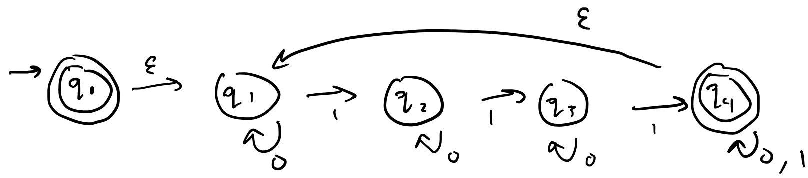
Homework 5

Monday, September 23, 2024 9:50 AM

1. 1.10 a) 1.6 b) $\{w \mid w$ contains at least three 1s$\} = A$

$A^* = \{w^* \mid w \in A\}$

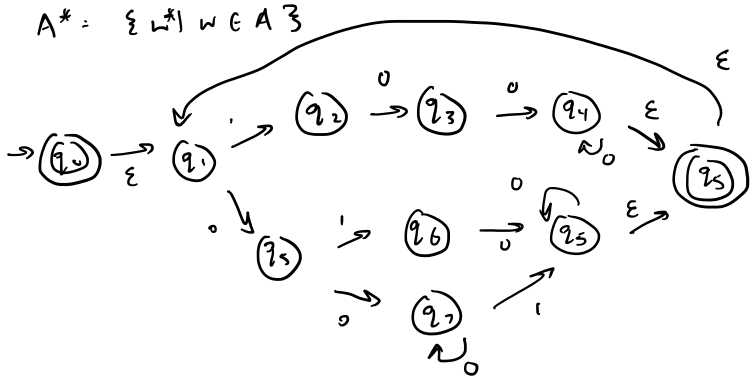


Example: $0011101 \in A$

$\rightarrow$ 00111010011101 Accept

2. 1.10 b) 1.6 j $\{w \mid w$ contains at least 2 0s, and at most one 1$\} = A$

$A^* = \{w^* \mid w \in A\}$



Either 00....1.... s.t. ....: 000....

100.... ...

010.....

Example 00011 reject ✓

001 → 001001 accept ✓

3. 1.29 b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Let $s = www = a^p a^p a^p \Rightarrow |s| = 3p$

Let $x = a^\alpha$ $y = a^\beta$ s.t. $\alpha + \beta \leq p$ and $\beta > 0$

$\Rightarrow |xy| \leq p$ and $|y| > 0$

$s = a^\alpha a^\beta a^{p-\alpha-\beta} a^p a^p$

Let $i = 2$

$s = xy^2z = a^{p+\beta} a^p a^p$

$\cup$

$p = p + B$ if $B = 0$

However, $B > 0$, contradiction.

Thus, $a^{p+B} a^p a^p$ is not

accepted because $a^{p+B} \neq a^p$ if $B > 0$

4. 1.46 a) $\{0^n 1^m 0^n \mid m, n \geq 0\}$

Let $s = 0^p 1^m 0^p$, $|s| = 2p + m > 0$

Let $x = 0^\alpha$, $y = 0^\beta$, s.t. $|xy| \leq p$, $\alpha + B \leq p$, $B = |y| > 0$

$s = 0^\alpha 0^\beta 0^{p-\alpha-B} 1^m 0^p$

For any i, $s = 0^\alpha 0^{Bi} 0^{p-\alpha-B} 1^m 0^p$

$= 0^{B(i-1)} 0^p 1^m 0^p$

$= 0^{p+B(i-1)} 1^m 0^p$

Thus, for some $i > 1$, such as 2;

$s^2 = 0^{p+B} 1^m 0^p$, $s^2 \in A$ if

$p + B = p \Rightarrow B = 0$ contradiction, $B = |y| > 0$

5. 1.46.b) $\{0^m 1^n \mid m \neq n\}$

Let $s = 0^p 1^{2p}$

Let $x = \varepsilon$ $y = 0^p$ s.t. $p > 0 \Rightarrow |y| = p > 0$ $|xy| = p \leq p$

Let $i = 2$

$s = xy^2z = 0^{2p} 1^{2p}$

Thus, $2p = 2p \Rightarrow m = n$, and

$xy^2z = 0^{2p} 1^{2p} \notin A$

A cannot be pumped, and is not regular

6. 1.46 c) $\{w \mid w \in \{0,1\}^*$ is not a palindrome $\}$

Let $s = 0^p 1^p 0^{p+1}$, $|s| \geq p$

Let $x = 0^{\alpha}$, $y = 0^{\beta}$ s.t. $\alpha + \beta \le p$ and $\beta > 0$

$\Rightarrow |xy| \le p$ and $|y| > 0$

$s = 0^{\alpha} 0^{\beta} 0^{p-\alpha-\beta} 1^{p} 0^{p+1}$

Let $i = 2$

$s = xy^2z = 0^{\alpha} 0^{2\beta} 0^{p-\alpha-\beta} 1^{p} 0^{p+1}$

$= 0^{p+\beta} 1^{p} 0^{p+1}$

Thus, if $p + \beta = p + 1$ i.e. if $\beta = 1$, then s is a palindrome and not in A.

7. 1.47) $Y = \{ w \mid w = x_1 \# y_2 \# \ldots \# x_k ,\ x_i \in 1^* \text{ and } x_i \ne x_j \text{ for } i \ne j \}$

Let $s = 1^{p} \# 1^{p+1} \Rightarrow |s| = 2p + 2$

Let $x = 1^{\alpha}$, $y = 1^{\beta}$, s.t. $\alpha + \beta \le p$ and $\beta > 0$

$\Rightarrow |xy| \le p$, $|y| > 0$

$s = 1^{\alpha} 1^{\beta} 1^{p-\alpha-\beta} \# 1^{p+1}$

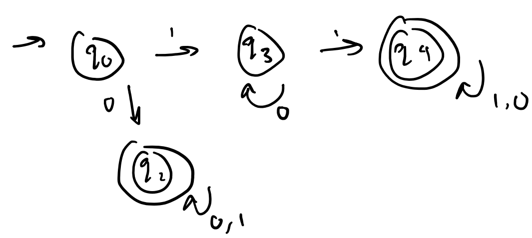
Let $i = 2$

$s = xy^2z = 1^{p+\beta} \# 1^{p+1}$

If $\beta = 1$, then $1^{p+\beta} = 1^{p+1}$ is not in A, since $x_i \ne x_j$ if $i \ne j$

Thus, A cannot be pumped

8. 1.47 a)



Accepts

1.47. b) $B = \{ 1^k y \mid y \in \{0,1\}^* \text{ and } y \text{ contains at most } k \text{ 1s for } k \ge 1 \}$

0 0 0 , p

$s = 1' \; y = 1' \, 0' \, 1'$

Let $x = 1^{\alpha} \quad y = 1^{\beta}$ s.t. $\alpha + \beta \leq p, \; \beta > 0$

$\Rightarrow |xy| \leq p$

$s = 1^{\alpha} 1^{\beta} 1^{p-\alpha-\beta} 0^{p} 1^{p}$

Let $i = 2$

$s = xy^2 z = 1^{p+\beta} 0^p 1^p$

$1^{p+\beta} 0^p 1^p \in A$ if $p + \beta \leq p \Rightarrow \beta = 0$

However, $\beta > 0$, contradiction

Further, for and $\beta > 0$, $p + \beta > p$

9. Show that $\{0^n 1^m 2^k \mid k \mid (m+n)\}$ not regular

Let $s = 0^p 1^p 2^{2p}$, thus, $|s| = 4p \geq p$

Let $x = 0^{\alpha}$, $y = 0^{\beta}$ s.t. $\alpha + \beta \leq p$, $\beta > 0$

$\Rightarrow |xy| \leq p$, $|y| > 0$

$s = 0^{\alpha} 0^{\beta} 0^{p-\alpha-\beta} 1^p 2^{2p}$

Let $i = 2$

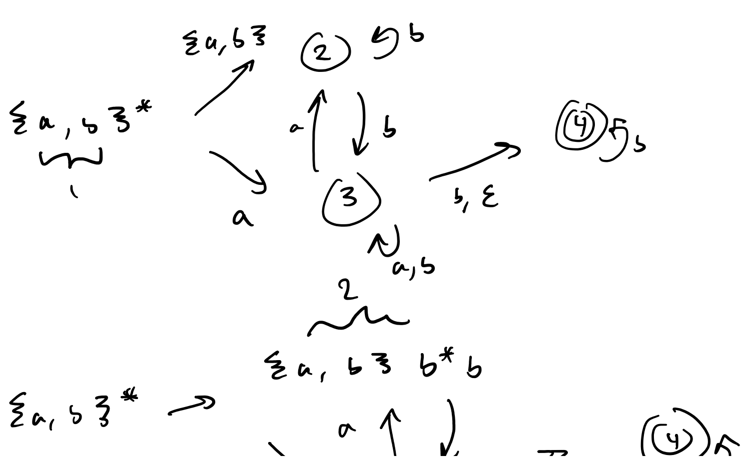
$s = xy^2 z = 0^{p+\beta} 1^p 2^{2p}$

$2p \mid (2p + \beta)$ only if $2p \mid \beta$.

If β is odd, $2p \nmid \beta$.

Thus, the string is not accepted

10.



a $\quad$ $(3)\ a,b$ $\quad$ b,ε $\quad$ b

$$\{a,b\}^* \to \{a,b\}\, b^*\, b\, (\overbrace{\{a,b\}^* \cup (\{a,b\}^* a b^* b)^*}^{3})^* \xrightarrow{b,\varepsilon} (4)\ \circlearrowleft b$$

$$\{a,b\}^* \to a\, (\underbrace{\{a,b\}^* \cup (\{a,b\}^* a b^* b)^*}_{3})^* \xrightarrow{b,\varepsilon}$$

$$\left[\{a,b\}^*((\{a,b\}b^*b) \cup a))(\{a,b\}^* \cup (\{a,b\}^* ab^* b)^*)^{\overset{4}{*}}\, b^*\right]$$