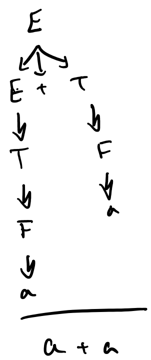
Homework 6

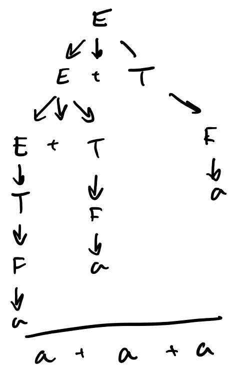
Wednesday, September 25, 2024 2:34 PM

1. a)

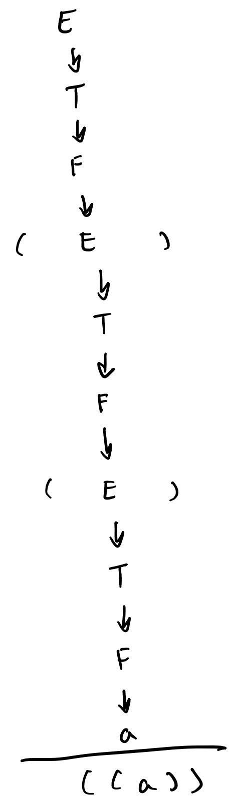
b)



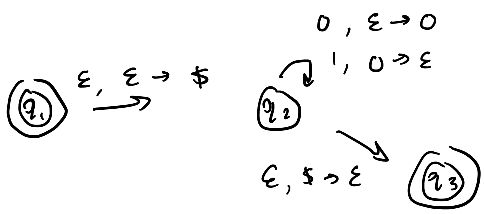
c)



d)



2.



3. a) $A \cap B = \{a^n b^n c^n \mid n \geq 0\}$

Let $s = a^p b^p c^p$, $|s| = 3p$

$s = uvxyz$

Given $|vxy| \leq p$, vxy either contains only one type of letter, s.t. $vxy = k^\alpha$ s.t. $0 < \alpha \leq p$, or, vxy contains two types of letters, s.t. $vxy = k^\alpha j^\beta$ s.t. $0 < \alpha + \beta \leq p$. Because $|vxy| \leq p$, and $s = a^p b^p c^p$, vxy cannot contain 3 types of letters.

With case 1, if vxy contains only 1 letter, if $i = 2$, then $a^n b^n c^n$ no longer holds, as only 1 letter's quantity increases.

With case 2, if vxy contains 2 letters, s.t. v and y contain different letters, with $\check{i}=2$, $a^n b^n c^n$ does not hold for the last letter.

Thus, $A \cap B$ is not CFL.

b) Let A, B, be context free. $\Rightarrow A \cup B$ is context free.

By DeMorgan's law, $(A \cup B)^c = A^c \cap B^c$

We know that CFL is not closed over intersection.

Thus, the complement of a union is equivalent to the intersection of 2 languages, which is NOT context-free.

4. 2.4b)

$S \rightarrow 0A0 \mid 1A1 \mid \varepsilon$

$A \rightarrow 0A \mid 1A \mid \varepsilon$

Accepts ε, 00, 11, 0....0, 1....1

5. $S \rightarrow 0W4F \mid VF \mid TF \mid AF \mid \varepsilon$

$W \rightarrow 0W4 \mid T \mid V$

$V \rightarrow 0V3 \mid U$

$T \rightarrow 1T4 \mid U$

$U \rightarrow 1U3 \mid \varepsilon \mid A$

$A \rightarrow A2 \mid \varepsilon$

$F \rightarrow F5 \mid \varepsilon$

This CFG allows for $0^a 1^b 2^c 3^d 4^e 5^f$ s.t. $b+c = d+e$, regardless of if $b > d$ or $b < d$.

In addition, any c of 3 and any f of 5 can be added in their respective positions.

Example: 00011 2 33344 555

$\rightarrow S \rightarrow 0W4F \rightarrow 00W44F \rightarrow 00V44F$

$\rightarrow 000V344F \rightarrow 000U344F \rightarrow 0001U3344F$

$\rightarrow 00011U33344F \rightarrow 00011A33344F$

$00011A233344F \rightarrow 00011233344F \rightarrow 00011233344F5$

$\rightarrow \ldots \rightarrow 00011233344555$

6. 2.4e) $S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

Accepts ε, 00, 11, and palindromes.

7. $S \to aD \mid aE \mid abc$
$A \to AA \mid Aa \mid ab$
$B \to aaD \mid BC$
$C \to a \mid bc$
$D \to AA$
$E \to BC$

$\Rightarrow$

$S \to FD \mid FE \mid FGH$
$A \to AA \mid AF \mid FG$
$B \to FFD \mid BC$
$C \to a \mid GH$
$D \to AA$
$E \to BC$
$F \to a$
$G \to b$
$H \to c$

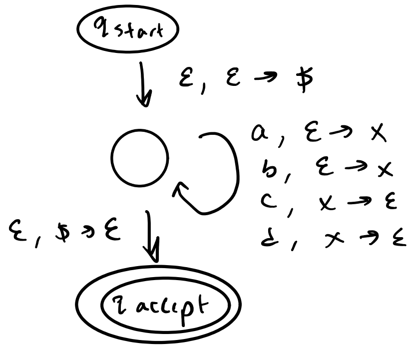
$\Rightarrow$

$S \to FD \mid FE \mid IH$
$A \to AA \mid AF \mid FG$
$B \to JD \mid BC$
$C \to a \mid GH$
$D \to AA$
$E \to BC$
$I \to FG$
$J \to FF$
$F \to a$
$G \to b$
$H \to c$

8. The * includes the empty string. If A did not accept the empty string, then adding $S \to SS$ to it's CFG would not allow it to generate empty strings.

9. $S \to 0A1 \mid 1A0$
$A \to 0A1 \mid 0A0 \mid 1A0 \mid 1A1 \mid \varepsilon$

This makes it so any string generated is even (≥ 2).
If it were empty $y \neq x^R$ would be violated.
Any string will be off the form $0\ldots1$ or $1\ldots0$, so the last symbol in x^R, the first in x, will always be different from the last of y.



q start
ε, ε → $
a, ε → x
b, ε → x
c, x → ε
d, x → ε
ε, $ → ε
q accept