

Competition with endogenous and exogenous switching costs

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Preliminary version and incomplete

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Abstract

This paper presents a general theoretical framework for a dynamic competition game in the presence of two types of switching costs: endogenous, which are set by providers, and exogenous that are specific to consumers. In a two-period game, the two providers compete in prices and switching fees, and can price discriminate between old (loyal) and new (switchers) consumers.

There are symmetric subgame perfect equilibria in pure strategies, where providers split the market equally in the first period. Equilibrium prices and switching fees are not uniquely determined, but providers' profits are. Endogenous switching costs only impact inter-temporal payoffs with countervailing effects, leaving multiperiod payoffs unaffected, whereas, exogenous switching costs affect consumer and social welfare. These results suggest that regulatory policies, in the telecommunications industry for example, should reduce exogenous switching costs (such as number portability, standardization or compatibility) rather than eliminate or regulate all switching fees.

Keywords: Dynamic competition, duopoly, switching costs, introductory offers.

1 Introduction

Switching costs (SC) affect consumers in their daily life, and may hinder their free decision to change providers of certain product or service. These costs may be related to learning (software usage), information (medical history of patients for example), transactions (paperwork to terminate and initiate the consumption of certain service), or due to direct firms' practices to keep consumers by charging early contract termination fees or by offering coupons and discounts to frequent consumers.

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Firms benefit from the presence of switching costs and use it strategically to keep their consumer base by locking them in. Consumers that stay for a long time may be more profitable than new ones in some markets. As an example, we can think of car insurance market vs. wholesale markets. In the former, consumers are profitable after one or two years, while in the latter they are immediately profitable. The literature suggests that firms strategically use switching costs to “lock-in” and “then-ripoff” consumers, which lessen competition and social welfare. Understanding the nature of switching costs and its impact on market outcomes and equilibrium conditions is important for researchers and policy makers, given their presumed detrimental effect, due to increasing average price or even changing price structures (NERA, 2003).

Switching costs can be considered endogenous when affect directly the profits of firms and these decide on their level; these costs include any fee or lock-in mechanism that directly affects firms’ profits.¹ On the other hand, switching costs are considered exogenous (to the firm) when affecting only to consumers and firms do not set their level. These include any other cost such as psychological costs of switching, learning costs, the opportunity cost of time spent on paperwork, among others.²

The relevant literature is vast, switching costs are usually explained by dynamic price models, from simple two-period models to infinite period models (Klemperer, 1983, 1987b,a; Farrel and Shapiro, 1988; Caminal and Matutes, 1990; Beggs and Klemperer, 1992; Padilla, 1995; To, 1996; Shaffer and Zhang, 2000; Dube et al., 2009; Pearcy, 2011; Cabral, 2012; Arie and Grieco, 2014; Fabra and Garcia, 2015; Cabral, 2016). Some explain market entry under the presence of switching costs (Klemperer, 1988; Farrel and Shapiro, 1988; Beggs and Klemperer, 1992; Wang and Wen, 1998). Most of the theoretical models use exogenous switching costs only, few as Caminal and Matutes (1990) and Shi (2013) add endogenous switching costs in the form of discounts. Also Chen (1997) explains poaching practices with exogenous switching costs.

This paper contributes to the literature by analyzing the effect of exogenous and endogenous switching costs. It is based on (Chen, 1997) but differs from it by distinguishing endogenous from exogenous switching costs, adding an individual taste shock and letting the endogenous switching costs level be set in the initial period and not in the switching period (second period). It also differs from (Shi, 2013) by considering introductory offers and making exogenous switching cost different across consumers. My ultimate aim is to understand what determines the endogenous component (switching fee), how its strategic use affects the market outcomes and consumer welfare, and how a reduction of the exogenous component (exogenous switching costs) impacts the market. This would also allow for improved policy recommendations.

I develop a theoretical framework for a dynamic competition game under the presence of switching costs, where two providers compete in prices and strategically use

¹As examples we can think of early termination fee in the telecommunications industry; the loyalty programs in the airline market; or cash rewards program in the credit card market. In this particular paper, I focus in the existence of switching fees in the form of ETF.

²As an example, we can think of the costs of unlocking a phone by a technician (locking phones are widely seen in mobile telecommunication market, and can be thought as given). Also incurred costs of security clearance paperwork relevant for the labor market in developing countries; or the time spent to get a medical examination and report to prove health condition in the insurance market.

endogenous switching costs, while consumers face additionally an exogenous switching cost. I consider a two-period game where providers simultaneously set prices and switching fees in the first period; in the second period, providers use introductory offers since they can distinguish between old consumers and newcomers, to attract new consumers (rival's consumers).

Using a discrete choice approach, the model allows for heterogeneity across consumers. Thus, focusing on equilibria in pure strategies, I find symmetric subgame perfect equilibria, where the market is split equally between providers and switching occurs in the second period (the size of its share depends on the exogenous switching costs parameter). Equilibrium tuples of prices and switching fees are not uniquely determined, but providers' profits are. An important result is that endogenous switching costs (in the form of switching fees) only impact intertemporal payoffs with countervailing effects, leaving multiperiod payoffs unaffected. Also, second-period prices are increasing in exogenous switching costs, and loyal consumers are charged higher than newcomers (switchers). Moreover, since both, social welfare and consumer surplus are negatively affected by the exogenous switching cost parameter, the model suggests that lowering exogenous switching costs (by a regulatory change for example) would lead to higher consumer surplus and bigger social welfare.

This paper is organized as follows: the related literature is presented in the following subsection, then Section 2 presents the model in detail, Section 3 presents the equilibrium analysis, and Section 4 presents the conclusions.

Related literature

Farrel and Klemperer (2007) discuss switching costs and network effect. Fudenberg and Tirole (2000) and Chen (1997) focuses more on the strategies used by firms to attract customers from their competitors. Toolsema (2009) adds an interesting approach by differentiating intra and interfirm switching costs, but she restricts her analysis to a static monopoly pricing structure. Shapiro (1999) deals directly with the exclusivity of services within industries with network effects.

Markets with switching costs are usually discussed in dynamic models. However, a static approach is also used in Klemperer (1988) and Shaffer and Zhang (2000). Klemperer (1988) analyzes firms' entry decisions in markets with switching costs. According to that model, when switching costs are unavoidable, entry is found to be socially undesirable due to the welfare losses caused by the switching costs that consumers have to face and the incumbents' output that would have been efficiently provided with no entry. Beggs and Klemperer (1992) show using two-period models, that switching costs seem to lead to higher equilibrium prices and higher profits, thus markets with switching costs become more attractive to the entry of new firms, and market shares would converge to the same rate if firms exhibit similar costs. This may be the reason why switching costs reduce demand elasticity.

The effect of switching costs on competition is ambiguous. By modeling a two-period economy that produces a homogeneous good, Klemperer (1987b) finds that switching costs lead to increased competition in the first period to get the larger portion of the

market in order to maximize second-period rents.³⁴ Competition intensity, however, is reduced in the following period, when also firms produce less. Thus due to switching costs, welfare is expected to fall. In a similar study, but with differentiated goods, Klemperer (1987a) finds that the effect on competition is ambiguous for the first period, but damaging in the second period due to the firms' incentive to take advantage of their loyal established consumers. Fabra and Garcia (2015) find switching costs becomes pro-competitive in the long-run when market shares tend to be symmetric, when market structure is asymmetric then switching costs lead to higher prices. Under the absence of price discrimination between loyal and non-loyal consumers, Arie and Grieco (2014) show switching costs have a significant compensating effect that lead firms to reduce prices, instead of increasing them, to attract switchers from the rival.

Caminal and Matutes (1990) present a duopoly model with endogenous switching costs and differentiated product. They consider pricing practices to retain customers as well pre-commitment to prices or coupons in the initial period. They find that price commitment enhances competition, while coupons shrink it. Firms would prefer switching costs to be absent, but since their next period rents depend on retained consumers, they would usually use switching costs in the form of coupons or discounts.

Some other researchers have used an infinite-period model with overlapping generations. Markets include established consumers and newcomers. Some of the models also include switchers and a replacement rate of established consumers (Farrel and Shapiro, 1988; Padilla, 1995; To, 1996; Cabral, 2012). In general, these studies solve for Markov perfect equilibria and get similar results. Farrel and Shapiro (1988) finds that incumbents supply only to their loyal/attached consumers and the entrants serve the newcomers. However, switching costs generate excessive entry, which creates inefficiencies in the market. In Padilla (1995), switching costs generate higher prices and profits in every period, and prices increase with firms' customer base, which also implies more difficulties in sustaining tacit collusion.⁵

Also, To (1996), based in Beggs and Klemperer (1992) where market shares evolve monotonically, finds that when consumers face finite horizon, market dominance and prices alternates among firms. With a different perspective, and also based on an infinite-period model, Cabral (2012) finds conditions for switching costs to affect prices in opposite directions. According to the study, switching costs in markets already competitive strengthen the competitive behavior by intensifying competition for new customers. However in markets with lower initial competition, switching costs make the market even less competitive because the switching costs' effect on reinforcing market power of larger firms dominates.

³Firms fiercely compete for attracting customers in the first period, even when that means setting prices below costs. This happens because they would charge monopoly prices in the second period to their loyal consumers.

⁴Farrel (1986) shows that firms with larger market share in the first period charge higher prices in the second period, up to the level that the firm still gets the larger market share in the second period.

⁵Switching costs would make punishments less severe in collusive agreements.

2 The Model

There is a unit mass of consumers, who are heterogeneous in their preferences and exogenous switching cost. There are two competing providers A and B , who offer substitute services to their consumers.

For simplicity, I assume that providers' marginal cost is zero, but they still face an entry cost, F . The providers operate in two periods and have the same discount rate $\delta \in (0, 1]$. A contract with provider $i \in \{A, B\}$ in period 1 is a pair (T_i, s_i) , where T_i is the fee a consumer has to pay for the first period unlimited service of i , and s_i is a switching fee a consumer of i will have to pay to provider i if he switches in the second period from i to j , $j \neq i$ (an early termination fee ETF). A contract in period 2 with provider $i \in \{A, B\}$ specifies a fee T_{ii} a consumer that chose i in both periods has to pay for the second period unlimited service of i ; and a fee T_{ji} a consumer that switched providers from j to i has to pay for the second period service of i .

From the demand side, and following a discrete choice approach, consumers have per period linear indirect utility (payoff) functions and have the same discount rate $\beta \in [0, 1]$.⁶ Consumer k has valuation $v + \sigma_{ik}$ for provider i 's service, where σ_{ik} is a relative idiosyncratic preference for provider i respect to provider j , which is uniformly distributed on the interval $[-1, 1]$ and has density function $h(\sigma_i)$. This relative taste is revealed in the first period and kept unchanged in the second period. Thus consumers do not change preferences between periods and there is no learning either.⁷ Consumers' valuation for the service v is assumed to be big enough so the market is covered. Providers only know the distribution of idiosyncratic variables.

In the second period, the source of heterogeneity comes from the presence of an individual specific exogenous switching cost, x_k and is uniformly distributed on the interval $[0, \omega]$ with density function $f(x)$. This exogenous switching cost x_k is learned by the consumers at the beginning of the second period and occurs independently of the first period shock. This exogenous cost refers to learning costs or costs (time, money, etc.) incurred by, for instance, canceling and account or unlocking a mobile phone set in the mobile telecommunications industry.

In the first period, after observing $((s_A, T_A), (s_B, T_B))$, every consumer chooses a provider from $\{A, B\}$. Then, given their chosen provider in the first period and the new prices in the second period, consumers decide either to stay with their provider i , or to switch to the other provider and pay a switching fee s_i , $i \in \{A, B\}$ to their previous provider and incur in additional switching costs x_k .

Timeline of the game

The game timeline is described below:

⁶I took the approach reviewed in the section 2.5 of Anderson et al. (1992), also used by Cabral (2016).

⁷This relative preference is such that $\sigma_{Ak} = -\sigma_{Bk}$.

$t = 1$		$t = 2$	
Stage 1	Stage 2	Stage 1	Stage 2
Providers set flat prices (T_A, T_B) and switching fees (s_A, s_B)	Consumers learn σ_{ik} , then given prices and preferences they choose their provider	Providers set new flat prices (T_{AA}, T_{BA}) and (T_{BB}, T_{AB})	Consumers learn x_k , then given new prices, they decide to stay or to switch

In the first stage of the first period, providers choose simultaneously (T_A, s_A) and (T_B, s_B) . In the second stage, consumers observe firms' choices and simultaneously choose provider (A, B) .

In the first stage of the second period, providers choose simultaneously their prices for newcomers, T_{BA} and T_{AB} , and for loyal consumers, T_{AA} and T_{BB} . Providers do not commit to keep previous period prices. Consumers observe the new prices, and simultaneously decide whether to stay with their providers or to switch providers.

The Payoffs

Consumers

In period 1, the payoff of a consumer k of firm $i \in \{A, B\}$ is

$$R_{1k}^i = v + \sigma_{ik} - T_i$$

In period 2, the payoff of a consumer that chose firm $i \in \{A, B\}$ in period 1 is

$$R_{2k}^i = \max \{R_{ii,k}, R_{ij,k}\}$$

where,

$$\begin{aligned} R_{ii,k} &= v + \sigma_{ik} - T_{ii} && \text{if the consumer chose firm } i \text{ also in the first period} \\ R_{ij,k} &= v - \sigma_{ik} - T_{ij} - s_i - x_k && \text{if the consumer switched providers from } i \text{ to } j \end{aligned}$$

The decision variable in the first period is given by the idiosyncratic relative taste parameter, and in the second period the decision variable is the exogenous switching cost.

Thus, the multi-period net payoff of a consumer k who chooses firm i in period 1 is

$$R_k^i = R_{1k}^i + \beta E_x[R_{2k}^i | \sigma_{ik} \geq \hat{\sigma}] \quad (1)$$

where $\hat{\sigma}$ is the taste shock threshold found in the first period, consumers with idiosyncratic relative taste shock above it choose provider i over j .

Providers

I denote α and $(1 - \alpha)$ the first period market shares of providers A and B , respectively. Likewise, in the second period, n_{ii} refers the share of consumers that consume from i

in both periods, and n_{ij} , to the share of consumers that switched from i to j .

In the second period, providers' profits come from loyal consumers, newcomers and the switching fee collected from switchers that left. Thus, second period profits are given by

$$\pi_2^i = n_{ii}T_{ii} + n_{ji}T_{ji} + n_{ij}s_i \quad \forall i, j \in \{A, B\}$$

The first period payoff is $\pi_1^i = \alpha T_i - F$, so provider i solves:

$$\max_{T_i, s_i} \pi^i = \pi_1^i + \delta \pi_2^i \quad (2)$$

I solve this game using backward induction, so I start finding the second-period equilibrium, and then continue with the first-period equilibrium in pure strategies.

2.1 The Second Period Equilibrium

Given their first period choices of provider and the second period new prices, consumers make their decision to switch or stay with their current provider.

A consumer k stays with his first-period provider if and only if his net payoff with this provider is at least as high as with the other (net of switching fee and costs), that means when $R_{ii} \geq R_{ij}$ (I omit the consumer index k for simplicity). Therefore, the probability that a consumer chooses to stay with his first period provider i is:⁸

$$\begin{aligned} P_{ii} &= Pr[\text{stay in } i|i] = Pr(R_{ii} \geq R_{ij} | \sigma_{ik} \geq \hat{\sigma}) \\ &= Pr(x + 2\sigma_i \geq T_{ii} - T_{ij} - s_i | \sigma_{ik} \geq \hat{\sigma}) \\ &= Pr(x + y \geq x_i | y \geq \hat{\sigma}) \\ P_{ij} &= Pr[\text{switch to } j|i] = 1 - P_{ii} \end{aligned}$$

First, for consumers that chose A as their first-period provider, we understand they revealed their relative preference for A , so they keep their σ_A (from now on I will refer only to σ_A , and also $y = 2\sigma_A$).⁹ Within this group, a consumer will be indifferent between staying in A (staying loyal) and switching to B if x is such that $R_{AA} = R_{AB}$ and $x \in [0, \omega]$, which means

$$\begin{aligned} v + \sigma_A - T_{AA} &= v - \sigma_A - T_{AB} - s_A - x \\ x &= (T_{AA} - T_{AB} - s_A) - 2\sigma_A \\ x + y &= x_A \end{aligned}$$

⁸I changed variable names, such that $y = 2\sigma_i$ and $y \sim U[-2, 2]$ with density function $g(y)$ and cumulative function $G(y)$. From now on I will refer only to the idiosyncratic relative taste of firm A over B , σ_A .

⁹I assume there is no learning, so consumers keep their relative provider taste in the second period. Since the relative taste is realized in the first period, in the second period this variable is not random anymore. The assumption is strong, but I add it to capture some consumer inertia due to habit formation or attachment to providers. We could even add an extra parameter $\eta \in [0, 1]$ that regulates how much of the relative taste is kept in the second period (we would have $\eta * \sigma_i$), thus η would measure the degree of firm attachment that consumers have, it could also be interpreted as network effect. If the network effect created in the first period is big, then consumers will keep most of their preference in favor of their first provider. For the purpose of simplicity, the model is developed as if $\eta = 1$.

Thus, $x_A - y$ represents exogenous switching cost level for the consumer in A who is indifferent between staying in A or switching to B , provided that $x_A \in [0, \omega]$ and $\sigma_A \geq \hat{\sigma}$. If $x_A > \omega + 2$ then every consumer of A prefers to stay with A . If $x_A < \hat{\sigma}$ then every consumer of A prefers to switch to B .¹⁰ If $\hat{\sigma} \leq x_A \leq \omega + 2$, then consumers with exogenous switching cost $x + y$ above x_A will stay with A and those consumers with exogenous switching costs below x_A will switch to B .

Similar analysis holds for the case of consumers that chose B as their first period provider. Hence, a consumer will be indifferent between staying and switching when $R_{BB} = R_{BA}$, thus $x_B = T_{BB} - T_{BA} - s_B$ and $x - y = x_B$.¹¹ In the case of consumers that chose B in the first period, the choice probabilities are conditioned on $\sigma_{Ak} \leq \hat{\sigma}$. Thus, consumers will stay or switch given that $\omega - 2 \leq x_A \leq \hat{\sigma}$.

In general, provided that $x_i \pm y \in [0, \omega] \times [-2, 2]$, and given that the first period relative taste shock σ_A and second period exogenous cost shock x are assumed independent, the choice probabilities are the following¹²

$$\begin{aligned} P_{AA} &= \int_{\hat{\sigma}}^2 \int_{x_A - y}^{\omega} f(x) \frac{g(y)}{1 - G(\hat{\sigma})} dx dy = \frac{1}{\omega} (\omega - [(T_{AA} - T_{AB} - s_A) - \frac{\hat{\sigma}}{2} - 1]) \\ P_{AB} &= \int_{\hat{\sigma}}^2 \int_0^{x_A - y} f(x) \frac{g(y)}{1 - G(\hat{\sigma})} dx dy = \frac{1}{\omega} [(T_{AA} - T_{AB} - s_A) - \frac{\hat{\sigma}}{2} - 1] \\ P_{BB} &= \int_{-2}^{\hat{\sigma}} \int_{x_B + y}^{\omega} f(x) \frac{g(y)}{G(\hat{\sigma})} dx dy = \frac{1}{\omega} (\omega - [(T_{BB} - T_{BA} - s_B) + \frac{\hat{\sigma}}{2} - 1]) \\ P_{BA} &= \int_{-2}^{\hat{\sigma}} \int_0^{x_B + y} f(x) \frac{g(y)}{G(\hat{\sigma})} dx dy = \frac{1}{\omega} [(T_{BB} - T_{BA} - s_B) + \frac{\hat{\sigma}}{2} - 1] \end{aligned}$$

Assuming α is the first period market share of provider A , and $(1 - \alpha)$, of provider B , we get the demands of loyal and switchers for each firm: $n_{AA} = \alpha P_{AA}$; $n_{AB} = \alpha P_{AB}$; $n_{BB} = (1 - \alpha) P_{BB}$; and $n_{BA} = (1 - \alpha) P_{BA}$.

Thus, the second period profits of the providers are

$$\begin{aligned} \pi_2^A &= n_{AA} T_{AA} + n_{BA} T_{BA} + n_{AB} s_A \\ \pi_2^B &= n_{BB} T_{BB} + n_{AB} T_{AB} + n_{BA} s_B \end{aligned}$$

Therefore, taking into account the values of choice probabilities, and provided that $x_A \in [\frac{\hat{\sigma}}{2} + 1, \omega + \frac{\hat{\sigma}}{2} + 1]$ and $x_B \in [1 - \frac{\hat{\sigma}}{2}, \omega + 1 - \frac{\hat{\sigma}}{2}]$, second period profits are

$$\pi_2^A = \frac{1}{\omega} [\alpha (T_{AA}(\omega - s_A - T_{AA}) + (T_{AA} - s_A)(T_{AB} + s_A + 1 + \frac{\hat{\sigma}}{2}) + (1 - \alpha) T_{BA}(T_{BB} - T_{BA} - s_B - 1 + \frac{\hat{\sigma}}{2}))] \quad (3)$$

$$\pi_2^B = \frac{1}{\omega} [(1 - \alpha) (T_{BB}(\omega - s_B - T_{BB}) + (T_{BB} - s_B)(T_{BA} + s_B + 1 - \frac{\hat{\sigma}}{2}) + \alpha T_{AB}(T_{AA} - T_{AB} - s_A - 1 - \frac{\hat{\sigma}}{2}))] \quad (4)$$

¹⁰In this case, we find the probability of staying or switching given the first period choice, which indicates that consumers with taste $\sigma_{Ak} \geq \hat{\sigma}$ chosen A over B in the first period.

¹¹Recall that $\sigma_B = -\sigma_A$.

¹²For these probabilities to exist, we need $x_A \in [\frac{\hat{\sigma}}{2} + 1, \omega + \frac{\hat{\sigma}}{2} + 1]$ and $x_B \in [1 - \frac{\hat{\sigma}}{2}, \omega + 1 - \frac{\hat{\sigma}}{2}]$.

Profit functions are quadratic and concave in their arguments (prices), so maximizing over prices (T_{ii} and T_{ji}) we expect an interior solution.¹³

Solving for the second-period equilibrium by using the first order conditions, the following is the unique solution. These results also satisfy the second order conditions:

$$T_{AA}^* = \frac{1}{3}(2\omega + 1 + \frac{\hat{\sigma}}{2}) + s_A \quad (5)$$

$$T_{BA}^* = \frac{1}{3}(\omega - 1 + \frac{\hat{\sigma}}{2}) \quad (6)$$

$$T_{BB}^* = \frac{1}{3}(2\omega + 1 - \frac{\hat{\sigma}}{2}) + s_B \quad (7)$$

$$T_{AB}^* = \frac{1}{3}(\omega - 1 - \frac{\hat{\sigma}}{2}) \quad (8)$$

Therefore, the equilibrium outcome for second-period prices do not depend on first-period market shares. These prices are increasing in the exogenous switching cost ω , and the endogenous switching fee s_i only affects the prices that loyal consumers face.¹⁴

Also, given these values, second-period shares are $n_{AA} = \frac{\alpha}{3\omega}(2\omega + 1 + \frac{\hat{\sigma}}{2})$, $n_{AB} = \frac{\alpha}{3\omega}(\omega - 1 - \frac{\hat{\sigma}}{2})$, similarly, $n_{BB} = \frac{(1-\alpha)}{3\omega}(2\omega + 1 - \frac{\hat{\sigma}}{2})$, $n_{BA} = \frac{(1-\alpha)}{3\omega}(\omega - 1 + \frac{\hat{\sigma}}{2})$. Switching will happen whenever $\omega > \max\{1 - \frac{\hat{\sigma}}{2}, 1 + \frac{\hat{\sigma}}{2}\}$. Thus, the total share of switchers, $n_{AB} + n_{BA} = \frac{1}{3\omega}(\omega - 1 + \frac{\hat{\sigma}}{2}(1 - 2\alpha))$, increases with the exogenous cost parameter. This result differs from Chen (1997), where a third of the population switched for the case of paying-consumers-to-switch.

Proposition 1. *Provided that $x_A \in [\frac{\hat{\sigma}}{2} + 1, \omega + \frac{\hat{\sigma}}{2} + 1]$ and $x_B \in [1 - \frac{\hat{\sigma}}{2}, \omega + 1 - \frac{\hat{\sigma}}{2}]$, there exists a unique Nash equilibrium of the second period subgame, where second-period prices do not depend on first-period market shares and the endogenous switching fee s_i only affects the prices that loyal consumers face. The share of the population that switch increases with the exogenous cost parameter, ω .*

The equilibrium in proposition 1 is a second period Nash equilibrium in the subgame and the proof is provided in the appendix. Now, using (5), (7), (8) and (6) into (3) and (4) profits are

$$\pi_2^{A*} = \frac{1}{9}[\omega(1 + 3\alpha) + \hat{\sigma}(1 + \alpha) + 2(3\alpha - 1)] + \frac{1}{9\omega}(1 + \frac{\hat{\sigma}^2}{4} - \hat{\sigma}(1 - 2\alpha)) + \alpha s_A \quad (9)$$

¹³The second derivatives are negative: $\frac{\partial^2 \pi_2^A}{\partial T_{AA}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_2^A}{\partial T_{BA}^2} = -\frac{2(1-\alpha)}{\omega} < 0$.

Likewise, $\frac{\partial^2 \pi_2^B}{\partial T_{AB}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_2^B}{\partial T_{BB}^2} = -\frac{2(1-\alpha)}{\omega} < 0$

¹⁴Second period prices are positively affected by exogenous switching cost: $\frac{\partial T_{ii}}{\partial \omega} = \frac{2}{3} > 0$ and $\frac{\partial T_{ij}}{\partial \omega} = \frac{1}{3} > 0$

$$\pi_2^{B*} = \frac{1}{9}[\omega(4 - 3\alpha) + \hat{\sigma}(\alpha - 2) + 2(2 - 3\alpha)] + \frac{1}{9\omega}(1 + \frac{\hat{\sigma}^2}{4} - \hat{\sigma}(1 - 2\alpha)) + (1 - \alpha)s_B \quad (10)$$

As expected, second period profits depend heavily in their first market share, which may imply higher incentives of providers to lock-in consumers with higher switching fees. It is straightforward to check that $\frac{\partial \pi_2^{A*}}{\partial \alpha} > 0$ if $s_A \geq -\frac{1}{9\omega}(\omega + 2)(3\omega + \hat{\sigma})$.¹⁵

2.2 The First Period

In the first period, consumers make a choice between providers, therefore, the payoff of a consumer k will be given by :

$$\begin{aligned} R_{1Ak} &= v + \sigma_{Ak} - T_A \\ R_{1Bk} &= v + \sigma_{Bk} - T_B \end{aligned}$$

where σ_{Ak} is the relative preference for firm A respect to firm B , and is uniformly distributed on the interval $[-1, 1]$. Recall that $\sigma_{Bk} = -\sigma_{Ak}$.

In the first period, consumers take decisions based on heir multiperiod payoffs. Thus, each consumer compare R^A vs. R^B

$$\begin{aligned} R_A &= R_{1A} + \beta E_x[R_{2A} | \sigma_A \geq \hat{\sigma}] \\ R_B &= R_{1B} + \beta E_x[R_{2B} | \sigma_A \leq \hat{\sigma}] \end{aligned}$$

We get the expected second period payoffs using the distribution of exogenous switching costs x_k and the distribution of $y = 2\sigma_{Ak}$.

$$E_x[R_{2A} | \sigma_A \geq \hat{\sigma}] = \int_{\hat{\sigma}}^2 \int_{x_A^* - y}^{\omega} R_{AA}^* f(x) \frac{g(y)}{(1 - G(\hat{\sigma}))} dx dy + \int_{\hat{\sigma}}^2 \int_0^{x_A^* - y} R_{AB}^* f(x) \frac{g(y)}{(1 - G(\hat{\sigma}))} dx dy$$

$$E_x[R_{2B} | \sigma_A \leq \hat{\sigma}] = \int_{-\hat{\sigma}}^2 \int_{x_B^* + y}^{\omega} R_{AA}^* f(x) \frac{g(y)}{G(\hat{\sigma})} dx dy + \int_{-\hat{\sigma}}^2 \int_0^{x_B^* + y} R_{AB}^* f(x) \frac{g(y)}{G(\hat{\sigma})} dx dy$$

Therefore,

$$\begin{aligned} R^A &= v + \sigma_A - T_A + \beta(v + \frac{(1-11\omega)}{18} + \frac{1}{18\omega}(4 + \frac{\hat{\sigma}}{2}(\omega + 2\hat{\sigma} - 4)) - s_A) \\ R^B &= v - \sigma_A - T_B + \beta(v + \frac{(1-11\omega)}{18} + \frac{1}{18\omega}(4 - \frac{\hat{\sigma}}{2}(\omega - 2\hat{\sigma} - 4)) - s_B) \end{aligned}$$

Thus,

$$P_A = Pr[\text{choose } A] = Pr[R_A \geq R_B] = Pr[\sigma_A \geq \hat{\sigma}]$$

A consumer is indifferent between A and B when $\hat{\sigma} = \frac{18\omega}{36\omega + \beta(\omega - 4)}(T_A - T_B + \beta(s_A - s_B))$, hence, provided that $\hat{\sigma} \in [-1, 1]$, and that $\sigma_A \sim U[-1, 1]$ with density function $h(\sigma_A)$ we can get the choice probabilities:¹⁶

$$P_A = \int_{\hat{\sigma}}^1 h(\sigma_A) d\sigma_A = \frac{1}{2} - \frac{9\omega(T_A - T_B + \beta(s_A - s_B))}{36\omega + \beta(\omega - 4)}$$

¹⁵The partial derivative is $\frac{\partial \pi_2^{A*}}{\partial \alpha} = \frac{1}{9\omega}(3\omega^2 + 3\omega(3s_A + 2) + \hat{\sigma}(\omega + 2))$.

¹⁶For the indifferent consumer $\sigma_A = \hat{\sigma}$, I used that resource to solve for $\hat{\sigma}$.

Similarly,

$$P_B = \int_{-1}^{\hat{\sigma}} h(\sigma_A) d\sigma_A = \frac{1}{2} + \frac{9\omega(T_A - T_B + \beta(s_A - s_B))}{36\omega + \beta(\omega - 4)}$$

Since we have a unit mass of consumers, these probabilities actually give us the first-period market shares of the providers, therefore $\alpha = P_A$. So, first-period profits for providers are $\pi_1^A = P_A T_A - F$ and $\pi_1^B = (1 - P_A) T_B - F$.

Providers maximize their multiperiod profits:

$$\begin{aligned} \max_{T_A, s_A} \pi^A(T_A, T_B, s_A, s_B) &= \pi_1^A + \delta \pi_2^{A*} \\ \max_{T_B, s_B} \pi^B(T_A, T_B, s_A, s_B) &= \pi_1^B + \delta \pi_2^{B*} \end{aligned}$$

I omit the detailed multiperiod profit functions, which are quadratic in their arguments (first-period prices and switching fees).¹⁷ Considering that consumers and providers are equally patient ($\delta = \beta$) – which is sensible when we think of stakeholders as owners of providers, who are ultimately consumers as well – and solving the system of equations, we find subgame perfect equilibria:¹⁸

$$T_A^* = 2 - \delta \left(\frac{5\omega + 4}{18\omega} + \frac{\omega}{3} + s_A \right) \quad (11)$$

$$T_B^* = 2 - \delta \left(\frac{5\omega + 4}{18\omega} + \frac{\omega}{3} + s_B \right) \quad (12)$$

There are not unique values for the switching fees, but they are bounded according to consumers constraints. Thus $\forall i \in \{A, B\}$

$$\frac{2 - v}{\delta} - \frac{\omega}{3} - \frac{5\omega + 4}{18\omega} \leq s_i^* \quad \text{thus, } R_{1i} \geq 0 \quad (13)$$

$$s_i^* \leq v - \frac{11\omega}{18} + \frac{\omega + 4}{18\omega} \quad \text{thus, } E[R_{2i}] \geq 0 \quad (14)$$

Given the boundaries for switching fees, let's call s^{min} and s^{max} to the boundaries. An increase of the exogenous switching cost parameter would displace the feasible region for switching fees at a lower level, which would imply a substitutability between exogenous and endogenous switching costs: lower exogenous switching costs imply higher upper bound for switching fees. It is important to highlight that switching fees, s_A and s_B , are not necessarily equal, but they should satisfy the above conditions.

Thus, second period prices are given by the following:

$$T_{AA}^* = \frac{(3s_A^* + 2\omega + 1)(\omega(\delta + 36) - 4\delta)}{108\omega - 3\delta(4\omega - 1)} \quad (15)$$

$$T_{BB}^* = \frac{(3s_B^* + 2\omega + 1)(\omega(\delta + 36) - 4\delta)}{108\omega - 3\delta(4\omega - 1)} \quad (16)$$

¹⁷The second derivatives are negative: $\frac{\partial^2 \pi^A}{\partial T_A^2} = \frac{\partial^2 \pi^B}{\partial T_B^2} = -\frac{18\omega(\omega(\beta + 2\delta + 36) - 4\beta + 3\delta)}{((\beta + 36)\omega + 4\beta)^2} < 0$, and $\frac{\partial^2 \pi^A}{\partial s_A^2} = \frac{\partial^2 \pi^B}{\partial s_B^2} = -\frac{18\beta\delta\omega(\beta - 3\omega(\beta + 12))}{((\beta + 36)\omega + 4\beta)^2} < 0$, given that $\omega > 1$ (condition found previously to have people switching in the second period).

¹⁸This assumption $\delta = \beta$, also guarantees that the Hessian matrix of the system of equations to be negative semi-definite, sufficient condition to get an interior solution.

$$T_{BA}^* = T_{AB}^* = \frac{(\omega - 1)(\omega(\delta + 36) - 4\delta)}{108\omega - 3\delta(4\omega - 1)} \quad (17)$$

First-period prices are decreasing in the exogenous cost parameter ($\frac{\partial T_i^*}{\partial \delta} < 0$) and in the discount factor ($\frac{\partial T_i^*}{\partial \omega} < 0$).¹⁹ Second-period prices are positively affected by exogenous switching cost parameter ($\frac{\partial T_{ii}^*}{\partial \omega} = \frac{2}{3}$ and $\frac{\partial T_{ij}^*}{\partial \omega} = \frac{1}{3}$) and are not affected by the discount factor ($\frac{\partial T_{ij}^*}{\partial \delta} = 0$). So an external reduction of exogenous switching costs would reduce second period prices, for both loyal consumers and switchers; but this reduction also would increase first period prices and both boundaries of endogenous switching fees (if the change is anticipated for the providers).

In the equilibrium, $\hat{\sigma} = 0$ and providers A and B split the market equally, $\alpha = \frac{1}{2}$, and make multiperiod profits²⁰ multiperiod profits

$$\pi_A^* = \pi_B^* = \frac{\delta}{36}(4\omega - 1) + 1 - F \quad (18)$$

second and first period profits, $i \in \{A, B\}$, are

$$\pi_{2i} = \frac{1}{18\omega}[\omega(2 + 9s_i^*) + 5\omega^2 + 2] \quad (19)$$

$$\pi_{1i} = 1 - \frac{\delta}{36\omega}[\omega(6\omega + 18s_i^* + 5) + 4] - F \quad (20)$$

An increase in switching fee affects positively to second-period profits but negatively to first-period profits. For the multiperiod profit maximizer firm, the effects cancel out and its multiperiod profits are independent on switching fee levels, $\frac{\partial \pi^{i*}}{\partial s_i} = 0$. Multiperiod profits do not depend on switching fees²¹ On the other hands, multiperiod profits are increasing in the exogenous switching costs parameter ω and in the discount factor δ . First period profits are decreasing in exogenous switching costs and second period profits are increasing in them.

The indifferent consumer has an idiosyncratic taste level of $\hat{\sigma} = 0$ and gets multiperiod payoff of

$$R_i = v(1 + \delta) + \frac{1}{18\omega}(\delta(8 + 6\omega - 5\omega^2) - 36\omega)$$

Notice also that this payoff does not depend on switching fee.

Additionally, second period market shares are $n_{ii} = \frac{2\omega+1}{6\omega}$, $n_{ij} = \frac{\omega-1}{6\omega}$. Thus the probability to stay loyal is $\frac{2\omega+1}{3\omega}$ and the probability to switch (the share of switchers) is $\frac{\omega-1}{3\omega}$. If $\omega \leq 1$ then no switching occurs. The results of the two-period model are summarized in the following proposition.

¹⁹ $\frac{\partial T_i^*}{\partial \omega} = -\frac{\delta(3\omega^2-2)}{9\omega^2} < 0$ since $\omega < 1$ and $\frac{\partial T_i^*}{\partial \delta} = -\frac{6\omega^2+5\omega+4}{18\omega} - s_i < 0$ because $\frac{6\omega^2+5\omega+4}{18\omega} > s_i^{min}$.

²⁰ The threshold levels to switch in the second period are $x_A^* = x_B^* = \frac{(\omega+2)(\omega(\delta+36)-4\delta)}{108\omega-3\delta(4\omega-1)}$.

²¹ Notice that $\frac{\partial \pi_1^*}{\partial s_i} < 0$, and $\frac{\partial \pi_2^*}{\partial s_i} > 0$

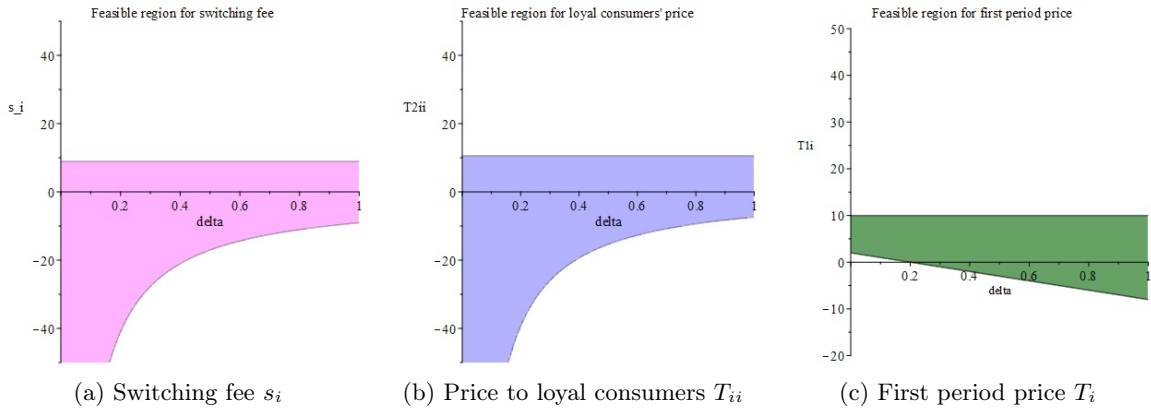
Proposition 2. *If $v \geq \frac{2}{1+\delta} - \frac{\delta}{18\omega(1+\delta)}(8 + 6\omega - 5\omega^2)$, then there are subgame perfect equilibria in pure strategies where each firm gets a half of the market in the first period, and switching occurs in the second period when $\omega > 1$. Multiperiod profits of both providers are non-negative whenever $F \leq \frac{\delta}{36}(4\omega - 1) + 1$.*

In these multiple equilibria, where different combinations of prices and switching fees reach the same providers' payoffs, multiperiod profits and multiperiod payoffs of consumers are independent of switching fees.

Negative switching fees, which mean providers care so much on the present and want to extract as much consumer surplus as possible so they would even promise to pay consumers if they decide to leave, are possible in this model. The feasible region expands to more negative values at lower discount factors.

3 Equilibrium analysis: implications

From the symmetric equilibrium conditions given in proposition 2, we can graphically observe the feasible region for switching fees depicted in figure 1. The figure shows that for lower patience level (low δ), the feasible region of negative switching fees becomes bigger. This lower bound increases as the discount factor approaches to one, but do not get to positive levels. Prices charged to loyal consumers depends positively on switching fees, and both its lower and upper bounds are higher than those of switching fees. First period prices depend negatively on switching fees, at most this price is set as high as consumers' valuation of the service v ; its lower bound increases slightly with the discount factor δ .

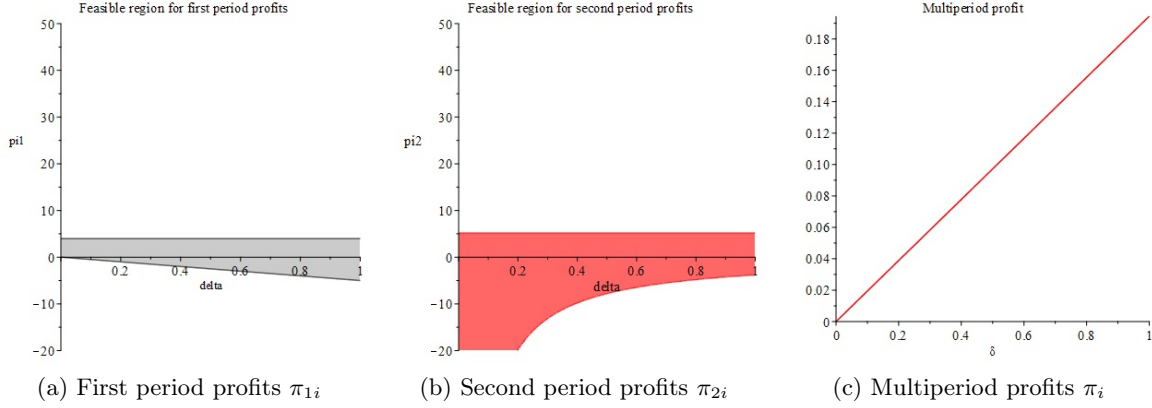


Parameters values: $v = 10$, $\omega = 2$, and $F = 1$

Figure 1: Feasible regions for switching fees optimal prices

Although there are many combinations of prices and switching fees, profits are set in a unique way; providers' profits are increasing in the discount rate and the exogenous switching cost parameter. 2 shows the feasible regions for first, second and multiperiod profits. Providers may risk and get negative first period profits as discount factor increases. Despite of the multiplicity of equilibrium outcomes for period profits, the

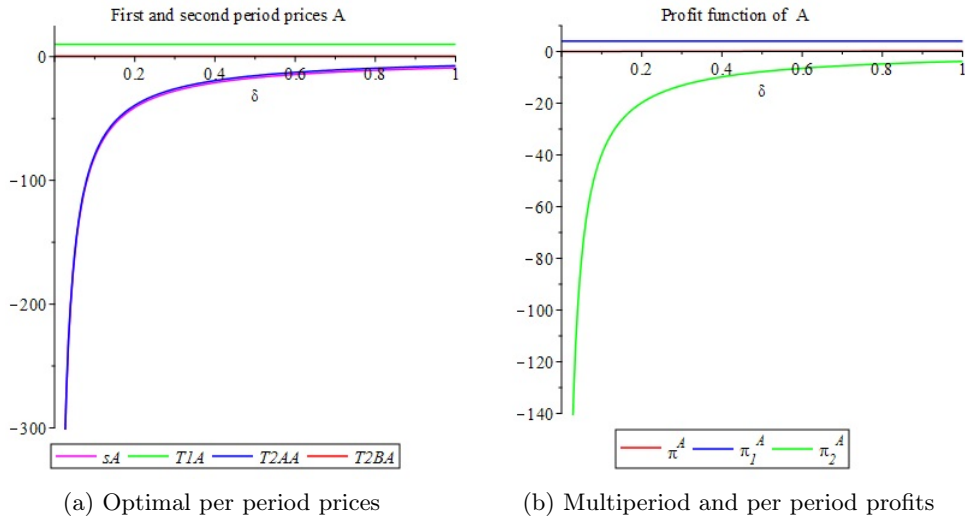
multiperiod or lifetime profits is uniquely determined.



Parameters values: $v = 10$, $\omega = 2$, and $F = 1$

Figure 2: Feasible regions for switching fees optimal prices

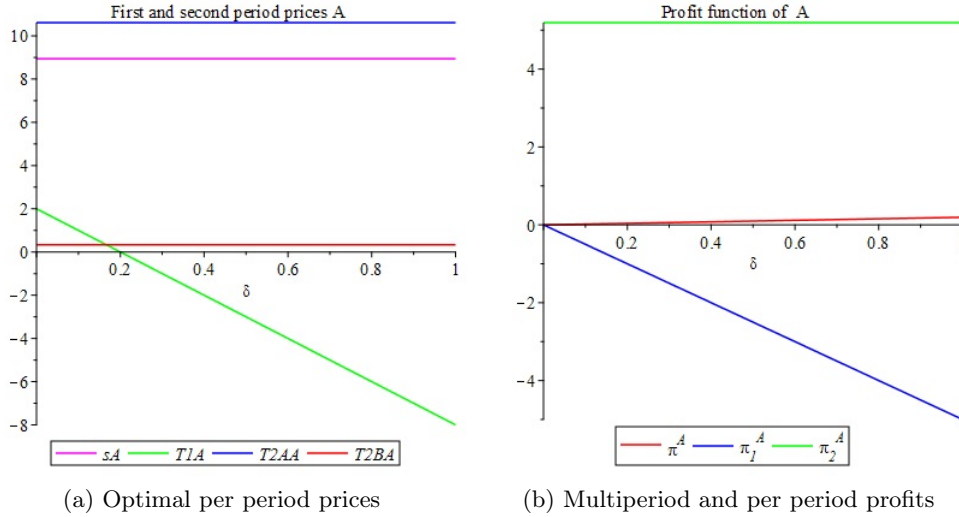
Figures 3 and 4 show the optimal prices and profits as functions of discount factor δ in different scenarios, when providers set minimum and maximum switching fee. Assuming providers always set s^{min} , Figure 3a shows that first-period prices are almost constant and always higher than second -period prices for loyal consumers and switchers; switchers are charged the same regardless of the discount factor. Likewise, figure 3b depicts the profit functions: multiperiod profit (red line) is always positive and increasing in δ ; first-period profits also are positive but they slightly decrease with patience level. Second-period profits are increasing in δ , but they are negative if $s_i = s^{min}$. This result indicates that the effect of a switching fee is inter-temporally compensated in providers' profits.



Parameters values: $v = 10$, $\omega = 2$, and $F = 1$.

Figure 3: Optimal prices and profits when both providers set $s_i = s^{min}$

When providers set a s^{max} , then second period prices and switching fees are positive, but first period prices quickly becomes negative as discount factor increases. Also first period profit are negative and keep decreasing with patience level, as shown by Figure 4. In this scenario, providers extract the entire consumer surplus in the second period and charge a low (even negative) first-period prices. However, profits in the first period can be negative due to the entry costs F (here fixed at 1); despite this, multiperiod profits are kept positive and increasing in δ , It is important to highlight that the mul-



Parameters values: $v = 10$, $\omega = 2$, and $F = 1$.

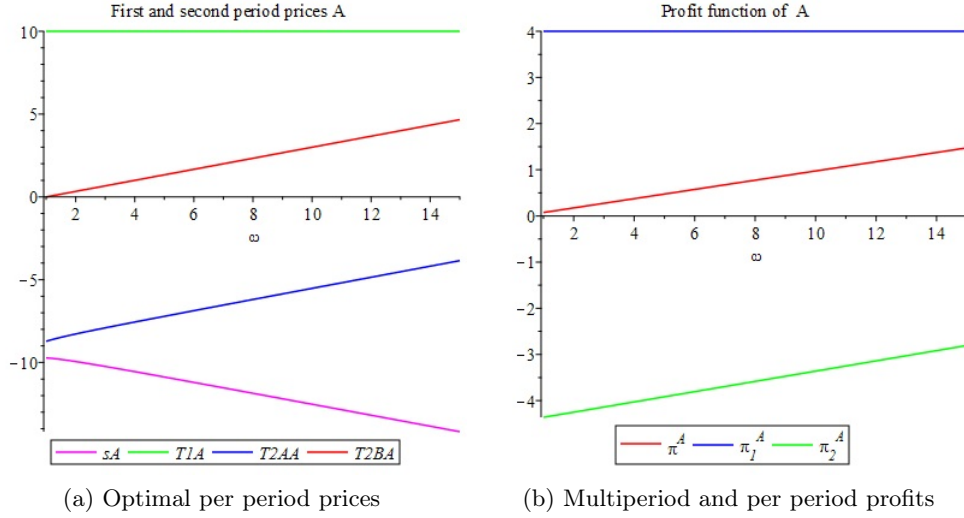
Figure 4: Optimal prices and profits when both providers set $s_i = s^{max}$

tiperiod profit function in both scenarios is the same, which is explained by the fact that switching fees do not affect multiperiod profits, their effect on period profits are compensated leaving multiperiod profits unaffected.

Figures 5 and 6 show the optimal prices and profits as functions of the switching cost parameter ω when providers set minimum and maximum switching fee (a positive amount). It is always the case that second period prices increases with ω , while switching fees decreases with ω . When switching fees are set at its minimum (a negative amount), first period prices equals the consumer valuation for the service v , and is independent of exogenous switching costs, when switching fee is set at its maximum, first period prices are negative but increasing in ω .

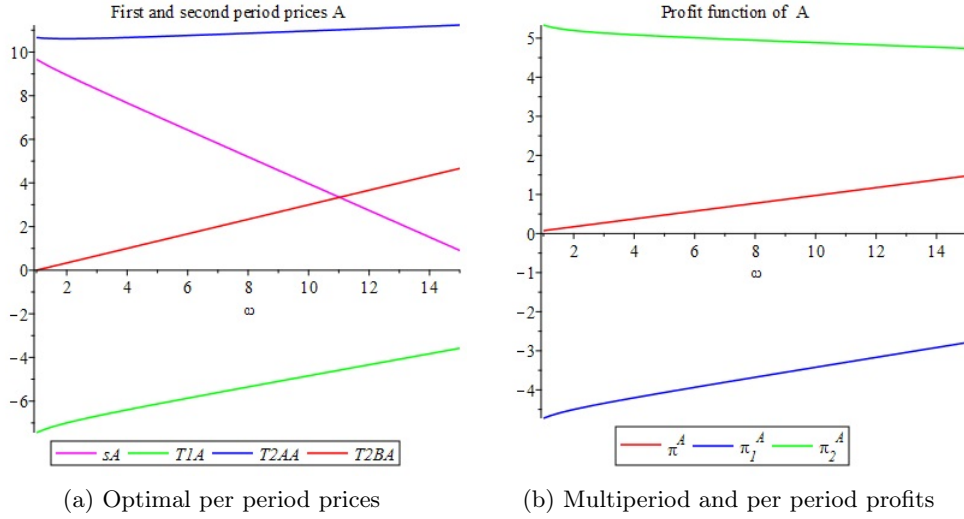
Multiperiod profits are always increasing in exogenous switching costs; when minimum switching fees are set, first period profits reach their maximum, while second period profits are negative but increasing in ω . At maximum switching fees, first period profits are negative (to compensate consumers firms set negative first period prices) but increasing in ω ; second period prices are positive but slightly decreasing in ω , this happens due to the effect of lower switching fees collected from more switchers. ²²

²²Recall that the switchers' share rise with ω).



Parameters values: $v = 10$, $\delta = 0.9$, and $F = 1$.

Figure 5: Optimal prices and profits when both providers set $s_i = s^{min}$



Parameters values: $v = 10$, $\delta = 0.9$, and $F = 1$.

Figure 6: Optimal prices and profits when both providers set $s_i = s^{max}$

Consumer surplus and social welfare

Let's now consider and depict the effect of the equilibrium outcomes on the consumer surplus and social welfare (producer plus consumer surplus). As mentioned before, endogenous switching costs or switching fees do not affect the multiperiod payoff of consumers (they affect per period payoff, and these effects that are canceled out in the total discounted multiperiod payoff), therefore multiperiod consumer surplus is also unaffected by switching fees. However, consumer's multiperiod and per period payoff are affected by exogenous switching costs.

Given that multiperiod profits of providers are also independent of switching fees, social welfare (defined as the summation of consumer surplus and providers' profits) is unaffected by switching fees (endogenous switching costs). This result may be striking, but it may explain why in some industries such as telecommunications, switching fees like ETF are being dismissed by some companies. It also agrees with the findings of Cullen et al. (2016) where equilibria where providers with and without switching fees may coexist. In the model presented in this paper that may happen because the effect of switching fees are compensated inter-temporally in such a way that they do not affect payoffs of consumers either providers.

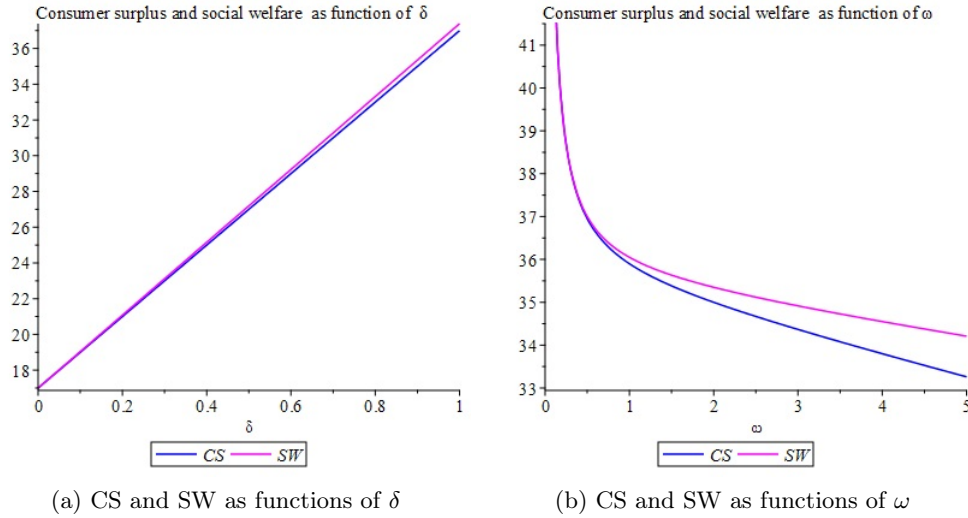


Figure 7: Consumer surplus and social welfare functions

Figures 7a and 7b show the consumer surplus (CS) and social welfare (SW) as functions of the discount factor δ and of the exogenous switching cost parameter ω . Both functions are clearly increasing in the patience level (δ), driven basically for greater consumer welfare as patience level increases.

On the other hand, consumer surplus and social welfare decrease with the exogenous switching cost parameter, more rapidly in the case of consumer surplus. Thus, less exogenous switching costs increases consumer surplus.

The model may suggest that regulatory policies that reduce exogenous switching costs such as number portability (in telecommunication industries, or banking industries), compatibility, standardization, or reduction of administrative barriers, would be more effective in increasing social welfare than policies that reduce endogenous switching costs such as switching fee (ETF in telecommunication industry).

4 Conclusions

The model developed in this paper shows that exogenous switching costs are more relevant than endogenous switching costs in the decision making of consumers. For the

providers, switching fees would not affect multiperiod profits but would accentuate a trade-off between present and future profits. Providers with high switching fees would compensate consumers with lower first period prices, but would charge higher second-period prices to loyal consumers; low switching fees would be associated with high first-period prices and lower second-period prices to loyals. Thus consumers with lower first-period surplus get compensated with a higher second-period surplus and vice versa.

Second-period prices are positively affected by exogenous switching cost parameter ω . Therefore an unanticipated external reduction of exogenous switching costs would reduce second-period prices, for both loyal consumers and switchers; however, if the providers anticipate the change, this reduction also would increase first-period prices and the possibility of higher switching fees. However, since the adverse effect of switching fees on first-period profits cancels out with their positive effect on the second-period profits, then regulatory policies should focus more on policy measures that reduce exogenous switching costs such as standardization, compatibility, number portability, red-tape reduction, etc.

On the other hand, since multiperiod profits are increasing in exogenous switching costs ω , therefore providers will have incentives to keep a high ω (opposing to regulatory changes such number portability or standardization or even by increasing searching costs). However, high exogenous switching costs induce firms to price very low or even negative in the first period to attract consumers, despite of charging a maximum switching fee; first period profits are decreasing in exogenous switching costs.

According to the model, providers charge higher to loyal consumers than to newcomers in the second period when patience level is high. When both providers charge a maximum switching fee, then they charge higher to loyal consumers in the second period respect to first-period prices.

The effect of switching fees in multiperiod payoffs is null, hence policies that target exogenous switching costs reduction may have a higher impact on social welfare than those that ban any existence of switching fees (endogenous SC); external reduction of exogenous switching costs increases social welfare, by increasing consumer surplus.

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Appendix

[Proofs in process]