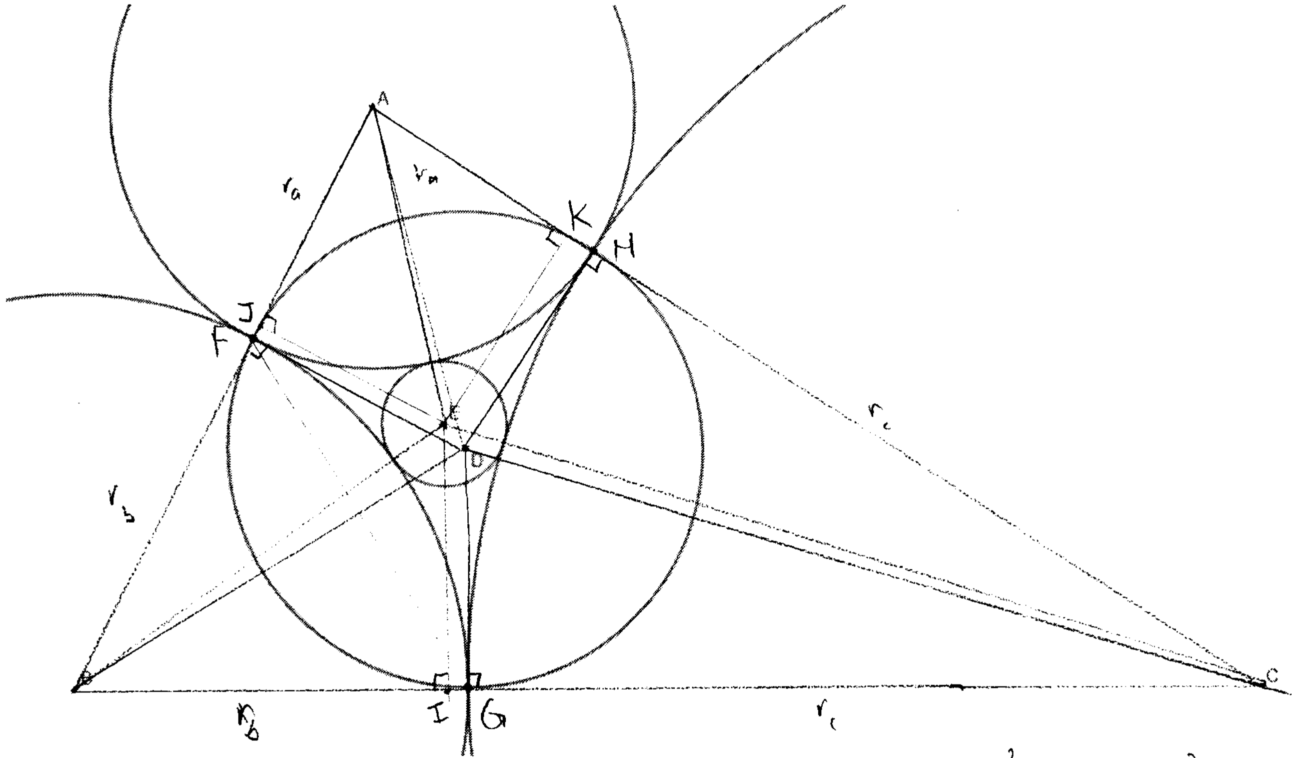

$1 \le r_a < r_b < r_c \le 100$

$$FG^2 = 2r_d^2 - 2r_d^2 \cos FDG$$
$$= 2r_b^2 - 2r_b^2 \cos FBG$$

$$r_b^2 + r_d^2 = \overline{BD}^2$$
$$r_a^2 + r_d^2 = AD^2$$

$$r_a^2 + r_b^2 + 2r_d^2 = BD^2 + AD^2$$

$$FDG + FBG = 180^\circ$$
$$-\cos FDG = +\cos FBG$$

$$(r_a + r_b)^2 = BD^2 + AD^2 - 2(AB)(AD)\cos(\angle ADB)$$

$$AB^2 + BC^2 = AC^2 + 2(AB)(AC)\cos FBG$$

have r_d, thus D

$$(r_b - GI)^2 + EI^2 = (r_b + r_e)^2$$
$$(r_c + GI)^2 + EI^2 = (r_c + r_e)^2$$

$$\cancel{r_b^2} + GI^2 - 2r_b GI + EI^2 = \cancel{r_b^2} + 2r_b r_e + r_e^2 \quad (1)$$
$$\cancel{r_c^2} + GI^2 + 2r_c GI + EI^2 = \cancel{r_c^2} + 2r_c r_e + r_e^2 \quad (2)$$

(2) − (1)

$$\cancel{2}(r_c + r_b) GI = \cancel{2}(r_c - r_b) r_e$$

$$r_e = \frac{r_c + r_b}{r_c - r_b} GI$$

Taking B at origin,

$$x_E^2 + y_E^2 = (r_b + r_e)^2 \qquad (1)$$

$$(x_E - x_A)^2 + (y_A - y_E)^2 = (r_a + r_e)^2 \qquad (2)$$

$x_c = r_b + r_c$

$$(x_C - x_E)^2 + y_E^2 = (r_c + r_e)^2 \qquad (3)$$

$$x_c^2 + x_E^2 - 2x_c x_E + y_E^2 = r_c^2 + r_e^2 + 2r_c r_e \qquad (3)$$

$$x_E^2 + y_E^2 = r_b^2 + r_e^2 + 2r_b r_e \qquad (1)$$

$$x_c^2 - 2x_c x_E = r_c^2 - r_b^2 + 2(r_c - r_b)r_e \qquad (3) - (1)$$

$$(r_b + r_c)^2 - 2(r_b + r_c)x_E = (r_b + r_c + 2r_e)(r_c - r_b)$$

Just use Descartes's thm instead

$$k_E = k_a + k_b + k_c \pm 2\sqrt{k_1k_2 + k_2k_3 + k_3k_1}$$

Take the larger solution because we want the circle with smaller radius (externally tangent).