Homework 1

Wednesday, August 28, 2024 9:54 AM

1. $\sum_{i=1}^{5} (4i-2) = 2 + 6 + 10 + 14 + 18 = 50$

$2(5)^2 = 50$

NTS: $\sum_{i=1}^{n+1} (4i-2) = 2(n+1)^2$

IA: $\sum_{i=1}^{m} (4i-2) = 2m^2$ for some $m \geq 1$

Base Case: $n=1$

$\sum_{i=1}^{1} (4i-2) = 2$

$2(1)^2 = 2$

Proof:

$$\sum_{i=1}^{n+1} (4i-2) = \sum_{i=1}^{n} 4i-2 + \sum_{i=n+1}^{n+1} (4i-2)$$

$$^{*}\text{IA} : 2n^2 + 4(n+1) - 2$$

$$= 2n^2 + 4n + 4 - 2$$

$$= 2n^2 + 4n + 2$$

$$= 2(n^2 + 2n + 1)$$

$$= 2(n+1)^2$$

2. $\sum_{i=1}^{5} (2i)^4 = 2^4 + 4^4 + 6^4 + 8^4 + 10^4$

$= 15664$

$(4(5))^4 = 160000$

$\sum_{i=1}^{n} (2i)^4 = 16 \sum_{i=1}^{n} i^4 \rightarrow$ At some point, $\sum_{i=1}^{n} i^4 \geq n^4$

$\sum^{n} i^4 \geq 4n^4$

$(4n)' = 64n'$ $\qquad i = 1$

IDK. $\hat{\ }$

3. $5^2 - 3(5) + 4 = 25 - 15 + 4 = 14$

NTS: $(n+1)^2 - 3(n+1) + 4$ is even

IA: $m^2 - 3m + 4$ is even for some $m \geq 0$, or,
$m^2 - 3m + 4 = 2j$, $j \geq 1$

If m even....
m^2 is even
$3m$ is even
even - even = even
even + 4 (even) = even

If m is odd...
m^2 = odd
$3m$ (odd · odd) = odd
odd - odd = even
even + 4 = even

Base Case: $n = 0$
$0 - 0 + 4 = 4$ $\qquad 4 = 2j = 2(2)$

Proof:
$(n+1)^2 - 3(n+1) + 4$
$= n^2 + 2n + 1 - 3n - 3 + 4$
$= (n^2 - 3n + 4) + 2n - 2$

IA $= 2j + 2(n-1)$

*j some integer n $= 2(j + (n-1))$
$= 2(j + j')$

Thus, $(n+1)^2 - 3(n+1) + 4$ is divisible

by 2, s.t. $\frac{(n+1)^2 - 3(n+1) + 4}{2} = j + j'$, s.t.

$j = \frac{n^2 - 3n + 4}{2}$ and $j' = n - 1$.

4. NTS : $m-1$ divides $m^{n+1} - 1$ $\forall n \geq 0$ and $m \geq 2$

IA : $(m-1) \mid (m^k - 1)$ $\forall k \geq 0$,

or $(m-1) = j(m^k - 1)$ s.t. $j \geq 1$

Base case : $n = 0$

$m-1$ Thus, $(m^1 - 1) = 1(m-1)$, $j = 1$

$m^1 - 1 = m - 1$

Proof :

$m^{n+1} - 1 = m\, m^n - 1$

$= m(m^n - 1 + 1) - 1$

* IA, $j \geq 1$ $= m(j(m-1) + 1) - 1$

$= j(m-1)m + m - 1$

$= \underbrace{j(m-1)m}_{\text{divisible by } m-1} + \underbrace{(m-1)}_{\text{divisible by } m-1}$

$= (m-1)(jm + 1)$

$= (m-1)j'$

Thus, $m^{n+1} - 1$, $n \geq 0$ is divisible by $m-1$,

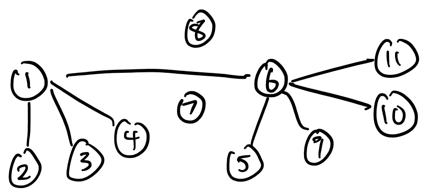
s.t. $m^{n+1} - 1 = (m-1)j'$ s.t. $j' = (jm + 1)$, $j = \frac{m^n - 1}{m-1}$

5. Let $G(2, E)$

In an undirected graph, each V can only have one edge w/ another V.

In the example above, each node either has 0 or 1 degrees.

6. a)



b) [5]

c) Current + 2 (to connect 7 and 8),

or , [10]

7. a) Yes

b) Yes

c) $]-\infty, \infty[$

because $x/_7 + 1$ can be expressed as a fraction

d) Yes. For $x \in \mathbb{Q}$, $g(x) = x/_7 + 1$, $g(x) \in \mathbb{Q}$.

Thus, $\forall x \in \mathbb{Q}$, $\exists! \, g(x) \in \mathbb{Q}$

e) Yes For $g(x) \in \mathbb{Q}$, $x = 7(g(x) - 1)$, $x \in \mathbb{Q}$

Thus, $\forall g(x) \in \mathbb{Q}$, $\exists! \, x \in \mathbb{Q}$

f) Yes

g) No. Let n be an even number, s.t.

$\lfloor n/2 \rfloor = n/2 \Rightarrow h(n) = n/2$

Let $n+1$ be odd...

$\Rightarrow \lfloor (n+1)/2 \rfloor = n/2 \Rightarrow h(n+1) = n/2$

why? Ex $3/2 = 1.5$

$\lfloor 3/2 \rfloor = 1$

Given an even n, $h(n) = h(n+1) = n/2$

Thus, h is not one-to-one because 2 inputs can [illegible] same output.

8. a) No

b) Yes

c) $\{x, y, z\}$

d) $\{x, y\}$

e) $\{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

f) 2^{13}: $\{\emptyset, \{x\}, \{y\}, \{x, y\}\}$

9. There is no guarantee that H_1 and H_2 have the same color. $h \in H_1$ has same color $\forall h \in H_1$, $h \in H_2$ has same color $\forall h \in H_2$, but this does not extend to the set H.

10. $a \equiv b \mod n$ if $n \mid (a-b)$

Reflexivity: $a \equiv a \mod n$

Let $a \in \mathbb{N}$

$a - a = 0$

because $n/0$ ONE

$n \mid 0$, or, $n = j(0)$ s.t. $j \in \mathbb{Z}$, $j \neq 0$

Symmetry: if $a \equiv b \mod n$, then $b \equiv a \mod n$

Let $a \equiv b \mod n$, $a, b \in \mathbb{N}$.

Thus $n \mid (a-b)$, or, $jn = (a-b)_j$, $j \in \mathbb{Z}$, $j \neq 0$

We know that $a-b = -(-a+b) = -(b-a)$

Thus, $nj' = (b-a)$, s.t. $j' \in \mathbb{Z}$, $j' \neq 0$, $j' = -j$

Thus, $b \equiv a \bmod n$ because $n \mid (b-a)$

Transitivity: if $a \equiv b \bmod n$ and $b \equiv c \bmod n$, then $a \equiv c \bmod n$

Let $a \equiv b \bmod n$ and $b \equiv c \bmod n$.

$\Rightarrow n \mid (a-b)$ and $n \mid (b-c)$,

or $jn = (a-b)$ and $j'n = (b-c)$

s.t. $j, j' \in \mathbb{Z}$, $j, j' \neq 0$

$a = b + jn$

$c = b - j'n$

$a - c = \cancel{b} + jn - (\cancel{b} - j'n)$

$a - c = (j' + j)(n)$

$\Rightarrow n \mid (a-c)$ s.t. $nj'' = (a-c)$, $j'' = j + j'$, $j'' \in \mathbb{Z}$, $j'' \neq 0$

Thus, $a \equiv c \bmod n$.