Spatial Statistics | Homework 4

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Problem 1

Part A

```r
library(sf)
library(spatstat)
library(ggplot2)
library(viridis)

eqData <- data.frame(
  longitude = c(140.123, 139.000, 141.250),
  latitude  = c(35.600, 36.500, 34.800),
  depth     = c(20, 12, 18),
  magnitude = c(5.2, 4.6, 5.0)
)
eqSf <- st_as_sf(eqData, coords = c("longitude", "latitude"), crs = 4326)
eqSfUtm <- st_transform(eqSf, crs = 32654)
summary(eqSfUtm)
```

```
     depth          magnitude                 geometry
 Min.   :12.00   Min.   :4.600   POINT         :3
 1st Qu.:15.00   1st Qu.:4.800   epsg:32654    :0
 Median :18.00   Median :5.000   +proj=utm ...:0
 Mean   :16.67   Mean   :4.933
 3rd Qu.:19.00   3rd Qu.:5.100
 Max.   :20.00   Max.   :5.200
```

Part B

```r
coords <- st_coordinates(eqSfUtm)
eqWindow <- owin(
  xrange = range(coords[, 1]),
  yrange = range(coords[, 2])
)
eqPpp <- ppp(
  x = coords[, 1],
  y = coords[, 2],
  window = eqWindow,
  marks = data.frame(
    depth     = eqSfUtm$depth,
    magnitude = eqSfUtm$magnitude
  )
)
summary(eqPpp)
```

```
Marked planar point pattern:  3 points
Average intensity 7.80156e-11 points per square unit

Coordinates are given to 10 decimal places

Mark variables: depth, magnitude
Summary:
     depth          magnitude    
 Min.   :12.00   Min.   :4.600  
 1st Qu.:15.00   1st Qu.:4.800  
 Median :18.00   Median :5.000  
 Mean   :16.67   Mean   :4.933  
 3rd Qu.:19.00   3rd Qu.:5.100  
 Max.   :20.00   Max.   :5.200  

Window: rectangle = [320878.7, 522868.3] x [3850893, 4041268] units
                    (202000 x 190400 units)
Window area = 38453800000 square units
```

```r
eqDataUtm <- data.frame(
  x         = coords[, 1],
  y         = coords[, 2],
  depth     = eqSfUtm$depth,
  magnitude = eqSfUtm$magnitude
```

```
)

ggplot(eqDataUtm, aes(x = x, y = y)) +
  geom_point(aes(color = depth, size = magnitude), alpha = 0.7) +
  scale_size_continuous(name = "Magnitude", range = c(1, 6)) +
  coord_fixed() +
  labs(title = "Earthquake Locations (UTM) by Depth and Magnitude ") +
  theme_minimal()
```

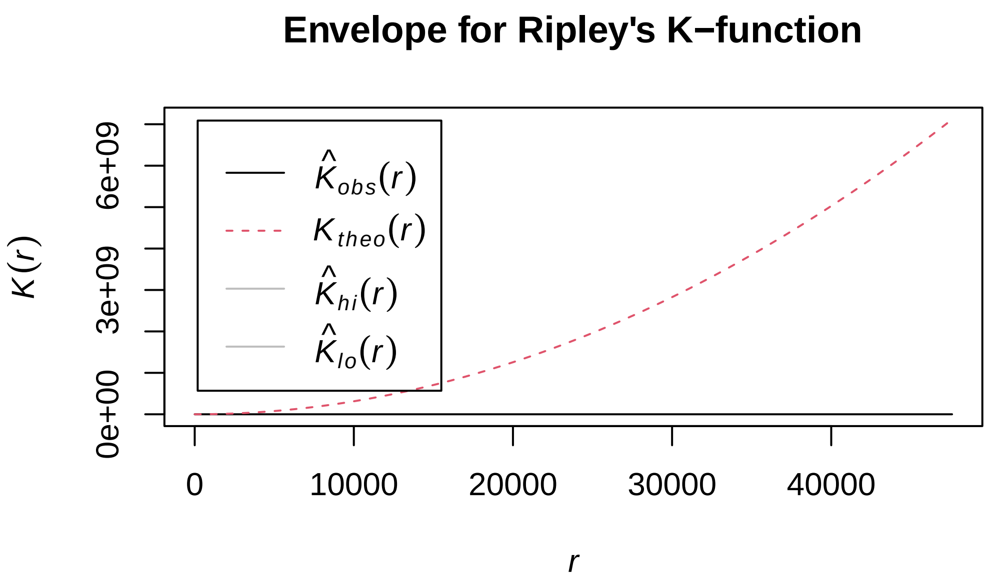
Part C

The envelope analysis shows that the observed K-function mostly stays inside the simulation envelope. This means the earthquake locations seem to occur in a completely random pattern. There's no clear sign of either clustering or a regular spacing out (inhibition), so overall, the spatial distribution of these earthquakes appears random.

```r
envEq <- envelope(eqPpp, fun = Kest, nsim = 39)
```

```
Generating 39 simulations of CSR  ...
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20,
21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38,
39.

Done.
```

```r
plot(envEq, main = "Envelope for Ripley's K-function")
```



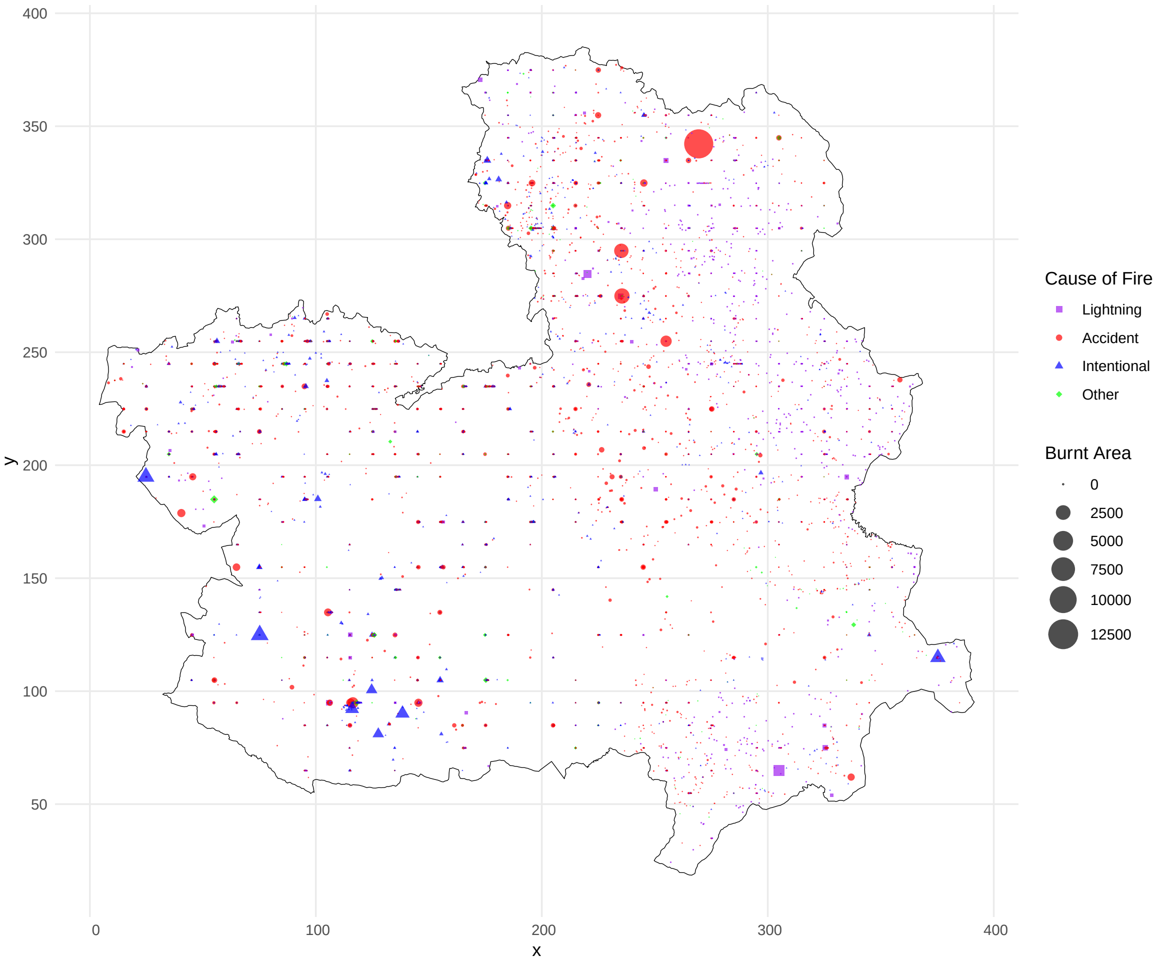
Problem 2

Part A

```r
data(clmfires)
dfClmFires <- data.frame(
  x         = clmfires$x,
  y         = clmfires$y,
  cause     = clmfires$marks$cause,
  burntArea = clmfires$marks$burnt.area
)

regionPoly <- as.polygonal(Window(clmfires))
regionSf   <- st_as_sf(regionPoly)

causeColors <- c(
  "lightning"   = "purple",
  "accident"    = "red",
  "intentional" = "blue",
  "other"       = "green"
)

causeShapes <- c(
  "lightning"   = 15,
  "accident"    = 16,
  "intentional" = 17,
  "other"       = 18
)

ggplot() +
  geom_sf(data = regionSf, fill = NA, color = "black") +
  geom_point(
    data = dfClmFires,
    aes(x = x, y = y, shape = cause, color = cause, size = burntArea),
    alpha = 0.7
  ) +
  scale_size_area(max_size = 8, name = "Burnt Area") +
  scale_color_manual(
    name   = "Cause of Fire",
    values = causeColors,
    breaks = names(causeColors),
```

```r
    labels = c("Lightning", "Accident", "Intentional", "Other")
  ) +
  scale_shape_manual(
    name   = "Cause of Fire",
    values = causeShapes,
    breaks = names(causeShapes),
    labels = c("Lightning", "Accident", "Intentional", "Other")
  ) +
  guides(
    color = guide_legend(order = 1, override.aes = list(shape = causeShapes)),
    shape = guide_legend(order = 1, override.aes = list(color = causeColors)),
    size  = guide_legend(order = 2)
  ) +
  labs(title = "Castilla-La Mancha Fires: Cause and Burnt Area") +
  theme_minimal()
```

Castilla−La Mancha Fires: Cause and Burnt Area

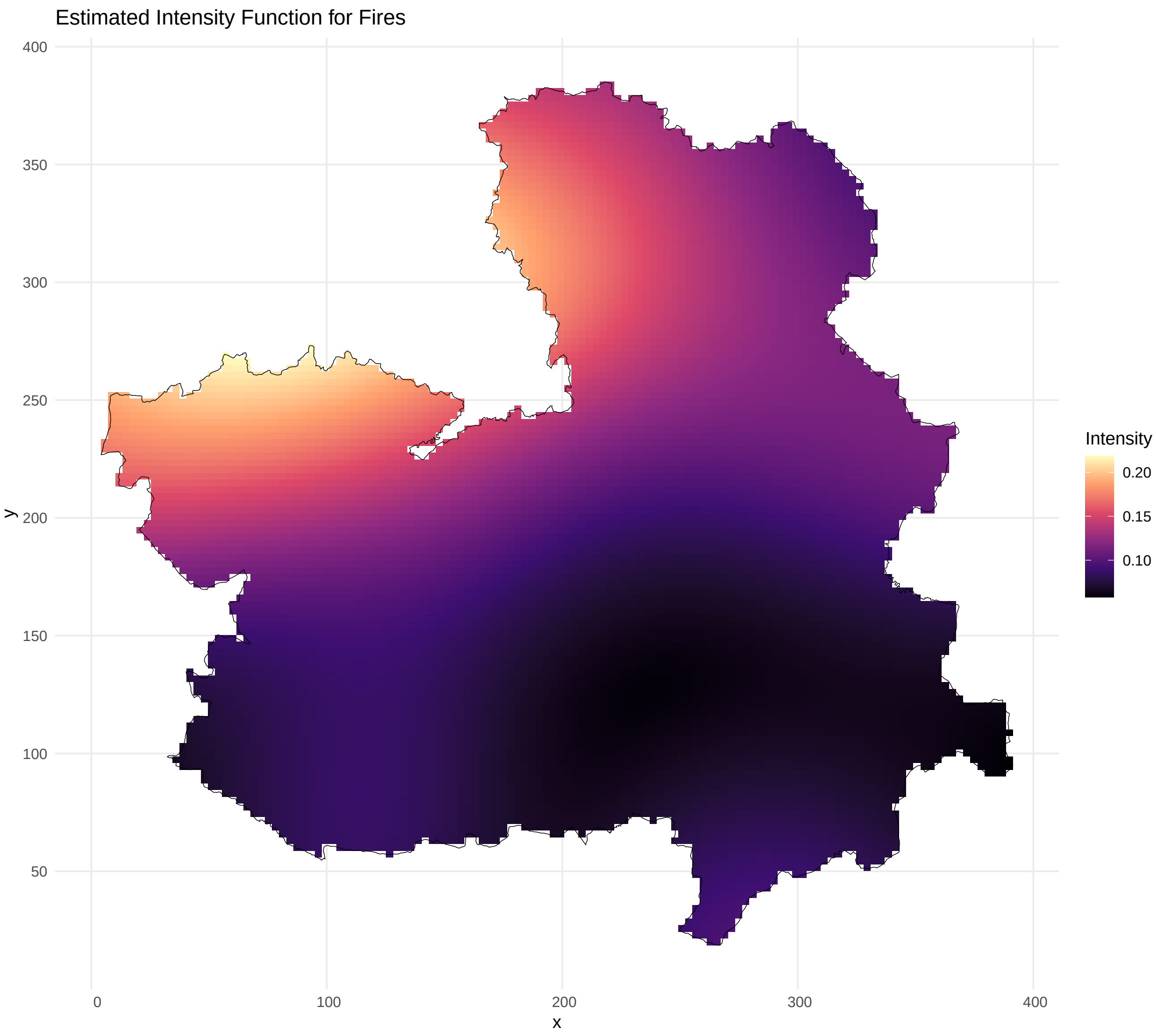


Part B

```r
clmFiresUnmarked <- unmark(clmfires)
summary(clmFiresUnmarked)
```

```
Planar point pattern:  8488 points
Average intensity 0.1069628 points per square kilometre

Coordinates are given to 14 decimal places

Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559
```

```r
densityEst <- density(clmFiresUnmarked)
densityDf <- as.data.frame(densityEst, xy = TRUE)
regionPolygon <- as.polygonal(Window(clmFiresUnmarked))
regionSf <- st_as_sf(regionPolygon)
ggplot() +
  geom_raster(data = densityDf, aes(x = x, y = y, fill = value)) +
  geom_sf(data = regionSf, fill = NA, color = "black") +
  scale_fill_viridis_c(name = "Intensity", option = "magma") +
  coord_sf() +
  labs(title = "Estimated Intensity Function for Fires") +
  theme_minimal()
```


Estimated Intensity Function for Fires
400
350
300
250
y 200
150
100
50
0
100
200
300
400
x
Intensity
0.20
0.15
0.10

Part C

```r
summary(clmfires$marks[["burnt.area"]])
```

```
    Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
   0.000     0.060     0.595    11.297     3.000 12887.370
```

Part D

```r
medianBurnt <- median(clmfires$marks[["burnt.area"]])
firesCut <- cut(clmfires, "burnt.area",
                breaks = c(-Inf, medianBurnt, Inf),
                labels = c("small", "large"))
summary(firesCut)
```

```
Marked planar point pattern:  8488 points
Average intensity 0.1069628 points per square kilometre

Coordinates are given to 14 decimal places

Multitype:
      frequency proportion  intensity
small      4244        0.5 0.05348142
large      4244        0.5 0.05348142

Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559
```

Part E

```r
firesSplit <- split(firesCut)
summary(firesSplit$small)
```

```
Planar point pattern:  4244 points
Average intensity 0.05348142 points per square kilometre

Coordinates are given to 14 decimal places

Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559
```

```r
summary(firesSplit$large)
```

```
Planar point pattern:  4244 points
Average intensity 0.05348142 points per square kilometre

Coordinates are given to 14 decimal places

Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559
```

```r
data(clmfires.extra)
modelLarge <- ppm(firesSplit$large ~ elevation,
                  data = clmfires.extra[["clmcov100"]])
summary(modelLarge)
```

```
Point process model
Fitted to data: firesSplit$large
Fitting method: maximum likelihood (Berman-Turner approximation)
```

```
Model was fitted using glm()
Algorithm converged
Call:
ppm.formula(Q = firesSplit$large ~ elevation, data = clmfires.extra[["clmcov100"]])
Edge correction: "border"
    [border correction distance r = 0 ]
--------------------------------------------------------------------------------
Quadrature scheme (Berman-Turner) = data + dummy + weights

Data pattern:
Planar point pattern:  4244 points
Average intensity 0.0535 points per square kilometre
Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559

Dummy quadrature points:
     140 x 140 grid of dummy points, plus 4 corner points
     dummy spacing: 2.766060 x 2.618743 kilometres

Original dummy parameters: =
Planar point pattern:  10948 points
Average intensity 0.138 points per square kilometre
Window: polygonal boundary
single connected closed polygon with 2325 vertices
enclosing rectangle: [4.1311, 391.3795] x [18.565, 385.189] kilometres
                     (387.2 x 366.6 kilometres)
Window area = 79354.7 square kilometres
Unit of length: 1 kilometre
Fraction of frame area: 0.559
Quadrature weights:
     (counting weights based on 140 x 140 array of rectangular tiles)
All weights:
    range: [0.135, 7.24]    total: 78700
Weights on data points:
    range: [0.135, 4.55]    total: 5380
Weights on dummy points:
    range: [0.137, 7.24]    total: 73300
--------------------------------------------------------------------------------
```

```
FITTED :

Nonstationary Poisson process

---- Intensity: ----

Log intensity: ~elevation
Model depends on external covariate 'elevation'
Covariates provided:
    elevation: im
    orientation: im
    slope: im
    landuse: im

Fitted trend coefficients:
 (Intercept)    elevation
-2.793168241 -0.000147325

                Estimate         S.E.       CI95.lo       CI95.hi Ztest
(Intercept) -2.793168241 5.280608e-02 -2.8966662530 -2.689670e+00   ***
elevation   -0.000147325 5.890025e-05 -0.0002627674 -3.188262e-05     *
                  Zval
(Intercept) -52.894824
elevation    -2.501262

----------- gory details -----

Fitted regular parameters (theta):
 (Intercept)    elevation
-2.793168241 -0.000147325

Fitted exp(theta):
(Intercept)   elevation
 0.06122692  0.99985269
```

The estimated intensity function is

$$\lambda(x, y) = \exp\Big(0.06122692 + 0.99985269 \cdot \text{elevation}(x, y)\Big).$$