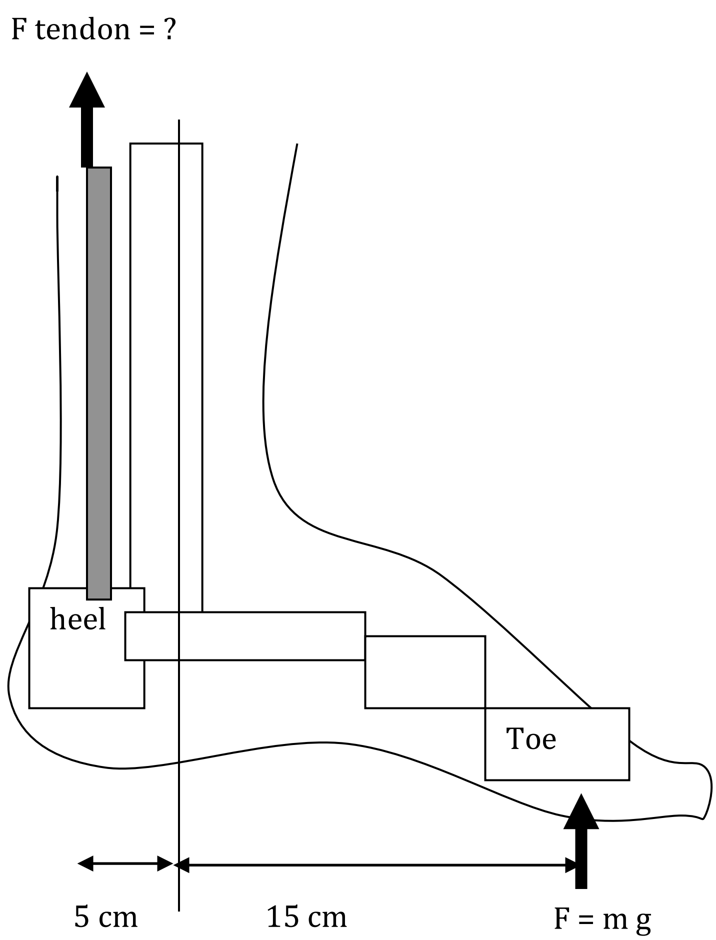
Biology 427: Homework 2

For this problem set assume that all biological materials are *Hookean* – that is they have a linear stress-strain relationship.

Some Data

Human mass = 100 kg
Mass of a train car = 1000 kg
Human Achilles tendon length = 10 cm
Human Achilles tendon cross sectional area = 1 cm^2
Earth's gravitational acceleration = 10 m/s^2

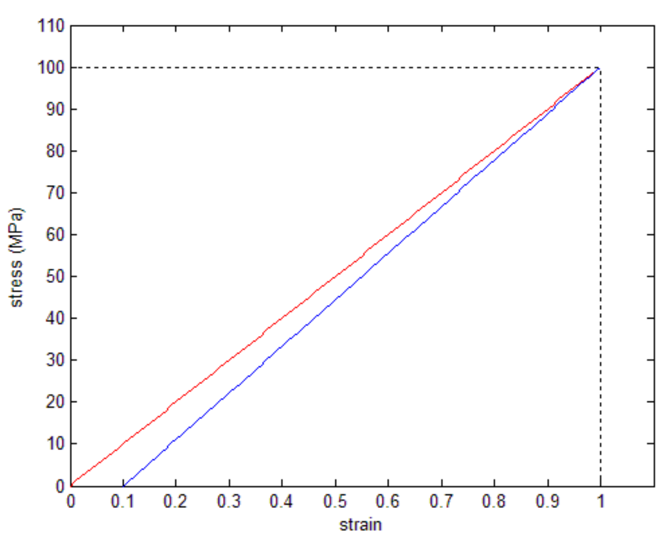
Material	Stiffness (GN/m^2)	Strength (MPa)	Density (kg/m^3)
Human bone	20.0	110	2000
Human tendon	0.6	82	1000
Human cartilage	0.02	1.4	1000
Clam shell	30	42	2700
Insect cuticle	9.5	95	1200
Crab cuticle	30	100	1900
Spider silk	5	350	1000



1. **During slow walking, or in standing on the toes** of one foot, your mass is supported by a small portion of the foot in contact with the ground (Diagram above). We are interested in computing the stress in the tendon of Achilles and the resultant strain, deformation and energy.

A. Use the diagram, data, and assumptions above to compute the stress in the Achilles tendon. To do so you should assume that the moment (torque) about the ankle created by the tendon force is equal to that created by the reaction force of the earth to your weight at the toe.
B. Compute the strain in the tendon from the stress you just calculated. Use this strain figure to compute the length change of the tendon.
C. Compute the strain energy per unit volume (area under a stress vs. strain plot) and the total strain energy (the area under a force vs. length plot) that this tendon absorbs.
D. During a vertical jump the force associated with your vertical acceleration adds increases the stress on your tendon. What is the maximum acceleration your tendon can withstand if you are jumping with one foot (as in the diagram)?
E. Which of the materials in the table above are best suited for energy storage (assuming similar resilience) in activities such as jumping?

2. You come across a new material called "Stuff" and set out to test some of its material properties in a stuff tester. The graph below summarizes your measurements made while stretching the Stuff nearly to its breaking point (red line), then allowing the material to relax back to an unstrained state (blue line):



Calculate the following material properties of Stuff:

a) Stiffness
b) Maximum stress (strength) and strain (extensibility)
c) Work of Extension (energy input)
d) Resilience

3. **The world wide web.** In the Youtube clip of spider man (http://www.youtube.com/watch?v=GYOYewO_Veg) , the protagonist attempts to stop a speeding passenger train consisting of three cars traveling at 80 miles/hour (caution on units). Each car has a mass of 500 kg. The protagonist shoots out 20 threads of spider silk (10 to the right and 10 to the left), each 30 meters long. As you are aware, spider silk is among the strongest biomaterials known. Assume that all 20 threads have exactly the same diameter. What is the minimum thread diameter that is needed to stop the train?

4. Muscle force (F) declines with increasing shortening velocity (v) according to Hill's equation (refer to both any outside reading or the lecture notes for a correct form of this equation):

$$F = \frac{bT_o - av}{v + b}$$

where v is the shortening velocity, T_o is the isometric (non-shortening) force, and the constants a and b have values that vary among muscle.

(a) For Hill's equation to be dimensionally correct, what must be the dimensions of the constants a, b, and T_o? (you can indicate either dimensions or specify units).

(b) Several authors (e.g. Y.C. Fung or T. McMahon) report that it is reasonable to assume that the non-dimensional ratios b/v_{max} and a/T_o are approximately equal to 1/4 for many muscles. Here, v_{max} is equal to the maximum shortening velocity for muscle (for zero force) and T_o is the maximum isometric tension (for zero shortening). Rewrite Hill's equation to express a dimensionless force (F/T_o) as a function of dimensionless shortening velocity (v/v_{max}).

(c) Like force, the mechanical power output of muscle also depends on shortening velocity. The mechanical power output is the rate at which mechanical work is done. Using your new form of Hill's equation, write an equation that expresses the mechanical power output of muscle in terms of shortening velocity (v), maximum shortening velocity v_{max} and isometric tension T_o.

(d) As shortening velocity tends to the limiting values of either 0 or v_{max} what happens to mechanical power output?

(e) Based on your answer to (d) does an intermediate value of shortening velocity maximize or minimized the mechanical power output? (1 pt. extra credit: find an expression for the optimal (max or min) power output).

5. Like the prior paper you read, there was an idea (hypothesis) raised that may not have been directly tested by the experiments and data presented. The paper by George et al., (2013) asserts that a temperature gradient leads to a functional gradient and the potential of energy storage in cross-bridges. In particular they state that

In addition to some attachment at the extrema of the length cycle. At these intermediate temperatures, cross-bridges that remain bound at the very end of lengthening or shortening can store energy in their axial or radial extension, respectively. This stored energy may return energy into the lattice when the crossbridges detach at the start of the subsequent phase. In doing so, the deformed cross-bridges could assist antagonistic muscles. Prior studies have shown that elastic energy storage is indeed crucial for meeting the high inertial power costs of flight (3, 4). If even a portion of these crossbridges facilitate elastic energy savings via a temperature gradient, they would contribute to the overall energy savings in locomotion. Because temperature gradients are an inevitable consequence of internal energy generation and heat dissipation in both vertebrates and invertebrates, this mechanism of energy storage could be a general phenomenon in locomotor systems (11, 12).

What experimental approach could you design to test the hypothesis that radial energy storage in cross-bridges contributes to locomotion efficiency?