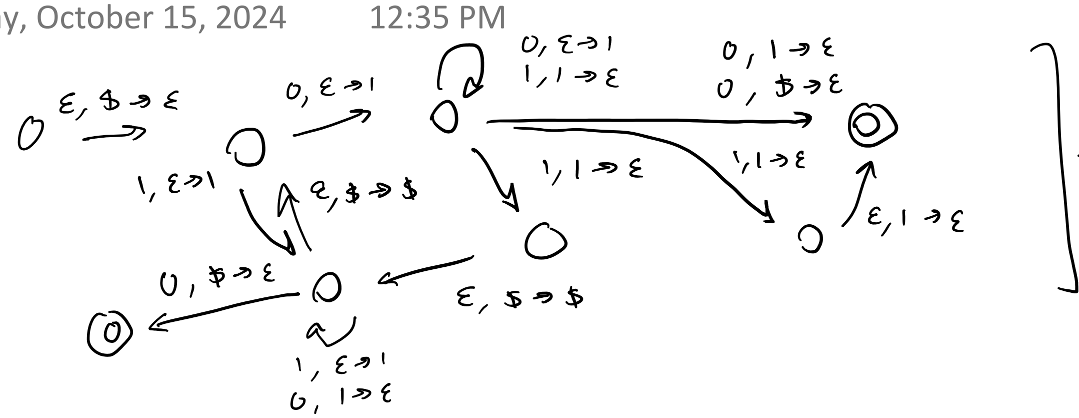
Homework 7

Tuesday, October 15, 2024 12:35 PM

1.



0s add 1 to stack,
1s remove.
If the last is 0
and the stack is at
$, it means one more 0 than
1. If stack is at 1, means
>1 more 0 than 1

If the last is 1, we need to
check specifically that the top
of the stack is 1 afterwards,
implying 0 > 1.

2. $A \to BAB \mid \varepsilon$ $\to$ $A \to BAB \mid BB \mid BA \mid AB \mid B$

$B \to 00 \mid \varepsilon$ $\qquad B \to 00$

$\to$ $A \to BC \mid BB \mid BA \mid AB \mid DD$

$B \to DD$

$C \to AB$

$D \to 0$

3. Let $G_1 = (V_1, \Sigma, R_1, S_1)$, $G_2 = (V_2, \Sigma, R_2, S_2)$,
$G_U = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 \mid S_2\}, S)$.

G_U contains the same terminals as G_1 and G_2. G_U contains the variables of both with the addition of S.
G_U contains the rules of G_1 and G_2, in addition to

$S \to S_1 \mid S_2$.

Thus, G_U has the variables and transitions to recognize any string G_1 and G_2 does, as it has the rules and transitions of both, and a rule to transition to either CFG's start state.

$L(G_U) = L(G_1) + L(G_2)$

4. Let $G_0 = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to S_1 S_2\}, S)$ and $G_1 = (V_1, \Sigma, R_1, S_1)$, $G_2 = (V_2, \Sigma, R_2, S_2)$.

G_0 contains the rules and variables of G_1 and G_2, and has a rule that connects the start states of each, allowing G_0 to accept strings $G_1 \circ G_2$.

Thus, $L(G_1 \circ G_2) = L(G_1) \circ L(G_2)$

5. Let $G_* = (V_1 \cup \{S'\}, \Sigma, R_1 \cup \{S' \to SS' \mid \varepsilon\}, S')$ and $G = (V_1, \Sigma, R_1, S)$.

$G*$ contains the rules and terminals of G, allowing it to create and accept strings from $G*$. The transition $\{S' \to SS' \mid \varepsilon\}$ allows $G*$ to accept any combination of strings that can be created by G through its start state.

Thus, $L(G_*) = L(G)^*$

6. The start states are the same, the final states have a rule to end the accepted string, and all rules correspond to an input and state in the DFA. Thus, they accept the same language.

7. Let $w = 0^p 1^p 0^p 1^p$ s.t. $|w| = 4p$

$w = uvxyz$ s.t. $|vy| > 0$ and $|vxy| \le p$

3 cases...

Case 1 vxy is all 0s

When we pump v and y, the resulting string will have a different number of 0s for either string, as vxy is only 0s implying it is within either 0^p, but not both.

Case 2 vxy is all 1s

The same applies as case 1 due to the length of 1^p and $|vxy| \le p$.

Case 3 vxy is both 1 and 0

vxy can only span from 0^p to another 1^p or vice versa, due to the length of vxy.

Thus, pumping v and y would change the length of one string of 0s and one string of 1s, but not the other 0^p and 1^p

Thus, no way of picking vxy for $w = uvxyz$ can be pumped, thus, L is not context free.

8. Let $w = a^p b^p c^p d^p$, s.t. $|w| = 4p > p$.

Let $w = uvxyz$ s.t. $|vy| > 0$ and $|vxy| \le p$.

3 cases...

Case 1, vxy is contained in a single character.

Pumping would violate either ~~$|a|=|b|$~~ or $|a| = |d|$

Case 2, vxy is contained in bc

Pumping would violate both $|a| = |b|$ and $|c| = |d|$

Case 3, vxy is contained in ab or cd

Pumping would only accept if $v = y$, however, this is not guaranteed and any other selection fails.

[illegible] $uvxyz$ can always be

Thus, no way of picking vxy for s = uvxyz can always be pumped, thus, L is not context free.

9. A, B regular $\Rightarrow$ A, B, context free.
Let $L(G_1) = A$, $G_1 = (V_1, \Sigma, R_1, S_1)$ and $L(G_2) = B$, $G_2 = (V_2, \Sigma, R_2, S_2)$.

$G_{\sim} = (V_1 \cup V_2 \cup \{S\}, \Sigma, R_1 \cup R_2 \cup \{S \to aA, A \to aA \mid ab$ $\forall a$

Thus, this CFG accepts string with $|a| > |b|$.

h

10. Let $w = a^p b^p c^p d^p e^p f^p$ s.t. $|w| = 6p$.
Let $w = uvxyz$ s.t. $|vy| > 0$ and $|vxy| \leq p$.

3 cases...

Case 1
If vxy contained in one letter...
Pumping violates $|a| + |b| = |c| + |d| = |e| + |f|$

Case 2
If contained in pairs ab, cd, or ef
Pumping violates $|a| + |b| = |c| + |d| = |e| + |f|$

Case 3
If contained in pairs bc, de
Pumping violates $|a| + |b| = |c| + |d| = |e| + |f|$,
regardless of if $|v| = |y|$, all 3 pairs
need the same length.

Thus, no way of picking vxy for s = uvxyz can always be pumped, thus, L is not context free.

$\in A,\ b \in B \},\ s)$