Math 4610 Lecture Notes

Types of Errors in Scientific Computing *

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Types of Errors: Summary.

Weather forcasting has improved over the past several decades due to significant increases in computer speeds and mathematical modeling techniques. Forcasts involve predicting future events based on present data and past experience. The accuracy of weather predictions can mean the difference between life and death in terms of major weather events like hurricanes and tornados. If the forcast is wrong it could mean that people will not have enough time to seek shelter. A typical weather model will involve systems of differential equations and input parameters based current conditions. In mathematical terms, this is called an initial value problem.
Problems in the real world typically involve making observations and posing questions about the physical processes being observed. For example, wildlife biologists may decide to fit collars equipped with GPS devices on migrating deet to find out how many times those deer cross highways or fences as they move from winter to summer feeding grounds. Locations for the deer are taken at discrete intervals to track the path deer take.
The following is a list of sources for error that need to be taken into account by compoutational scientists.

1. **Modeling Errors** These errors can occur when assumptions are made about the phenomena being studied. For example, one may consider a model of the solar system where the planets are assumed to be spheres, which is not the case.

2. **Measurement Errors:** These errors occur when instruments are used to measure physical quantities. For example, the temperature of molten lava might be measured to within one or two degrees based on the magnitude of the exact temperature. The neglected fractional part of the measurement would characterize the error.

3. **Discretization Error:** In order to compute solutions to real world mathematical problems using computers, it necessary that the model be finite or discretized. For example, weather models based on systems of partial differential equations require a discretization of the continuous model to fit in the discrete framework of a computer simulation. The discretization in this case requires computing averages over finite volumes of the physcial space which is a source of error.

4. **Roundoff Errors:** These errors occur due to finite precision of numbers on any computer. Computers are limited by the finite amount of memory and storage. Most real numbers require an infinite number of digits (at least all irrational numbers). So the numerical values we work with will produce errors.

5. **Arithmetic Operations on Machine Numbers:** We need to be able to perform binary operations like addition, subtraction, multiplication, and division. If we are using finite precision machine numbers, the output will also have errors that can accumulate.

Of these errors, only one can be controlled by computational mathematicians. Modeling errors are typically beyond the control of computational scientists since the model is typically handed off to the computational scientist. Measurements are completely dependent on the measurement instrumentation. Roundoff error and accumulation of errors due to arithmetic are all dependent on the computers we use. The only error computational scientists can work on is on discretization error. We can change the methods used to improve approximations for mathematical problems.