ST565 Homework 1

Brian Cervantes Alvarez
February 2, 2024
ST565: Time Series

Question 1

Part A

Finding $E(\bar{x})$

- $E(\bar{x}) = E\left(\frac{x_1+x_2}{2}\right)$
- This becomes $\frac{E(x_1)+E(x_2)}{2}$.
- Given $E(x_1) = E(x_2) = \mu$, it simplifies to $E(\bar{x}) = \frac{\mu+\mu}{2} = \mu$.

Finding Var$(\bar{x})$

- Var$(\bar{x}) =$ Var $\left(\frac{x_1+x_2}{2}\right)$
- The variance becomes $\frac{1}{4}$Var$(x_1 + x_2)$
- Since Var$(x_1 + x_2) =$ Var$(x_1) +$ Var$(x_2) + 2$Cov(x_1, x_2) and Cov$(x_1, x_2) = 0$ because cor$(x_1, x_2) = 0$, it simplifies to $\frac{1}{4}(2\sigma^2)$ because Var$(x_1) =$ Var$(x_2) = \sigma^2$.
- Thus, Var$(\bar{x}) = \frac{\sigma^2}{2}$.

Part B

Finding $E(\bar{x})$

- $E(\bar{x})$ does not depend on the correlation; hence, $E(\bar{x}) = \mu$.

Finding Var$(\bar{x})$

- Var$(\bar{x}) = \frac{1}{4}$Var$(x_1 + x_2)$ still holds.
- Now, Var$(x_1 + x_2) =$ Var$(x_1) +$ Var$(x_2) + 2$Cov(x_1, x_2), but Cov$(x_1, x_2) \neq 0$.
- Given Cov$(x_1, x_2) = \rho\sigma^2$ because cor$(x_1, x_2) = \rho$ and Cov$(x_1, x_2) =$ cor$(x_1, x_2) \cdot \sigma_{x_1} \cdot \sigma_{x_2}$, the variance becomes $\frac{1}{4}(2\sigma^2 + 2\rho\sigma^2)$.
- Therefore, Var$(\bar{x}) = \frac{\sigma^2}{2}(1 + \rho)$.

Part C

Effect of Correlation on Variance of Sample Mean

- When cor$(x_1, x_2) = 0$, the variance of the sample mean is $\frac{\sigma^2}{2}$.
- When cor$(x_1, x_2) = \rho$, the variance of the sample mean becomes $\frac{\sigma^2}{2}(1 + \rho)$.
- The correlation affects the variance of the sample mean. Positive correlation increases the variance, whereas negative correlation decreases it.
- Ignoring correlation in inference can lead to underestimating or overestimating the variability in the data.

Question 2

Part A

Show $n = 3, 4$

- For $n = 3, 4$, can be described by the formula $x_t = \delta t + \sum_{j=1}^{t} w_j$ for $t = 1, \ldots, n$.
- Let's show it for $t = 1, 2, 3, 4$:
  - When $t = 1$, $x_1 = \delta + x_0 + w_1 = \delta + w_1$.
  - When $t = 2$, $x_2 = \delta + x_1 + w_2 = \delta + (\delta + w_1) + w_2 = 2\delta + w_1 + w_2$.
  - When $t = 3$, $x_3 = \delta + x_2 + w_3 = \delta + (2\delta + w_1 + w_2) + w_3 = 3\delta + w_1 + w_2 + w_3$.
  - When $t = 4$, $x_4 = \delta + x_3 + w_4 = \delta + (3\delta + w_1 + w_2 + w_3) + w_4 = 4\delta + w_1 + w_2 + w_3 + w_4$.

Part B

Determining the Mean

- $E[x_t] = E[\delta + x_{t-1} + w_t]$.
- Since $E[w_t] = 0$ for white noise, $E[x_t] = \delta + E[x_{t-1}]$.
- If you continue the pattern, the mean becomes δt.

Determining the Variance

- Var$(x_t) =$ Var$(\delta + x_{t-1} + wt)$.
- Since Var$(w_t) = \sigma_w^2$ and w_t is independent, the variance converges to $t\sigma_w^2$.

Determining Stationarity

- The process is not stationary as the mean and variance depend on time t.

Part C

- Autocovariance function is given as $\gamma(s, t)$ of x_t for $\delta = 0$ where $\gamma(s, t) =$ Cov(x_s, x_t).
- For $\delta = 0$, $x_t = \sum_{j=1}^{t} w_j$.
- $\gamma(s, t)$ depends on the overlap of the sums of w_j. This means that if $s \leq t$, then the ACF is $\gamma(s, t) = s\sigma_w^2$.

Part D

- The autocovariance function does the following compared to the random walk and moving average processes:
  - In the random walk process, the autocovariance function increases with time lag.
  - In a moving average process, the autocovariance function usually becomes zero after a certain lag, indicating no correlation beyond a specific period.

Question 3

Part A: $x_t = w_t - w_{t-3}$

Stationarity

- The process is stationary.
- Each w_t is independent and identically distributed, and the structure of x_t does not change over time.

Mean

- The mean is $E[x_t] = E[w_t - w_{t-3}] = E[w_t] - E[w_{t-3}] = 0 - 0 = 0$.

Autocovariance Function

- The autocovariance function depends on the lag k:
  - For $k = 0$, Cov$(x_t, x_t) =$ Var$(w_t - w_{t-3}) =$ Var$(w_t) +$ Var$(w_{t-3}) = 2$.
  - For $k = 3$ or $k = -3$, Cov$(x_t, x_{t-3}) = -1$.
  - Otherwise, it is 0.

Part B: $x_t = w_1^3$

Stationarity

- The process is not stationary.
- x_t is constant over time, which violates the requirement for stationarity that the mean and variance must not depend on time.

Part C: $x_t = t + w_1^3$

Stationarity

- The process is not stationary.
- The term t introduces a time-dependent trend.

Part D: $x_t = w_t^2$

Stationarity

- The process is stationary.
- Each w_t^2 follows a Chi-squared distribution with 1 degree of freedom and is independent of time t.

Mean

- The mean is $E[w_t^2] = 1$ (since the variance of w_t is 1).

Autocovariance Function

- The autocovariance function depends on the lag k:
  - For $k = 0$, it is the variance of w_t^2, which is 2.
  - For $k \neq 0$, it is 0 since w_t are independent.

Part E: $x_t = w_t w_{t-2}$

Stationarity

- The process is stationary.
- The multiplication of two independent, identically distributed normal random variables at different times does not depend on time t.

Mean

- The mean is $E[w_t w_{t-2}] = E[w_t] \cdot E[w_{t-2}] = 0$.

Autocovariance Function

- The autocovariance function depends on the lag k:
  - For $k = 2$ or $k = -2$, Cov$(x_t, x_{t-2}) = 1$.
  - Otherwise, it is 0.

Question 4

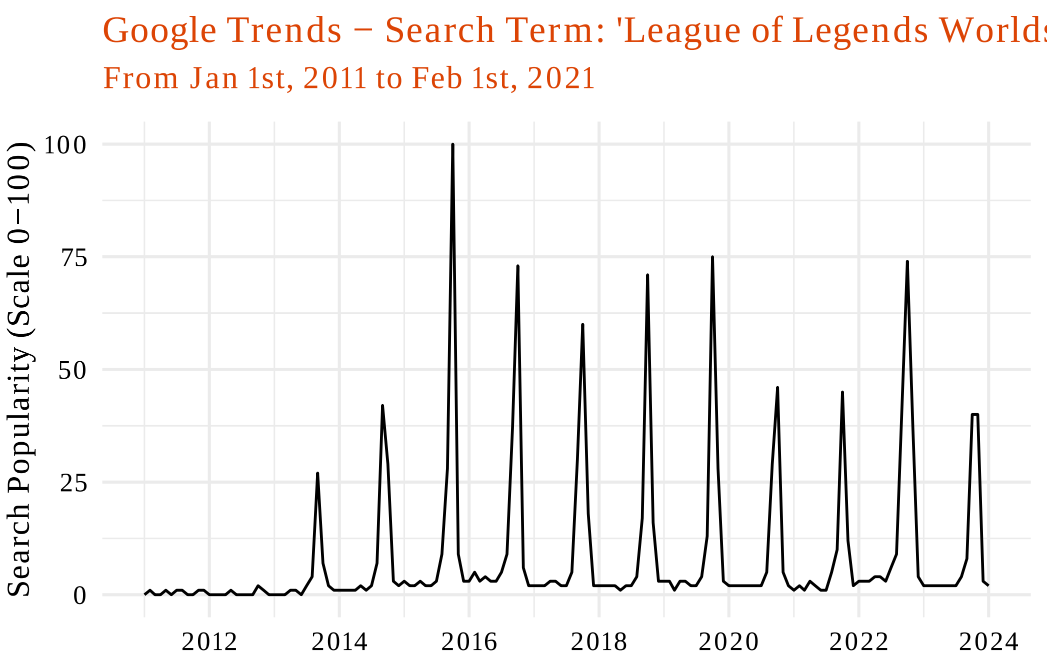
Part A

Initial Popularity (2011-2013):

- **Low Baseline Interest:** During this period, the interest in "League of Legends Worlds" was considerably low, reflecting poor interest early on.

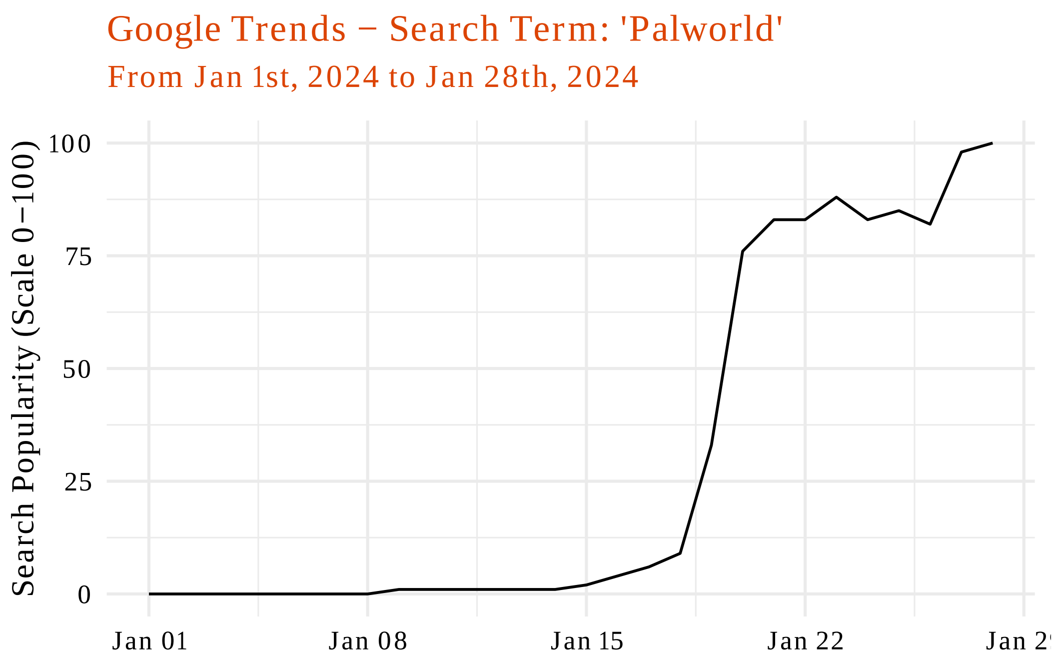
Growth Phase (2014 Onwards):

- **Rapid Increase in Popularity:** This could be attributed to several factors, including the rising popularity of esports, increased marketing efforts by Riot Games, and the event's growing reputation as a premier competitive gaming spectacle.
- **Annual Peaks:** These peaks are likely more pronounced and have a higher amplitude compared to earlier years, indicating heightened anticipation and engagement from the global gaming community.



Part B

- **Here's Some Context:** Palworld, a video game blending elements from Ark: Survival Evolved, Legend of Zelda: Breath of the Wild, and incorporating Pokémon-like mechanics, saw exponential growth in search interest post its early access launch starting on January 19th 2024, showcasing a dramatic spike in public attention.
- **Potential Cause in Popularity:** Top streamers from Twitch began to stream Palworld, causing a major influx of gamer attention. Even a few of my friends started playing it. In other words, they got on board of what I like to call the "current streamer game train".
- **Where it is trending?** After the initial surge, the game stabilized and then entered a phase of steady linear growth from January 22, 2024, onwards, reflecting a sustained interest in the game's unique gameplay experience. Maybe it's a good Pokémon-like game?



Part C

- **Event-Driven Spike:** A sharp increase in search interest for "Anita Max Wynn" was triggered by a mention from rapper Drake, peaking sharply on December 25th due to viral spread and memeing.
- **Rapid Decline and Leveling Off:** After peaking, interest in the term quickly declined from late January, showing the nature of viral meme culture, yet it leveled off slightly above pre-spike levels, indicating some residual interest-probably due to streamers referencing that meme joke.
- **Absence of Seasonality and Trend:** It does not follow predictable seasonal patterns or a long-term trend, highlighting its dependence on being a viral meme.