**MAT0236 - Funções Diferenciáveis e Séries**
**Lista 3 - 2019**

1. Decidir se a série converge absolutamente, condicionalmente ou diverge.

   (a) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{1}{\sqrt{n}}$ (b) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^{\frac{3}{2}}}$ (c) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{2n^2+1}{n^3+3}$ (d) $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{1}{\ln n}$

   (e) $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{\ln n}{n}$ (f) $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{n(\ln n)^2}$ (g) $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{\sqrt{n}}{\ln n}$ (h) $\sum\limits_{n=1}^{\infty} (-1)^{2n+1} \dfrac{1}{\sqrt{n}}$

   (i) $\sum\limits_{n=1}^{\infty} (-1)^n \mathrm{sen} \dfrac{1}{n^p}$, $p > 0$ (j) $\sum\limits_{n=2}^{\infty} \dfrac{\ln n}{n^2}$ (k) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{\ln n}{\sqrt{n}}$ (l) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n+1}{n+5}$

2. Se $\sum a_n$ e $\sum b_n$ convergem absolutamente, mostre que $\sum(a_n \pm b_n)$ e $\sum a_n b_n$ também convergem absolutamente.

3. Se $\sum a_n^2$ e $\sum b_n^2$ convergem, mostre que $\sum a_n b_n$ converge absolutamente. [**Dica:** $(a \pm b)^2 \geq 0$.]

4. Mostre que se $\sum a_n^2$ converge, então $\sum \dfrac{a_n}{n}$ converge.

5. Verifique que $1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \cdots = \frac{3}{2} \ln 2$.

6. Mostre que $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{n!}$ converge absolutamente para todo $x \in \mathbb{R}$. Deduza que $\lim\limits_{n\to\infty} \dfrac{x^n}{n!} = 0$ para todo $x \in \mathbb{R}$.

7. Determine os valores de $x \in \mathbb{R}$ para os quais as séries convergem.

   (a) $\sum\limits_{n=1}^{\infty} x^n(1+x^n)$ (b) $\sum\limits_{n=1}^{\infty} x^n \cos\left(\frac{n\pi}{2}\right)$ (c) $\sum\limits_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n^{\ln x}}$ (d) $\sum\limits_{n=1}^{\infty} n! x^n$

   (e) $\sum\limits_{n=1}^{\infty} \left(x^n + \frac{1}{2^n x^n}\right)$ (f) $\sum\limits_{n=0}^{\infty} (-1)^{n+1} e^{-n \mathrm{sen} x}$ (g) $\sum\limits_{n=0}^{\infty} \frac{2n+1}{(n+1)^5} x^{2n}$

8. Determine o intervalo máximo de convergência de cada uma das séries de potências abaixo:

   (a) $\sum\limits_{n=1}^{\infty} \dfrac{n}{4^n} x^n$ (b) $\sum\limits_{n=1}^{\infty} n!\, x^n$ (c) $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{n^3+1}$

   (d) $\sum\limits_{n=1}^{\infty} \dfrac{(3n)!}{(2n)!} x^n$ (e) $\sum\limits_{n=1}^{\infty} (-1)^{n-1} \dfrac{(x-5)^n}{n3^n}$ (f) $\sum\limits_{n=1}^{\infty} \dfrac{(x+1)^n}{(n+1)\ln^2(n+1)}$

   (g) $\sum\limits_{n=1}^{\infty} \dfrac{10^n}{(2n)!}(x-7)^n$ (h) $\sum\limits_{n=1}^{\infty} \dfrac{\ln n}{e^n}(x-e)^n$ (i) $\sum\limits_{n=1}^{\infty} \dfrac{n!}{n^n}(x+3)^n$

   (j) $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{(x-3)^n}{(2n+1)\sqrt{n+1}}$ (k) $\sum\limits_{n=0}^{\infty} \dfrac{n^2}{4^n}(x-4)^{2n}$ (l) $\sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{n(\ln n)^2} x^n$

   (m) $\sum\limits_{n=1}^{\infty} 2^n x^{n^2}$ (n) $\sum\limits_{n=1}^{\infty} \dfrac{3^n}{n4^n} x^n$

9. Obtenha o raio de convergência para as séries seguintes.

   (a) $\sum\limits_{n=1}^{\infty} x^n \dfrac{(2n)!}{(n!)^2}$ (b) $\sum\limits_{n=1}^{\infty} x^{2n} \dfrac{(2n)!}{(n!)^2}$ (c) $\sum\limits_{n=1}^{\infty} x^{n^2} \dfrac{(2n)!}{(n!)^2}$ (d) $\sum\limits_{n=1}^{\infty} x^n \dfrac{n!}{n^n}$ (e) $\sum\limits_{n=1}^{\infty} x^{3n} \dfrac{n!}{n^n}$ (f) $\sum\limits_{n=1}^{\infty} x^{n!} \dfrac{n!}{n^n}$.

10. Determine o intervalo de convergência de:

    (a) $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{(2+(-1)^n)^n}$ (b) $\sum\limits_{n=1}^{\infty} \left(\dfrac{3n+2}{5n+7}\right)^n x^n$ (c) $\sum\limits_{n=1}^{\infty} \left(\dfrac{2^n+3}{3^n+2}\right) x^n$ (d) $\sum\limits_{n=2}^{\infty} \dfrac{x^n}{\ln n}$

    (e) $\sum\limits_{n=1}^{\infty} \dfrac{(x+1)^n}{a^n+b^n}$, com $b > a > 0$.

**RESPOSTAS**

1. (a) converge condicionalmente, (b) converge absolutamente, (c) converge condicionalmente, (d) converge condicionalmente, (e) converge condicionalmente, (f) converge absolutamente, (g) diverge, (h) diverge, (i) converge absolutamente se $p > 1$ e converge condicionalmente se $p \leq 1$, (j) converge absolutamente, (k) converge condicionamente, (l) diverge.

7. (a) $\{x \in \mathbb{R} : |x| < 1\}$, (b) $\{x \in \mathbb{R} : |x| < 1\}$, , (c) $\{x \in \mathbb{R} : x > 1\}$, (d) $\{x = 0\}$, (e) $\{x \in \mathbb{R} : 1/2 < |x| < 1\}$, (f) $\{x \in \mathbb{R} : 2k\pi < x < (2k+1)\pi,\ k \in \mathbb{Z}\}$, (g) $\{x \in \mathbb{R} : |x| \leq 1\}$.

8. (a) $]-4, 4[$; (b) $\{0\}$; (c) $[-1, 1]$; (d) $\{0\}$; (e) $]2, 8]$; (f) $[-2, 0]$; (g) $\mathbb{R}$; (h) $]0, 2e[$; (i) $]-3-e, -3+e[$; (j) $[2, 4]$; (k) $]2, 6[$; (l) $[-1, 1]$; (m) $]-1, 1[$; (n) $[-4/3, 4/3[$.

9. (a) $R = 1/4$; (b) $R = 1/2$; (c) $R = 1$; (d) $R = e$; e) $R = \sqrt[3]{e}$; f) $R = 1$.

10. (a) $]-1, 1[$; (b) $]-5/3, 5/3[$; (c) $]-3/2, 3/2[$; (d) $[-1, 1[$; (e) $]-b-1, b-1[$.