Homework 4

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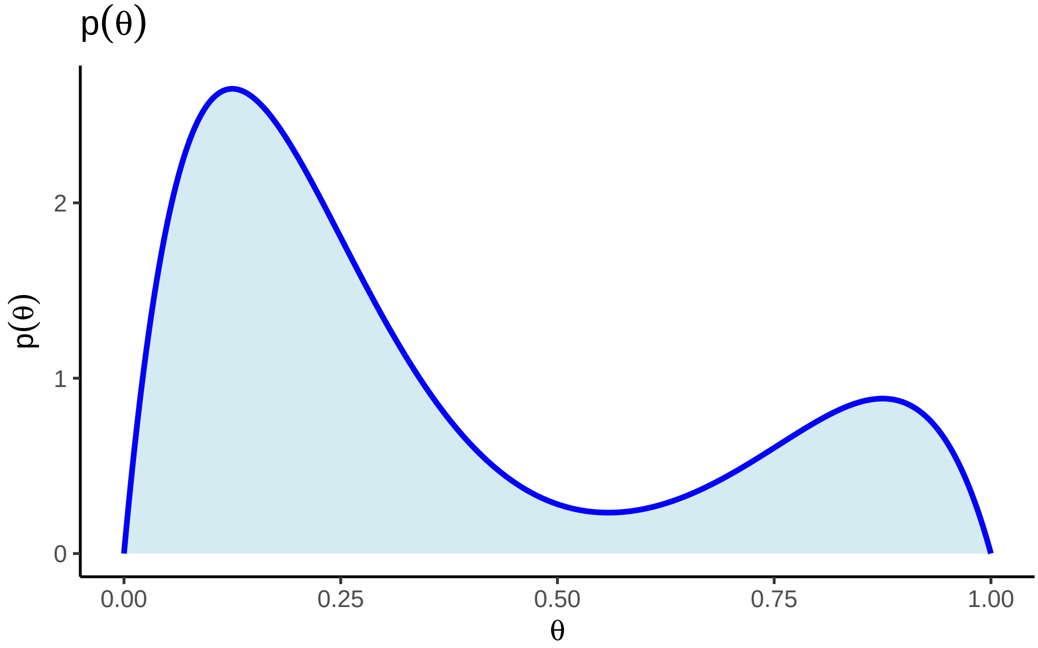
ST 559 Bayesian Statistics

Problem 1

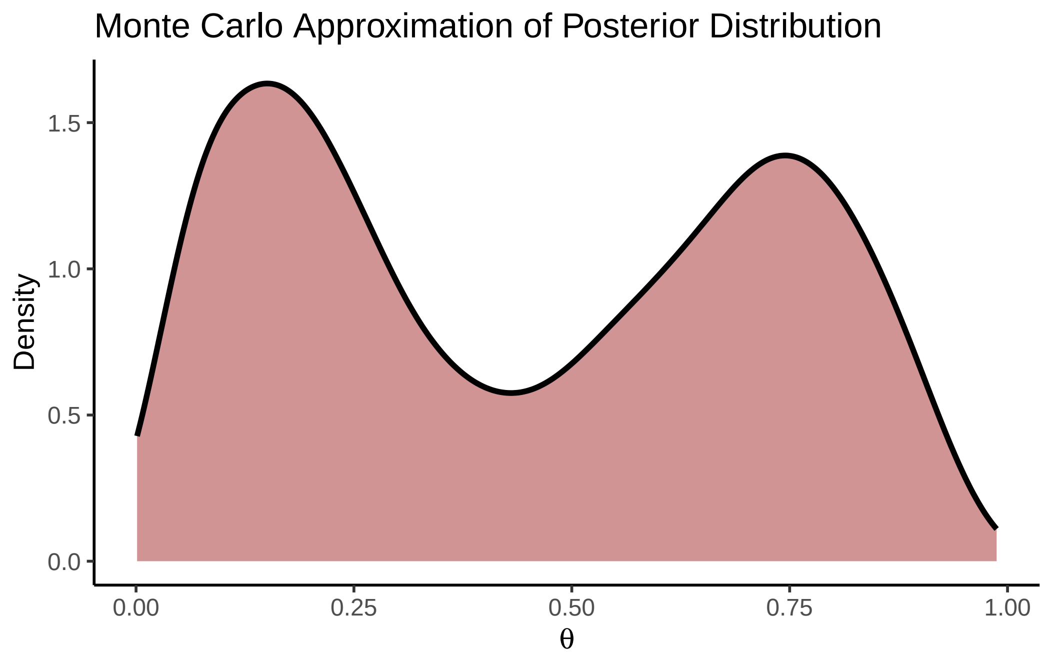
Part A

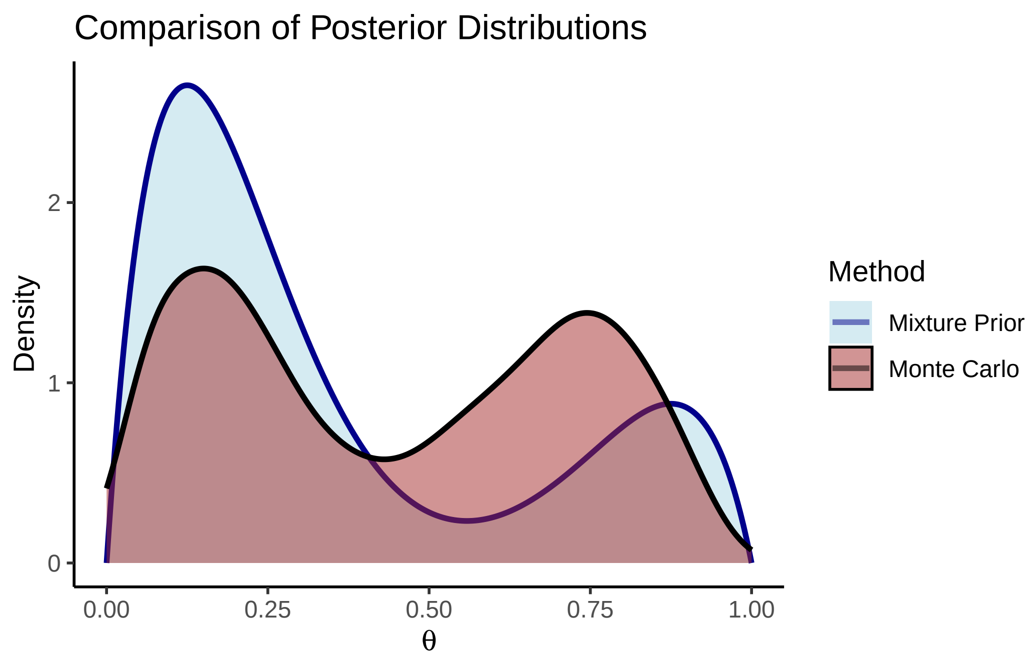
I plotted $p(\theta|y)$ using the mixture prior distribution over a dense sequence of θ values. By calculating the cumulative sum of the posterior, I was able to derive a 95% posterior credible interval for θ as [0.03203203, 0.9389389].



Part B

Using Monte Carlo sampling, we sampled z from the mixture distribution $wp_1(z)+(1-w)p_0(z)$. I was able to derive a 95% posterior credible interval for θ as [0.04108905, 0.9023689].The credible interval is slightly narrower compared to the interval derived from the cumulative sum of the posterior [0.03203203, 0.9389389] from part a. This difference indicates that the Monte Carlo approximation might provide a slightly more conservative estimate of the posterior distribution's uncertainty. Both methods, however, generally agree and support similar conclusions about the distribution of θ.





Problem 2

Using different prior parameters (κ_0, ν_0), we calculated the probability $Pr(\theta_A < \theta_B | y_A, y_B)$ via Monte Carlo sampling. The probabilities that were calculated are,

- For $\kappa_0 = \nu_0 = 1$, $Pr(\theta_A < \theta_B) = 0.7978$
- For $\kappa_0 = \nu_0 = 2$, $Pr(\theta_A < \theta_B) = 0.7874$
- For $\kappa_0 = \nu_0 = 4$, $Pr(\theta_A < \theta_B) = 0.7768$
- For $\kappa_0 = \nu_0 = 8$, $Pr(\theta_A < \theta_B) = 0.7474$
- For $\kappa_0 = \nu_0 = 16$, $Pr(\theta_A < \theta_B) = 0.7289$
- For $\kappa_0 = \nu_0 = 32$, $Pr(\theta_A < \theta_B) = 0.6816$

Additionally, the plot below shows a decreasing trend. We can see that as more weight is placed on the prior, the probability that $\theta_A < \theta_B$ decreases. This plot helps to convey the evidence that $\theta_A < \theta_B$ by showing how sensitive the posterior probability is to different prior opinions. For those who are skeptical about the prior data, the higher probabilities with smaller κ_0 and ν_0 suggest that the data strongly supports $\theta_A < \theta_B$. Hence, those who trust the prior data more may see the lower probabilities with larger κ_0 and ν_0 as an indication of less certainty.