Probability Theory

MTH 664

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1.3.2. Prove Theorem 1.3.6 when $n = 2$ by checking $\{X_1 + X_2 \leq x\} \in \mathcal{F}$.

Proof

Let $q \in \mathbb{Q}$. Then,

$$A = \bigcup_{q \leq x} \left(\{\omega \in \Omega : X_1(\omega) \leq q\} \cap \{\omega \in \Omega : X_2(\omega) \leq x - q\} \right).$$

Since $\mathbb{Q}$ is countable, the union is countable. The sets $\{\omega : X_1(\omega) \leq q\}$ and $\{\omega : X_2(\omega) \leq x - q\}$ are in $\mathcal{F}$ because X_1 and X_2 are measurable. Intersections and countable unions of sets in $\mathcal{F}$ are also in $\mathcal{F}$.

Therefore, $\{X_1 + X_2 \leq x\} \in \mathcal{F}$.