Methods of Data Analysis

Problem 2 - Part A

2025-01-16

Goal

Find the **change in the difference of mean depression scores** between education categories (iii) and (i) for a one-unit increase in age.

Write the Regression Equation

$$\text{Depression Score} = \beta_0 + \beta_1 \text{Age} + \beta_2 \text{Edu}_{ii} + \beta_3 \text{Edu}_{iii} + \beta_4 (\text{Age} \cdot \text{Edu}_{ii}) + \beta_5 (\text{Age} \cdot \text{Edu}_{iii}) + \epsilon$$

1. β_0 : The mean depression score for individuals with a college degree (category (i)) at age 0.

2. β_1 : The change in depression score per year of age for individuals with a college degree (category (i)).

3. β_2 : The difference in mean depression scores between individuals with some college (category (ii)) and those with a college degree (category (i)) at age 0.

4. β_3 : The difference in mean depression scores between individuals with only a high school degree (category (iii)) and those with a college degree (category (i)) at age 0.

5. β_4 : The difference in the effect of age (slope) between individuals with some college (category (ii)) and those with a college degree (category (i)).

6. β_5 : The difference in the effect of age (slope) between individuals with only a high school degree (category (iii)) and those with a college degree (category (i)).

7. ϵ : The residual error, representing the variability in depression scores not explained by the model.

Plug in the Values for Categories (i) and (iii)

1. **Category (i):** This is the baseline, so $\text{Edu}_{ii} = 0$ and $\text{Edu}_{iii} = 0$.
   The equation for the mean depression score becomes:

$$\text{Depression Score for (i)} = \beta_0 + \beta_1 \text{Age}$$

2. **Category (iii):** Here, $\text{Edu}_{iii} = 1$ and $\text{Edu}_{ii} = 0$.
   The equation for the mean depression score becomes:

$$\text{Depression Score for (iii)} = \beta_0 + \beta_1 \text{Age} + \beta_3 + \beta_5 \text{Age}$$

Find the Difference Between Categories (iii) and (i)

Subtract the depression score equation for category (i) from that for category (iii):

$$\text{Difference} = (\beta_0 + \beta_1 \text{Age} + \beta_3 + \beta_5 \text{Age}) - (\beta_0 + \beta_1 \text{Age})$$

Simplify:

$$\text{Difference} = \beta_3 + \beta_5 \text{Age}$$

Thus, the difference in mean depression scores between categories (iii) and (i) is:

$$\text{Difference} = \beta_3 + \beta_5 \text{Age}$$

Find the Change in Difference with Respect to Age

The "Change in Difference" is how this difference changes with one unit increase in age. To calculate this, take the derivative of the difference with respect to age:

$$\frac{d}{d\text{Age}}(\beta_3 + \beta_5 \text{Age}) = \dots$$

This detailed explanation is meant to guide you through the problem, as many students found it challenging. Your final solution can be much shorter and more focused. I hope this clarifies the process and helps you reach the correct answer!