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## Math 4610 Fundamentals of Computational Mathematics - Topic 17.

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Computers are essential tools in science and engineering as well as in entertainment in video streaming and gaming. Over time, the capabilities of computers have grown due to computer engineering advances. However, the resources will always be limited to finite storage and memory capacities.

With all of these advancements, one might think that issues related to numerical precision would be taken care of. However, this is not the case. There are a couple of sites that document catastrophic failures of real systems that can be attributed to errors in numerical values due to finite precision of arithmetic. The sites are:

1. <http://www.ima.umn.edu/~arnold/disasters/patriot.html>
2. <https://www5.in.tum.de/persons/huckle/bugse.html>

Scientists and engineers use continuous models that are defined for all real numbers including irrational numbers. Irrational numbers cannot be represented with a finite amount of information. These numbers can only be approximated on computers.

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### Standardizing Number Representation:

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If we represent a real number,  $x$ , by an approximate number,  $fl(x)$ , then the difference between the two numbers is

$$E = |x - fl(x)|$$

We will see a more precise definition of error in the next topic when we cover absolute and relative errors.

In this section we will focus on the limitations of number representation on computers. As mentioned above, the capabilities of computers have advanced rapidly. Over the years computer architecture has increased resolution by using more bits to account for smaller and larger numbers. Starting with 8-bit, 16-bit, and then onto 32-bit and 64-bit numbers, the resolution has gotten better, but will never be able to represent an infinite number of digits needed to store irrational numbers.

Even a number like  $1/3$  must be truncated after a finite number of digits due to the way computers store numbers using bits. Data is stored in a binary format on all computers. So, the number  $1/3$  becomes

$$\frac{1}{3} = 0.10101010101010 \dots$$

The representation repeats, but it is necessary to use an infinite number of digits to exactly represent the number. As an example, consider a 64 bit representation of a number. The 64 bits are split into three chunks. The bits are used to represent the mantissa or

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### Displaying Mathematics in HTML:

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### Github Pages and Markdown:

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Talk about accessing html pages through github pages ... usually not a problem.