Introduction to Mobile Robotics

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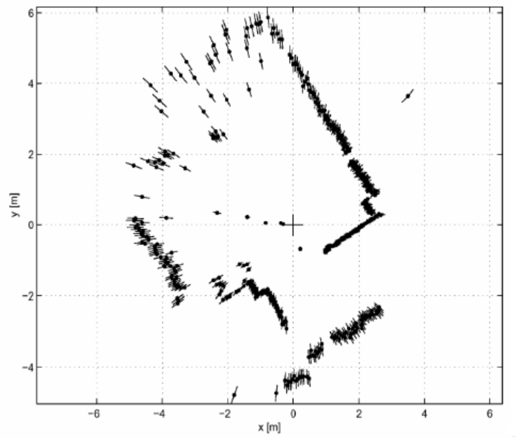
LIDAR Map Construction

Typical range image of a 2D laser range sensor with a rotating mirror. The length of the lines through the measurement points indicate the uncertainties.

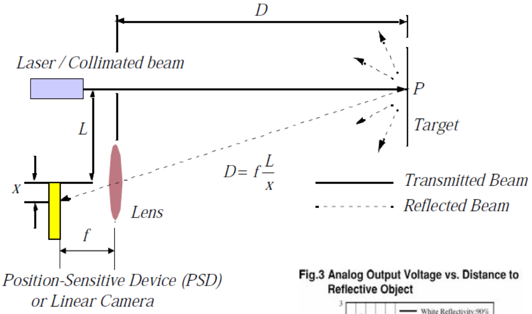


Triangulation

Geometrical Properties of the image or object used to establish a distance measurement.

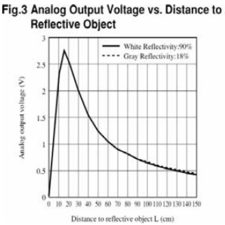
- Project a well defined light pattern (points, lines) onto the environment
  - reflected light is then captured by a photo-sensitive line or matrix (camera) sensor device
  - simple triangulation allows computation of distance
  - the standard IR range sensor uses this approach
  - Kinect and ASUS sensors use arrays of project IR dots - size of dots in IR image indicates distance
- size of object known
  - triangulation can be done without projecting light
  - computer vision techniques can recover relative image size

Laser Triangulation



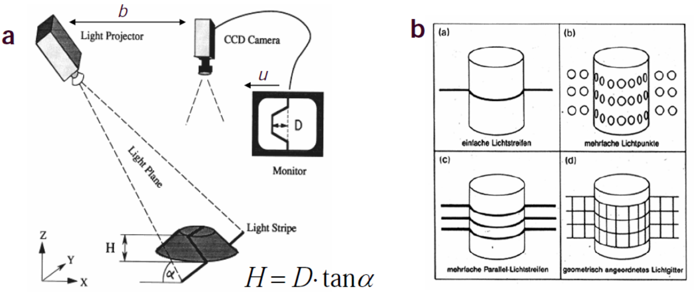
Principle of 1D laser triangulation.

$$D = f\frac{L}{X}$$



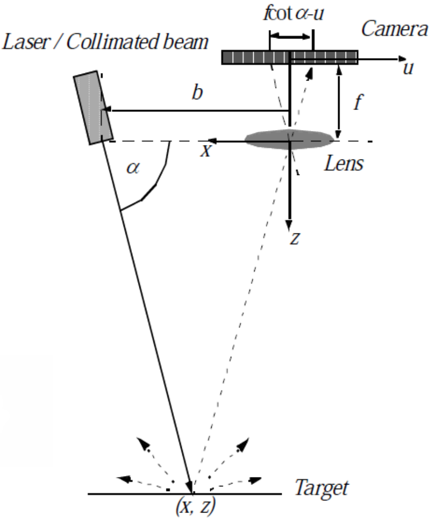
'H Zurich - ASL

Structured Light



- Eliminate the correspondence problem by projecting structured light on the scene.
- Light stripes (laser via rotating mirror)
- Camera records stripes
- Range to object can be determined by geometry

Common Industrial Computer Vision Setup



Structured Light Optics

Look at the diagram and see what formulas can we derive. Note that:

$$z/x = f/u, \quad \text{and} \quad \tan(\alpha) = z/(b - x).$$

Flip the second formula:

$$\cot(\alpha) = (b - x)/z.$$

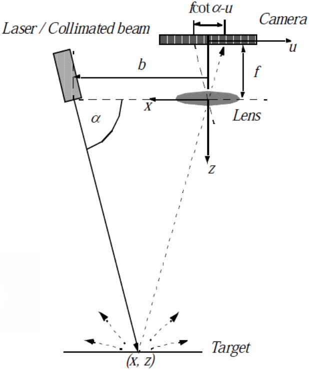
Then multiply by z:

$$z \cot(\alpha) = b - x$$

and move the x over:

$$z \cot(\alpha) + x = b.$$

From the first ratio: $z = fx/u$.



Structured Light Optics

Plug this in for z:

$$(fx/u)\cot(\alpha) + x = b.$$

Factor out the x and divide the rest over:

$$x = \frac{b}{(f/u)\cot(\alpha) + 1} \Rightarrow x = \frac{bu}{f\cot(\alpha) + u}.$$

Then using

$$z = (f/u)x = \left(\frac{f}{u}\right)\frac{bu}{f\cot(\alpha) + u} \Rightarrow z = \frac{bf}{f\cot(\alpha) + u}.$$

Structured Light Optics

What are x & z if b = 20cm, f = 2cm, α = 60deg, and u = 7mm?

So, using these formulas:

$$x = 20 * 0.7/(2\cot(60) + 0.7) = 7.55cm,$$

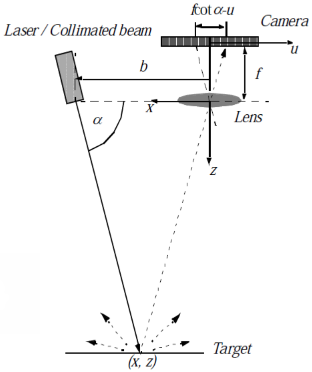
$$z = 20 * 2/(2\cot(60) + 0.7) = 21.57cm.$$

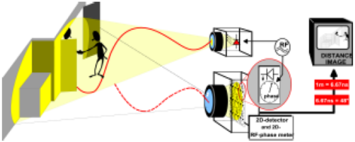
Structured Light Optics

Note that we obtain

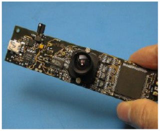
$$x = \frac{b \cdot u}{f \cot \alpha + u}$$

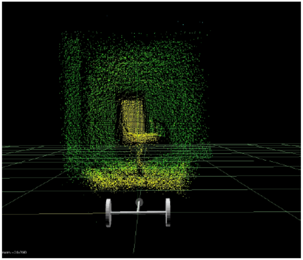
$$z = \frac{b \cdot f}{f \cot \alpha + u}$$



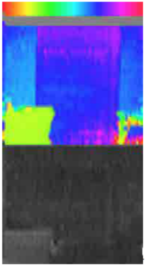


Swiss Ranger (CSEM)

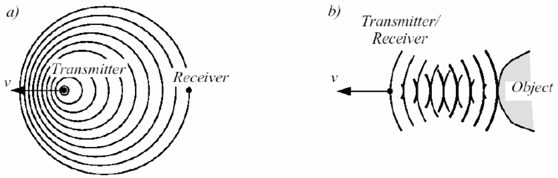




Video CSEM



Measuring Velocity



- $f_r = f_t(1 + v/c)$ if the transmitter is moving
- $f_r = f_t/(1 + v/c)$ if the receiver is moving
- $\Delta f = f_t - f_r = (2 f_t v \cos\theta)/c$
- θ is relative angle between direction of motion and beam axis.

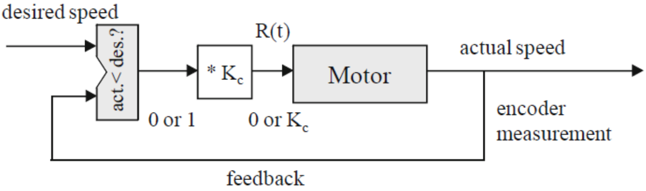
Control Systems

- Open Loop
- Closed Loop
- On-off Control
- P Control
- PD Control
- PID Control
- Nonlinear, Optimal, Fuzzy, etc controls

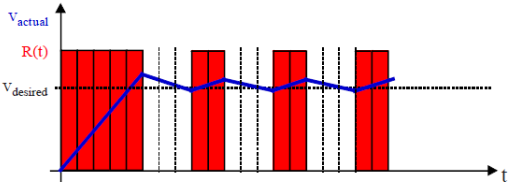
Bang-Bang Control

$R(t)$	motor output function over time *t*
$v_{act}(t)$	actual measured motor speed at time *t*
$v_{des}(t)$	desired motor speed at time *t*
K_C	constant control value

$$R(t) = \begin{cases} K_C & \text{if } v_{act}(t) < v_{des}(t) \\ 0 & \text{otherwise} \end{cases}$$

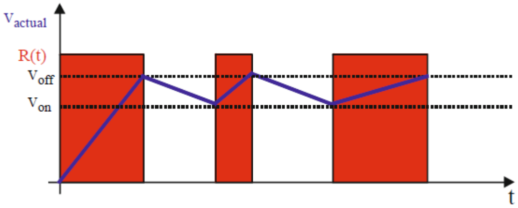


Bang-Bang Control output



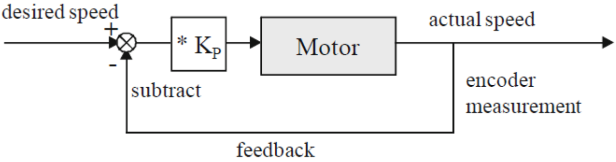
On-Off Control with Hysteresis

$$R(t+\Delta t) = \begin{cases} K_C & \text{if} \quad v_{act}(t) < v_{on}(t) \\ 0 & \text{if} \quad v_{act}(t) > v_{off}(t) \\ R(t) & \text{otherwise} \end{cases}$$



Proportional Control

Control: $R(t) = K_P(v_{des}(t) - v_{act}(t))$



This seems like it should work rather well ..

Proportional Control

From Wikipedia:

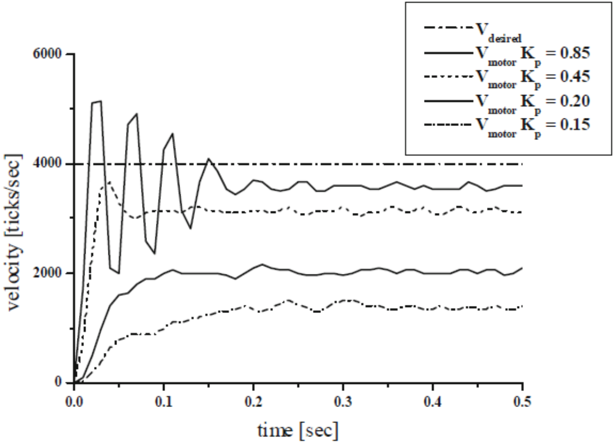
The proportional control system is more complex than an on-off control system like a bi-metallic domestic thermostat, but simpler than a proportional-integral-derivative (PID) control system used in something like an automobile cruise control.

On-off control will work where the overall system has a relatively long response time, but will result in instability if the system being controlled has a rapid response time.

Proportional control overcomes this by modulating the output to the controlling device, such as a continuously variable valve.

P Control

Here are some sample curves for the response:



Not what we expected … why?

System Analysis

Motor control with a P-Controller:

- Dynamics via a second order differential equation

$$\frac{D^2 w}{dt^2} + 2\frac{dw}{dt} + cw = k(w^* - w)$$

$$\frac{D^2 w}{dt^2} + 2\frac{dw}{dt} + (c + k)w = kw^*$$

- Steady state solution:

$$w = \frac{kw^*}{c + k}$$

- Real solutions: $ck < 1$

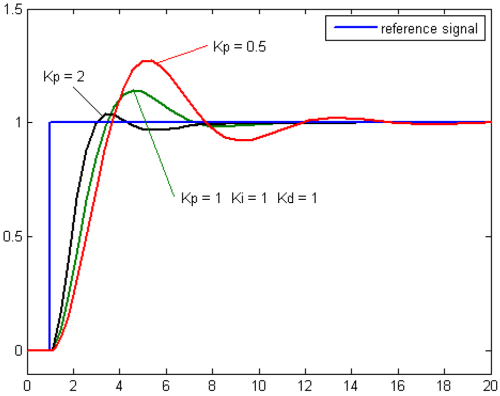
PID Control Overview

$$R(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

- $e(t)$ - Error $= v_{des}(t) - v_{act}(t)$
- K_P - Proportional gain
- K_I - Integral gain
- K_D - Derivative gain

PID - Proportional Term

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error, and a less responsive or less sensitive controller.



PID - Integral Term

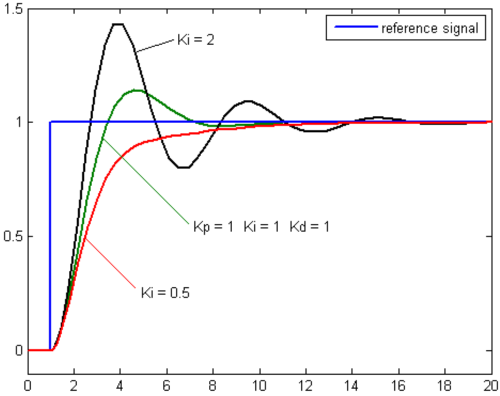
The contribution from the integral term is proportional to both the magnitude of the error and the duration of the error.

The integral in a PID controller is the sum of the instantaneous error over time and gives the accumulated offset that should have been corrected previously.

The accumulated error is then multiplied by the integral gain (Ki) and added to the controller output.

PID - Integral Term

The integral term accelerates the movement of the process towards setpoint and eliminates the residual steady-state error that occurs with a pure proportional controller. However, since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the setpoint value.



PID - Derivative Term

The derivative of the process error is calculated by determining the slope of the error over time and multiplying this rate of change by the derivative gain Kd.

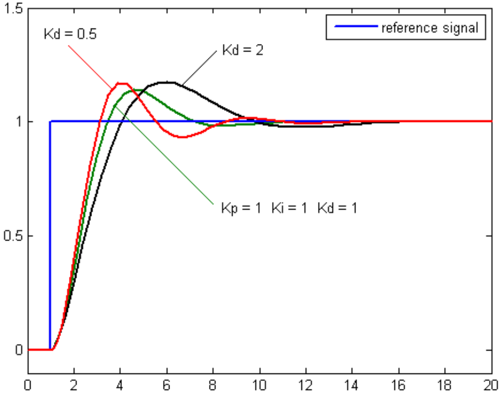
The magnitude of the contribution of the derivative term to the overall control action is termed the derivative gain, Kd.

The derivative term slows the rate of change of the controller output.

Derivative control is used to reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability. However, the derivative term slows the transient response of the controller.

PID - Derivative Term

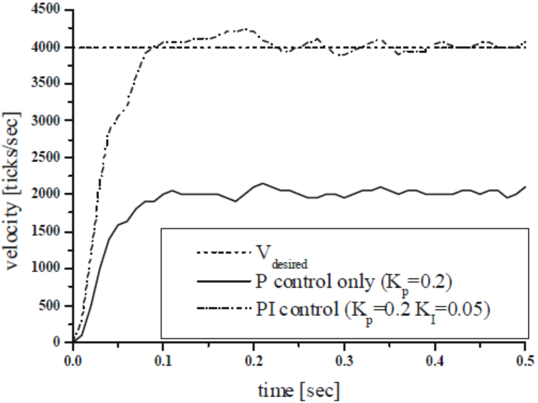
Also, differentiation of a signal amplifies noise and thus this term in the controller is highly sensitive to noise in the error term, and can cause a process to become unstable if the noise and the derivative gain are sufficiently large.



P vs PI Control

Set $K_D = 0$

$$R(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau$$



PI Recursive Control Formulas

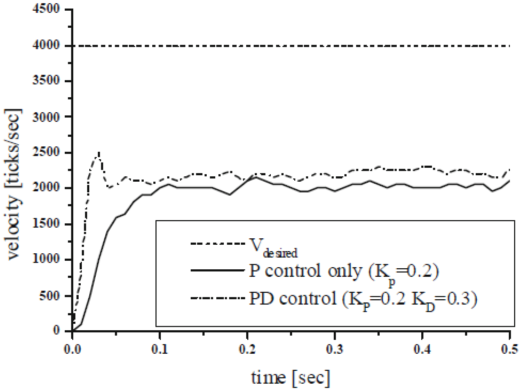
$$R_n = K_P e_n + Q_I \Delta t \sum_{i=1}^{n} \frac{e_i + e_{i-1}}{2}$$

$$R_n - R_{n-1} = K_P(e_n - e_{n-1}) + Q_I \Delta t \left(\frac{e_n + e_{n-1}}{2} \right)$$

$$R_n = R_{n-1} + K_P(e_n - e_{n-1}) + K_I(e_n + e_{n-1})$$

PD Control

$$R(t) = K_P \left(e(t) + T_D \frac{de(t)}{dt} \right)$$



PID Recursive Control

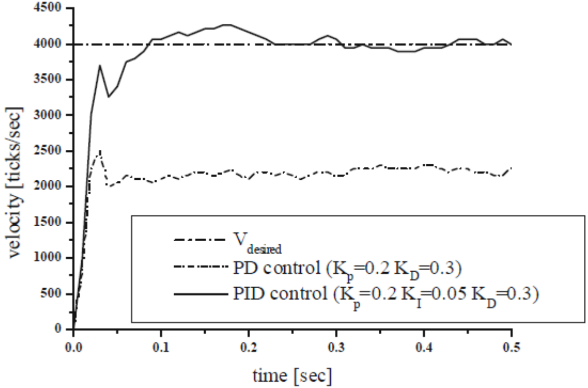
$$R(t) = K_P e(t) + Q_I \int_0^t e(\tau) d\tau + Q_D \frac{de(t)}{dt}$$

$$R_n = K_P e_n + Q_I \Delta t \sum_{i=1}^{n} \frac{e_i + e_{i-1}}{2} + Q_D \frac{e_n - e_{n-1}}{\Delta t}$$

$$R_n - R_{n-1} = K_P(e_n - e_{n-1}) + Q_I \Delta t \left(\frac{e_n + e_{n-1}}{2} \right) + Q_D \frac{e_n - 2e_{n-1} + e_{n-2}}{\Delta t}$$

$$R_n = R_{n-1} + K_P(e_n - e_{n-1}) + K_I(e_n + e_{n-1}) + K_D(e_n - 2e_{n-1} + e_{n-2})$$

PID Control Response



PID Parameter Tuning

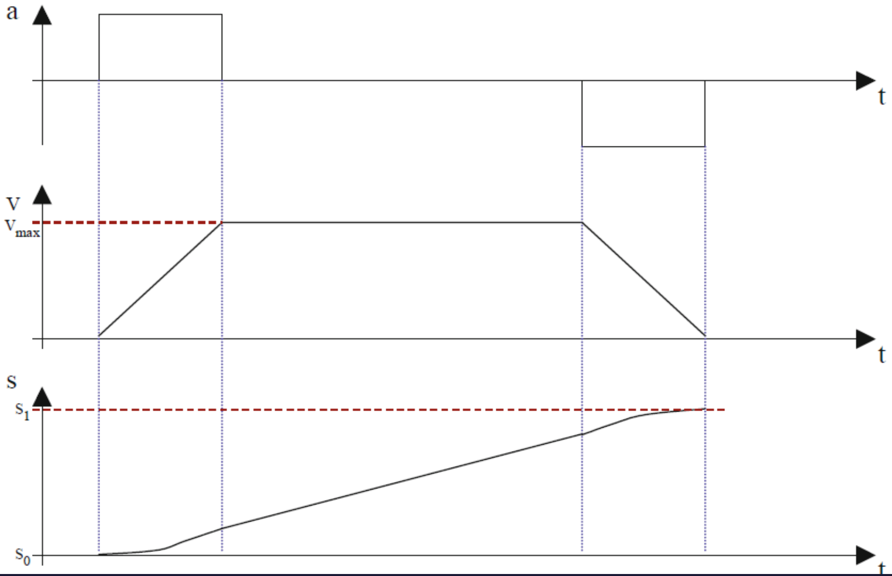
1. Select a typical operating setting for the desired speed, turn off integral and derivative parts. Increase K_P to maximum or until oscillation occurs.
2. If system oscillates, divide K_P by 2.
3. Increase K_D and observe behaviour when increasing or decreasing the desired speed by 5%. Select a value of K_D which gives a damped response.
4. Slowly increase K_I until oscillation starts. Then divide K_I by 2 or 3.
5. Check overall controller performance under typical conditions.

PID Parameter Tuning: ZieglerNichols method

Another heuristic tuning method is formally known as the ZieglerNichols method, introduced by John G. Ziegler and Nathaniel B. Nichols in the 1940s. As in the method above, the Ki and Kd gains are first set to zero. The P gain is increased until it reaches the ultimate gain, Ku, at which the output of the loop starts to oscillate. Ku and the oscillation period Pu are used to set the gains as shown:

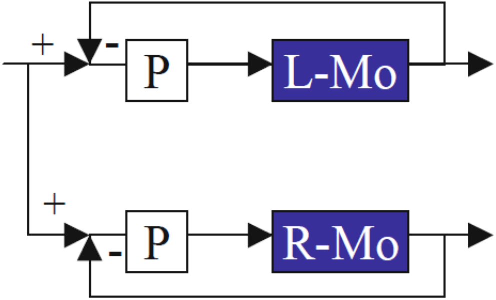
Control Type	Kp	Ki	Kd
P	0.50Ku	-	-
PI	0.45Ku	1.2Kp / Pu	-
PID	0.60Ku	2Kp / Pu	KpPu / 8

Speed Ramp

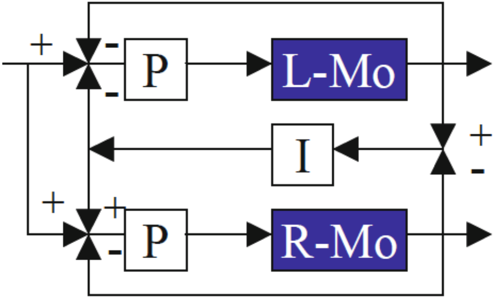


Multiple Motors

Simple approach



Simple approach



Simple approach