

# Statistical Analysis of Repeated Measurements Data

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# What is this Course About

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*Grouped data* arise in a wide range of disciplines

- Typical examples of grouped data
  - ▷ *repeated measurements*: measuring the same outcome multiple times on the same sample unit (e.g., biomarkers in patients)
  - ▷ *multilevel data*: outcomes measured on sample units that are organized in different levels (e.g., patients in medical centers or students in schools)

# What is this Course About (cont'd)

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- Statistical analysis of clustered/grouped data
  - ▷ Features of grouped data
  - ▷ describe their distribution
  - ▷ inference using suitable regression models

# Lexical convention

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- The following terms are used interchangeably to denote multivariate outcomes
  - ▷ clustered data
  - ▷ repeated measurements data
  - ▷ multilevel data
  - ▷ grouped data

# Learning Objectives

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- Goals: After this course participants will be able to
  - ▷ identify settings in which a repeated measurements model is required,
  - ▷ construct and fit an appropriate model to the data at hand, and
  - ▷ correctly interpret the results
- Even though the course will be primarily explanatory
  - ▷ sufficient mathematical detail will be provided in order participants to obtain a clear view on the different modeling approaches, and how they should be used in practice

# Agenda

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- **Chapter 1:** Motivating Data Sets

- ▷ Data sets that we will use throughout the course
- ▷ General repeated measurements settings
- ▷ Formulation of possible research questions

- **Chapter 2:** Marginal Models for Continuous Data

- ▷ Features of repeated measurements data
- ▷ Naive approaches
- ▷ Review linear regression
- ▷ Marginal models

# Agenda (cont'd)

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- **Chapter 3:** The Linear Mixed Effects Model

- ▷ Intuition behind mixed models
- ▷ Mixed models with correlated errors
- ▷ Nested and cross random effects
- ▷ Time-varying covariates

- **Chapter 4:** Marginal Models for Discrete Data

- ▷ Review generalized linear models
- ▷ Generalized estimating equations

# Agenda (cont'd)

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- **Chapter 5:** Mixed Models for Discrete Data

- ▷ Generalized linear mixed effects models
- ▷ interpretation of parameters
- ▷ approximations of the integrand & integral

- **Chapter 6:** Statistical Analysis with Incomplete Grouped Data

- ▷ Problems with incomplete data
- ▷ Missing data mechanisms
- ▷ Valid inferential approaches

# Structure of the Course & Material

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- Lectures & software practicals using R
- Material:
  - ▷ Course Notes
  - ▷ R code in soft format
- Within the course notes there are several examples of R syntax – these are denoted by the symbol ‘**R**> ’



# Software Requirements

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- The up-to-date versions of R and Rstudio; downloadable from
  - ▷ <https://cran.r-project.org/>
  - ▷ <https://www.rstudio.com/>
- Additional required packages
  - ▷ **nlme, lme4, GLMMadaptive, geepack,**
  - ▷ **MASS, lattice, shiny, corrplot**

# Software Requirements

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- Up-to-date versions of these packages and their dependencies can be installed using the command

```
install.packages(c("shiny", "nlme", "lattice", "lme4",  
                  "GLMMadaptive", "geepack", "MASS", "corrplot"),  
dependencies = TRUE)
```

- Up-to-date version of a modern web browser, e.g.,
  - ▷ Mozilla Firefox (<https://www.mozilla.org/firefox/>)
  - ▷ Google Chrome (<https://www.google.com/chrome/>)

# Software Requirements

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- We will use a [shiny](#) web app that replicates all analyses in the course including also some additional illustrations

- The app is available on GitHub and can be invoked using the following two-step procedure (assuming internet connection is available and you have installed the aforementioned packages)

1. Start R

2. Run the command

```
shiny::runGitHub("Repeated_Measurements", "drizopoulos")
```

this will open a new web browser window (or tab) with the app

- Note: in order the app to be functional you should **not** close R

# References

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  - ▷ Diggle, P., Heagerty, P., Liang, K.-Y., and Zeger, S. (2002). *Analysis of Longitudinal Data*, 2nd edition. New York: Oxford University Press.
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  - ▷ Molenberghs, G. and Verbeke, G. (2005). *Models for Discrete Longitudinal Data*. New York: Springer-Verlag.
  - ▷ Fitzmaurice, G., Laird, N., and Ware, J. (2011). *Applied Longitudinal Analysis*, 2nd Ed. Hoboken: John Wiley & Sons.
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  - ▷ Lindsey, J. (1993). *Models for Repeated Measurements*. Oxford: Oxford University Press.
  - ▷ Pinheiro, J. and Bates, D. (2000). *Mixed Effects Models in S and S-plus*. New York: Springer-Verlag.
  - ▷ Verbeke, G. and Molenberghs, G. (2000). *Linear Mixed Models for Longitudinal Data*. New York: Springer-Verlag.

*... the megalomaniacal strategy of fitting a grand unified model, supposedly capable of answering any conceivable question that might be posed, is, in our view, dangerous, unnecessary and counterproductive.*

Drum and McCullach (1993, *Statistical Science* **8**, 300–301)