Dynamic price competition with endogenous and exogenous switching costs

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Abstract

Switching costs affect consumers in their daily life and may hinder their free decision to change providers of a certain product or service. This paper presents a general theoretical framework for a dynamic competition game under the presence of two types of switching costs: endogenous, which are set by the firms, and exogenous costs that are specific to consumers. In this two-period game, the two providers compete in prices and switching fees, and they can price discriminate in the second period between old (loyal) and new (switchers) consumers.
There are symmetric subgame Nash equilibria in pure strategies, in which the market is split equally between firms and a third of the population switch in the second period. Equilibrium prices and switching fees are not uniquely determined, but firms' profits are. An important result is that endogenous switching costs (in the form of penalties or fees, e.g. early termination fees (ETF)) do not play a role, and only impact inter-temporal payoffs with countervailing effects, leaving lifetime payoffs unaffected. Whereas, exogenous switching costs affect consumer and social welfare. My model suggests that regulatory policies, in the telecommunications industry for example, should reduce exogenous switching costs (such as number portability, standardization or compatibility) rather than eliminate or regulate any switching fees.

Keywords: Dynamic competition, duopoly, switching costs, introductory offers.

1 Introduction

Switching costs (SC) affect consumers in their daily life, and may hinder their free decision to change providers of certain product or service. These costs may be related to other costs, such as learning costs (software usage), information costs (medical history of patients for example), transactions costs (paperwork to terminate and initiate the consumption of certain service), or due to direct firms' practices to keep consumers by charging early contract termination fees, by offering coupons and discounts to praise frequent consumers.

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Firms benefit from the presence of switching costs and use it strategically to keep their consumer base by locking them in. Consumers that stay for a long time may be more profitable than acquiring new ones for some markets. As an example, we can think of car insurance market vs. wholesale markets, in the first case, consumers are profitable after one or two years, while in the latter, they are immediately profitable. Thus, as the literature suggests firms strategically use switching costs to "lock-in" and "then-ripoff" consumers, which lessen competition and social welfare. Understanding the nature of switching costs and its impact on market outcomes and equilibrium conditions is important for researchers and policy makers, given their presumed detrimental effect by increasing average price or even changing price structures (NERA, 2003).

This paper contributes to the literature by analyzing the effect of exogenous and endogenous switching costs. It is closely related to (Chen, 1997) but differs from it by endogenizing the switching costs and clearly distinguishing from exogenous switching costs. It also differs from (Shi, 2013) by considering introductory offers and making exogenous switching cost different across consumers.

Switching costs are considered endogenous when directly affect firms' profits, and firms decide on their level, which includes any fee or lock-in mechanism that directly affects firms' profits.[1] On the other hand, switching costs are considered exogenous (exogenous to the firm) when affecting only to consumers that switch and firms do not set their level. That means that these include any other cost incurred by the consumer when switching providers, such as psychological costs of switching, learning costs, the opportunity cost of time spent on paperwork, among others.[2] My ultimate aim with this paper is to understand what determines the endogenous component (switching fee), how is its strategic use affect to the market outcomes and consumer welfare, and how a reduction of the exogenous component (exogenous switching costs) impacts the market. This would also allow for improved policy recommendations.

The relevant literature is vast, switching costs are usually explained by dynamic price models, from simple two-period models to infinite period models (Klemperer, 1983, 1987b,a; Farrel and Shapiro, 1988; Caminal and Matutes, 1990; Beggs and Klemperer, 1992; Bensaid and Lesne, 1996; Padilla, 1995; To, 1996; Dube et al., 2009; Pearcy, 2011; Cabral, 2012, 2016; Fabra and Garcia, 2012, 2015). Some explain market entry under the presence of switching costs (Klemperer, 1988; Farrel and Shapiro, 1988; Beggs and Klemperer, 1992; Wang and Wen, 1998). Most of the theoretical models use exogenous switching costs only, few as Caminal and Matutes (1990) and Shi (2013) add endogenous switching costs in the form of discounts, also Chen (1997) explains poaching practices with exogenous switching costs.

I develop a theoretical framework for a dynamic competition game under the pres-

[1] As examples we can think of Early termination fee in the telecommunications industry; the loyalty programs in the airline market; or cash rewards program in the credit card market. In this particular paper, we focus in the existence of switching fees in the form of ETF.

[2] As an example, we can think of the costs of unlocking a phone by a technician (locking phones are widely seen in mobile telecommunication market, and can be thought as given). Also incurred costs of security clearance paperwork relevant for the labor market in developing countries; or the time spent to get a medical examination and report to prove health condition in the insurance market.

ence of switching costs, where two providers compete in prices and strategically use switching costs (SC are endogenous and set by the firms). I consider a two-period game where firms simultaneously compete and set prices and switching fees in the first period; in the second period, firms use introductory offers since they can distinguish between old and newcomers consumers, to attract new consumers (rival's consumers).

Using a discrete choice approach, the model allows for heterogeneity across consumers. Thus, by backward induction and focusing on finding equilibria in pure strategies, I found symmetric Nash equilibria in the subgame, where the market is split equally between firms and a third of the population switch in the second period. Equilibrium tuples of prices and switching fees are not uniquely determined, but firms' profits are. An important result is that endogenous switching costs (in the form of switching fees) do not play a role, and only impact intertemporal payoffs with countervailing effects, leaving multiperiod payoffs unaffected. According to these results, second-period prices are increasing in exogenous switching costs, and loyal consumers are charged higher than newcomers (switchers). Moreover, since both, social welfare and consumer surplus are negatively affected by the exogenous switching cost parameter, the model suggests that lowering exogenous switching costs (by a regulatory change for example) would lead to higher consumer surplus and bigger social welfare.

This paper is organized as follows: the related literature is presented in the following subsection, then Section 2 presents the model in detail, Section 3 presents the equilibrium analysis, and Section 4 presents the conclusions.

Related literature

A major source of information about switching costs and network effects is found in Farrel and Klemperer (2007), where the topic is broadly explained. Fudenberg and Tirole (2000) and Chen (1997) offer friendly approach to the strategies used by firms to attract customers from their competitors. Toolsema (2009) adds an interesting approach by differentiating intra and interfirm switching costs, but she restricts her analysis to a static monopoly pricing structure. Shapiro (1999) deals directly with the exclusivity of services within industries with network effects.

Markets with switching costs are usually explained by dynamic models, being the two-period model the simplest one. However, a static approach was also used, Klemperer (1988) analyzes firms' entry decisions in markets with switching costs and its welfare effect. According to that model, when switching costs are unavoidable, entry is found to be socially undesirable due to the welfare losses caused by the switching costs that entrant's consumers have to face and the incumbents' output that would have been efficiently provided with no entry. Following two-period models, these costs seem to lead to higher equilibrium prices and higher profits, therefore markets with switching costs become more attractive to the entry of new firms, and market shares would converge to the same rate if firms exhibit similar costs (Beggs and Klemperer, 1992). This may be the reason why switching costs reduces demand elasticities.

In theory, the total effect of switching costs in competition is not clear. By modeling a two-period economy that produces a homogeneous good, Klemperer (1987b) finds that price competition in a market with switching costs leads to increased competition in the first period to get the larger portion of the market in order to maximize second-period rents. [3] [4] Competition intensity, however, is reduced in the following period, when also firms produce much less. Thus due to switching costs, welfare is expected to lessen. In a similar study, but with differentiated goods, the effect on competition is found ambiguous for the first period, but damaging in the second period due to the firms' incentive to take advantage of their loyal established consumers Klemperer (1987a).

Some interesting extensions have been worked to the two-period model, for example, Caminal and Matutes (1990) present a duopoly model with endogenous switching costs and differentiated product, in this case, pricing practices to retain customers are considered, as well as firms are allowed to pre-commit to prices or to give coupons in the initial period. They found that pricing commitment improves competition, while coupons policies shrink it. Since in their model firms endogenize the switching costs, they would prefer switching costs to be absent, but since their next period rents depend on retained consumers, they would usually generate switching costs in the form of coupons or discounts.

Likewise, some other researchers used an infinite-period model with an overlapping generation approach. Thus the markets would have established consumers and newcomers; some of the models also include switchers and a replacement rate of established consumers (Farrel and Shapiro, 1988; Padilla, 1995; To, 1996; Cabral, 2012). In general, these studies solve for Markov Perfect equilibria and get nearly similar results. Farrel and Shapiro (1988) finds that incumbents would supply only to their loyal/attached consumers and the entrants would have the newcomers' market to serve; however, switching costs generate excessive entry, which creates inefficiencies in the market. In the equilibrium found by Padilla (1995), switching costs generate higher prices and profits in every period, and prices are found to be increasing with firms' customer base; which also implies more difficulties in sustaining tacit collusion. [5]

Also, To (1996) (based in Beggs and Klemperer (1992) where market shares evolve monotonically) finds that when consumers face finite horizon, the market dominance alternates among firms, as well as prices do. With a different perspective, and also based on an infinite-period model, Cabral (2012) finds conditions for switching costs to affect prices in opposite directions. According to the study, switching costs in markets already competitive, strengthen the competitive behavior by intensifying competition for new customers; however in market with lower initial competition degree, switching costs would make the market even less competitive because the switching costs' effect on reinforcing market power of larger firms dominates.

[3]Firms fiercely compete for attracting customers in the first period, even when that means setting prices below costs. This happens because they would charge monopoly prices in the second period to their loyal consumers.

[4]Farrel (1986) shows that firms with larger market share in the first period charge higher prices in the second period, up to the level that the firm still gets the larger market share in the second period.

[5]Switching costs would make punishments less severe in collusive agreements.

2 The Model

2.1 Price competition with introductory offers

There is a unit mass of a continuum of consumers, who are heterogeneous in their random firm preference and random exogenous switching cost. There are two competing providers A and B, who offer substitute services to their consumers.

For simplicity, I assume that firms' marginal service cost is zero, but they still face an entry cost F. The providers operate in two periods only and have the same discount rate $\delta \in (0, 1]$. A contract with provider $i \in \{A, B\}$ in period 1 is a pair (T_i, s_i), where T_i is the fee a consumer has to pay for the first period unlimited service of i, and s_i is a switching fee a consumer of i will have to pay to provider i if he switches in the second period from i to provider j, $j \neq i$ (let's think of a cost such as an early termination fee ETF). A contract in period 2 with provider $i \in \{A, B\}$ specifies a fee T_{ii} a consumer that chose i in both periods has to pay for the second period unlimited service of i; and a fee T_{ji} a consumer that switched providers from j to i has to pay for the second period unlimited service of i.

From the demand side, and following a discrete choice approach, consumers have per period linear indirect utility functions and have the same discount rate $\beta \in [0, 1)$.[6] Every consumer k has valuation $v + \sigma_{ik}$; v is the valuation for the service and is constant and identical across consumers, and σ_{ik} is a relative idiosyncratic preference for provider i respect to provider j, which is uniformly distributed on the interval $[-\theta_1, \theta_2]$. This relative taste is revealed only in the first period and kept unchanged in the second period, consumers do not change preferences between periods and there is no learning either. [7]

Consumers' outside option is assumed to worth $-\infty$, so the market is covered. Consumers know their preferences and exogenous switching costs, firms only know the distribution of those idiosyncratic variables.

In the second period, the source of heterogeneity comes from the presence of an exogenous switching cost x_k that is individual specific and is uniformly distributed on the interval $[0, \omega]$, thus $f(x_k) = \frac{1}{\omega}$. This exogenous switching cost x_k refers to learning costs or costs (time, money, etc.) incurred by canceling and account or unlocking a mobile phone set in the mobile cellular industry for example.

In the first period, after observing $((s_A, T_A), (s_B, T_B))$, every consumer chooses a provider from $\{A, B\}$. Then, given their chosen provider in the first period, in the second period, consumers decide either to stay with their provider i, or to switch to the other provider and pay a switching fee s_i, $i \in \{A, B\}$ to their previous provider and incur in additional switching costs x_k.

In the second period, given exogenous switching costs, prices and switching fees, consumers choose to stay with or to switch to the network provider the gives them the highest indirect utility. In the first period, given preferences, prices and assuming con-

[6] I took the approach reviewed in the section 2.5 of Anderson et al. (1992), also used by Cabral (2016).

[7] This relative preference are such that $\sigma_{Ak} = -\sigma_{Bk}$.

sumers have rational expectations, they choose the network provider that gives them the highest lifetime indirect utility.

Consumers
In period 1, the indirect utility of a consumer k of firm $i \in \{A, B\}$ is

$$R^i_{1k} = v + \sigma_{ik} - T_i$$

In period 2, the indirect utility of a consumer that chose firm $i \in \{A, B\}$ in period 1 is

$$R^i_{2k} = \max\{R_{ii,k}, R_{ij,k}\}$$

where,

$$\begin{array}{ll} R_{ii,k} = v + \sigma_{ik} - T_{ii} & \text{if the consumer chose firm } i \text{ also in the first period} \\ R_{ij,k} = v + \sigma_{ik} - T_{ij} - s_i - x_k & \text{if the consumer switched firms from } i \text{ to } j \end{array}$$

The decision variable in the first period is given by the idiosyncratic relative taste parameter, and in the second period the decision variable is the exogenous switching cost.

Firms do not know consumers' idiosyncratic preferences and exogenous costs, but they know their distribution.

The lifetime net indirect utility of a consumer k who chooses firm i in period 1 is

$$R^i_k = R^i_{1k} + \beta E[R^i_{2k}] \tag{1}$$

I denote α and $(1-\alpha)$ the first period market shares of firms A and B, respectively. Likewise, in the second period, n_{ii} refers the share of consumers that consume from i in both periods, and n_{ij}, to the share of consumers that switched from i to j.

Firms
In the second period, firms‘ profits come from loyal consumers, newcomers and the switching fee collected from switchers that left. Thus, second period profits are given by

$$\pi^i_2 = n_{ii}T_{ii} + n_{ji}T_{ji} + n_{ij}s_i \quad \forall i, j \in \{A, B\}$$

where, n_{ij} denotes the share of consumers that chose firm i in the first period, and switched to firm j in the second period.

In the first period, $\pi^i_1 = \alpha T_i - F$, but firms maximize their total lifetime profits:

$$\max_{T_i, s_i} \pi^i = \pi^i_1 + \delta\pi^i_2 \qquad \forall i \in \{A, B\} \tag{2}$$

Timeline of the game

The game timeline is described below:

$t=1$		$t=2$	
Stage 1	Stage 2	Stage 1	Stage 2
Firms set flat prices (T_A,T_B) and switching fees (s_A,s_B)	Given prices and preferences, consumers choose their provider	Firms set new flat prices ((T_{AA},T_{BA}),(T_{BB},T_{AB}))	Given new prices, consumers decide to stay or to switch

In the first stage of the first period, firms choose simultaneously (T_A, s_A) and (T_B, s_B). In the second stage, consumers observe firms' choices and simultaneously choose provider (A, B).

In the first stage of the second period, firms choose simultaneously their prices for newcomers, T_{BA} and T_{AB}, and for loyal consumers, T_{AA} and T_{BB}. Firms do not commit to keep previous period prices. Consumers observe the new prices, and simultaneously decide whether to stay with their providers or to switch providers.

I solve this game using backward induction, so I start finding the second-period equilibrium, and then continue with the first-period equilibrium in pure strategies.

2.1.1 The Second Period Equilibrium

Given their first period choices of provider and the second period new prices, consumers make their decision to switch or stay with their current provider.

A consumer k stays with his first-period provider if and only if his net payoff with this provider is at least as high as with the other (net of switching fee and costs), that means when $R_{ii} \geq R_{ij}$ (I omit the consumer index k for simplicity).

Therefore, the probability that a consumer choose to stay with his first period provider i is:

$$P_{ii} = Pr[stay\ \ in\ \ i|i] = Pr(R_{ii} \geq R_{ij}) = Pr(x \geq T_{ii} - T_{ij} - s_i)$$
$$P_{ij} = Pr[switch\ \ to\ \ j|i] = 1 - P_{ii}$$

First, for consumers that chose A as their first-period provider, we understand they revealed their relative preference for A, so they have σ_{Ak}. Within this group, a consumer will be indifferent between staying in A (staying loyal) and switching to B if x is such that $R_{AA} = R_{AB}$ and $x \in [0, \omega]$, which means

$$v + \sigma_A - T_{AA} = v + \sigma_A - T_{AB} - s_A - x$$
$$x_A = T_{AA} - T_{AB} - s_A$$

Thus, x_A represents exogenous switching cost level for the consumer in A who is indifferent between staying in A or switching to B, provided that $x \in [0, \omega]$. If $x_A > \omega$ then every consumer of A prefers to stay with A. If $x_A < 0$ then every consumer of A prefers to switch to B. If $0 \leq x_A \leq \omega$, then consumers with exogenous switching cost x above x_A will stay with A and those consumers with exogenous switching costs below x_A will switch to B.

Similar analysis holds for the case of consumers that chose B as their first period provider. Hence, a consumer will be indifferent between staying and switching when $R_{BB} = R_{BA}$, thus, $x_B = T_{BB} - T_{BA} - s_B$

Provided that $x_i \in [0, \omega]$, the choice probabilities are the following

$$\begin{aligned}
P_{AA} &= \int_{x_A}^{\omega} \frac{1}{\omega} dx = \frac{\omega - (T_{AA} - T_{AB} - s_A)}{\omega} \\
P_{AB} &= \int_{0}^{x_A} \frac{1}{\omega} dx = \frac{T_{AA} - T_{AB} - s_A}{\omega} \\
P_{BB} &= \int_{x_B}^{\omega} \frac{1}{\omega} dx = \frac{\omega - (T_{BB} - T_{BA} - s_B)}{\omega} \\
P_{BA} &= \int_{0}^{x_B} \frac{1}{\omega} dx = \frac{T_{BB} - T_{BA} - s_B}{\omega}
\end{aligned}$$

Assuming α is the first period market share of provider A, and $(1-\alpha)$, of provider B, we get the demands of loyal and switchers for each firm: $n_{AA} = \alpha P_{AA}$; $n_{AB} = \alpha P_{AB}$; $n_{BB} = (1-\alpha)P_{BB}$; and $n_{BA} = (1-\alpha)P_{BA}$.
Thus, the second period profits of the providers are

$$\pi_2^A = n_{AA}T_{AA} + n_{BA}T_{BA} + n_{AB}s_A$$

$$\pi_2^B = n_{BB}T_{BB} + n_{AB}T_{AB} + n_{BA}s_B$$

Therefore, taking into account the values of choice probabilities, and provided that $x_i \in [0, \omega]$, second period profits are

$$\pi_2^A = \alpha T_{AA} - \frac{\alpha}{\omega}(T_{AA} - T_{AB} - s_A)(T_{AA} - s_A) + \frac{(1-\alpha)}{\omega} T_{BA}(T_{BB} - T_{BA} - s_B) \tag{3}$$

$$\pi_2^B = (1-\alpha)T_{BB} - \frac{(1-\alpha)}{\omega}(T_{BB} - T_{BA} - s_B)(T_{BB} - s_B) + \frac{\alpha}{\omega} T_{AB}(T_{AA} - T_{AB} - s_A) \tag{4}$$

Provided that $x_A \in [0, \omega]$ and $x_B \in [0, \omega]$, profit functions are quadratic and concave in their arguments (prices), so we expect an interior solution.[8]

Solving for the second-period equilibrium by using the first order conditions, the following is the unique solution, which also satisfies the second order conditions:

[8]The second derivatives are negative: $\frac{\partial^2 \pi_2^A}{\partial {T_{AA}}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_2^A}{\partial {T_{BA}}^2} = -\frac{2(1-\alpha)}{\omega} < 0$. Likewise, $\frac{\partial^2 \pi_2^B}{\partial {T_{AB}}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_2^B}{\partial {T_{BB}}^2} = -\frac{2(1-\alpha)}{\omega} < 0$

$$T^*_{AA} = \frac{2}{3}\omega + s_A \tag{5}$$

$$T^*_{BB} = \frac{2}{3}\omega + s_B \tag{6}$$

$$T^*_{BA} = T^*_{AB} = \frac{\omega}{3} \tag{7}$$

Therefore, the equilibrium outcome for second-period prices do not depend on first-period market shares. These prices are increasing in the exogenous switching cost ω, and the endogenous switching fee s_i only affects the prices that loyal consumers face.[9]

Also, given these values, second-period shares are $n_{AA} = \frac{2}{3}\alpha$, $n_{AB} = \frac{1}{3}\alpha$, similarly, $n_{BB} = \frac{2}{3}(1-\alpha)$, $n_{BA} = \frac{1}{3}(1-\alpha)$, which clearly indicates that a third of first period consumers switch in the second period. This is a similar result to Chen (1997) for the case of paying-consumers-to-switch.

Proposition 1. *Provided that $x_A, x_B \in [0, \omega]$, there exists a unique Nash equilibrium of the second period subgame, where second-period prices do not depend on first-period market shares and the endogenous switching fee s_i only affects the prices that loyal consumers face. A third of the population switch.*

The equilibrium in proposition 1 is a second period Nash equilibrium in the subgame and the proof is provided in the appendix. Now, using (5), (6) and (7) into (3) and (4) profits are

$$\pi_2^{A*} = \frac{\omega}{9}(1 + 3\alpha) + \alpha s_A \tag{8}$$

$$\pi_2^{B*} = \frac{\omega}{9}(1 + 3(1-\alpha)) + (1-\alpha) s_B \tag{9}$$

As expected, second period profits depend heavily in their first market share, which may imply higher incentives of firms to lock-in consumers with higher switching fees. It is easy to check that $\frac{\partial \pi_2^{A*}}{\partial \alpha} = \frac{\omega}{3} + s_A > 0$ if $s_A \geq -\frac{\omega}{3}$.

2.1.2 The First Period

In the first period, consumers make a choice between providers, therefore, the indirect utility of a consumer k will be given by :

$$R_{1k}^A = v + \sigma_{Ak} - T_A$$
$$R_{1k}^B = v - \sigma_{Ak} - T_B$$

[9] Second period prices are positively affected by exogenous switching cost: $\frac{\partial T_{ii}}{\partial \omega} = \frac{2}{3} > 0$ and $\frac{\partial T_{ij}}{\partial \omega} = \frac{1}{3} > 0$

where σ_{Ak} is the relative preference for firm A respect to firm B , and is uniformly distributed on the interval $[-\theta_1, \theta_2]$. Recall that $\sigma_{Bk} = -\sigma_{Ak}$.

However in the first period, consumers do not take decisions only based on their current period indirect utilities, but based on their lifetime utilities. Thus, each consumer compare R^A vs. R^B

$$\begin{array}{c} R^A = R_1^A + \beta E[R_2^A] \\ R^B = R_1^B + \beta E[R_2^B] \end{array}$$

where

$$E[R_2^i] = P_{ii}R_{ii} + P_{ij}R_{ij} \qquad \forall i, j \in \{A, B\}$$

Therefore, we get $E[R_2^A]$ and $E[R_2^B]$ using the distribution of exogenous switching costs x_k

$$E[R_2^A] = v + \sigma_A - \left(\int_{x_A}^{\omega} T_{AA}^* \frac{1}{\omega} dx + \int_0^{x_A} (T_{AB}^* + s_A + x) \frac{1}{\omega} dx \right) = v + \sigma_A - \tfrac{11}{18}\omega - s_A$$

$$E[R_2^B] = v - \sigma_A - \left(\int_{x_B}^{\omega} T_{BB}^* \frac{1}{\omega} dx + \int_0^{x_B} (T_{BA}^* + s_B + x) \frac{1}{\omega} dx \right) = v - \sigma_A - \tfrac{11}{18}\omega - s_B$$

Therefore,

$$\begin{array}{c} R^A = v + \sigma_A - T_A + \beta(v - \frac{11}{18}\omega - s_A + \sigma_A) \\ R^B = v - \sigma_A - T_B + \beta(v - \frac{11}{18}\omega - s_B - \sigma_A) \end{array}$$

Thus,

$$P_A = Pr[choose \;\; A] = Pr[R^A \geq R^B] = Pr[\sigma_A \geq \tfrac{1}{2(1+\beta)}(T_A - T_B + \beta(s_A - s_B))]$$

A consumer is indifferent between A and B when $\sigma_A^* = \frac{1}{2(1+\beta)}(T_A - T_B + \beta(s_A - s_B))$, hence, provided that $\sigma_A^* \in [-\theta_1, \theta_2]$ we can get the choice probabilities:

$$P_A = \int_{\sigma_A^*}^{\theta_2} \frac{1}{\theta_2 + \theta_1} d\sigma_A = \frac{1}{2(\theta_2 + \theta_1)(1 + \beta)} (2\theta_2(1 + \beta) - (T_A - T_B + \beta(s_A - s_B)))$$

Similarly,

$$P_B = \int_{-\theta_1}^{\sigma_A^*} \frac{1}{\theta_2 + \theta_1} d\sigma_A = \frac{1}{2(\theta_2 + \theta_1)(1 + \beta)} (2\theta_1(1 + \beta) + (T_A - T_B + \beta(s_A - s_B)))$$

Since we have a unit mass of consumers, these probabilities actually give us the first-period market shares of the providers, therefore $\alpha = P_A$. So, first-period profits for providers are $\pi_1^A = P_A T_A - F$ and $\pi_1^B = (1 - P_A)T_B - F$.

Providers maximize their lifetime profits:

$$\begin{array}{c} \max_{T_A, s_A} \pi^A(T_A, T_B, s_A, s_B) = \pi_1^A + \delta\pi_2^{A*} \\ \max_{T_B, s_B} \pi^B(T_A, T_B, s_A, s_B) = \pi_1^B + \delta\pi_2^{B*} \end{array}$$

Plugging all the second-period optimal values and the value for first-period market share ($\alpha = P_A$), providers A and B maximize the following profits over their first-period prices and switching fees.

$$\pi^A = \frac{T_A\theta_2}{\theta_1+\theta_2} + \frac{1}{2(\theta_1+\theta_2)(1+\beta)}[T_B - T_A + \beta(s_B - s_A)][T_A + \delta(\tfrac{\omega}{3} + s_A)] + \frac{\delta(4\omega\theta_2+9s_A\theta_2+\omega\theta_1)}{9(\theta_1+\theta_2)} - F$$
$$\pi^B = \frac{T_B\theta_1}{\theta_1+\theta_2} + \frac{1}{2(\theta_1+\theta_2)(1+\beta)}[T_A - T_B + \beta(s_A - s_B)][T_B + \delta(\tfrac{\omega}{3} + s_B)] + \frac{\delta(4\omega\theta_1+9s_B\theta_A+\omega\theta_2)}{9(\theta_1+\theta_2)} - F$$

First order conditions give us the following reaction functions:

$$\begin{aligned}
T_A(T_B, s_A, s_B) &= \frac{1}{2}(\theta_2(1+\beta) + T_B + \beta s_B - s_A(\beta+\delta) - \frac{\delta\omega}{3}) \\
s_A(T_A, T_B, s_B) &= \frac{\theta_2(1+\beta)}{\beta} + \frac{T_B + \beta s_B}{2\beta} - \frac{T_A(\beta+\delta)}{2\delta\beta} - \frac{\omega}{6} \\
T_B(T_A, s_A, s_B) &= \frac{1}{2}(\theta_1(1+\beta) + T_A + \beta s_A - s_B(\beta+\delta) - \frac{\delta\omega}{3}) \\
s_B(T_A, T_B, s_A) &= \frac{\theta_1(1+\beta)}{\beta} + \frac{T_A + \beta s_A}{2\beta} - \frac{T_B(\beta+\delta)}{2\delta\beta} - \frac{\omega}{6}
\end{aligned}$$

Since profit functions are quadratic in their arguments (first-period prices and switching fees), we expect to get an interior solution.[10] Considering that consumers and providers are equally patient ($\delta = \beta$) – which is sensible when we think of stakeholders as owners of firms, who are ultimately consumers as well– and solving the system of equations, we find subgame perfect equilibria: [11]

$$T_A^* = \frac{2(1+\delta)}{3}(\theta_1 + 2\theta_2) - \delta(\frac{\omega}{3} + s_A) \tag{10}$$
$$T_B^* = \frac{2(1+\delta)}{3}(2\theta_1 + \theta_2) - \delta(\frac{\omega}{3} + s_B) \tag{11}$$

$$\frac{1}{3\delta}(2\delta(\theta_1 + 2\theta_2) + 3(\theta_1+\theta_2)) - \frac{v}{\delta} - \frac{\omega}{3} \leq s_A^* \leq v - \frac{2\omega}{3} - \frac{(\theta_1 - \theta_2)}{3} \tag{12}$$
$$\frac{1}{3\delta}(2\delta(2\theta_1 + \theta_2) + 3(\theta_1+\theta_2)) - \frac{v}{\delta} - \frac{\omega}{3} \leq s_B^* \leq v - \frac{2\omega}{3} + \frac{(\theta_1 - \theta_2)}{3} \tag{13}$$

The lower bound of switching fees comes from satisfying the constraint $R_1^i \geq 0$ and the upper bound, from satisfying $R_{ii} \geq 0$ and $R_{ij} \geq 0$, so some consumers switch and some stay. [12]

Thus, second period prices are still given by (5), (6) and (7), where switching fees are s_A^* and s_B^*. It is easy to check that second-period prices are positively affected by exogenous switching cost parameter ω, so an external reduction of exogenous switching costs would reduce second period prices, for both loyal consumers and switchers; but this

[10] The second derivatives are negative: $\frac{\partial^2\pi^A}{\partial {T_A}^2} = \frac{\partial^2\pi^B}{\partial {T_B}^2} = -\frac{1}{(\theta_1+\theta_2)(1+\beta)} < 0$, and $\frac{\partial^2\pi^A}{\partial s_A^2} = \frac{\partial^2\pi^B}{\partial s_B^2} = -\frac{\beta\delta}{(\theta_1+\theta_2)(1+\beta)} < 0$.

[11] This assumption $\delta = \beta$, also guarantees that the Hessian matrix of the system of equations to be negative semi-definite, sufficient condition to get an interior solution.

[12] If s_i is very large such that $R_{2i}^j < 0$ then nobody switches, moreover all forward looking consumers will be better off out of the market.

reduction also would increase first period prices and the upper bound of endogenous switching fees (if the change is anticipated for the firms).

First period market share of A, $\alpha = \frac{\theta_1+2\theta_2}{3(\theta_1+\theta_2)}$, is increasing in θ_2 and decreasing in θ_1, conversely for the case of market share of provider B. [13] Finally, using (10) to (13) into the profit functions of the firms, we get second period profits

$$\pi_2^A = \frac{1}{9(\theta_1+\theta_2)}[\omega(2\theta_1+3\theta_2)+3s_A(\theta_1+2\theta 2)] \tag{14}$$
$$\pi_2^B = \frac{1}{9(\theta_1+\theta_2)}[\omega(3\theta_1+2\theta_2)+3s_B(2\theta_1+\theta 2)] \tag{15}$$

and lifetime profits

$$\pi^{A*} = \frac{\delta\omega}{9} + \frac{2(1+\delta)(\theta_1+2\theta_2)^2}{9(\theta_1+\theta_2)} - F$$
$$\pi^{B*} = \frac{\delta\omega}{9} + \frac{2(1+\delta)(2\theta_1+\theta_2)^2}{9(\theta_1+\theta_2)} - F$$

Lifetime profits do not depend on switching fees, $\frac{\partial\pi^{i*}}{\partial s_i} = 0\ \forall i \in \{A, B\}$, but profits are increasing in the exogenous switching costs parameter ω and the taste parameter that favors the provider (for instance $\frac{\partial\pi^{A*}}{\partial\theta_2} > 0$ and $\frac{\partial\pi^{A*}}{\partial\theta_1} < 0$).

The indifferent consumer has an idiosyncratic taste level of $\tilde{\sigma_A} = \frac{\theta_2-\theta 1}{3}$ and gets lifetime indirect utility of

$$R^i = v(1+\delta) - \frac{5\omega\delta}{18} - (\theta_1+\theta_2)$$

Increasing the taste parameter, either θ_1 or θ_2 will determine consumers' choice of a provider. Therefore providers have the incentive to invest in changing the magnitude of these preferences. The following section extends the model.

2.2 Providers invest on marketing

Given that firms increases market share and profits with relative preference parameter, we enable firms to invest on it. We assume this 'marketing' cost is convex. So firms' first period profit changes to:

$$\pi_1^A = \alpha T_A - \phi\theta_2^2 - F$$
$$\pi_1^B = (1-\alpha)T_B - \phi\theta_1^2 - F$$

In the stage zero of period one, providers decide how much to invest to increase the relative taste that favors them. Thus, the timeline of this extended model is depicted in the following diagram.

[13] It is easy to verify that $\frac{\partial\alpha}{\partial\theta_1} = -\frac{\theta_2}{3(\theta_1+\theta_2)^2} < 0$ and $\frac{\partial\alpha}{\partial\theta_2} = \frac{\theta_1}{3(\theta_1+\theta_2)^2} > 0$.

	$t=1$		$t=2$	
Stage 0	Stage 1	Stage 2	Stage 1	Stage 2
Firms invest on increasing taste preferences towards them, set level (θ_1,θ_2)	Firms set flat prices (T_A,T_B) and switching fees (s_A,s_B)	Given prices and preferences, consumers choose their provider	Firms set new flat prices (T_{AA},T_{BA}), (T_{BB},T_{AB})	Given new prices, consumers decide to stay or to switch

Second-period results still apply, but first-period results differ from previous analysis due to the introduction of convex costs of advertising.

Thus, solving by backward induction, firms' marketing investment lead to equal maximum relative preferences level,

$$\theta_1^* = \theta_2^* = \frac{5(1+\delta)}{12\phi} \tag{16}$$

Therefore, each firm get a half of the market in the first period ($\alpha = \frac{1}{2}$). Likewise, equilibrium prices are

$$\begin{aligned} T_A^* &= \frac{5(1+\delta)^2}{6\phi} - \delta(\frac{\omega}{3} + s_A) \qquad & T_{AA}^* &= \frac{2}{3}\omega + s_A \\ & & T_{BB}^* &= \frac{2}{3}\omega + s_B \\ T_B^* &= \frac{5(1+\delta)^2}{6\phi} - \delta(\frac{\omega}{3} + s_B) \qquad & T_{BA}^* = T_{AB}^* &= \frac{\omega}{3} \end{aligned} \tag{17}$$

and switching fees are such that $\forall i \in \{A, B\}$

$$\frac{5(1+\delta)^2}{6\delta\phi} - \frac{v}{\delta} - \frac{\omega}{3} \leq s_i \qquad \text{thus, } R_1^i \geq 0 \tag{18}$$

$$s_i \leq v - \frac{2\omega}{3} \qquad \text{thus, } R_{ii} \geq 0, R_{ij} \geq 0 \tag{19}$$

Given the boundaries for switching fees, lets call $s^{max} = v - \frac{11\omega}{18}$. An increase of the exogenous switching cost parameter would displace the feasible region for switching fees at a lower level, which would imply a substitutability between exogenous and endogenous switching costs: lower exogenous switching costs imply higher upper bound for switching fees. It is important to highlight that switching fees, s_A and s_B, are not necessarily equal, but they should satisfy the above conditions.

Lifetime profits of providers are

$$\pi^{A*} = \pi^{B*} = \frac{\delta\omega}{9} + \frac{35(1+\delta)^2}{144\phi} - F \tag{20}$$

Second period profits are

$$\pi_2^{i*} = \frac{5\omega}{18} + \frac{s_i}{2}$$

and first period profits are

$$\pi_1^{i*} = \frac{35(1+\delta)^2}{144\phi} - \delta(\frac{s_i}{2} + \frac{\omega}{6}) - F \qquad \forall i \in \{A, B\}$$

Notice that $\frac{\partial \pi_1^{i*}}{\partial s_i} < 0$, $\frac{\partial \pi_2^{i*}}{\partial s_i} > 0$, and $\frac{\partial \pi^{i*}}{\partial s_i} = 0$.

An increase in switching fee affects positively to second-period profits but negatively to first-period profits. For the lifetime profit maximizer firm, the effects cancel out and its lifetime profits are independent on switching fee levels. [14]

Now, the indifferent consumer has an idiosyncratic taste level of $\tilde{\sigma_A} = 0$ and gets lifetime indirect utility of

$$R^i = v(1+\delta) - \frac{5\omega\delta}{18} - \frac{5}{6\phi}(1+\delta)^2 \qquad \forall i \in \{A, B\}$$

The results of the game with the small change are summarized in the following proposition.

Proposition 2. *If $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{1}{6\phi}(1+\delta)$, then there are subgame perfect equilibria in pure strategies where each firm gets a half of the market in the first period, and a third of the population switches in the second period. Lifetime profits of both firms are non-negative whenever $F \leq \frac{\delta\omega}{9} + \frac{35(1+\delta)^2}{144\phi}$.*

In these multiple equilibria, where different combinations of prices and switching fees reach the same providers' payoffs, lifetime profits are independent of switching fees.

Corollary Switching fee may be negative for low discount factor, specifically

$$s_i^{min} < 0 \qquad \text{if} \qquad \delta < \tfrac{1}{5}(\phi\omega + \sqrt{\omega^2\phi^2 + 10\phi(3v-\omega)}) - 1 \text{ and } \delta \in [0,1]$$

Thus, depending on the values of the parameters, a negative switching fee may exist for low values of discount factor *delta*. Negative switching fees mean providers care so much on the present and want to extract as much consumer surplus as possible, so they would even promise to pay consumers if they decide to leave.

Proposition 3. *The subgame-perfect equilibrium outcome presented in proposition 2 is unique.*

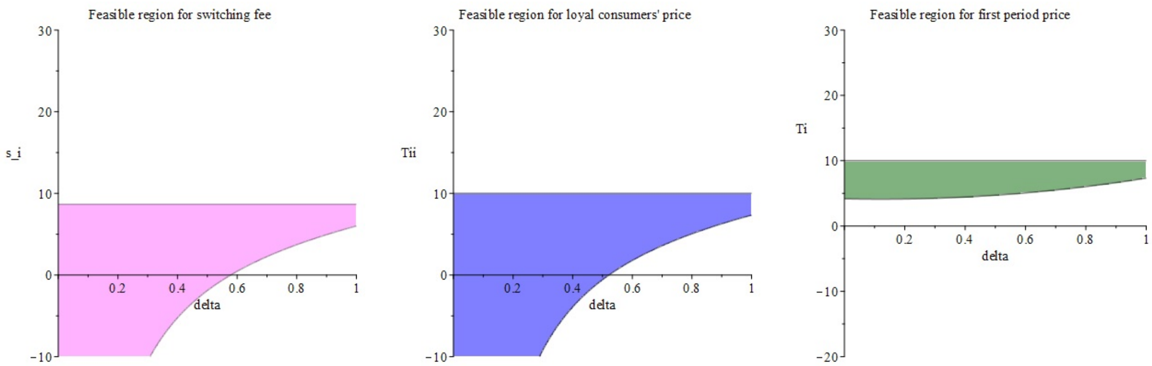
To prove this last proposition, we have to prove that there is no equilibrium where none switches, and there is no equilibrium where everyone switches. The proof is shown in the appendix.

3 Equilibrium analysis: implications

From the symmetric equilibrium conditions given in proposition 2, we can graphically observe the feasible region for switching fees depicted in figure 1. The figure shows that for lower patience level (low δ), providers may offer negative switching fees. This lower

[14] Notice that $\frac{\partial \pi_1^{i*}}{\partial s_i} < 0$, $\frac{\partial \pi_2^{i*}}{\partial s_i} > 0$, and $\frac{\partial \pi^{i*}}{\partial s_i} = 0$.

bound tends to a positive level as the discount factor approaches to one. Prices charged to loyal consumers depends positively on switching fees, and both its lower and upper bounds are higher than those of switching fees. First period prices depend negatively on switching fees, at most this price is set as high as consumers' valuation of the service v; its lower bound increases slightly with the discount factor δ.



(a) Switching fee s_i (b) Price to loyal consumers T_{ii} (c) First period price T_i

Parameters values: $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.

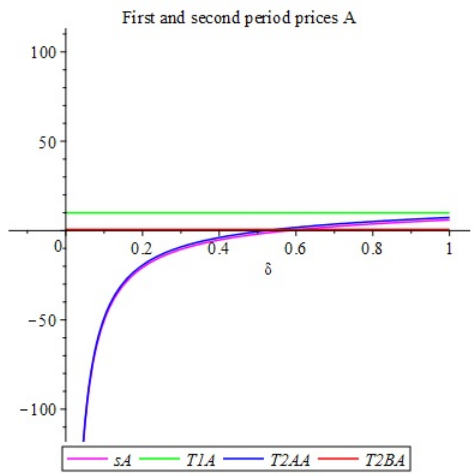
Figure 1: Feasible regions for switching fees optimal prices

Although there are many combinations of prices and switching fees, profits are set in a unique way; firms' profits are increasing in the discount rate and the exogenous switching cost parameter. [15]

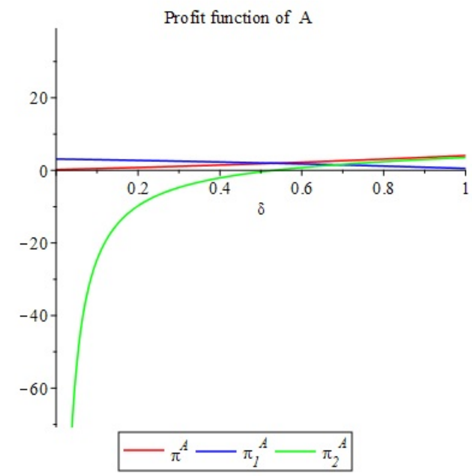
Figures 2 and 3 show the optimal prices and profits as functions of discount factor δ in different scenarios, when providers set minimum and maximum switching fee. Assuming providers always set s^{min}, Figure 2a shows that first-period prices are almost constant and always higher than second -period prices for loyal consumers and switchers; switchers are charged the same ($\omega/3$) regardless of the discount factor. However, given the parameter values, prices for loyal consumers are positive for $\delta > 0.522$ and switching fees are positive for $\delta > 0.5787$. Likewise, figure 2b depicts the profit functions: lifetime profit (red line) is always positive and increasing in δ; first-period profits also are positive but they decrease with patience level. Second-period profits are increasing in δ, but they are positive only for high level of patience ($\delta > 0.53$). This result indicates that the effect of a positive or negative switching fee is inter-temporally compensated in providers' profits.

When providers set a s^{max}, then all prices and switching fees are positive, and first-period profit decreases with patience level and quickly gets negative values, as shown by Figure 3. In this scenario, providers extract the entire consumer surplus in the second period and charge a positive but low first-period prices. However, profits in the first period can be negative due to the entry costs F and the cost of investment in marketing; despite this, lifetime profits are kept positive and increasing in δ, It is important to highlight that the lifetime profit function in both scenarios is the same,

[15] It is easy to check from equation (20) $\frac{\partial \pi_i^*}{\partial \delta} = \frac{\omega}{9} + \frac{35(1+\delta)}{72\phi} > 0$ and $\frac{\partial \pi_1^*}{\partial \omega} = \frac{\delta}{9} > 0$



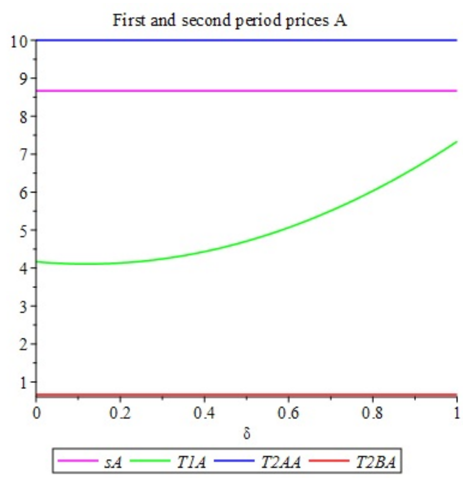
(a) Optimal per period prices



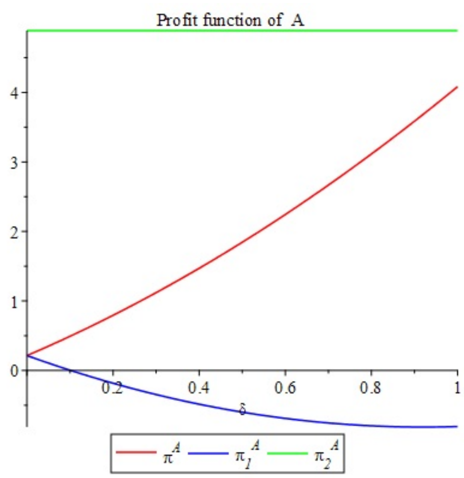
(b) Lifetime and per period profits

Parameters values: $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.

Figure 2: Optimal prices and profits when both providers set $s_i = s^{min}$



(a) Optimal per period prices



(b) Lifetime and per period profits

Parameters values: $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.

Figure 3: Optimal prices and profits when both providers set $s_i = s^{max}$

which is explained by the fact that switching fees do not affect lifetime profits, their effect on period profits are compensated leaving lifetime profits unaffected.

This harmless feature of switching fee is also observed in the consumer problem. Consumer's lifetime indirect utility is unaffected by switching fees; this happens because prices compensate for the presence of switching fees, so the consumer's lifetime indirect utility is independent of switching fees. By using some numerical exercises; table 1 and table 2 present the different calculated values for lifetime profits (for A and B), lifetime consumer surpluses, and lifetime indirect utility of a typical consumer, as well as per period profits and typical consumer's indirect utility under different scenarios.

Table 1 shows how the values of our variables change with different levels of the discount factor, under the scenario where both providers set minimum switching fee versus setting maximum switching fee. For the same discount factor, lifetime profits, lifetime indirect utilities, consumer surplus and social welfare are kept unchanged regardless of the level of switching fee. When switching fees are set to its minimum value, providers extract all the consumer surplus in the first period (charging the highest first-period price). On the other hand, when switching fees are set to their maximum level, providers extract all the consumer surplus in the second period (charging the highest price to loyal consumers).

	A and B set $s_i = s^{min}$		A and B set $s_i = s^{max}$	
	$\delta = 0.5$	$\delta = 0.9$	$\delta = 0.5$	$\delta = 0.9$
Lifetime profit π^i	1.85	**3.59**	1.85	**3.59**
π_1^i	2.05	0.86	**-0.59**	**-0.81**
π_2^i	-0.40	3.02	4.89	4.89
T_i	10.00	**10.00**	4.71	6.64
s_i	-1.92	4.94	8.67	8.67
T_{ii}	-0.58	6.27	10.00	**10.00**
Cost of switching	0.75	7.60	11.33	11.33
R^i	5.354	3.46	5.35	3.46
R_1^i	0	**0**	5.29	3.36
ER_i	10.69	3.84	0.11	0.11
$R_{ii} = R_{ij}$	10.58	3.73	0	**0**
CS^i	3.84	3.61	3.85	3.61
CS	7.69	**7.22**	7.69	**7.22**
SW	11.38	**14.39**	11.38	**14.39**

Parameters values: $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.
Prices to switchers are $T_{2j}^i = 0.67$ for all the cases
Cost of switching includes switching fee, maximum exogenous switching cost ω and switcher's price.

Table 1: Providers set same switching fee, s^{min} or s^{max}

On the other hand, for the same discount factor ($\delta = 0.9$), Table 2 shows again that for any combination of maximum or lowest switching fees used by the providers, lifetime payoffs (profits, indirect utilities, and consumer surpluses) are kept unchanged. The observed differences come from the existing trade-off between inter-temporal payoffs when a low or high switching fee is used.

Consumer surplus and social welfare

Let's now consider and depict the effect of the equilibrium outcomes on the consumer surplus and social welfare (producer plus consumer surplus). As mentioned before, endogenous switching costs or switching fees do not affect the lifetime indirect utility of consumers (they affect per period indirect utility, and these effects that are canceled out in the total discounted lifetime utility), therefore lifetime consumer surplus is also unaffected by switching fees. However, consumer's lifetime and per period indirect utility are affected by exogenous switching costs.

	s^{min} vs. s^{max}		A and B set $s_i = 0$	$s_i = 0$ vs. s^{max}		$s_i = 0$ vs. s^{min}	
	A (s^{min})	B (s^{max})	Firm i	A ($s_i = 0$)	B (s^{max})	A ($s_i = 0$)	B (s^{min})
Lifetime profit π^i	**3.59**	**3.59**	**3.59**	**3.59**	**3.59**	**3.59**	**3.59**
π_1^i	0.86	-0.81	3.09	3.09	-0.81	3.09	0.87
π_2^i	3.02	4.89	0.56	0.56	4.89	0.55	3.02
T_i	**10.00**	**6.64**	**14.44**	14.44	**6.64**	14.44	**10.00**
s_i	4.94	8.67	0	0	8.67	0	4.94
T_{ii}	6.27	10.00	**1.33**	1.33	10	1.33	6.27
Cost of switching	7.60	11.33	2.67	2.67	11.33	2.67	7.60
R^i	**3.46**	**3.46**	**3.46**	**3.46**	**3.46**	**3.46**	**3.46**
R_1^i	0	**3.36**	**-4.44**	-4.44	3.36	-4.44	0
ER_i	**3.84**	**0.11**	**8.78**	8.78	0.11	8.78	3.84
$R_{ii} = R_{ij}$	3.73	**0**	8.67	8.67	0	8.67	3.73
CS^i	**3.61**	**3.61**	**3.61**	**3.61**	**3.61**	**3.61**	**3.61**
CS	**7.22**	**7.22**	**7.22**	**7.22**	**7.22**	**7.22**	**7.22**
SW	**14.39**	**14.39**	**14.39**	**14.39**	**14.39**	**14.39**	**14.39**

Parameters values: $\delta = 0.9$, $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.
Prices to switchers are $T_{2j}^i = 0.67$, and $\theta_1 = \theta_2 = 3.96$ for all the cases.
Cost of switching includes switching fee, maximum exogenous switching cost ω and switcher's price.

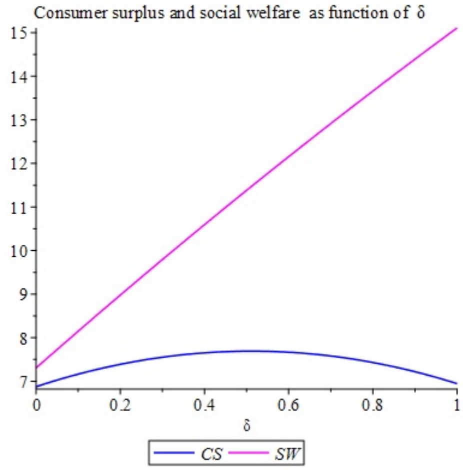
Table 2: Providers set different switching fee, s^{min}, $s_i = 0$ or s^{max}

Given that lifetime profits of providers are also independent of switching fees, social welfare (defined as the summation of consumer surplus and firms' profits) is unaffected by switching fees (endogenous switching costs). This result may be striking, but it may explain why in some industries such as telecommunications, switching fees like ETF are being dismissed by some companies. It also agrees with the findings of Cullen et al. (2016) where equilibria where firms with and without switching fees may coexist. In the model presented in this paper that may happen because the effect of switching fees are compensated inter-temporally in such a way that they do not affect payoffs of consumers either providers.
Figures 4a and 4b show the consumer surplus (CS) and social welfare (SW) as functions of the discount factor δ and of the exogenous switching cost parameter ω. Social welfare is clearly increasing in the patience level (δ), driven basically for greater firms' profits as patience level increases. Meanwhile, social welfare is slightly decreasing on the exogenous switching cost parameter ω.

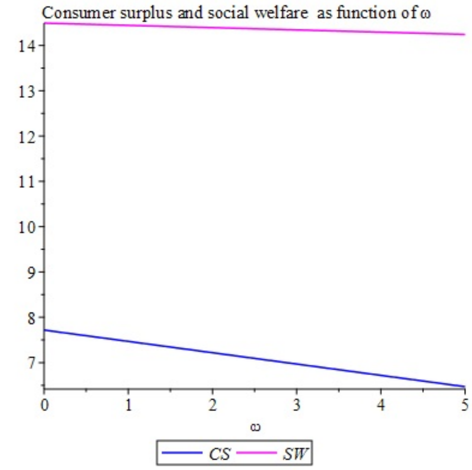
Consumer welfare increases with patience (δ) up to certain level then decreases. This might be associated with the different levels first-period prices have according to the discount factor.[16] On the other hand, consumer welfare decreases with the exogenous switching cost parameter, therefore less exogenous switching costs increases consumer surplus.

The model suggests regulatory policies that reduce exogenous switching costs such as number portability (in telecommunication industries, or banking industries), compatibility, or standardization, would be more effective in increasing social welfare than policies that reduce endogenous switching costs such as switching fee (ETF in telecommunication industry).

[16] $\frac{\partial T_i^*}{\partial \delta} = \frac{5(1+\delta)}{3\phi} - \frac{\omega+3s_i}{3}$, which is positive if $\frac{5(1+\delta)}{3\phi} - \frac{\omega}{3} > s_i$ and negative if $\frac{5(1+\delta)}{3\phi} - \frac{\omega}{3} < s_i \quad \forall i \in \{A, B\}$.



(a) CS and SW as functions of δ



(b) CS and SW as functions of ω

Parameters values: $v = 10$, $\omega = 2$, $F = 1$, and $\phi = 0.2$.

Figure 4: Consumer surplus and social welfare functions

4 Conclusions

The model developed in this paper shows that exogenous switching costs are more relevant than endogenous switching costs in the decision making of consumers. For the providers, switching fees would not affect lifetime profits but would accentuate a trade-off between present and future profits. Providers with high switching fees would compensate consumers with lower first period prices, but would charge higher second-period prices to loyal consumers; low switching fees would be associated with high first-period prices and lower second-period prices to loyals. Thus consumers with lower first-period surplus get compensated with a higher second-period surplus and vice versa.

Second-period prices are positively affected by exogenous switching cost parameter ω. Therefore an unanticipated external reduction of exogenous switching costs would reduce second-period prices, for both loyal consumers and switchers; however, if the providers anticipate the change, this reduction also would increase first-period prices and the possibility of higher switching fees. However, since the adverse effect of switching fees on first-period profits cancels out with their positive effect on the second-period profits, then regulatory policies should focus more on policy measures that reduce exogenous switching costs such as standardization, compatibility, number portability, red-tape reduction, etc.

On the other hand, since profits are increasing in exogenous switching costs ω, therefore providers will have incentives to keep a high ω (opposing to regulatory changes such number portability or standardization or even by increasing searching costs). Also, given that profits are increasing in relative taste parameters, providers have greater incentives to invest in advertising to influence consumer preferences, when they do, in a symmetric case, firms invest until they both get same relative taste level.

According to the model, firms charge higher to loyal consumers than to newcomers in the second period when patience level is high. When both providers charge a maximum switching fee, then they charge higher to loyal consumers in the second period respect to first-period prices.

The effect of switching fees in lifetime payoffs is null, hence policies that target exogenous switching costs reduction may have a higher impact on social welfare than those that ban any existence of switching fees (endogenous SC); external reduction of exogenous switching costs increases social welfare, by increasing consumer surplus.

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Appendix

Proof Proposition 1:
No firm will profitable deviate from the equilibrium prices.

Proof. Suppose provider A deviates and use prices $\widehat{T_{AA}}$ and T^*_{BA} , where $\widehat{T_{AA}} = T^*_{AA}+\Delta$, while provider B keep using equilibrium prices T^*_{BB} and T^*_{AB}. We can check, using (8) that new profits of provider A after deviation are

$$\begin{aligned}
\widehat{\pi^A_2} &= \alpha\widehat{T_{AA}} - \frac{\alpha}{\omega}(\widehat{T_{AA}} - T^*_{AB} - s^*_A)(\widehat{T_{AA}} - s^*_A) + \frac{(1-\alpha)}{\omega}T^*_{BA}(T^*_{BB} - T^*_{BA} - s^*_B) \\
&= \pi^{A*}_2 + \frac{\alpha}{\omega}[\Delta\omega - \Delta(T^*_{AA} - T^*_{AB} - s^*_A) + \Delta(T^*_{AA} - s^*_A) - \Delta^2] \\
&= \pi^{A*}_2 - \frac{\alpha\Delta^2}{\omega}
\end{aligned}$$

Then, $\widehat{\pi^A_2} < \pi^{A*}_2$.

Now, *ceteris paribus*, suppose A deviates to $\widehat{T_{BA}} = T^*_{BA} + \Delta$; then in similar fashion and using (8) that new profits are

$$\begin{aligned}
\widehat{\pi^A_2} &= \alpha T^*_{AA} - \frac{\alpha}{\omega}(T^*_{AA} - T^*_{AB} - s^*_A)(T^*_{AA} - s^*_A) + \frac{(1-\alpha)}{\omega}\widehat{T_{BA}}(T^*_{BB} - \widehat{T_{BA}} - s^*_B) \\
&= \pi^{A*}_2 + \frac{1-\alpha}{\omega}[-\Delta T^*_{BA} + \Delta(T^*_{BB} - T^*_{BA} - s^*_B) - \Delta^2] \\
&= \pi^{A*}_2 - \frac{\alpha\Delta^2}{\omega}
\end{aligned}$$

Once again, $\widehat{\pi^A_2} < \pi^{A*}_2$.

Therefore, regardless of the deviation ($\Delta > 0$ or $\Delta < 0$), profits are always lower than the profit achieved with equilibrium prices, and providers do not have any profitable deviation. □

Proof Proposition 3
Claim 1: There is no equilibrium where nobody switches.

Proof. Let's suppose $x_A > \omega$ & $x_B > \omega$ and analyze the game in the second period. In this case, consumers prefer to stay with their provider, which means that the indirect utilities of a consumer that chose A in the first period are as follows:

$$\begin{aligned}
R_{AA} \geq 0 \quad &\Rightarrow \quad v \geq T_{AA} \\
R_{AB} \leq 0 \quad &\Rightarrow \quad v - s_A - x \leq T_{AB}
\end{aligned}$$

Likewise, the indirect utilities of a consumer that chose B in the first period are

$$\begin{aligned} R_{BB} \geq 0 &\quad\Rightarrow\quad v \geq T_{BB} \\ R_{BA} \leq 0 &\quad\Rightarrow\quad v - s_B - x \leq T_{BA} \end{aligned}$$

Since consumers are better off staying than switching, then $R_{AA} \geq R_{AB}$ and $R_{BB} \geq R_{BA}$. Therefore the following must hold:

$$\begin{aligned} T_{AB} + s_A + x \geq T_{AA} \\ T_{BA} + s_B + x \geq T_{BB} \end{aligned}$$

Given consumers preferences, firms set their second period prices that maximize their profits assuming the rival provider charges zero to newcomers; thus $T_{ii} > 0$ to loyal consumers and $T_{ji} = 0$ for $i \neq j \quad i, j \in \{A, B\}$ to rival's consumers. Therefore, firm A solves the following problem:

$$\max_{T_{AA}} \pi_2^A = \alpha T_{AA} - F$$

$$s.t. \quad \substack{R_{AA} \geq 0 \\ R_{AA} \geq R_{AB} \\ T_{BA} = 0 \\ x \sim U[0,\omega]}$$

which is reduced to the following:

$$\max_{T_{AA}} \pi_2^A = \alpha T_{AA} - F$$

$$s.t. \quad T_{AA} \leq min\left\{v, s_A + x^{min}\right\}$$

Given the distribution of x, then $x^{min} = 0$. Also, v is the reservation value of any consumers, by construction s_A cannot be higher than v, otherwise any consumer would consume the service. Thus, since profits are increasing in T_{AA}, firms will price as high as possible, which means the maximizing price T_{AA} for firm A is s_A.

$$T_{AA}^* = s_A$$

Similarly for firm B, then

$$T_{BB}^* = s_B$$

Therefore, firms' profits in the second period are given by:

$$\begin{aligned} \pi_2^{A*} =& \alpha s_A \\ \pi_2^{B*} =& (1 - \alpha) s_B \end{aligned}$$

Now, suppose firm A, *ceteris paribus*, increase its price T_{AA}^* to $\widehat{T_{AA}} = s_A + \epsilon \quad \forall \epsilon \in (0, \frac{\omega}{2})$. Since it is increasing its price a bit (by ϵ), there will be some consumers that switch. We can check that by looking at the preferences or indirect utilities of consumers. At the new price $\widehat{T_{AA}}$, consumers will stay when $R_{AA} \geq R_{AB}$, i.e.

$$v - (s_A + \epsilon) \geq v - s_A - x$$
$$x \leq \epsilon$$

thus, provided that $\epsilon \in [0, \omega]$, the new choice probabilities are:

$$\widehat{P_{AA}} = \int_{\epsilon}^{\omega} \frac{1}{\omega} dx = \frac{\omega - \epsilon}{\omega}$$
$$\widehat{P_{AB}} = \int_{0}^{\epsilon} \frac{1}{\omega} dx = \frac{\epsilon}{\omega}$$

And the shares of loyal consumers to A and switchers from A to B are $n_{AA} = \alpha \widehat{P_{AA}}$ and $n_{AB} = \alpha \widehat{P_{AB}}$, respectively. Then, new profits become:

$$\begin{aligned} \widehat{\pi_2^A} &= \alpha(1 - \frac{\epsilon}{\omega})(s_A + \epsilon) + \alpha \frac{\epsilon}{\omega} s_A \\ &= \alpha s_A + \frac{\alpha \epsilon}{\omega}(\omega - \epsilon) \\ &= \pi_2^{A*} + \frac{\alpha \epsilon}{\omega}(\omega - \epsilon) \end{aligned}$$

Thus, since $\epsilon < \omega$ by construction, firm A would deviate to $\widehat{T_{AA}}$, increasing its price and getting higher profits ($\widehat{\pi_2^A} > \pi_2^{A*}$). Therefore, there is no an equilibrium where nobody switches. $\square$

<u>*Claim 2*</u>: There is no equilibrium where everyone switches.

Proof. Let's suppose $x_A < 0$ & $x_B < 0$ and as before, I analyze the game in the second period. In this case, consumers prefer to switch rather than stay with their provider, which means that the indirect utilities of a consumer that chose A in the first period are as follows:

$$\begin{aligned} R_{AA} \leq 0 \quad &\Rightarrow \quad v \leq T_{AA} \\ R_{AB} \geq 0 \quad &\Rightarrow \quad v - s_A - x \geq T_{AB} \end{aligned}$$

Likewise, the indirect utilities of a consumer that chose B in the first period are

$$\begin{aligned} R_{BB} \leq 0 \quad &\Rightarrow \quad v \leq T_{BB} \\ R_{BA} \geq 0 \quad &\Rightarrow \quad v - s_B - x \geq T_{BA} \end{aligned}$$

Since consumers are better off switching than staying, then $R_{2A}^A \leq R_{2A}^B$ and $R_{2B}^B \leq R_{2B}^A$. Therefore the following must hold:

$$T_{AA} - s_A - x \geq T_{AB}$$
$$T_{BB} - s_B - x \geq T_{BA}$$

Given the preferences of consumers, firms set their second period prices that maximize their profits assuming the rival provider charges zero to their consumers in order to retain them; thus $T_{ii} = 0 \ \forall i \in \{A, B\}$.
Therefore, firm A solves the following problem:

$$\max_{T_{BA}} \pi_2^A = \alpha T_{AA} + (1-\alpha)T_{BA} + \alpha s_A$$
$$s.t. \ \substack{R_{BA} \geq 0 \\ R_{BB} \leq R_{BA} \\ T_{BB}=0 \\ x \sim U[0,\omega]}$$

which is reduced to the following:

$$\max_{T_{BA}} \pi_2^A = (1-\alpha)T_{BA} + \alpha s_A$$
$$s.t. \quad T_{BA} \leq min\left\{v - s_B - x^{min}, -s_B - x^{min}\right\}$$

Given the distribution of x, then $x^{min} = 0$. Also, $v \leq T_{AA}$, but since firm A also want to retain its loyal consumers, it will charge them $T_{AA} = 0$, which imply zero reservation value of consumers for the service, $v = 0$ because it cannot be negative. Therefore, $T_{BA} = -s_B$, firms would make losses in the second period. Also, since reservation value of consumers does not change between periods, consumers would not be interested in buying the service if the first period prices are positive, $v = 0$. Thus, firms would need to price zero in both periods, and finally they would just make losses by operating under this case, therefore firms would be better off by not operating. Hence, there is not an equilibrium where everyone switches.

□