



# ST561: HOMEWORK 1

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## Problems

### Question 1

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1. Textbook, Page 37, Exercise 1.1

- (a)
- (b)
- (c)
- (d)



## Question 2

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2. Suppose that you own 6 different Spanish textbooks, 4 different History textbooks, 4 different Geology textbooks, and 5 different English textbooks. You have reserved a shelf on a bookcase on which you place all these textbooks. How many different ways are there to place all 19 textbooks on the shelf so that all the textbooks in a given subject are grouped together?



### Question 3

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3. Four cards are drawn from a deck of 52 ordinary playing cards, without replacement.
- (a) What is the probability that all 4 cards are aces?
  - (b) What is the probability of drawing, in order, the aces of clubs, diamonds, hearts, and spades?
  - (c) Answer (a) and (b) above if each card is replaced (and the deck shuffled) after it is drawn.



## Question 4

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4. Find the union  $A_1 \cup A_2$  and the intersection  $A_1 \cap A_2$  of the two sets  $A_1$  and  $A_2$  where

- (a)  $A_1 = \{x : 0 < x < 1\}$ ,  $A_2 = \{x : 1 \leq x < 3\}$ ;
- (b)  $A_1 = \{(x, y) : 0 \leq x < 3, 0 < y \leq 3\}$ ,  $A_2 = \{(x, y) : 1 < x \leq 4, 1 \leq y < 4\}$ ;  
(Draw a picture to show the set.)
- (c)  $A_1 = \{(x, y) : x + y \leq 1, x \geq 0, y \geq 0\}$ ,  $A_2 = \{(x, y) : x^2 + y^2 \leq \frac{1}{2}, x \geq 0, y \geq 0\}$ .

You are not required to show your work for this problem.



## Question 5

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5. (Urn Model)  $r$  different balls  $B_1, \dots, B_r$  are to be randomly put into  $n$  different urns  $U_1, \dots, U_n$  ( $r \leq n$ ). What is the probability of the following events?
- (a) Urns  $U_1, \dots, U_r$  each contains exactly one ball;
  - (b) No urn contains more than one ball;
  - (c) Urn  $U_1$  contains exactly  $m$  balls ( $m \leq r$ ).



## Question 6

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6. Suppose  $n > 0, 0 \leq k \leq n$ . Show that

(a)  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

(b)  $\binom{n}{0} = \binom{n}{n} = 1$

(c)  $\binom{n}{1} = n$

(d) If  $k > 1$ ,  $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ .

Can you give intuitive explanations for the results above? (You do not have to turn in your explanations.)



## Question 7

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7. Consider the experiment of dealing a five-card poker hand at random from a deck of 52 ordinary playing cards. Let  $A$  be the event that you obtain a full house (3 cards of one rank and 2 cards of a second rank, e.g., 3 kings and 2 eights). Let  $B$  be the event that you obtain two pairs (2 cards of one rank, 2 cards of a second rank, and a 5th card of a third rank, e.g., 2 tens, 2 threes, and the Jack of diamonds). One of the following two probabilities is correct, and the other probability is incorrect.
- (a) Determine which probability is correct.
  - (b) Correct the incorrect probability.
  - (c) Explain clearly how the correct probability is obtained and why the incorrect probability is wrongly calculated.