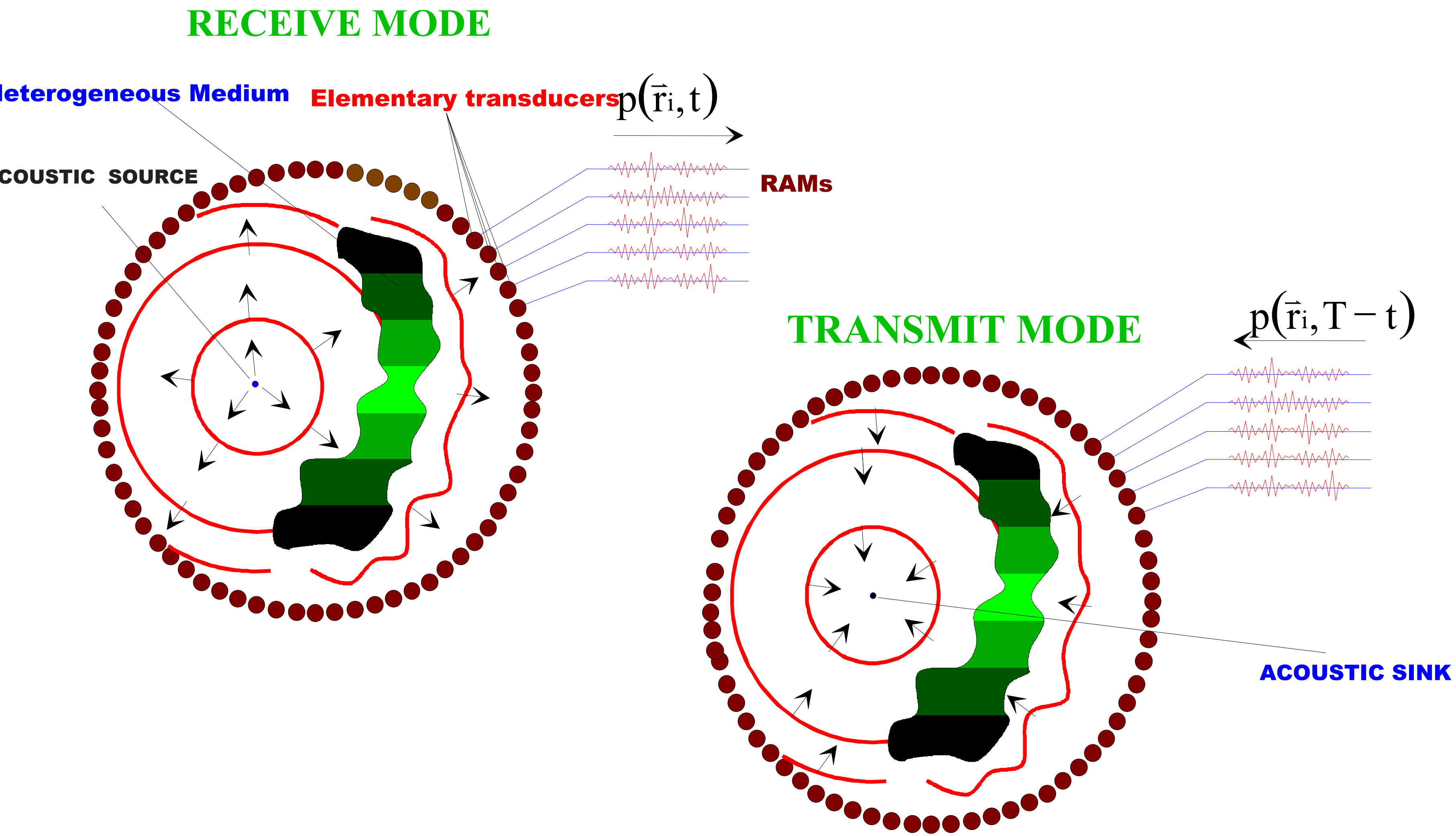
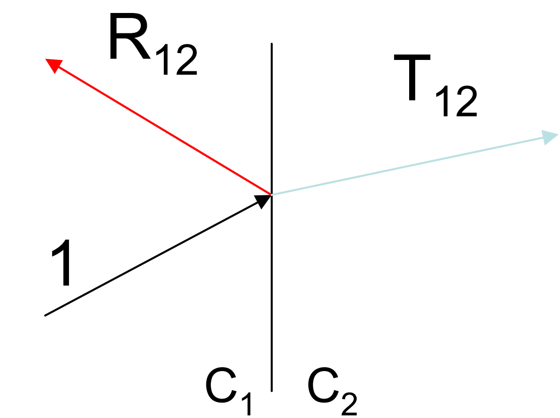
Time Reversal Cavity



Stokes Formula : A plane wave approach



One incident plane wave : amplitude 1 :

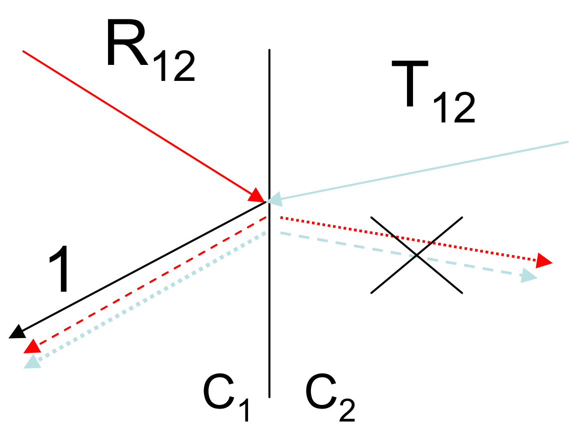
One reflected wave : R_{12}

One transmitted wave : T_{12}

$$\boxed{T_{12}T_{21}+R_{12}R_{12}=1}$$

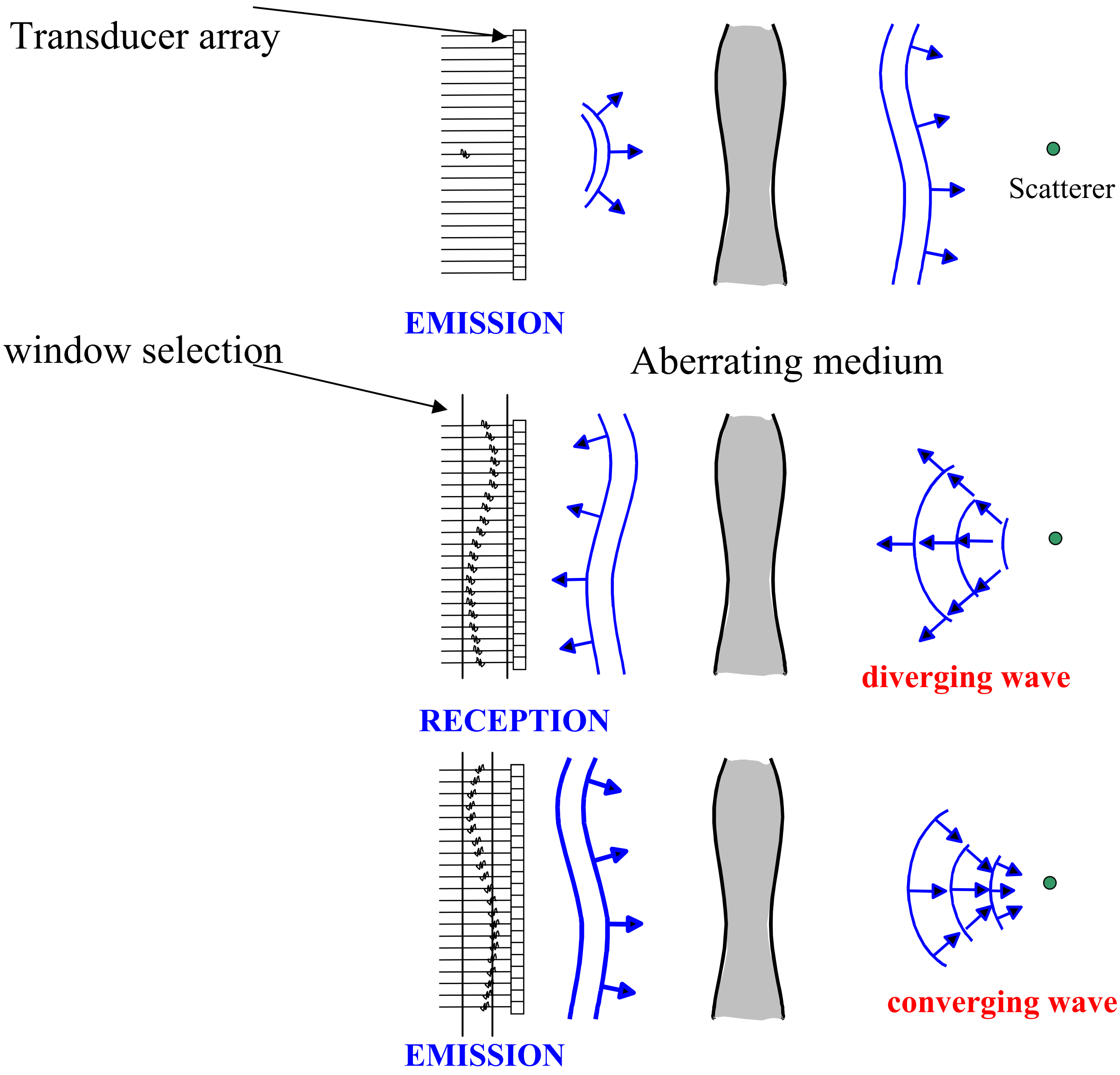
$$\boxed{T_{12}R_{21}+R_{12}T_{12}=0}$$

The Time-Reversed Situation



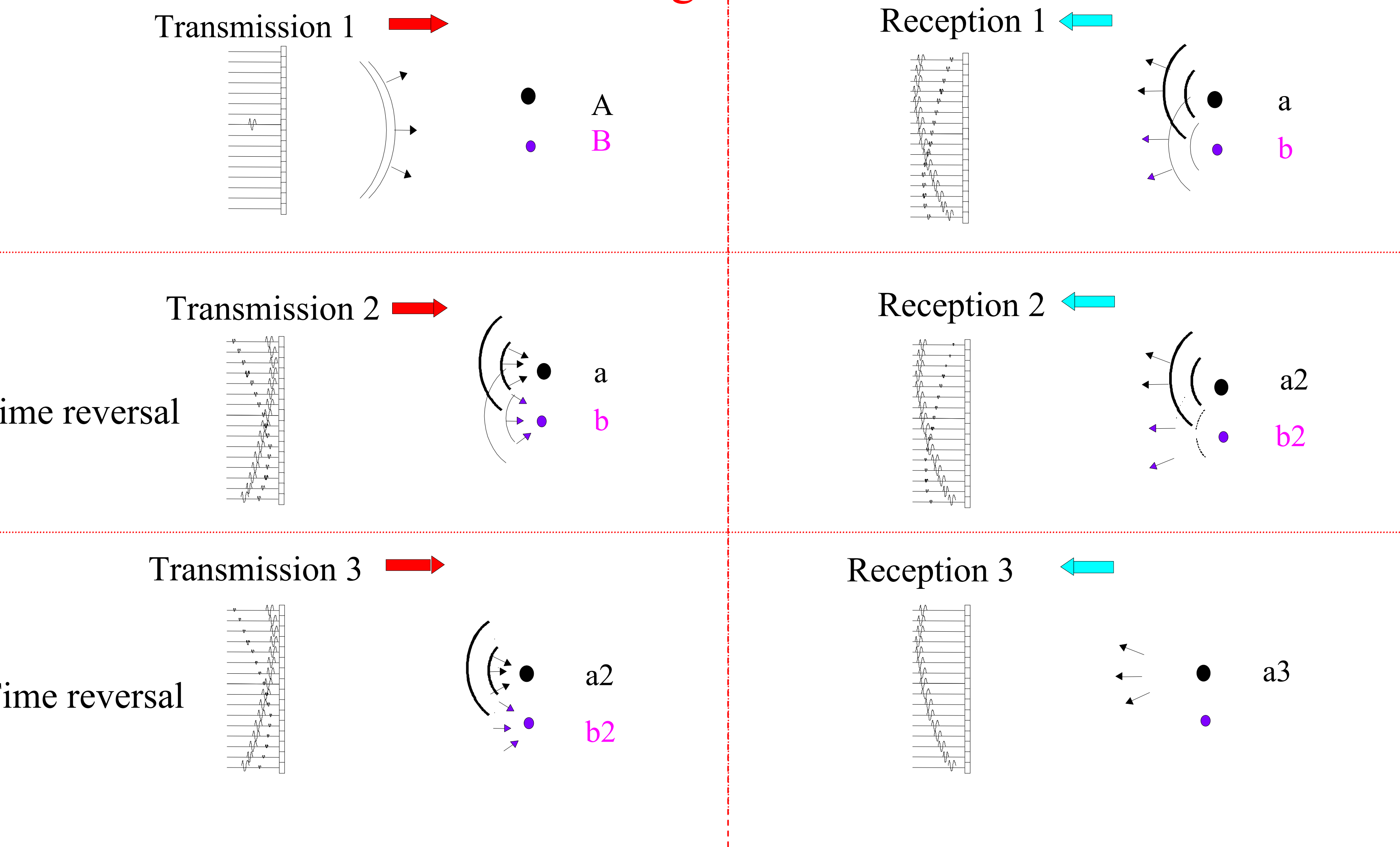
$$\boxed{\begin{array}{l} R_{12}{}^2 = T_{12}T_{21} \\ R_{12} = -R_{21} \end{array}}$$

Time Reversal in Pulse Echo mode : 1 target

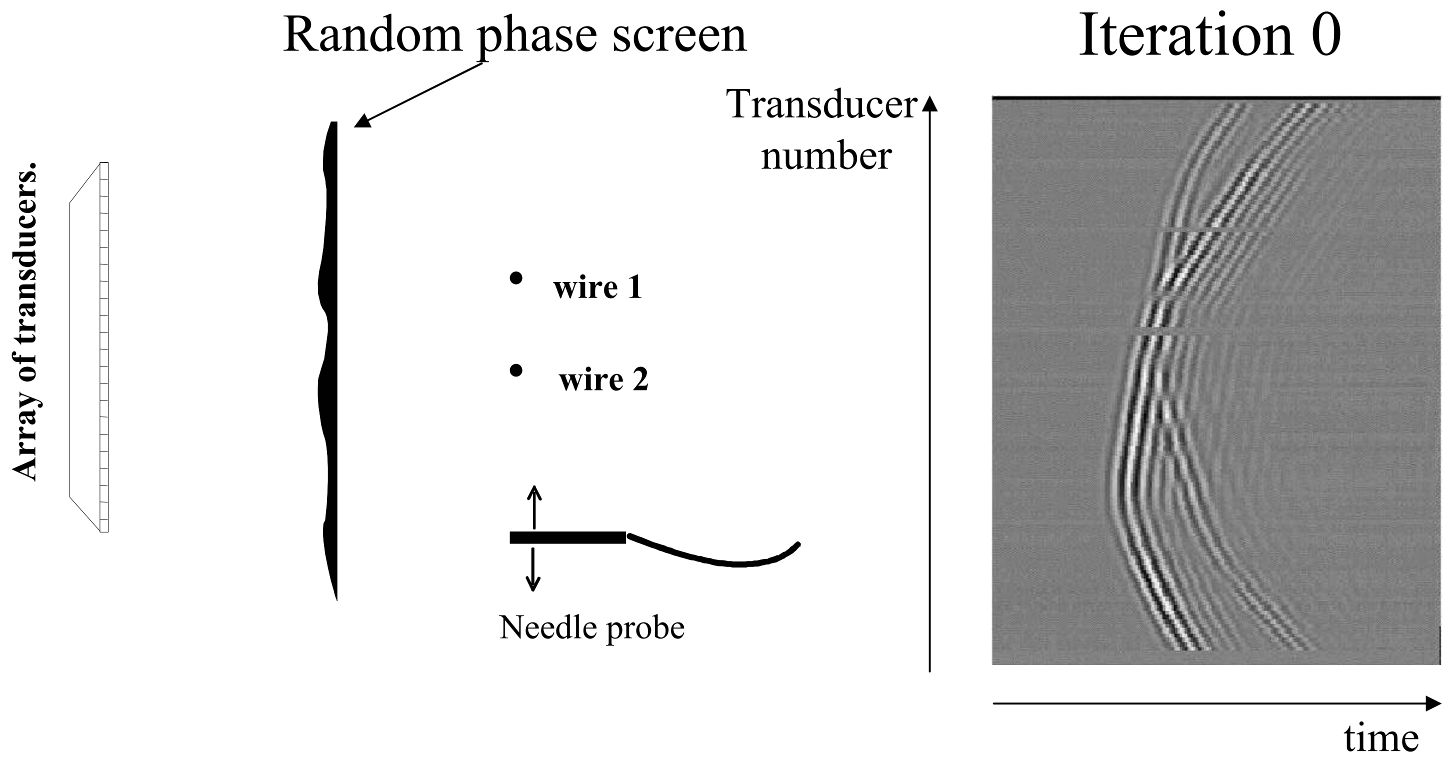


Iterative Time Reversal on multi target medium

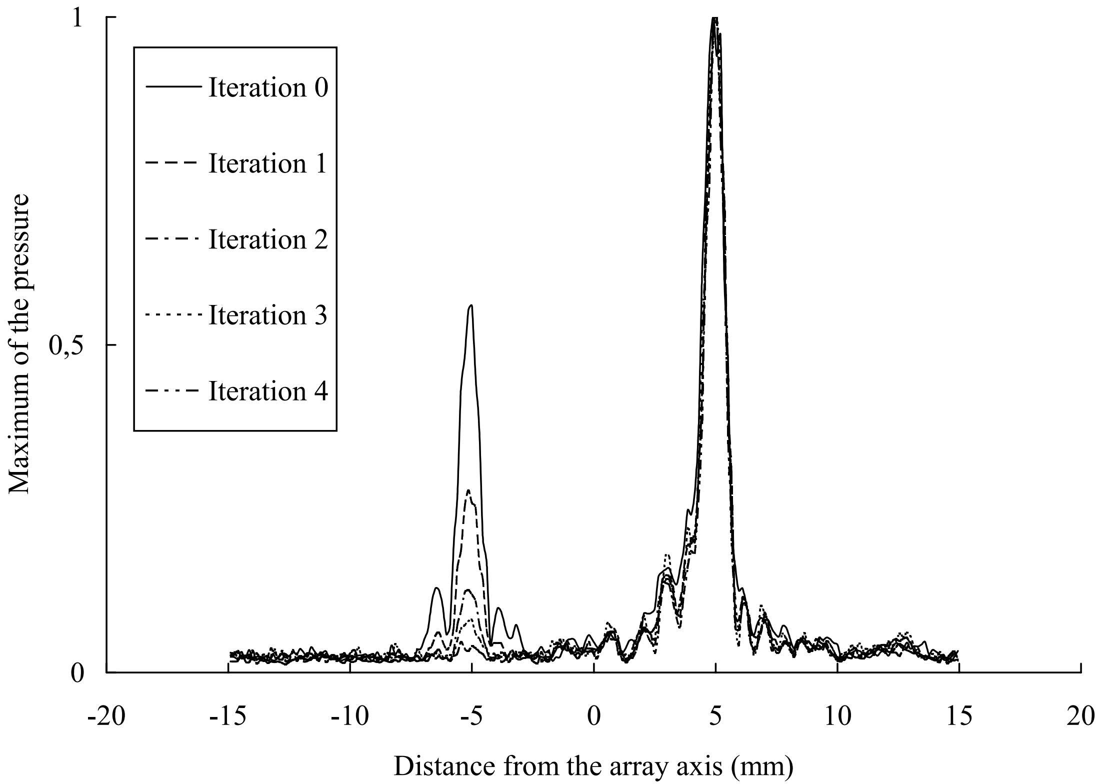
Multi target medium



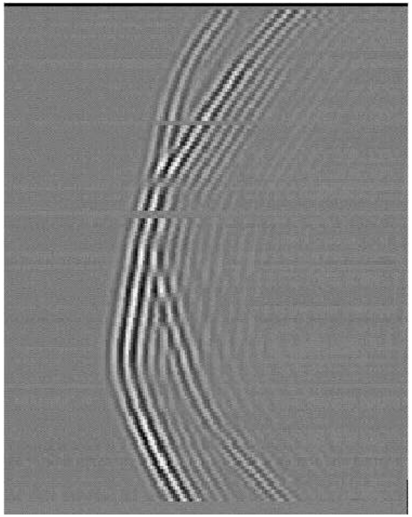
Iterative Time Reversal on multi target medium



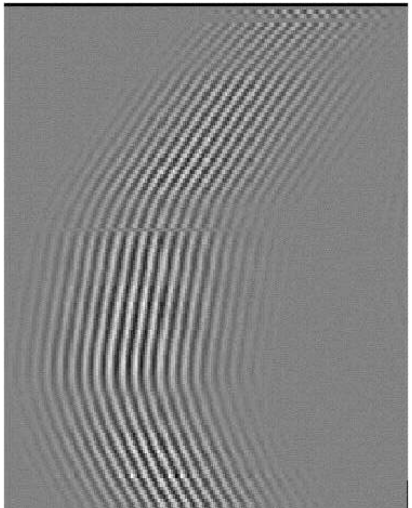
Iterative Time Reversal on multi target medium



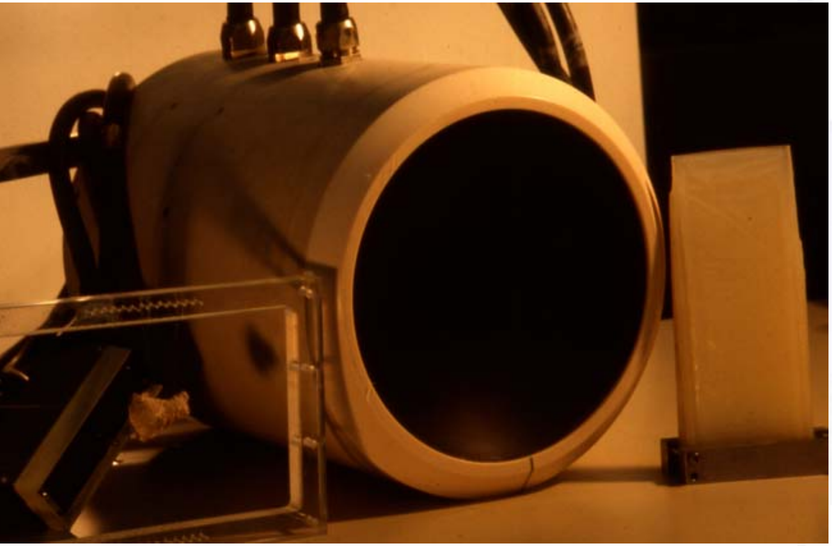
Iteration 0



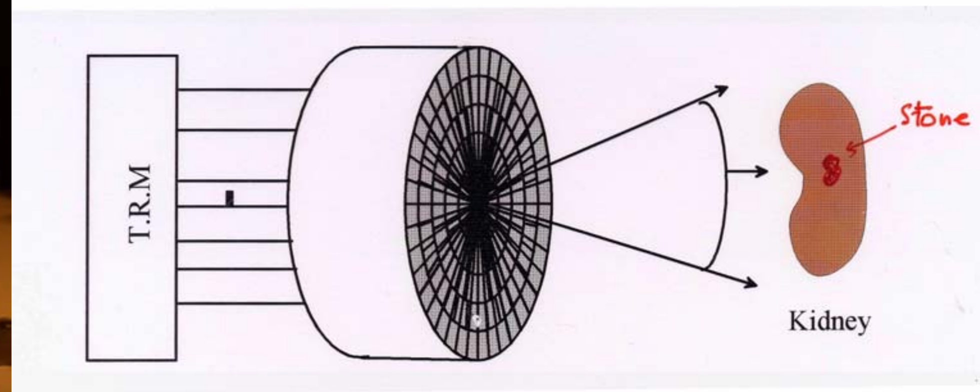
Iteration 4

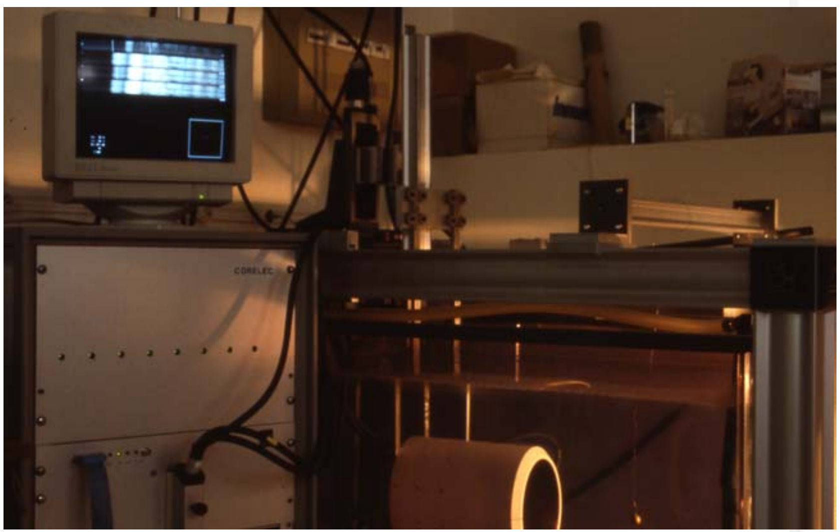


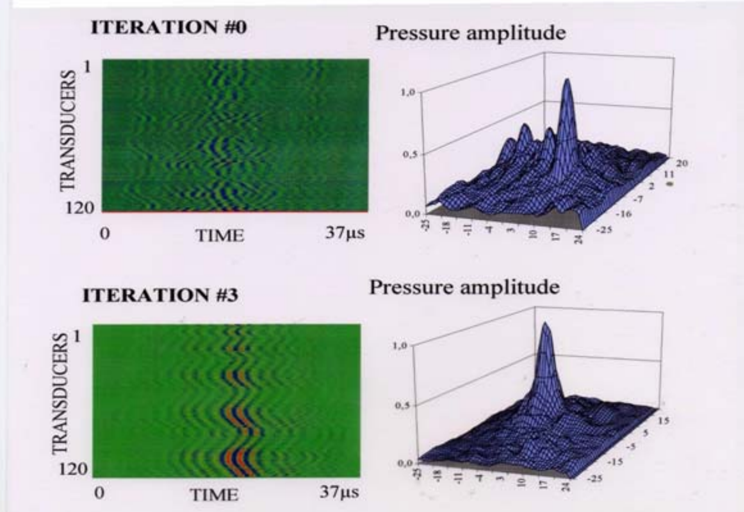
Time reversal and kidney stone destruction



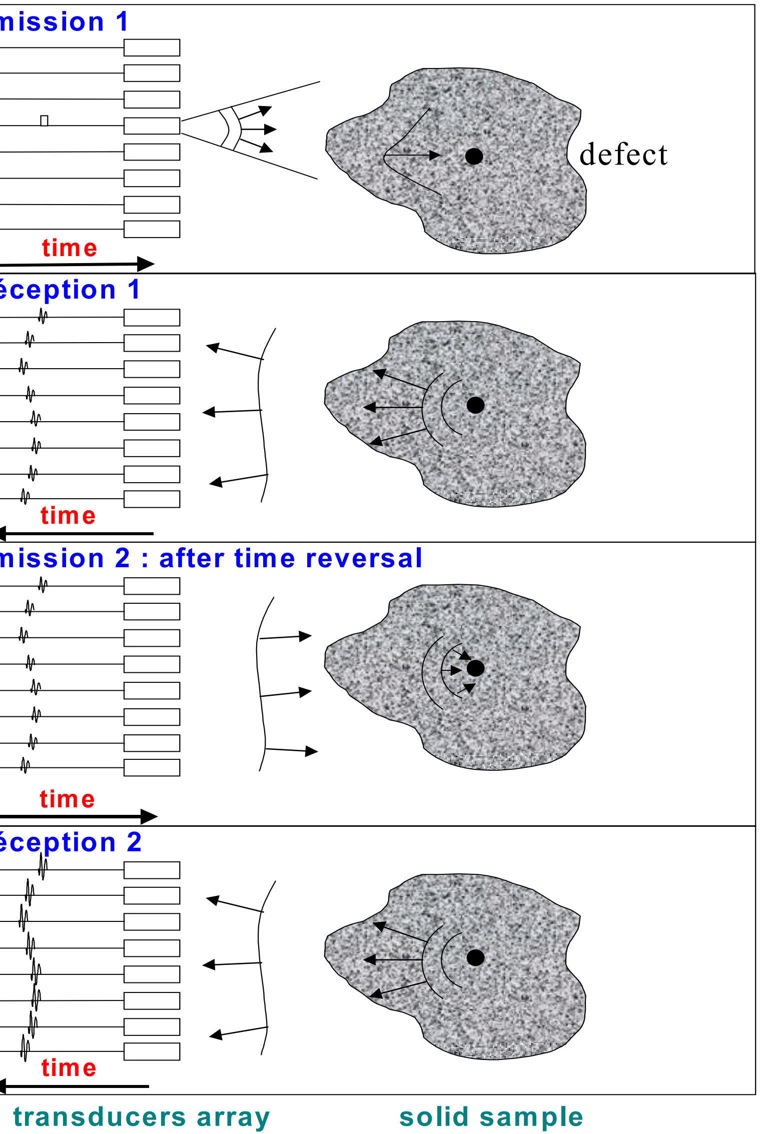
LITHOTRIPSY with TRM

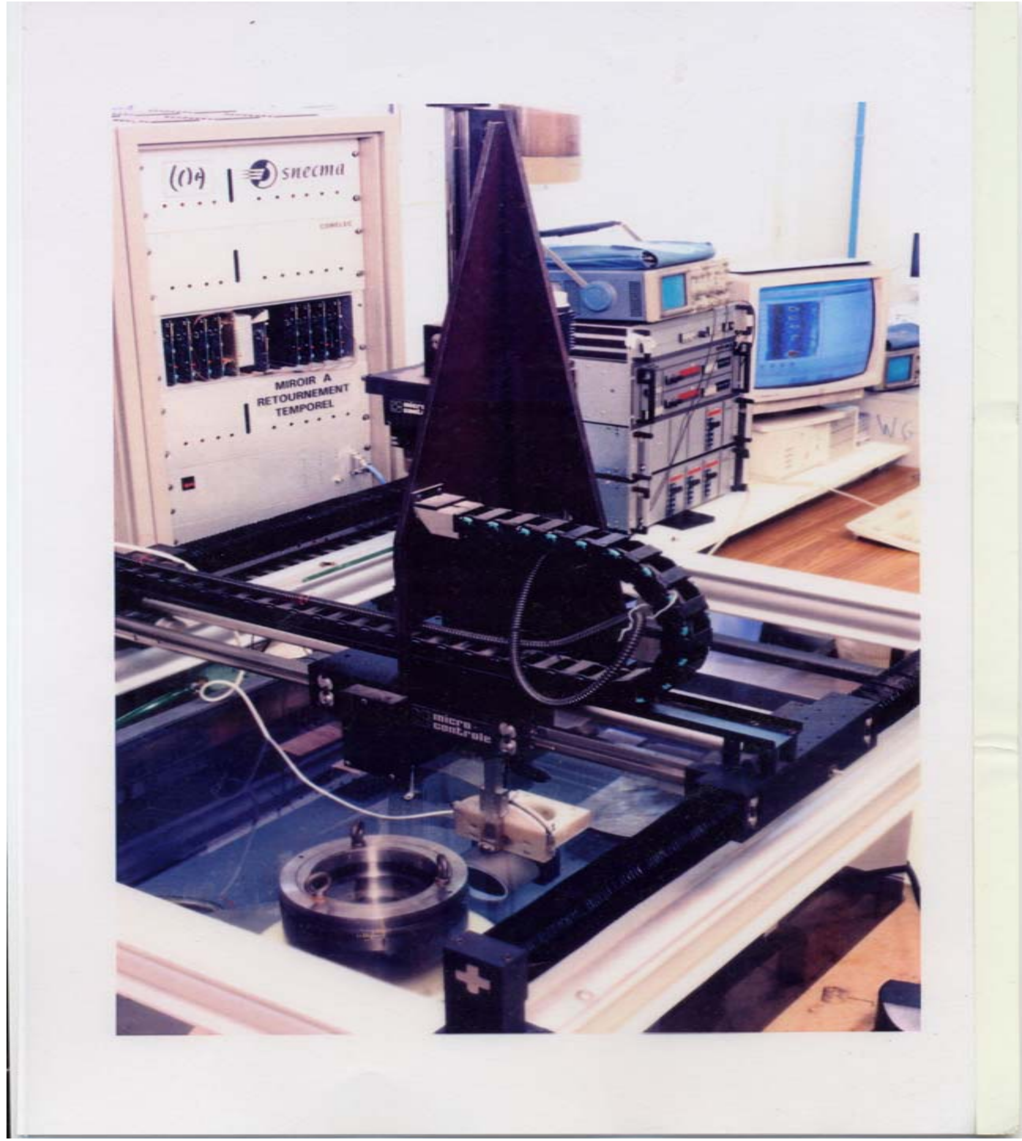






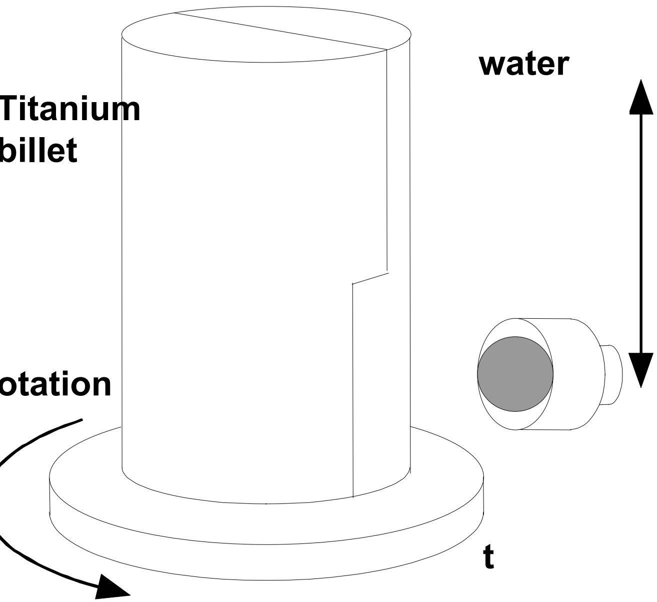
Time reversal and non-destructive testing

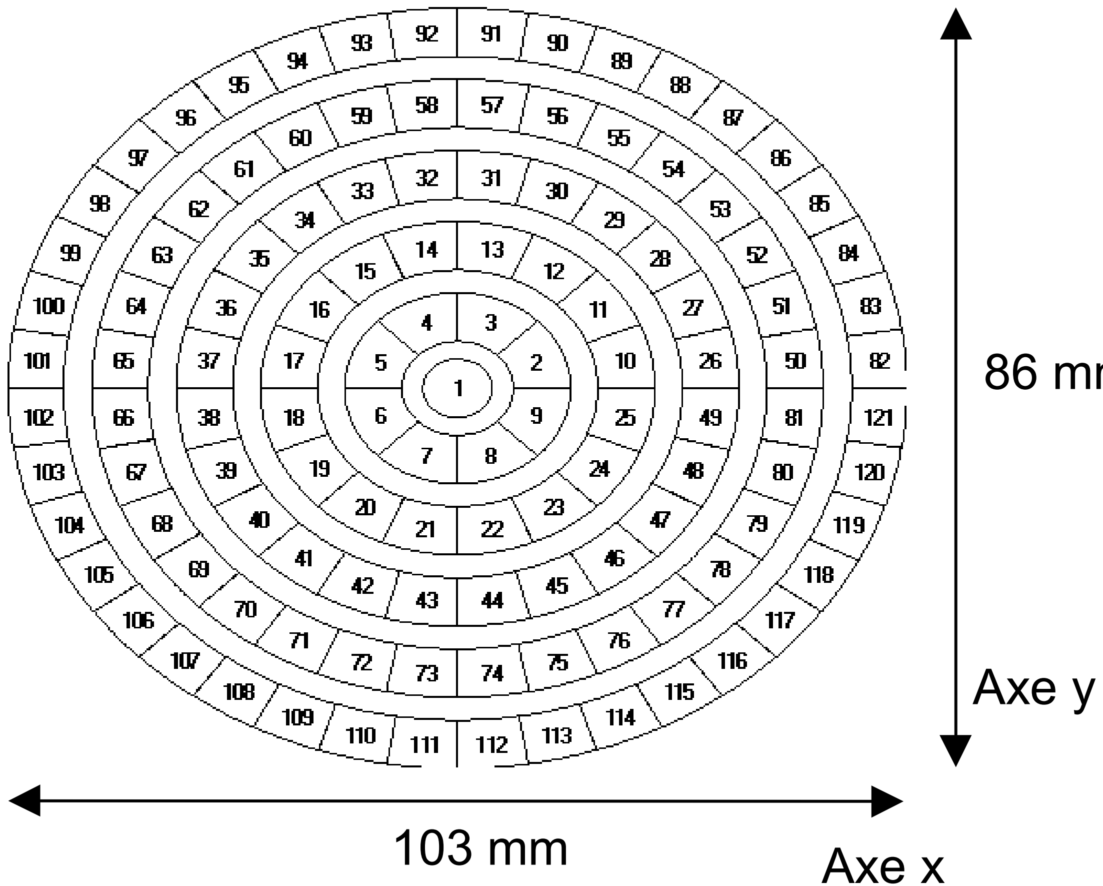




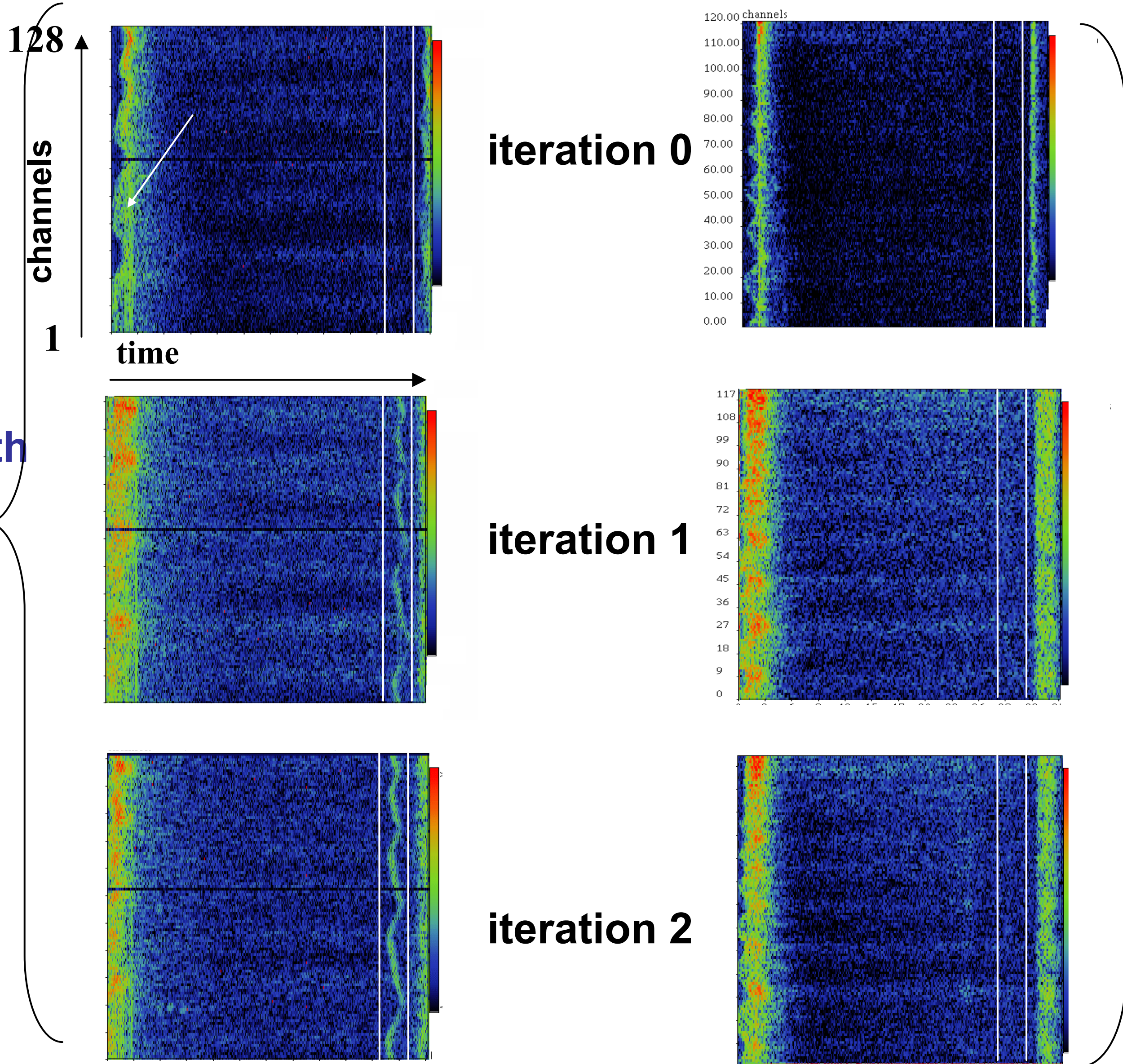
2D Time Reversal Mirror for NDT

Array of 128 transducers
5 MHz central frequency





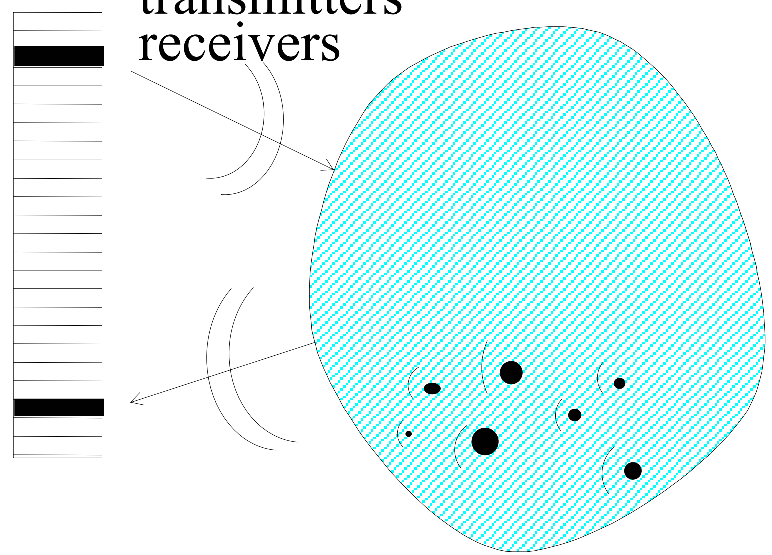
Iterative time reversal on titanium alloy



Zone with a flat bottom hole at 140mm depth

Zone without defect (speckle)

A general approach : Backscattering Operator



NxN inter element impulse responses : $k_{lm}(t)$

Transmitted signals: $e_m(t)$

Received signals:

$$r_l(t) = \sum_{m=1}^{N} k_{lm}(t) \otimes e_m(t), \quad 1 \le l \le L$$

In the frequency domain

$\mathbf{R}(\omega)=\mathbf{K}(\omega)\mathbf{E}(\omega)$

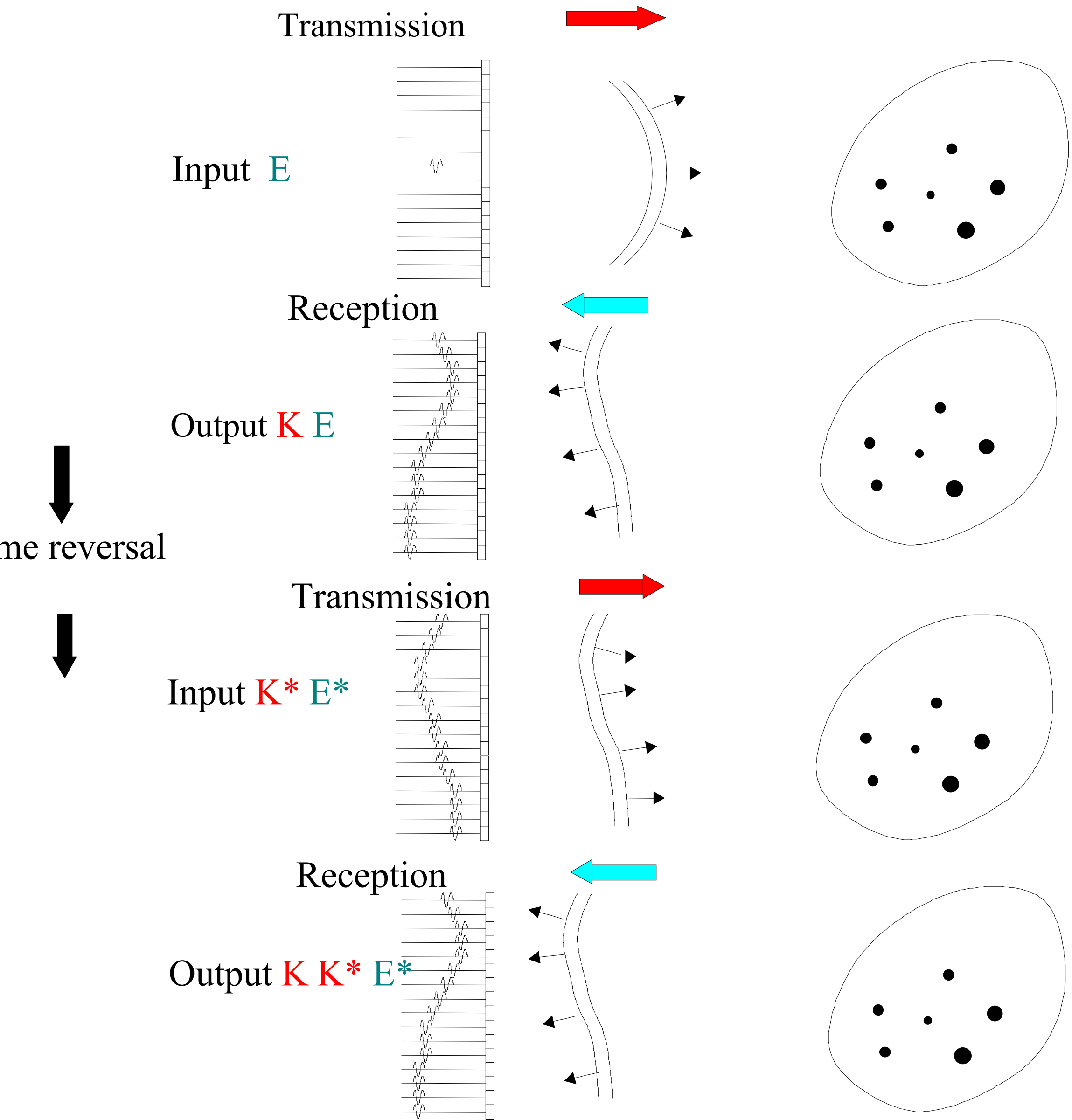
$\mathbf{E}(\omega)$ and $\mathbf{R}(\omega)$ vector signals,

$\mathbf{K}(\omega)$ is the $N \times N$ transfer matrix.

Spatial Reciprocity => $\mathbf{K}(\omega)$ is symmetrical

C. Prada, M. Fink

The Backscattering Time Reversal Operator

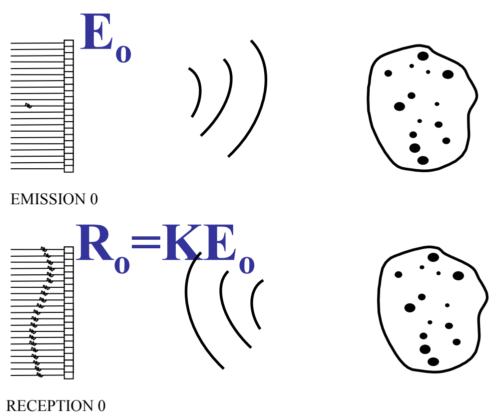


K* K

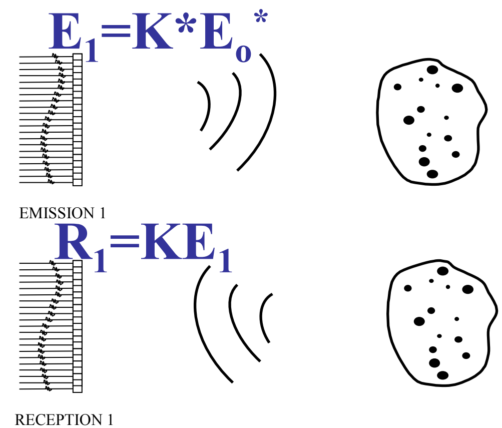
Time Reversal Operator

Iterations of the Time Reversal Operation

Iteration 0



Iteration 1



…

… Iteration 2n: $\Rightarrow E_{2n}=[K^*K]^n E_o$

Eigenvalues : **depend on target reflectivities**

Eigenvectors : **waveforms transmitted by the array to focus on each target**

D.O.R.T
Decomposition of the Time Reversal Operator

Diagonalization of $K^{*}K$

Hermitian
Positive eigenvalues

$$K^{*}K = V^{-1}\,\Delta\,V$$

Eigenvectors of $K^{*}K$

$\updownarrow$

Invariants of time reversal process

What are these invariants ?

1. First invariant : limit of an iterative time reversal process

$$(K^{*}K)^{n}\,E_0 \sim \lambda_1^{\,n}\,V_1$$

2. Other invariants ….. Later ..

Singular Value Decomposition of K

$$\Delta = \Lambda^2 \,, \qquad \mathbf{K} = \mathbf{U} \ \Lambda \ \mathbf{V}$$

Λ real diagonal matrix of singular values
U and **V** are unitary matrices

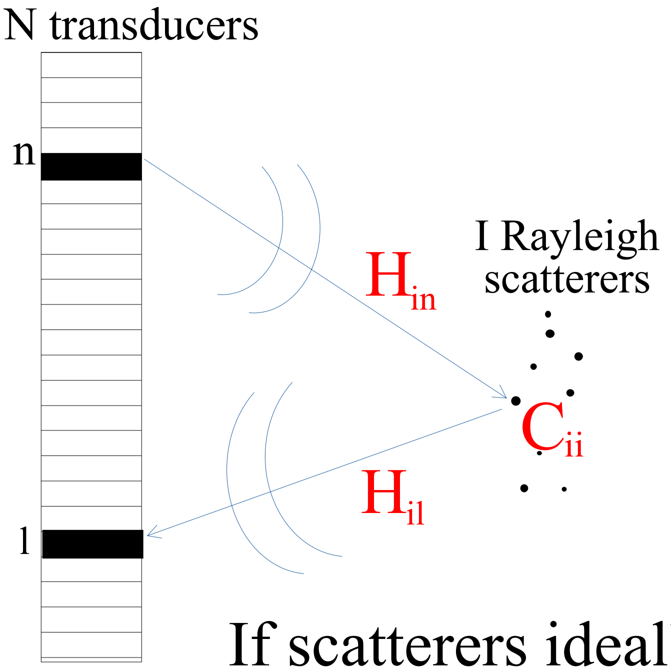
eigenvalues of K^*K <=> squares of the singular values of K
eigenvectors of K^*K <=> columns of V

Remark : You transmit a singular vector V_i
$\Rightarrow$ You receive $\mathbf{K}\ V_i = \lambda_i V_i{}^*$
The scattering medium acts as a time reversal mirror on the singular vectors

D.O.R.T method in practice:

- measure the inter element impulse responses $k_{lm}(t)$
- calculated the SVD of $K(\omega)$ at chosen frequencies
- analysis of singular values and vectors

Singular Value Decomposition of **K** for scatterers



$$\mathbf{K} = {}^t\mathbf{H}\ \ \mathbf{C}\ \ \mathbf{H}$$

Each line H_l of **H** focuses on scatterers i

C diagonal in single scattering process

$$\mathbf{SVD} \Rightarrow \mathbf{K} = {}^t\mathbf{V}\ \ \mathbf{S}\ \ \mathbf{V}$$

If scatterers ideally resolved $\Rightarrow$, the lines H_i are orthogonal : Focal spots do not overlap

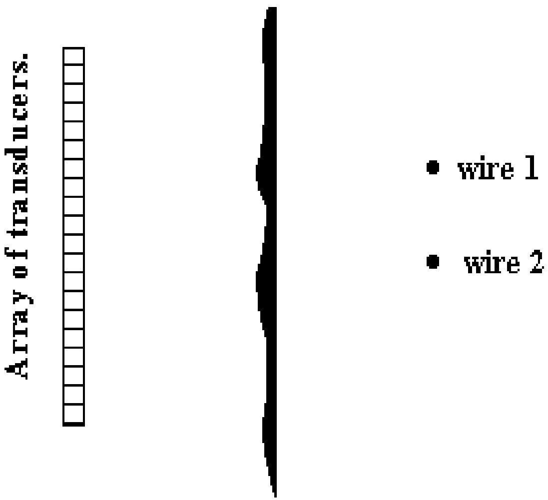
Unicity of SVD $\Rightarrow$ **V** $\rightarrow$ **H** and **S** $\rightarrow$ **C**

One scatterer $\Leftrightarrow$ one singular vector

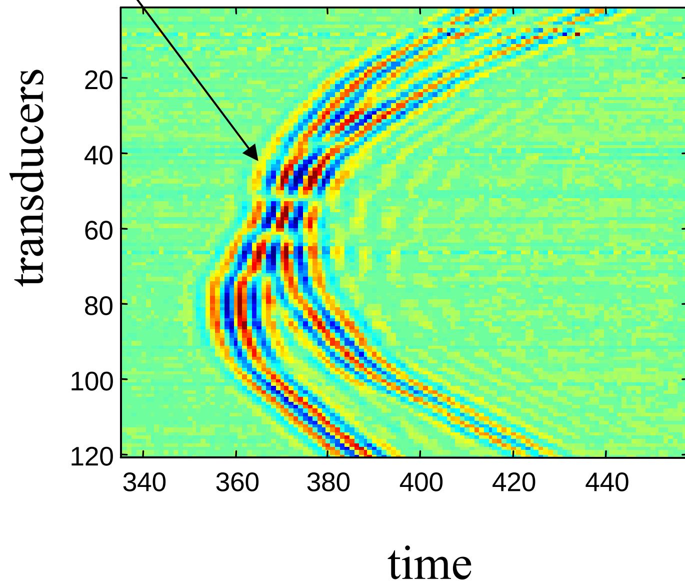
Remark : It is not true in general

D. Chambers JASA 109 (6) (2001)
Time reversal for a single spherical scatterer

Decomposition of **K** for 2 scatterers



Echoes of the scatterers after a pulse is emitted by transducer 64



2 eigenvalues values

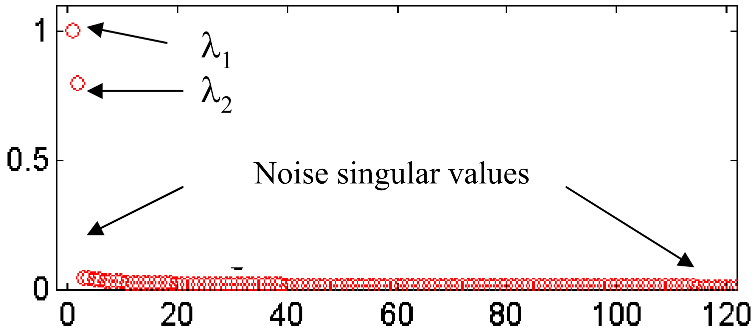
$$\lambda_i = C_i^2 \left\{ \sum_{n=1}^{N} \left| H_{in} \right|^2 \right\}^2 \quad i = 1,2.$$

2 eigenvectors vectors

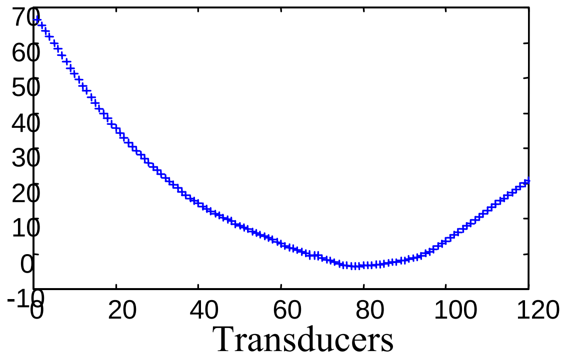
$$V_i \approx H_i^* \quad \text{pour} \quad i = 1,2\ .$$

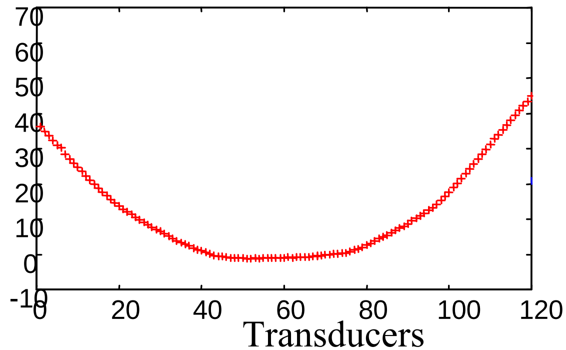
Result of the decomposition at 3.5 Mhz

Eigenvalues

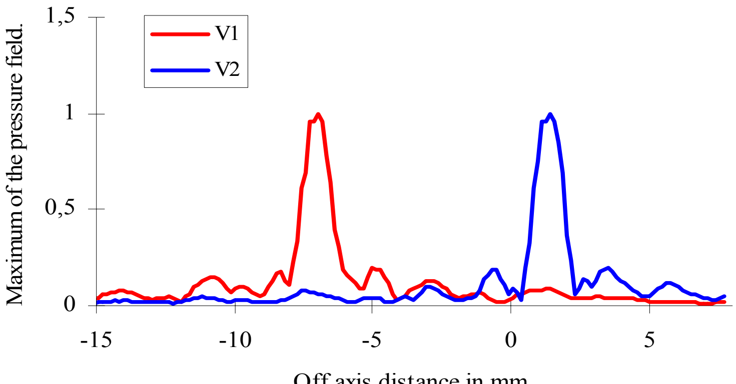


Phase of the first eigenvector and of the second eigenvector

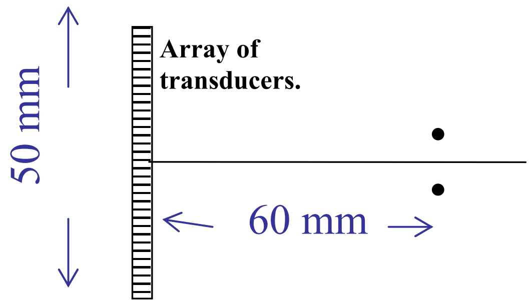




Transmission of the eigenvectors



2 Symmetrical Scatterers ?



Experiment

$\lambda \sim 1$ mm
Copper wires $\varnothing$ 0.1 mm

Eigenvectors

H_1 response from scatterer 1 to the array

$$V_+ = H_1 + H_2 \quad \text{and} \quad V_- = H_1 - H_2$$

H_2 response from scatterer 2 to the array

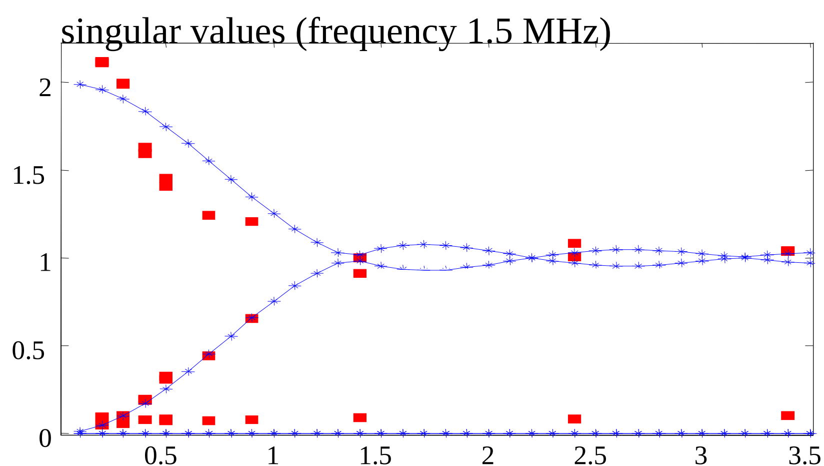
Singular values

$$\lambda_+ = \|H_1\|^2 + \langle H_1 | H_2 \rangle \quad \text{and} \quad \lambda_- = \|H_1\|^2 - \langle H_1 | H_2 \rangle$$

When the wires become closer

$$\lambda_+ \to 2\|H_1\|^2$$

$$\lambda_- \to 0$$



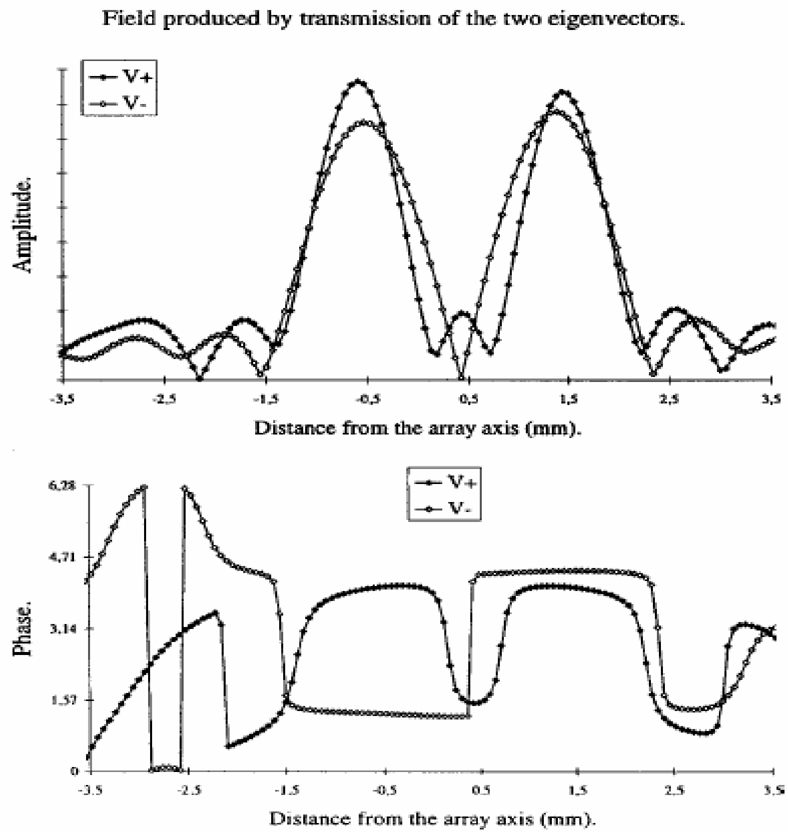
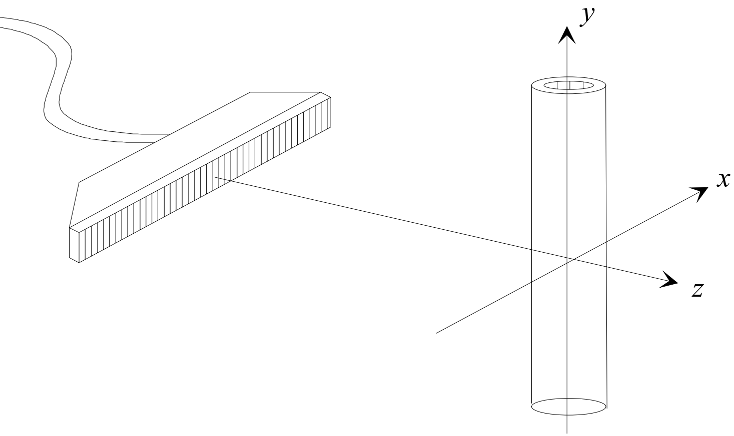
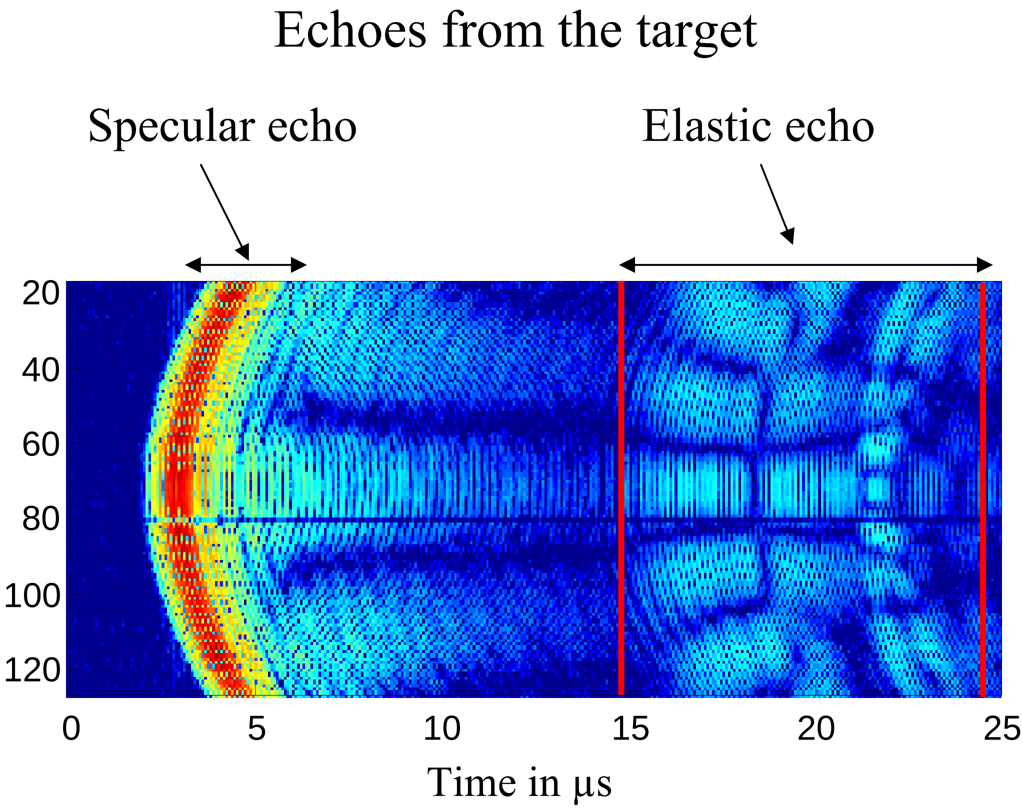


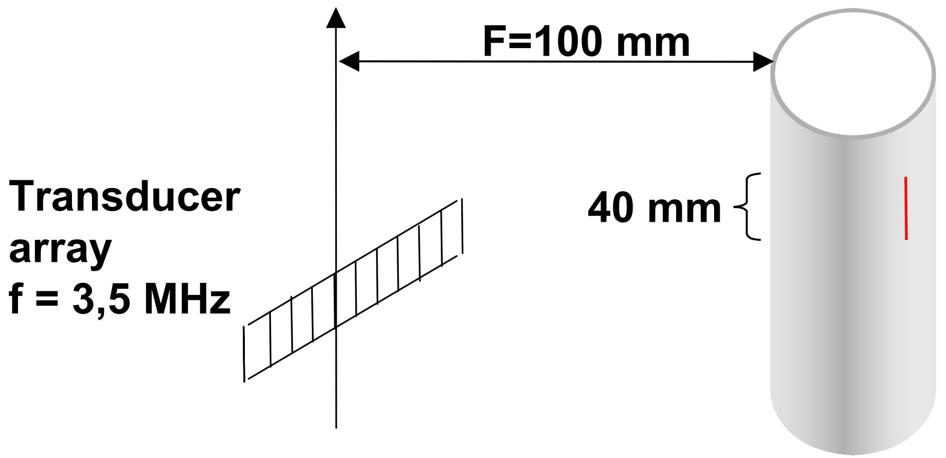
FIG. 3. Amplitude and phase of the field produced by transmission of the two eigenvectors.

How to put an extended target in resonance with a TRM ?

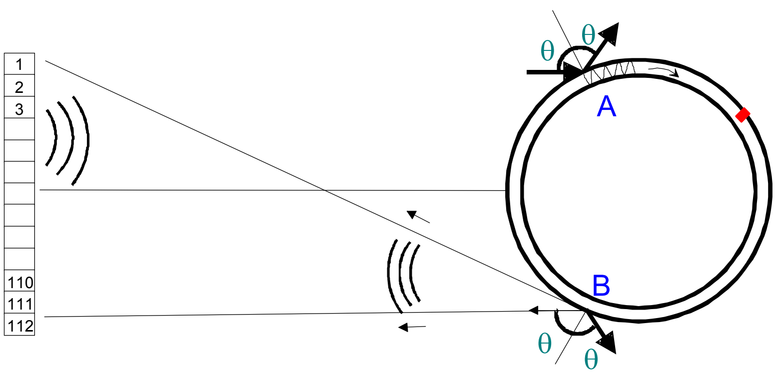




Origin of the Elastic Echo : Lamb waves



$$\sin(\theta)=\frac{c_0}{v_\varphi}$$

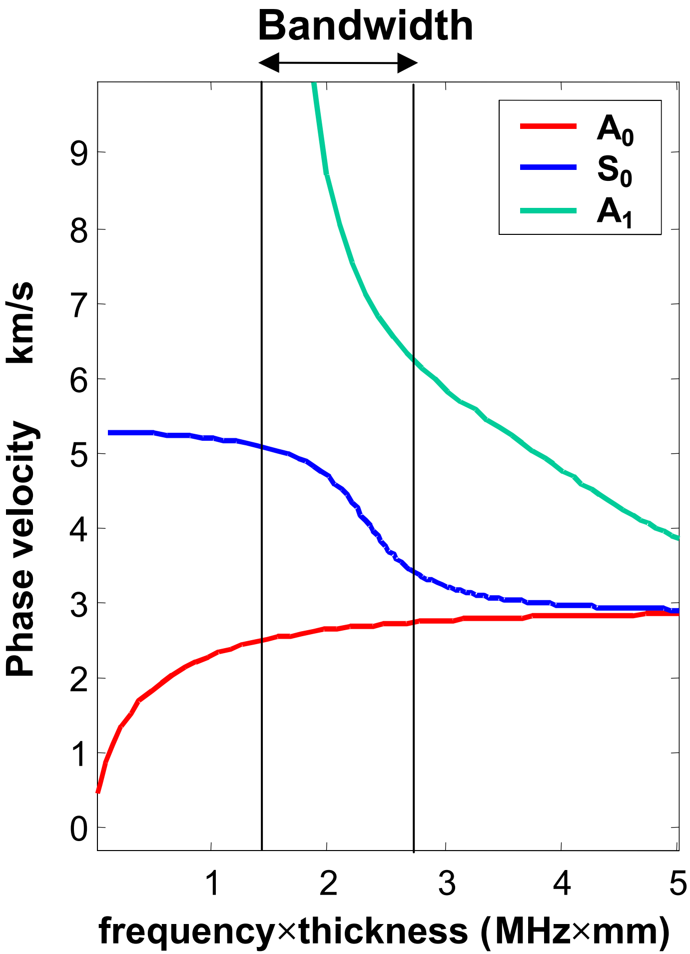


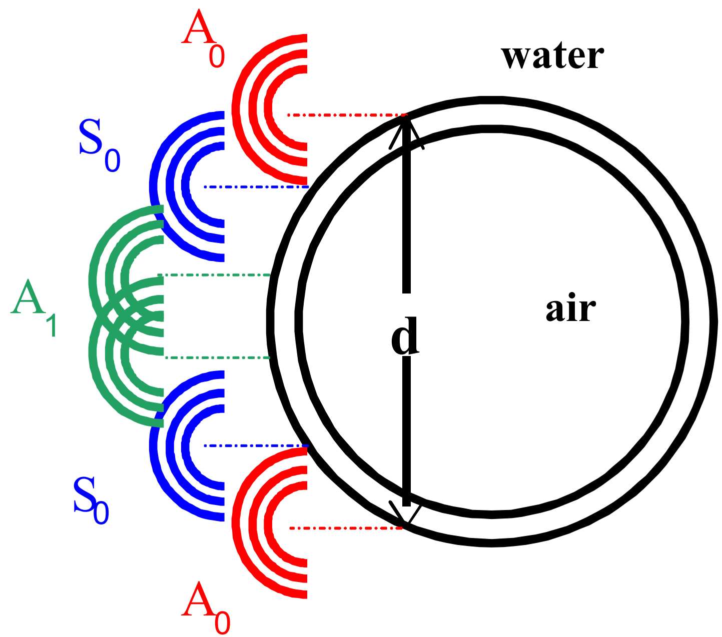
tube :
diameter : D = 20 mm
thickness e = 0,54 mm

A and B : 2 source point generating lamb waves

θ : incident angle
c_0 : sound speed in water
v_ϕ : phase velocity of the generated Lamb wave

Dispersion curve for Lamb Waves





$$d = \frac{Dc_0}{v_\varphi}$$

d : distance between 2 sources

D : tube diameter

Lamb Waves as TR invariants

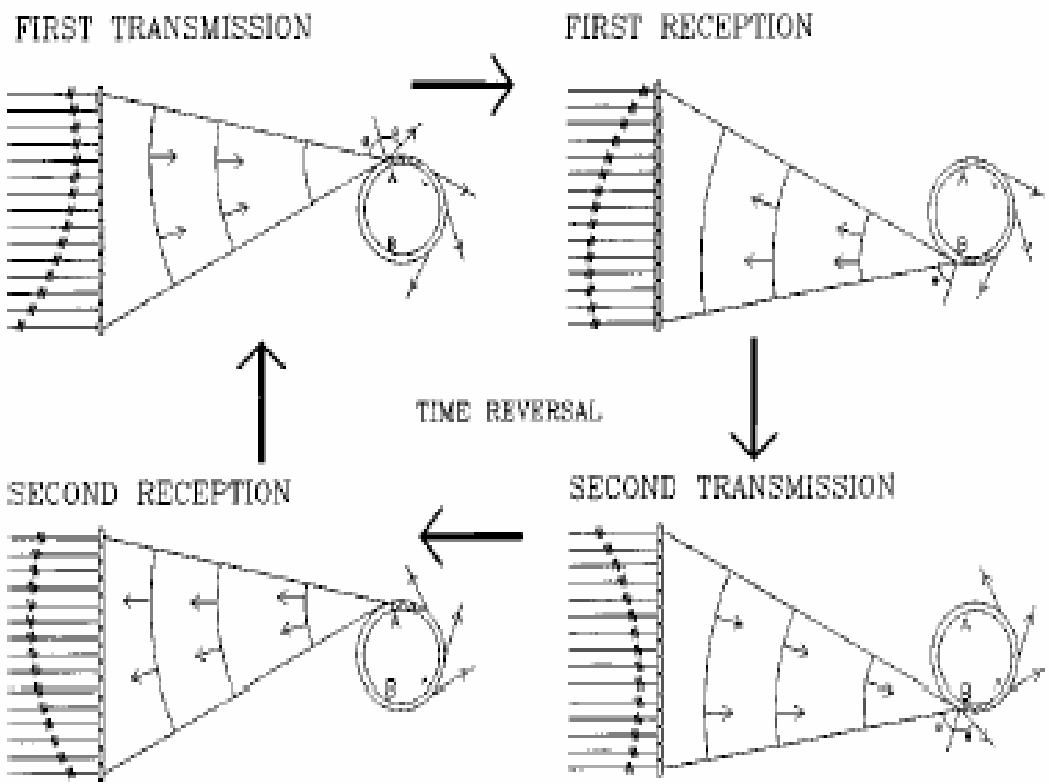
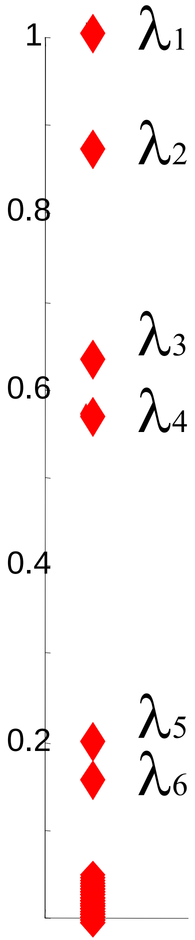


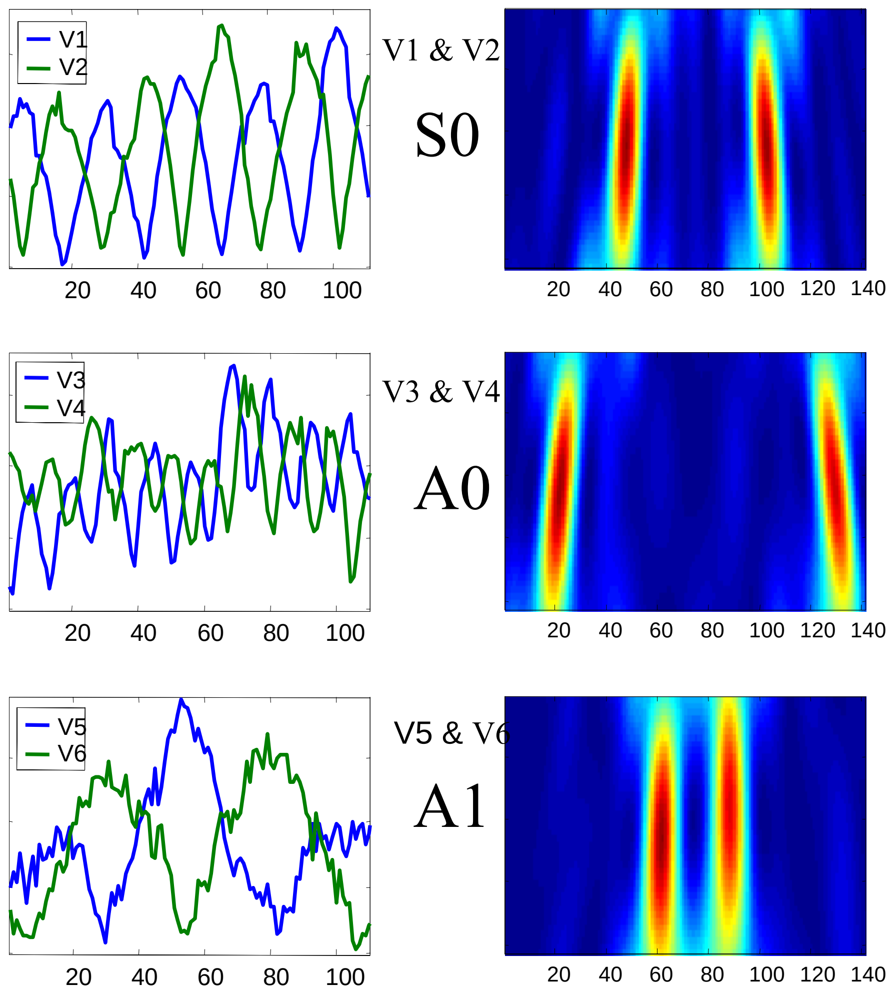
FIG. 5. Lamb waves are invariants of the time-reversal process: A wave focused on point A generates a Lamb wave which radiates towards the array from point B. After two successive time-reversal processes of this Lamb wave, the transmitted wave is similar to the first one, consequently this wave is associated to an invariant of the time-reversal process.

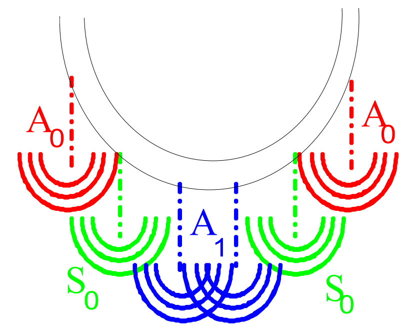
Six main eigenvectors and backpropagation

Singular values
at f = 3.1 MHz



Moduli of
the eigenvectors





For high phase velocity : the distance between radiation points is small so that high resolution method is required.

Time reversal on an extended target

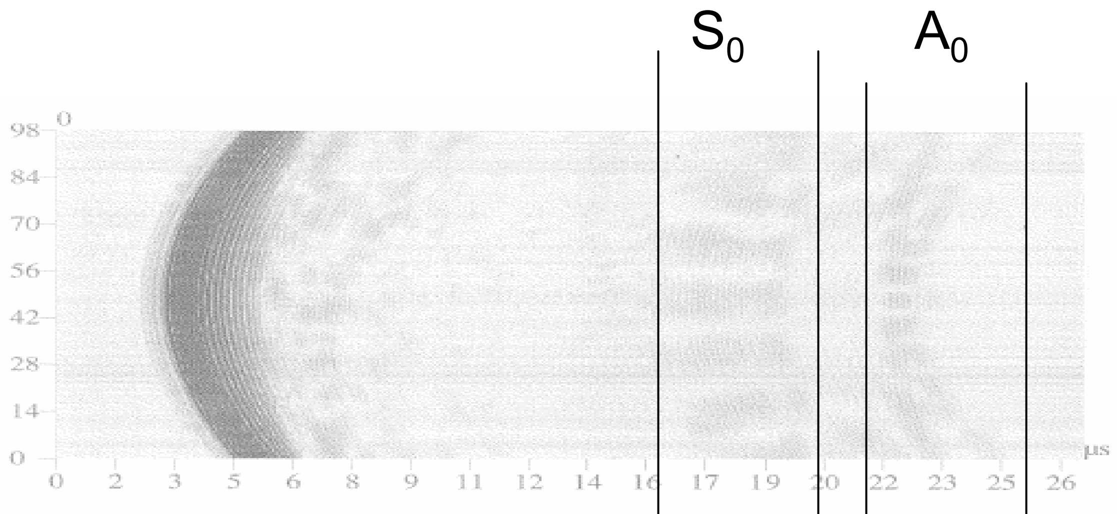


Figure 27. Echo of the shell received by the 128 transducers after transmission of a short pulse by the centre element of the array. The first wavefront is the specular echo, the second is the contribution of the S_0 Lamb wave and the third the contribution of the A_0 Lamb wave.

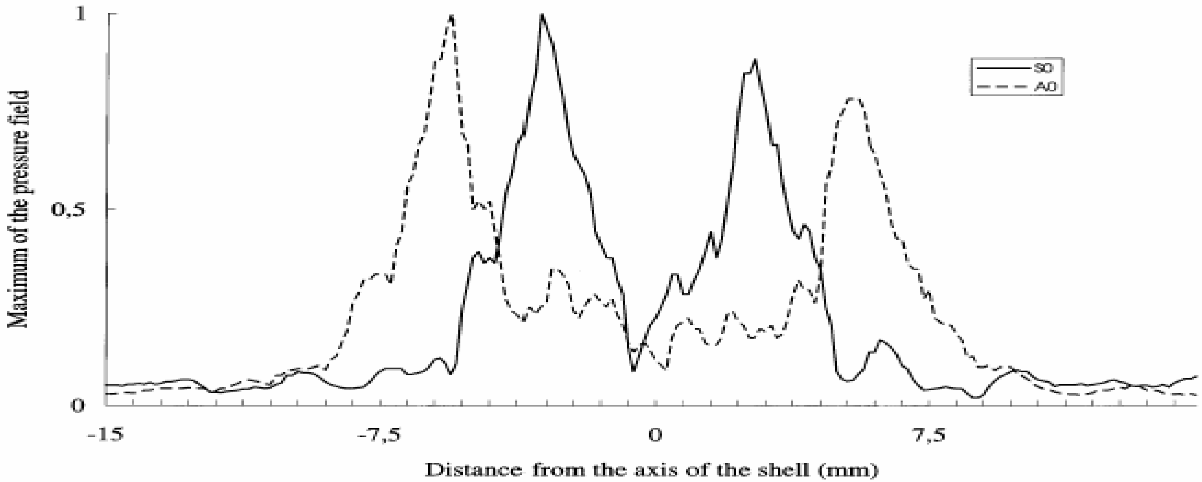
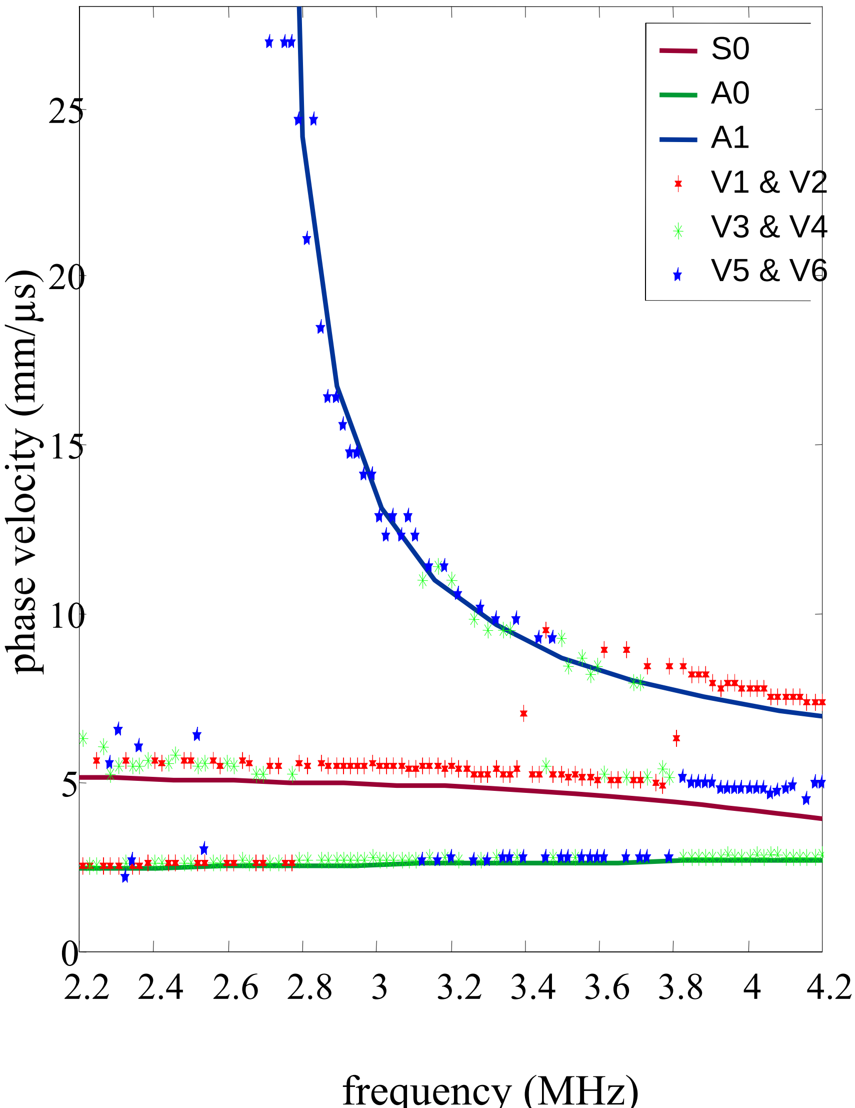


Figure 28. Directivity pattern measured in the plane of the shell after TR of the echo corresponding to the A_0, S_0 Lamb waves.

Phase velocity deduced from the eigenvectors

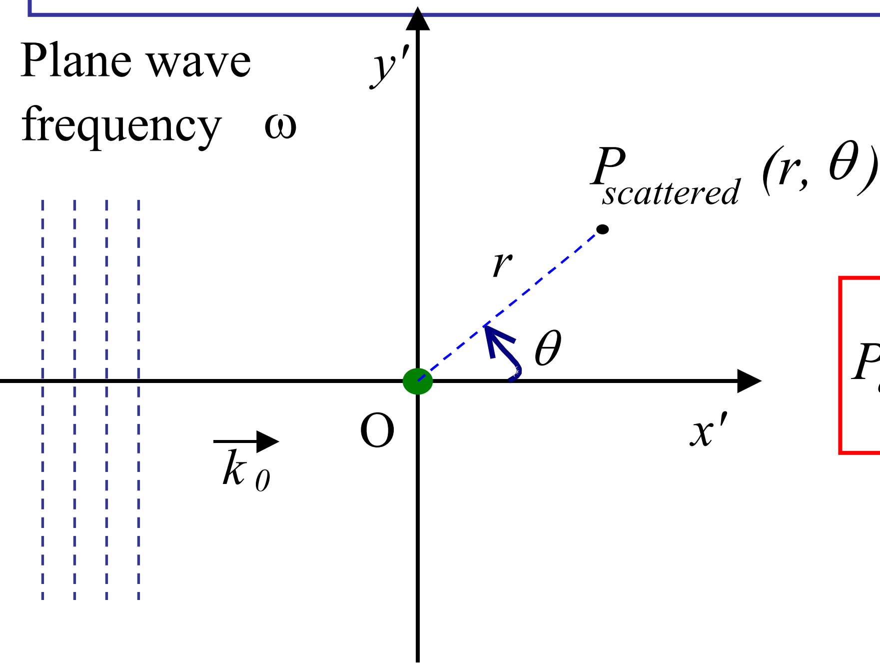


For each frequency, the Measurement of d gives the corresponding phase velocity

The cut-off frequency of A_1 provides the thickness of the shell

Partial Wave Decomposition of an extended target

Cylindrical target



cylindrical wave

$\varepsilon_n = 0$, 1 or 2

$$P_{diff}(r,\theta) = p_0 \sqrt{\frac{2}{i\pi k_0 r}}\, e^{ik_0 r} \sum_{n=0}^{\infty} \varepsilon_n R_n \cos(n\theta)$$

$\varepsilon_n R_n$: mode « weight »

mode radial distribution

Series of normal modes :

monopole, dipole, quadrupole ...

D. Chambers

Small cylinder limit

In acoustics, the series has **two terms** : R_0 et $2R_1$

$$\boxed{R_0 \propto \alpha = \frac{\kappa_0}{\lambda + \mu} - 1}$$

α : monopole, compressibility contrast

$$\boxed{R_1 \propto \beta = 2\frac{\rho_{fil} - \rho_0}{\rho_{fil} + \rho_0}}$$

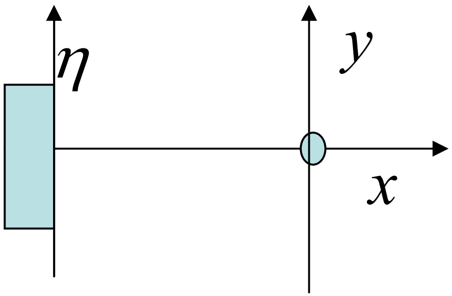
β : dipole, density contrast

Reduced TR matrix for a small cylinder :

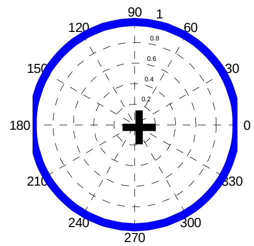
$$\boxed{K_{ij}^{réduite}(\omega) = \frac{1}{\sqrt{r_i r_j}}\left(\alpha + \beta\frac{F^2 + \eta_i \eta_j}{r_i r_j}\right)}$$

$\Rightarrow$ **SVD exact calculation**

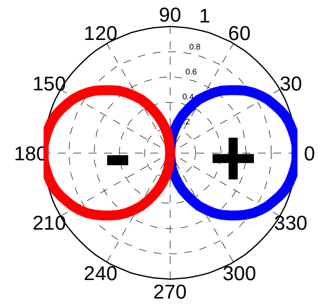
Small cylinder limit



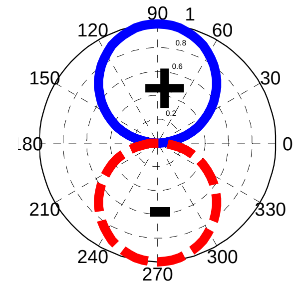
Normal modes



s



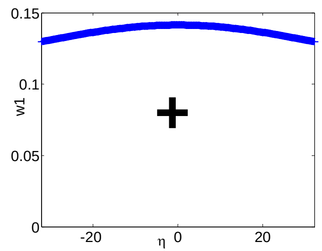
dx

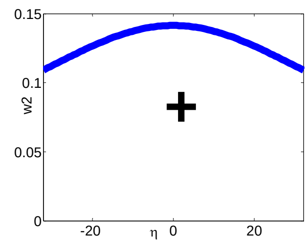


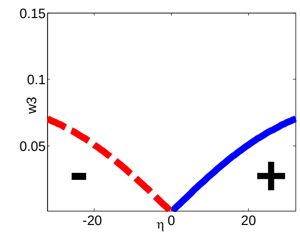
dy

Projection onto the array (non orthogonal)

⇒ expression of $K^{reduced}$







$$w_s(\eta_i) = \frac{1}{r_i^{1/2}} \qquad w_{dx}(\eta_i) = \frac{F}{r_i^{3/2}} \qquad w_{dy}(\eta_i) = \frac{\eta_i}{r_i^{3/2}}$$

Singular values and singular vectors

$$\begin{bmatrix} \alpha W_{ss} & \alpha W_{sx} & 0 \\ \beta W_{sx} & \beta W_{xx} & 0 \\ 0 & 0 & \beta W_{yy} \end{bmatrix}$$

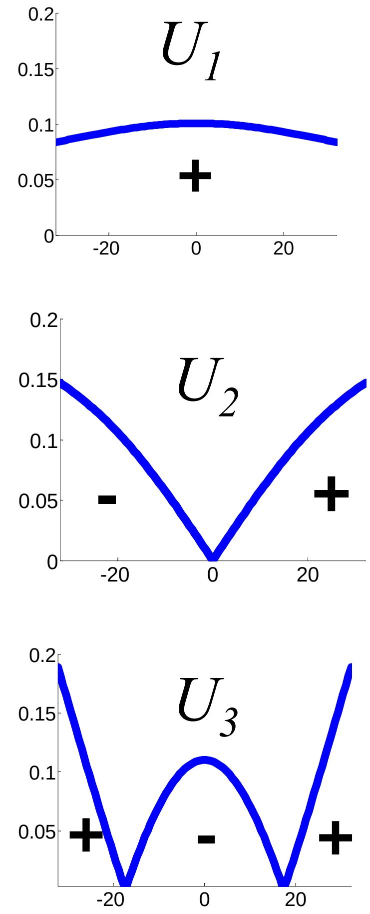
Diagonalization : 3 singular values

$$\lambda_1 \approx \alpha W_{ss} + \beta W_{xx}$$

$$\lambda_2 = \beta W_{yy}$$

$$\lambda_3 \approx \alpha\beta \frac{W_{ss} W_{xx} - W_{sx}^{\;2}}{\alpha W_{ss} + \beta W_{xx}} << \alpha\beta\lambda_1$$

and 3 singular vectors

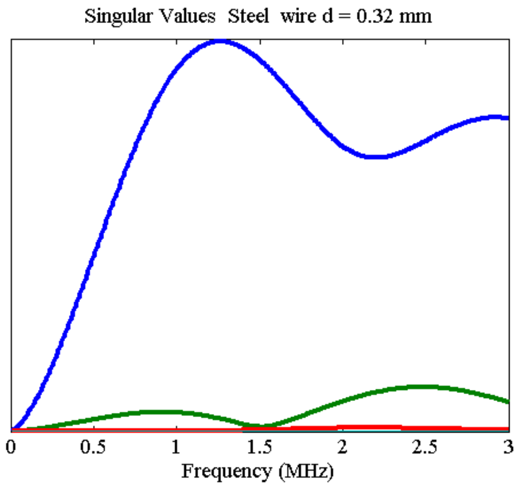


3 singular values for a unique target

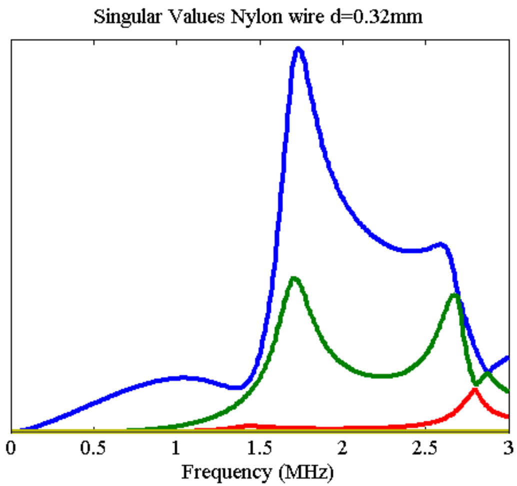
Theoretical singular values

- array aperture 65 mm
- distance array-wire 50 mm
- wire diameter = 0.32 mm

Steel :
c_L=5.75 mm/µs
c_T=3 mm/µs
ρ= 7.8



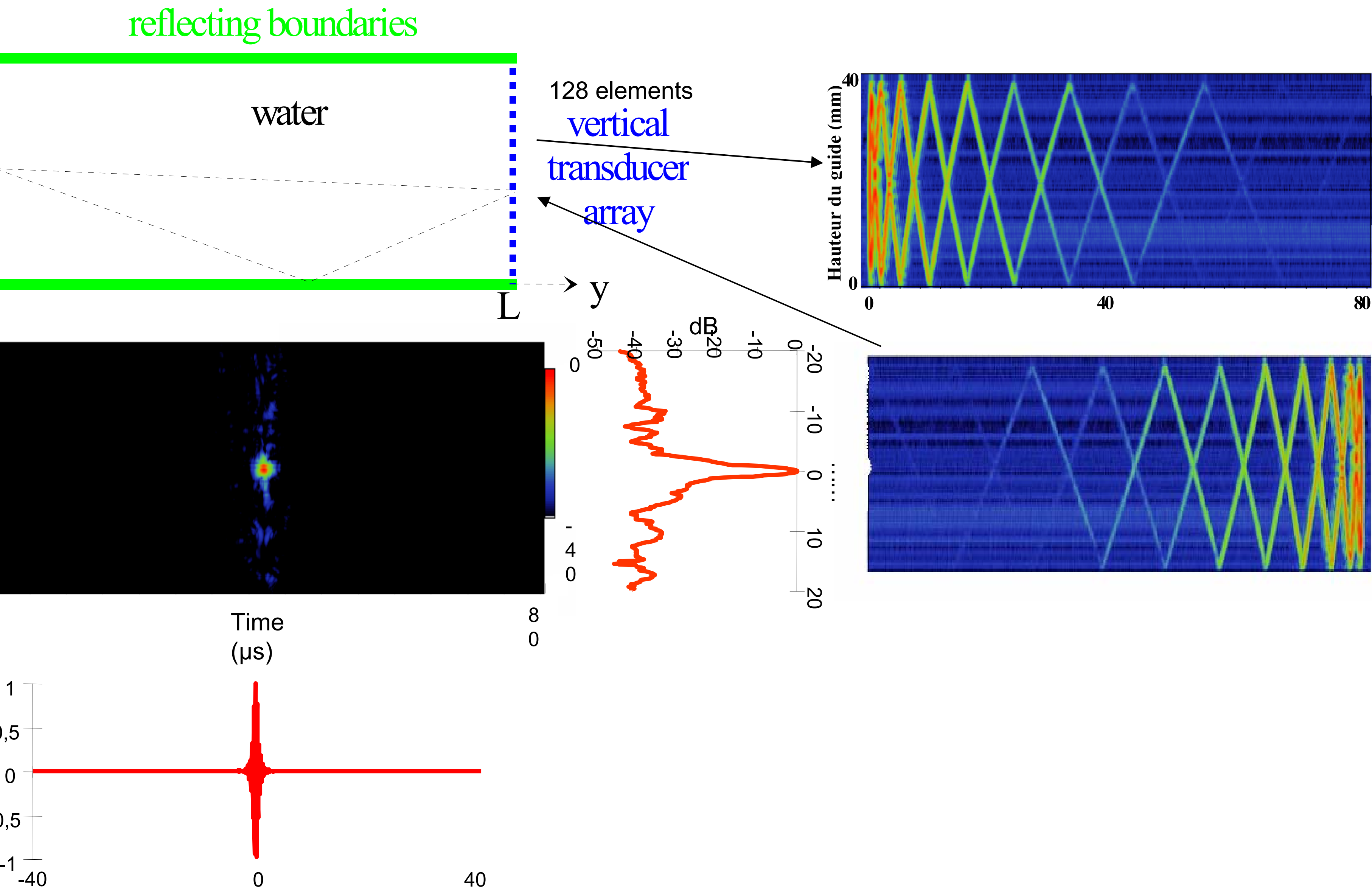
Nylon :
c_L=2.6 mm/µs
c_T=1.1 mm/µs
ρ=1.1



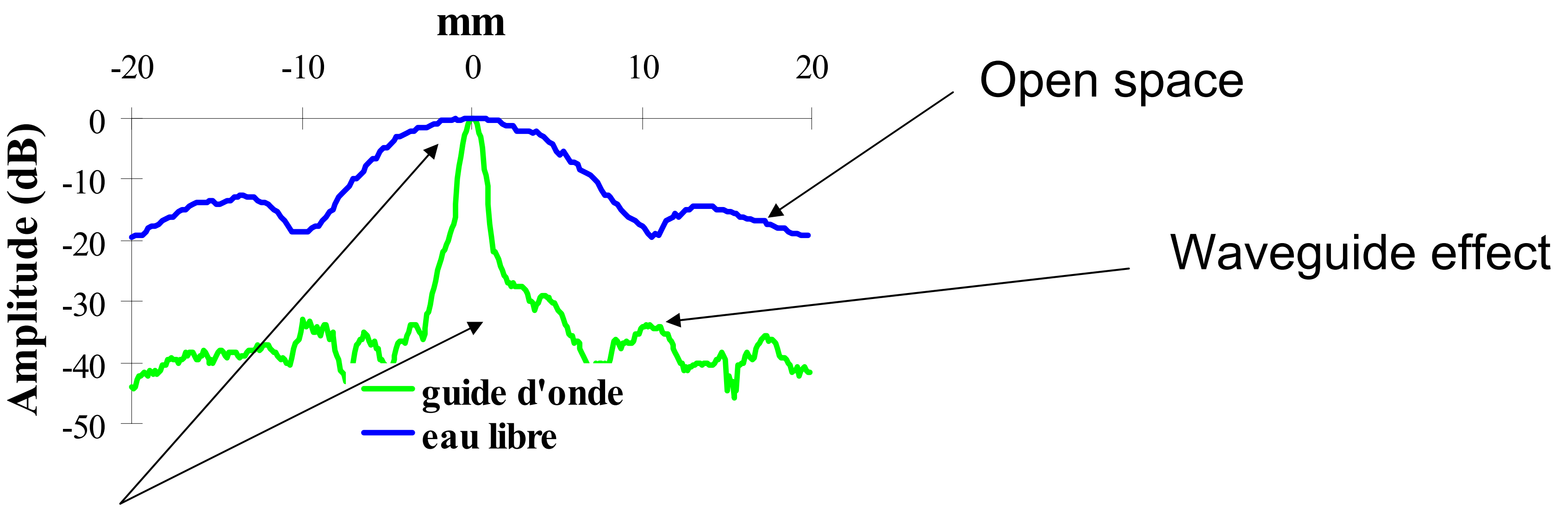
D. Chambers JASA 109 (6) (2001)

Time Reversal in a Waveguide

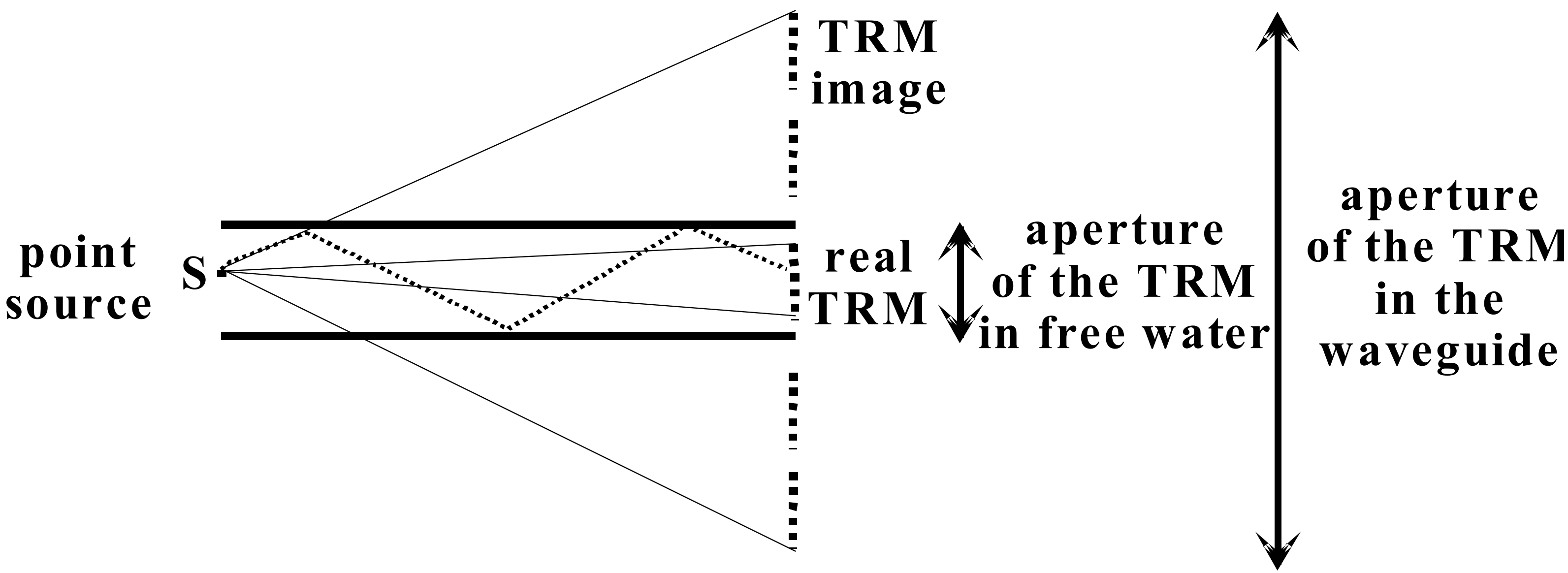
P. Roux, M. Fink



The Kaleidoscopic Effect : Virtual Transducers

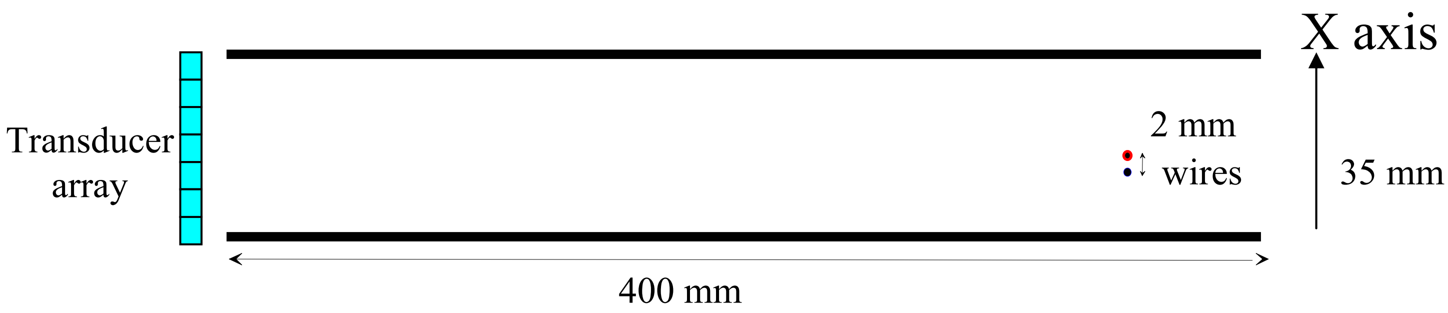


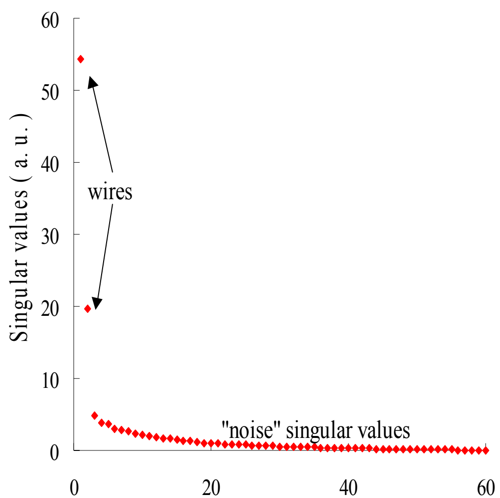
A comparison between the focal spot with and without the waveguide



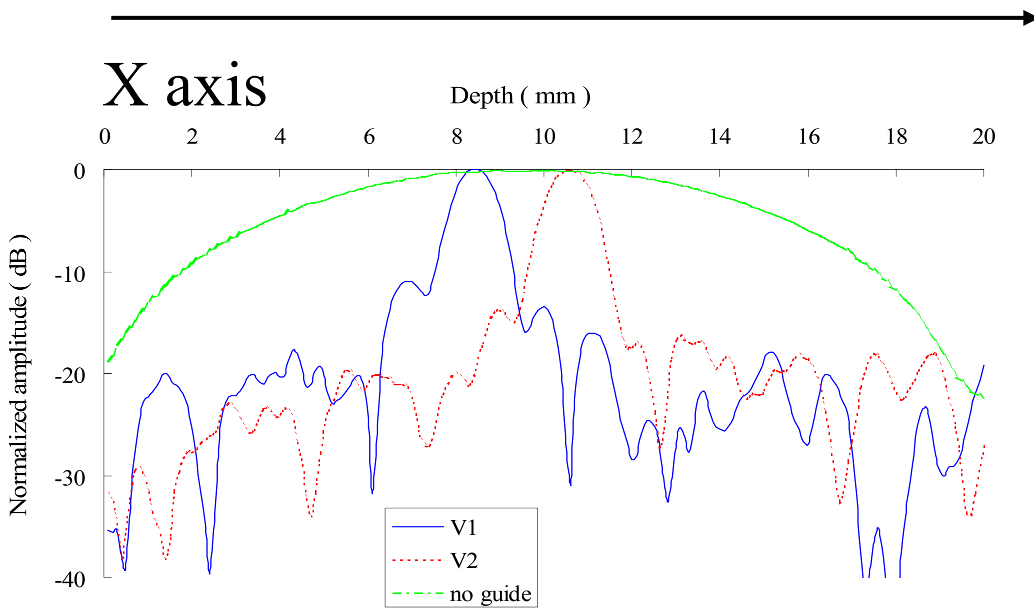
the pitch is to large : grating lobes

D.O.R.T in a waveguide



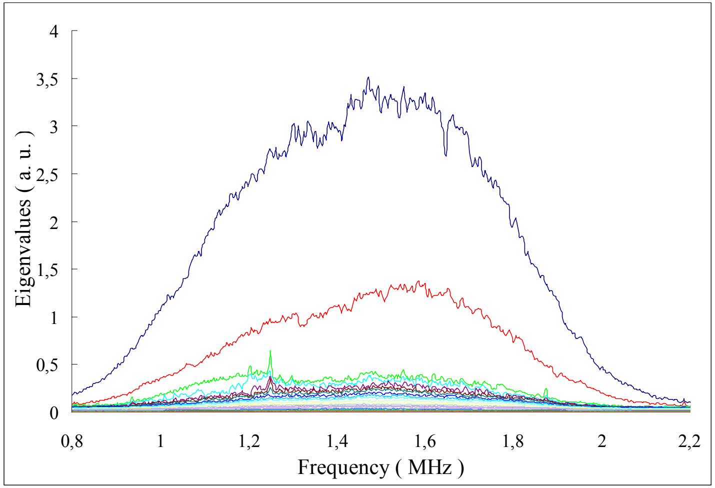


Diagonalization of the TRO at 1.5 MHz

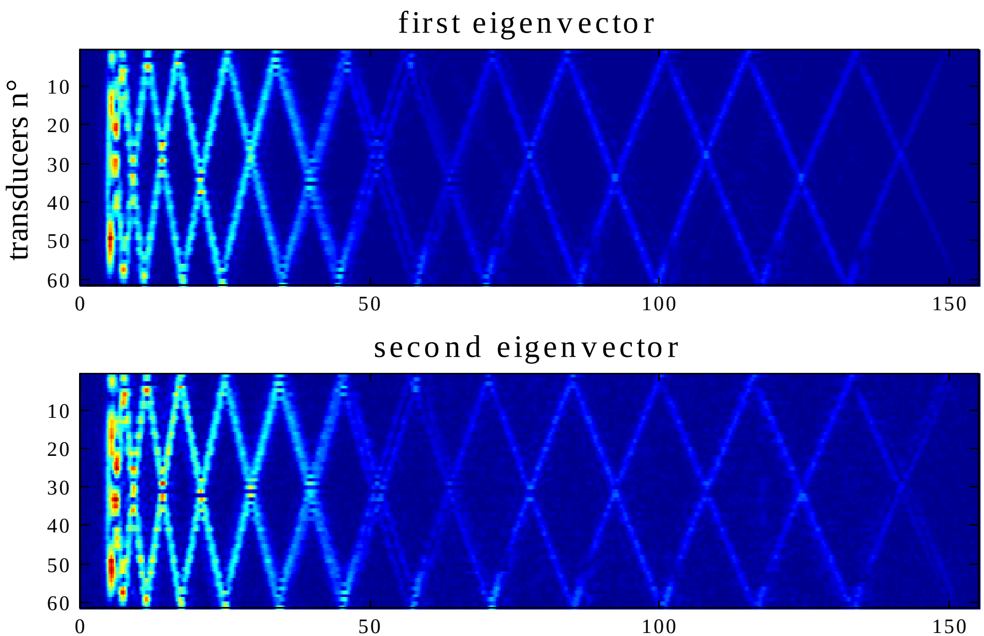


Backpropagation of the two main eigenvectors

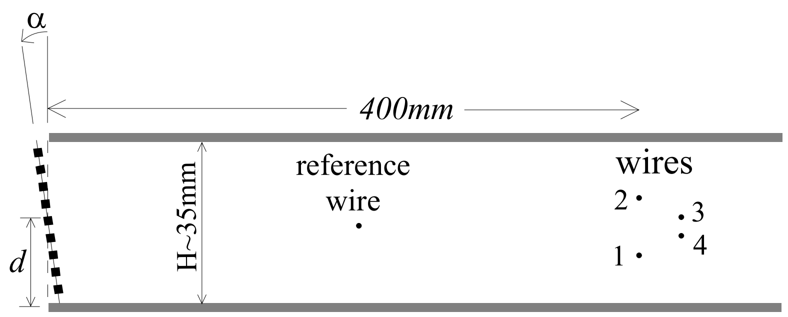
Building temporal eigenvectors



Frequency dependence of the eigenvalues



Imaging in a waveguide



(D.O.R.T. in a wave guide JASA 1999, Mordant & al.)

f =1.5MHz

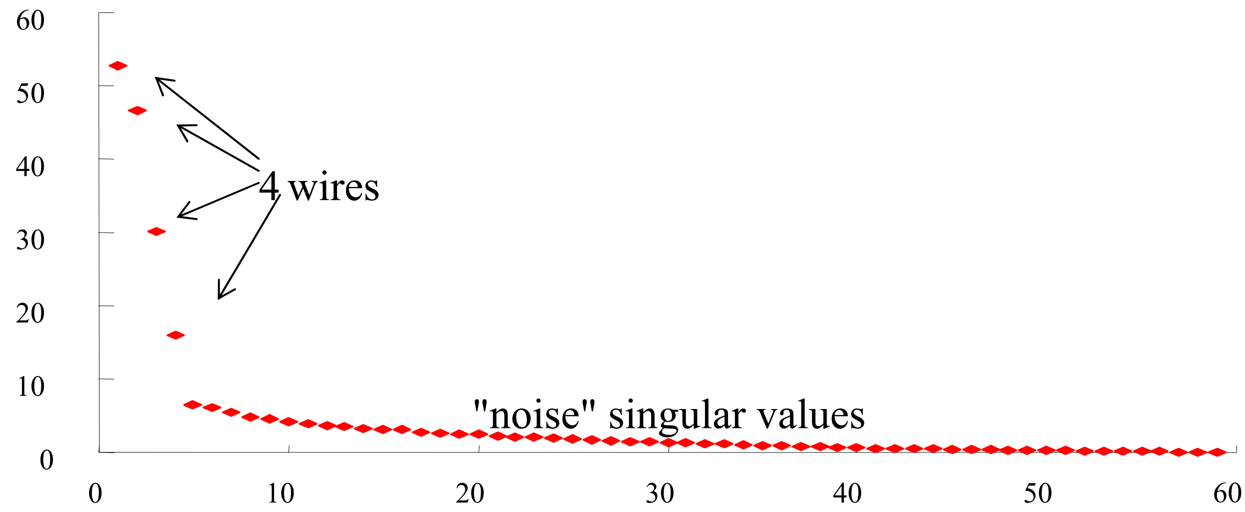
$\lambda f/2d$ =11mm

An estimate model of the waveguide :

The Green function is estimate with a simple model

A reference is use to optimize the parameters α, d, h

Singular values



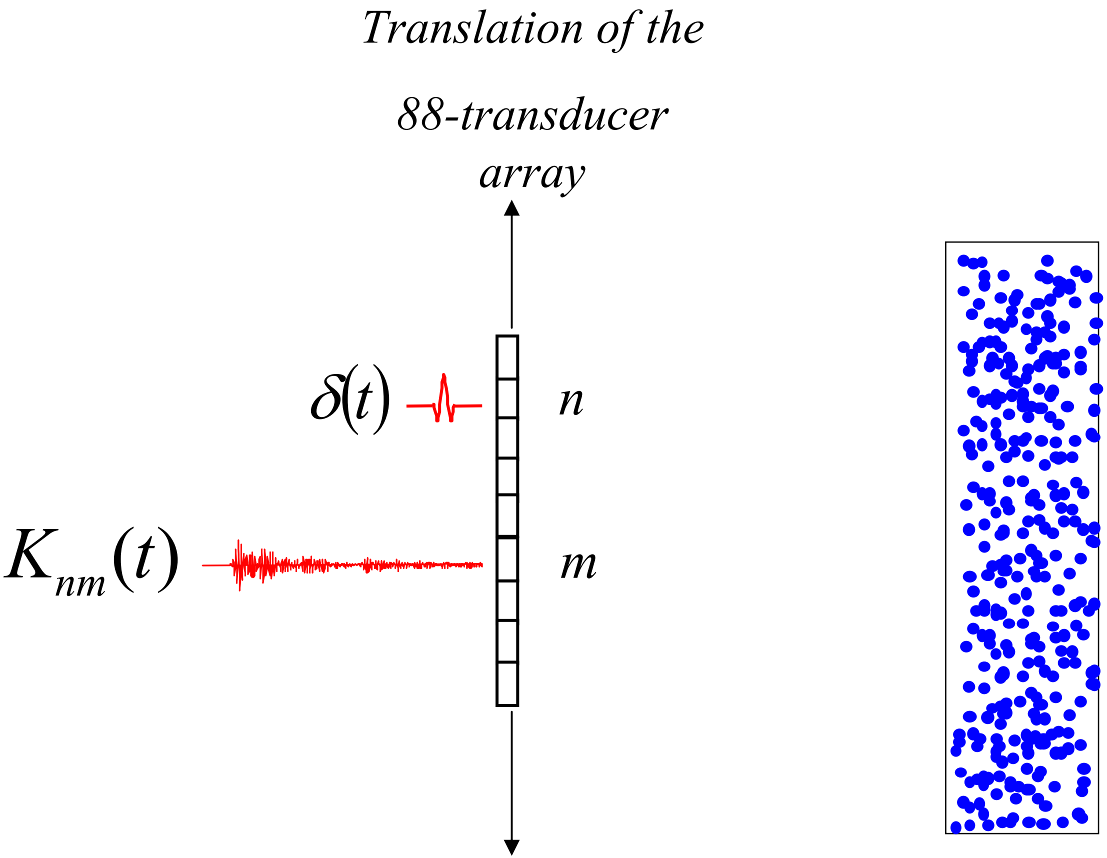
Multiple scattering of ultrasound and Random Matrix Theory

Diameter	: 0.8 mm
Density :	19 tiges / cm²
ℓ^*	5 mm
ℓ_a	~300 mm
D	2.5 mm²/µs

5mm < L < 80mm

A.Tourin , M. Fink

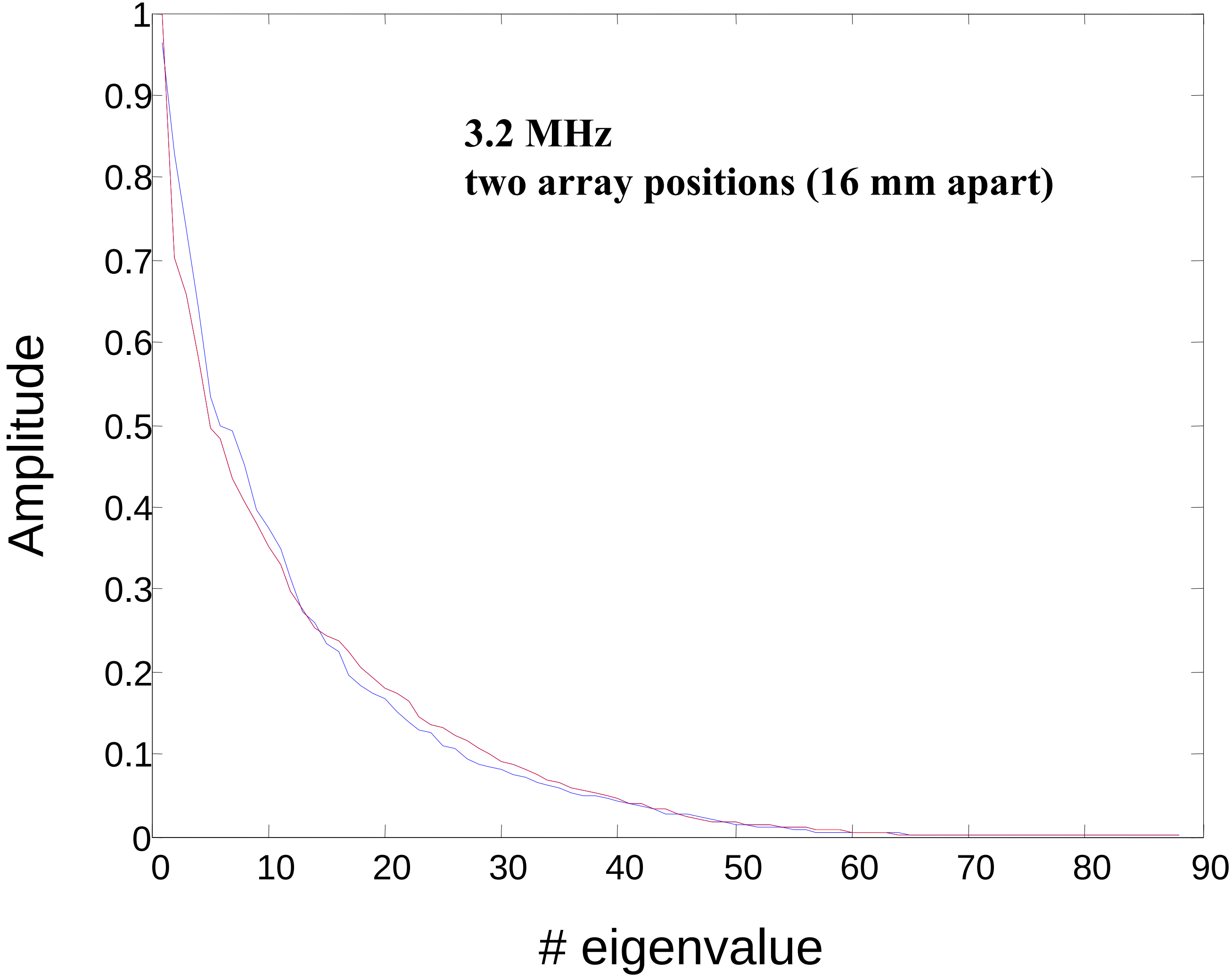
Recording the Bacscattering Matrix



FT

$$K_{nm}(t) \Rightarrow K(\omega) \Rightarrow K^{*}K$$

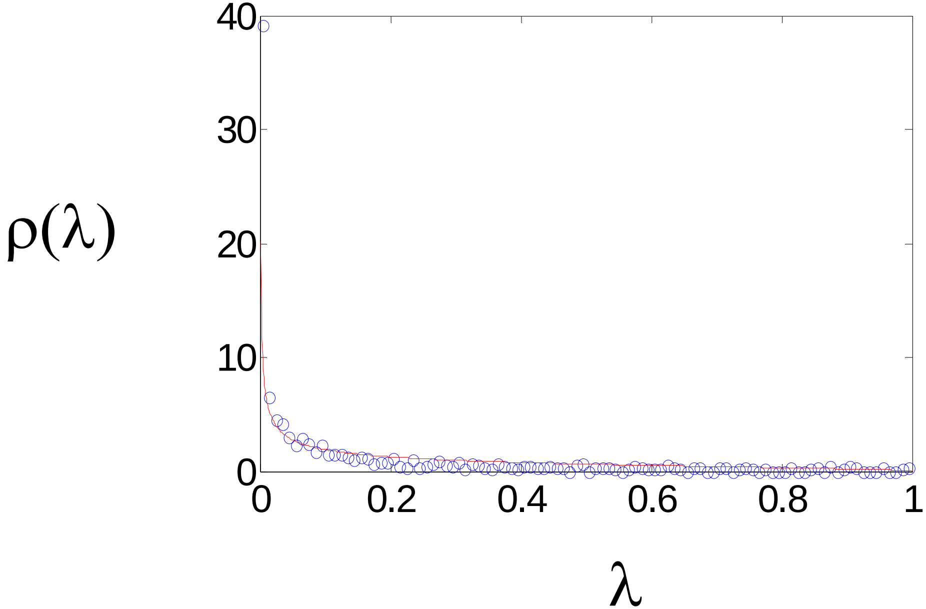
Eigenvalues of the Bacscattering TR operator



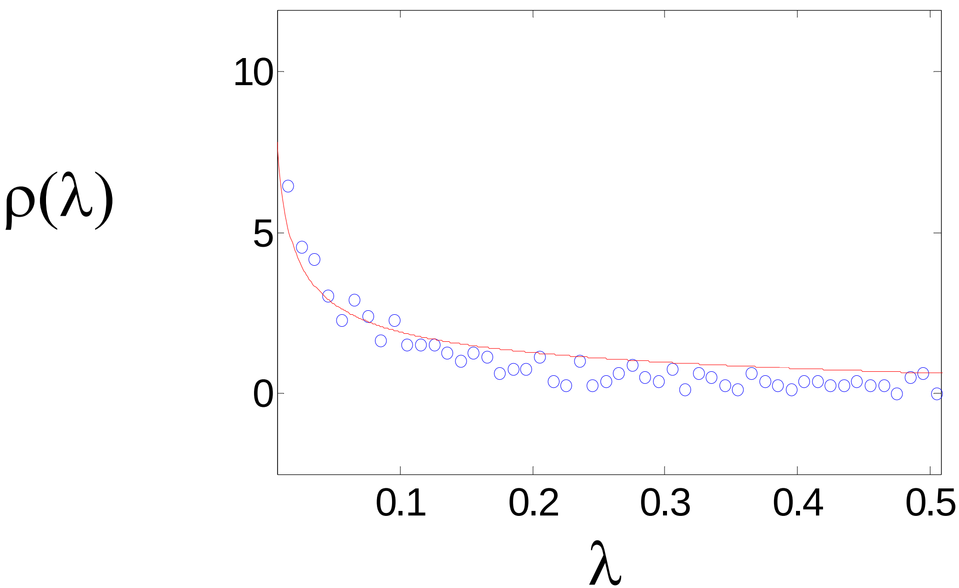
Assuming that the entries are complex gaussian uncorrelated random variables, the eigenvalues density of K*K follows:

$$\rho(\lambda) = \frac{2}{\lambda_{\max}\pi}\sqrt{\lambda_{\max} - \lambda}/\sqrt{\lambda}$$

The so-called quarter Circle Law

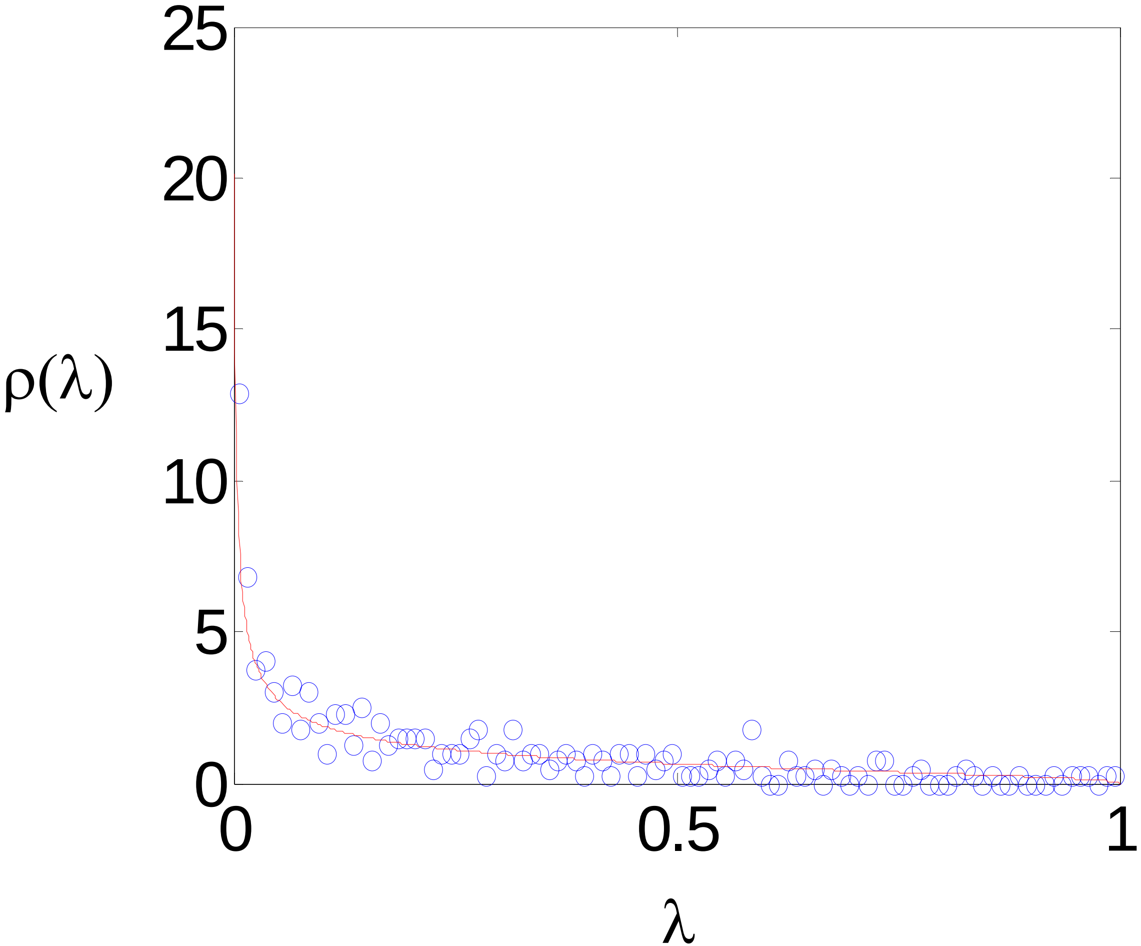


ZOOM



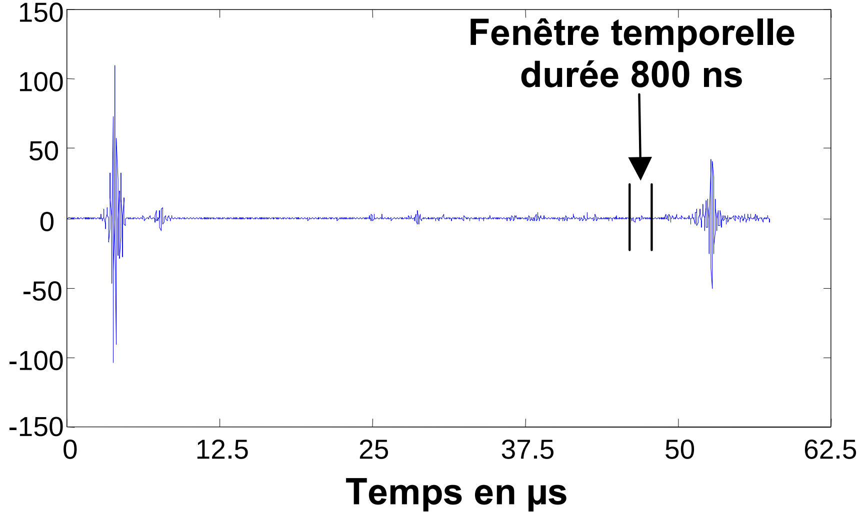
Correlations ?

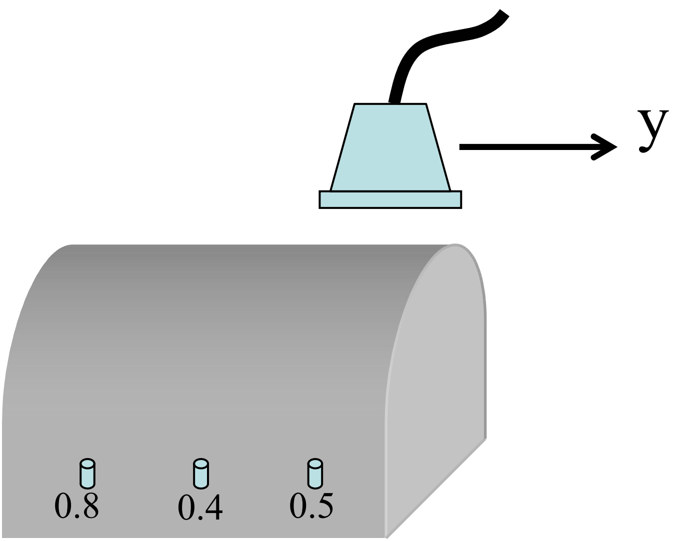
The fit is better when taking into account a larger spacing between transducers



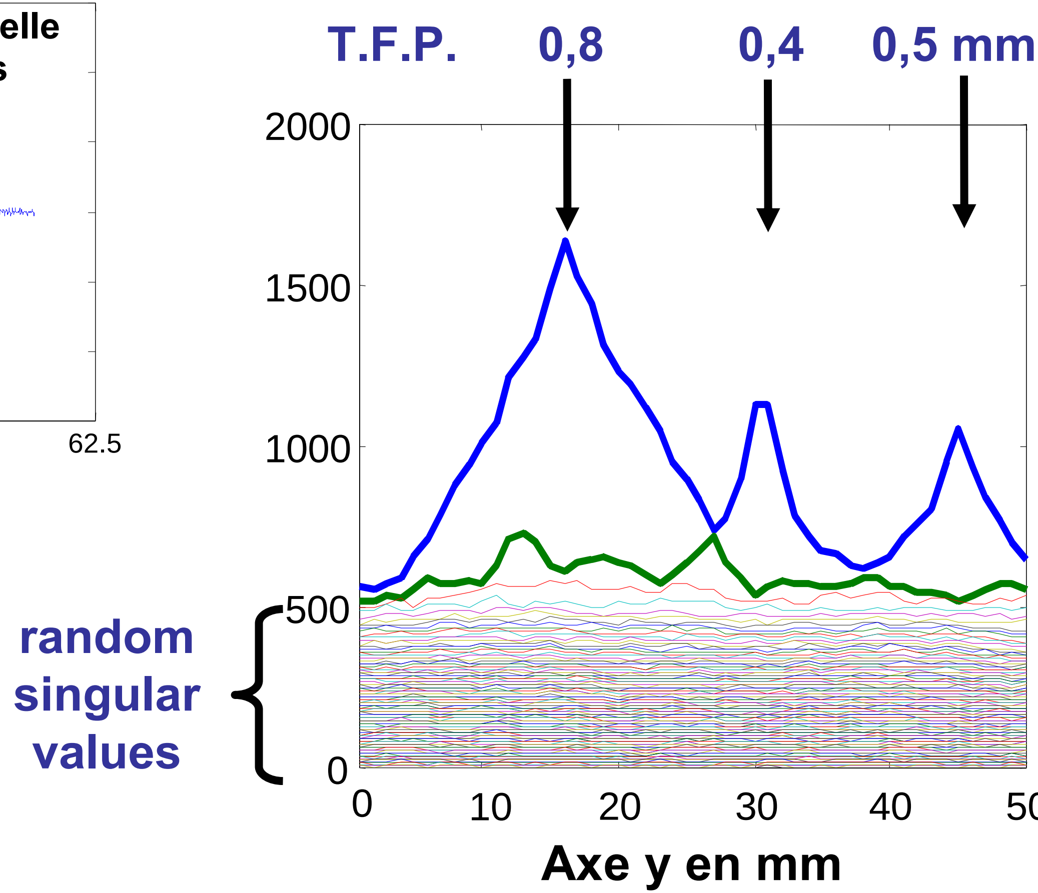
DORT Analysis in NDT (titanium alloy with defect)

$k_{40,10}(t)$



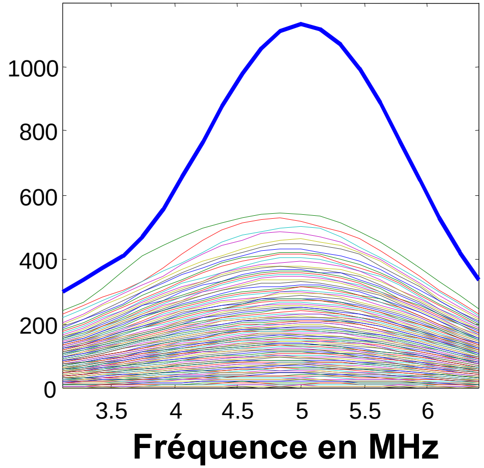


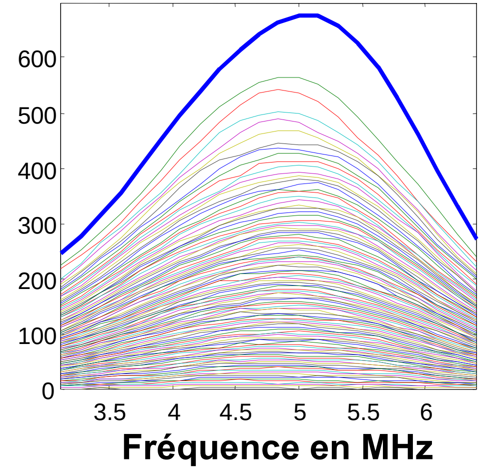
Singular values at 5 MHz



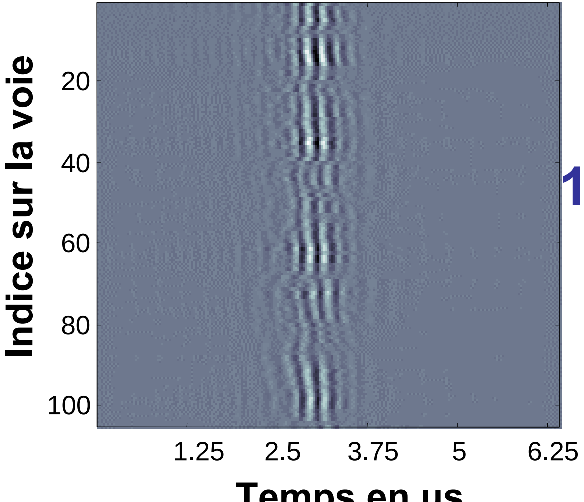
The DORT method in the time domain

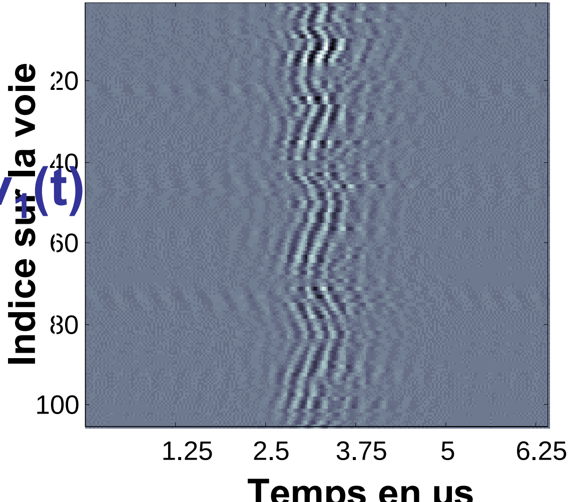
In front of a defect T.F.P. 0,4 mm at 5 mm off axis





Singular values



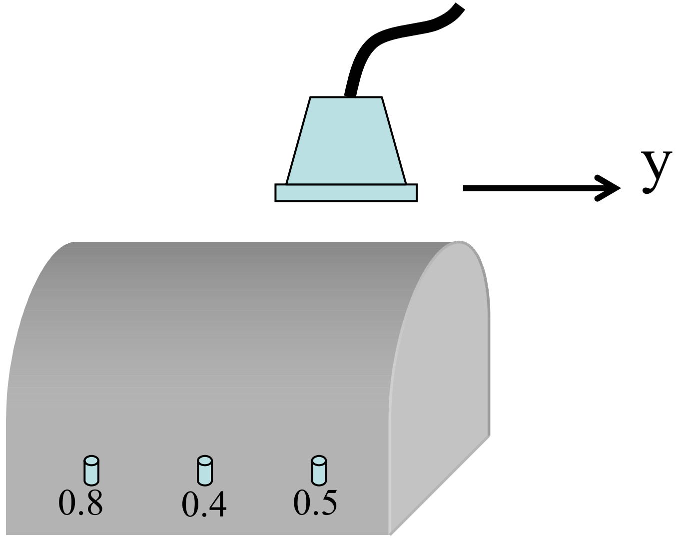


1er temporal eigenvector $v_1(t)$

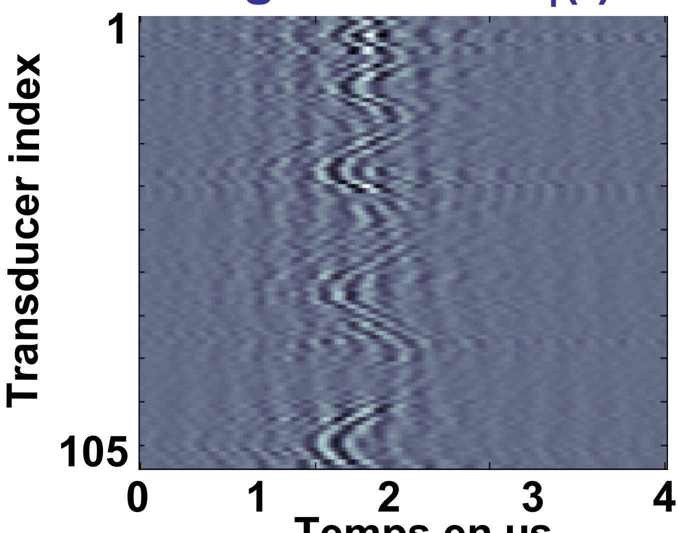
Imaging with DORT method

Backpropagation of $v_1(t)$ with Simul-PA

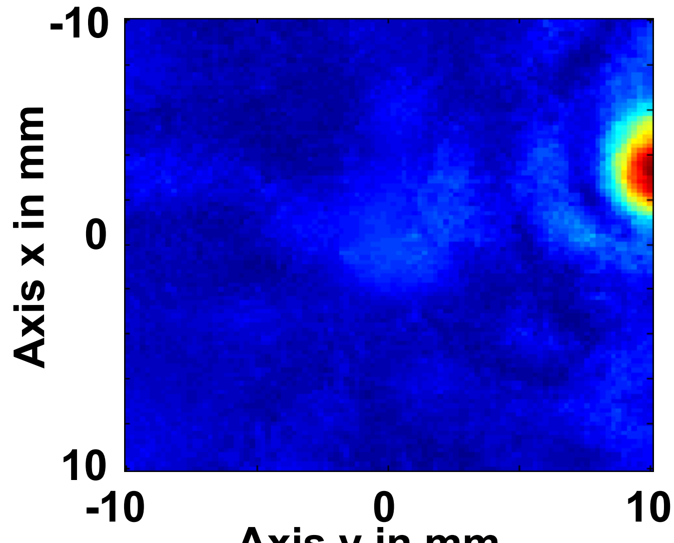
50 positions



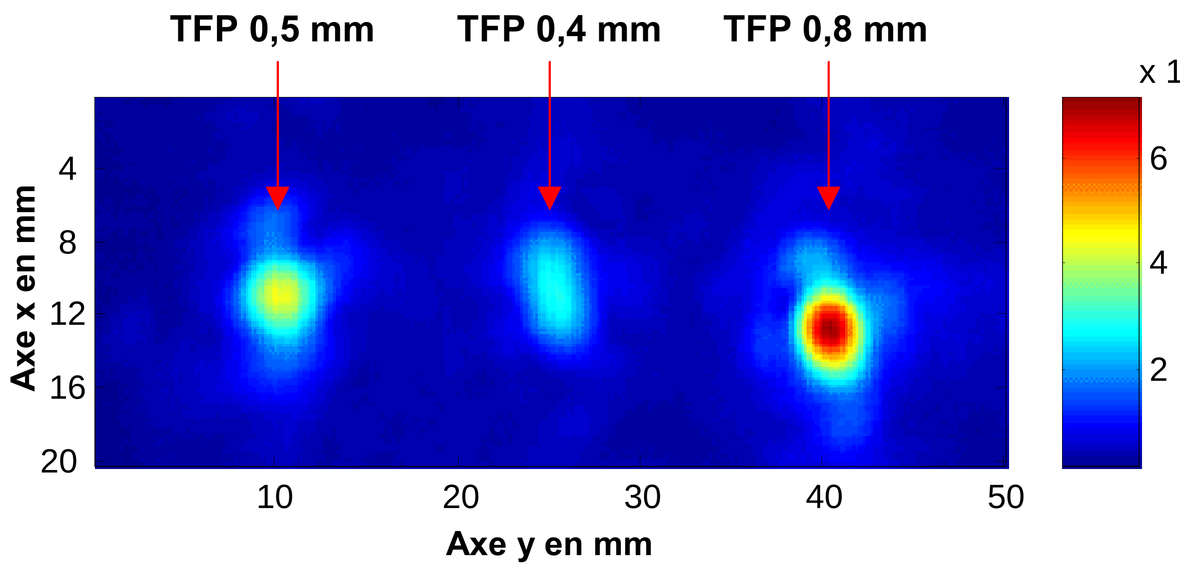
1er temporal Eigenvector $v_1(t)$

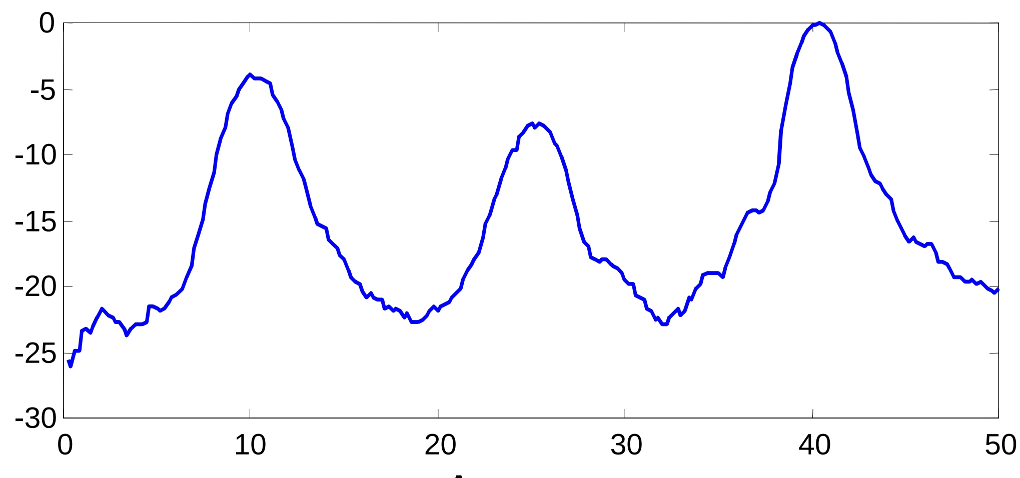


Field in z = 140 mm,



Incoherent Summation of 50 fields, DORT





(dB)

TFP 0,4 mm : SNR 15 dB