u_{\min} \leq u \leq u_{\max}\}$

motion generation benchmark

nonlinear program formulation

$$
\begin{aligned}
\min_{\substack{\xi_0,\dots,\xi_N,\\ u_0,\dots,u_{N-1},\\ \epsilon_0,\dots,\epsilon_N}} \quad & \frac{1}{2}\sum_{i=0}^{N-1} \|\eta(\xi_i,u_i)\|_W^2 + \frac{1}{2}\|\eta_N(\xi_N)\|_{W_N}^2 + \frac{\mu}{2}\sum_{i=0}^{N}\|\epsilon_i\|_1 \\
\text{s.t.} \quad & \xi_0 - \bar{\xi}_0 = 0, && && \text{(initial state)}, \\
& \xi_{i+1} - F(\xi_i,u_i) = 0, && i = 0,\dots,N-1, && \text{(dynamics)}, \\
& u_i \in \mathcal{U}, && i = 0,\dots,N-1, && \text{(control box constraint)}, \\
& \epsilon_i + d_{\mathcal{O}}(p_i) \geq d_{\mathcal{C}}, && i = 0,\dots,N, && \text{(collision avoidance constraint)}, \\
& \epsilon_i \geq 0, && i = 0,\dots,N, && \text{(slack constraint)}
\end{aligned}
$$

- state $\xi_i \in \mathbb{R}^{13}$, controls $u_i \in \mathbb{R}^4$
- residuals of the least-squares cost $\eta(\xi,u) := (\xi_i - \xi_i^r, u_i - u_i^r)^T$ and $\eta_N(\xi_N) := \xi_N - \xi_N^r$
- weighting matrices $W \in \mathbb{R}^{17\times 17}$ and $W_N \in \mathbb{R}^{13\times 13}$
- gradient wrt the lower $\mu_l \in \mathbb{R}^2$ and upper $\mu_u \in \mathbb{R}^2$ slack penalty values
- prediction horizon $N = 50$, sampling time $t_s = 15$ ms
- implemented in `acados`, using the solver `HPIPM`

video


KU LEUVEN
UNI FREIBURG
STVDIVM VRBIS
Least Conservative Linearized Constraint Formulation for Real-Time Planning
Bárbara Barros Carlos[1], Tommaso Sartor[2], Andrea Zanelli[3], Moritz Diehl[3] and Giuseppe Oriolo[1]
[1] Robotics Lab, DIAG Sapienza Università di Roma
[2] MECO Research Team KU Leuven
[3] SYSCOP Lab, IMTEK University of Freiburg
May 2020

motion generation benchmark

simulation 1 – dynamic obstacles

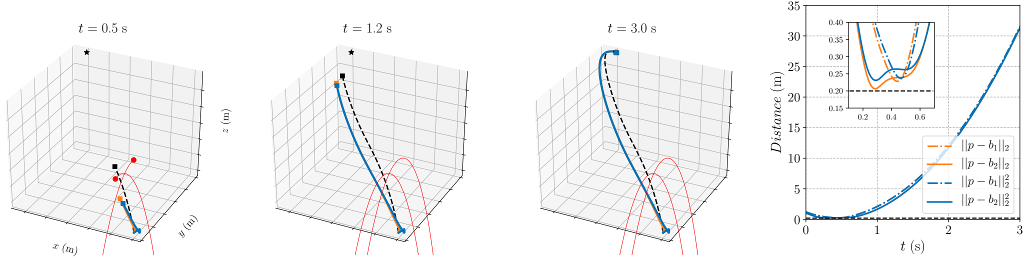


Figure: Generated trajectories among moving balls (red): real-time planner with LCL constraint formulation (orange), real-time planner with $\|\cdot\|_2^2$ constraint formulation (blue); reference trajectory is dashed. The $\star$ is the final goal. On the farthest right the distance between the quadrotor and the balls b_i.

average computational effort $t_{AVG} = 4.76$ ms

motion generation benchmark

simulation 2 – dynamic obstacles

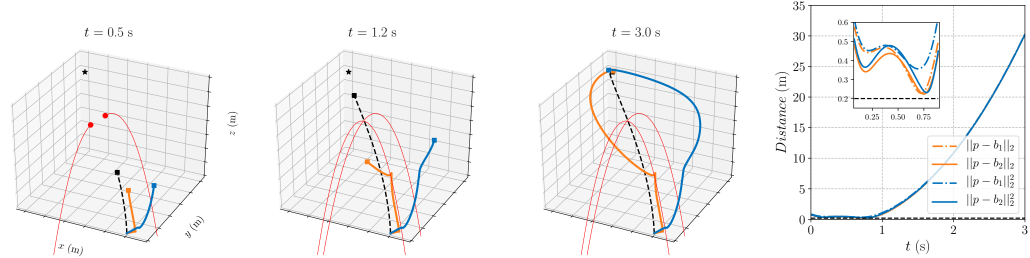


Figure: Generated trajectories among moving balls (red): real-time planner with LCL constraint formulation (orange), real-time planner with $\|\cdot\|_2^2$ constraint formulation (blue); reference trajectory is dashed. The $\star$ is the final goal. On the farthest right the distance between the quadrotor and the balls b_i.

average computational effort $t_{AVG} = 7.97$ ms

motion generation benchmark

simulation 3 – static obstacles, cluttered environment

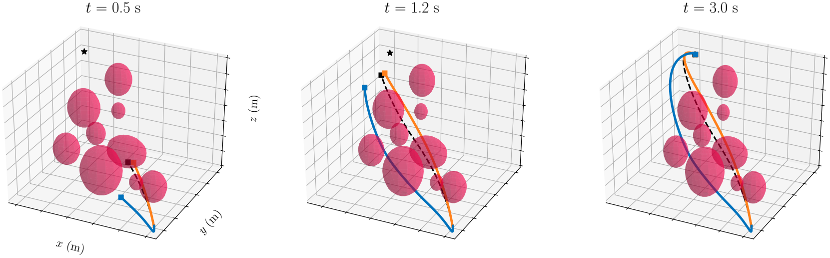


Figure: Generated trajectories among static obstacles (red): real-time planner with LCL constraint formulation (orange), real-time planner with $\|\cdot\|_2^2$ constraint formulation (blue); reference trajectory is dashed. The $\star$ is the final goal.

average computational effort $t_{AVG} = 8.5$ ms

conclusions and future work

- new scheme for real-time motion generation for robotic systems has been presented
- effectiveness of our method stem from the algorithmic ideas tied to the RTI scheme to formulate a least conservative linearized constraint
- the approach guarantees less conservative trajectories for planners whose optimization algorithm is based on Newton-type method
- ensure both constraint satisfaction and feasibility of the optimization problem
- efficient computational performance based on the numerical simulations
- future work: experimental validation with the real quadrotor