Basics of Nonlinear Time Reversal Acoustics

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Abstract.
An investigation of time reversal of nonlinear and non-dissipative or dissipative plane sound waves was made. Propagation and backpropagation is time reversal invariant only when the dissipation is zero and the wave has not been shocked. A wave that has shocked has irretrievably lost part of its content. Still, if the wave and its derivatives are considered continuous there remain in theory forever information about the original signal. Factors like numerical accuracy and noise naturally set limits in practical situations.

Keywords: Acoustic time reversal, acoustic back propagation, nonlinear attenuation, shock front reversal
PACS: 43.60.Tj, 43.25.+y

INTRODUCTION

The results in this paper were presented at the Acoustical Society of America meeting at Penn State in June 1997 [1]. Some years have passed since then, but the approach is different from papers up till now and permits a different point of view of nonlinear time reversal acoustics.

Linear time reversed acoustics takes a received signal, and retransmits the time reversed signal in the direction it came from. This simple technique turns out to have several applications in medicine, undersea communications, hydrodynamics and material analysis [2]. The linear time reversal started to gain momentum in the beginning of the 1990s where it was shown how multiple scattering media are better at refocusing than homogeneous as the effective focusing aperture is wider and that the metod is also extraordinary insensitive to initial conditions [3, 4, 5]. As a way of examining objects, e.g. underwater [6] or medical ultrasound, a three-step sequence gives the illumination of specific point reflectors. First a pulse is sent and reflected from a point. This signal is then received by an array of transducers, time reversed and re-emitted. This new pulse will focus on the point reflector, automatically compensating for irregularities (refraction, mode conversion and anisotropy) in the medium, and thus giving a strong response [7]. This type of experimental use of the term time reversal is a forward propagation using the time inverted reflected wave as boundary condition.

Backpropagation in time is more like a true type of time reversal, and can of course only be done in the virtual world. Sometimes, the propagation by using negative time yields approximately the same as the reflections (no damping and no shocks with pressure release surface at reflection) and sometimes the same as the inverse problem. Information may have been lost.

Work on nonlinear time reversal propagation has been published by, for example, Hedberg [8], Brysev *et al.*, [9], Hallaj *et al.* [10], Tanter *et al.* [11] and Cunningham

CP1106, *Mathematical Modeling of Wave Phenomena,* edited by B. Nilsson, L. Fishman, A. Karlsson, and S. Nordebo

et al. [12]. Recently there has been work on the combination of time reversal with a nonlinear acoustic material characterization [13, 14, 15] - the concept being proposed in 1998 [8]. It should be noted that in this context there exist two types of nonlinear time reversal. One type is where the wave propagation through the medium is nonlinear, and the other is where the target region itself produces a nonlinear response, as is the case for a crack in solids.

The time invariant property of linear, non-dissipative waves has been accentuated. This invariant property is when a wave may be received, time reversed, retransmitted and the nearly original wave will be received [16] at the point of the transmission of the original signal. This time reversal invariance is violated when dissipation is taken into account and certainly to a higher degree when nonlinear effects are present. The signal received at the point of transmission will not be the original signal. The nonlinear effects are accumulated with distance, just as the dissipative effects, and are not reversed by hard reflections which explains the existence of standing shock waves in tubes [17, 18]. A soft reflection, on the on the hand reverses the wave shape at the reflection and the nonlinear influence may be more or less reversed on the return to the starting point [11].

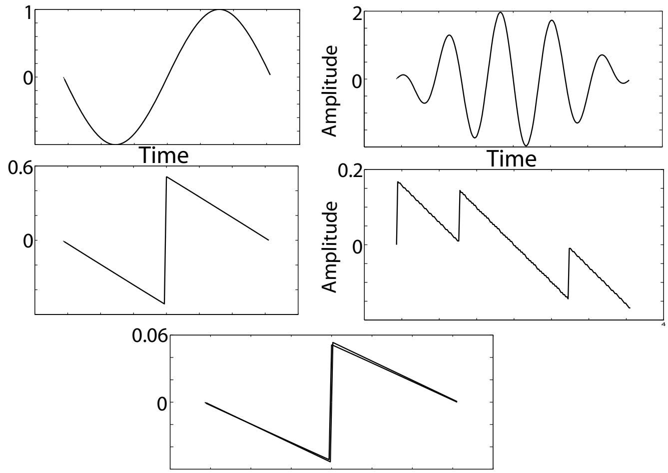

FIGURE 1. Two completely different waves evolve nonlinearly for certain distances so that they are indistinguishable.

For strong nonlinearity with zero dissipation one always obtain saw-tooth waves in the lowest frequency of the original signal. In Figure 1 a single frequency wave evolves on the left side column and a double frequency on the right. In the last frame they are not indistinguishable to the eye, but which is which ? They would have been practically indistinguishable if one of the distances σ was to be slightly different.

Without dissipation, the condition in Figure 1 does not come into consideration. The situation presently depicted in Figure 2, raises other questions. A formerly shocked sinus is compared to a pure sinus. Has the near sinus recently been created or is it traced from a formerly shocked wave which has had much higher amplitude ? How can we see if a

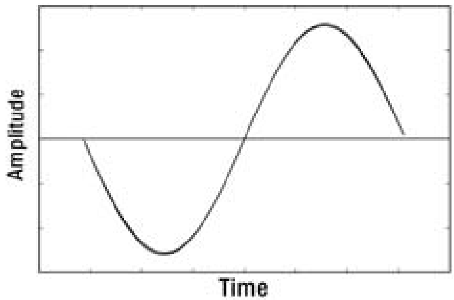

FIGURE 2. A formerly shocked wave is compared to a pure sinus. How can it be shown that one of these waves has been shocked and the other not - which is which ?

wave has been through a nonlinear shock stage ?

Nonlinear sound waves lose information through its shocks and after some time the details of the original wave may be completely lost. This is what it seems like. The question that will be investigated in this work is a simple one: Can the information lost in strongly nonlinear evolution of acoustic waves be recovered ? The short answer is, yes - in principle, but not always in practice.

CONCEPT OF BACK-PROPAGATION OF NONLINEAR WAVES

Nonlinear and dissipative one-dimensional sound waves are described by the Burgers equation.

$$\frac{\partial V}{\partial \sigma} - V\frac{\partial V}{\partial \theta} - \nu\frac{\partial^2}{\partial \theta^2} = 0 \tag{1}$$

Here V is the velocity amplitude, σ the distance from the source, θ the retarded time and ν is a ratio between dissipation and nonlinearity, all of which are dimensionless.

The back-propagation for zero dissipation is made through a simple analytical method where points on the wave are propagated negatively in time θ in proportion to their amplitude V multiplied by the distance σ. The method is the same as can be used for the Burgers equation before the shocks are formed, except that higher amplitudes propagate slower (instead of faster) and lower amplitudes faster (instead of slower). For the Burgers equation there is the problem with shocks appearing which makes this simple method result in multi-valuedness, and therefore un-physical, waves. This problem is not an issue for back-propagation. The wave is propagated away from shocks and if new multi-valued solutions are obtained, the wave has been back-propagated too far for the solutions to be correct - a shorter back-propagation distance must be chosen.

The method for dissipative waves is to propagate backwards the distance σ to see what information is still the same. This is the same as true time reversal. The method is outlined in the following four steps:

i) Propagate the signal V backwards the distance σ,which gives V_B.

ii) Identify the parts including relevant information.
iii) If necessary, adapt a Fourier series to these parts (choose period and number of harmonics) which results in V_{0B}.
iv) Compare V_{0B} to the original signal V_0, if known.

NON-DISSIPATIVE NONLINEAR WAVES

In Figure 3 we have a single frequency propagated the distances $\sigma = 0.5$ and $\sigma = \sigma_{shock} = 1$. We see how the back-propagated wave is exactly the original wave. No Fourier series approach need to be made to approximate V_B. No information is lost as long as the wave has not shocked.

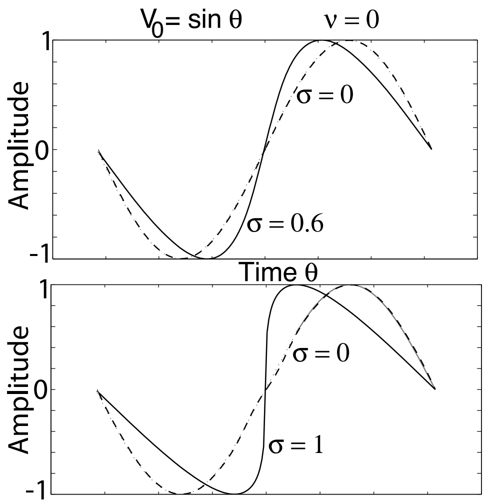

FIGURE 3. Before shock, a wave can be backpropagated and the original wave will be obtained.

The wave can really be backpropagated twice as long, all the way to when the negative shock is reached at $\sigma = -1$. Then the wave can be propagated forward again getting the same wave shapes as before [19]. In Figure 4 top row is depicted again how the wave can be propagated within positive and negative shock distance getting the same shapes. When the wave is propagated past the shock formation it is deformed, and part of the wave is lost. After this the wave can be propagated within the negative shock distance (again), and forward to the furthest point without being more deformed as shown in Figure 4 bottom row.

The method outlined in section 2 is applied through the example in Figure 5. A single sinusoidal is the original signal V_0 (solid line). It is propagated to one and a half shock distances i.e. $\sigma = 1.5$. The result V (also a solid line) has a sharp shock (Figure 5 a and a close-ups in b-c). This wave can be seen as the input to our back-propagation algorithm. In practical applications this would be a measured signal. This wave V is back-propagated the distance $\sigma = 1.5$, which in this case is known to be correct. Parts of

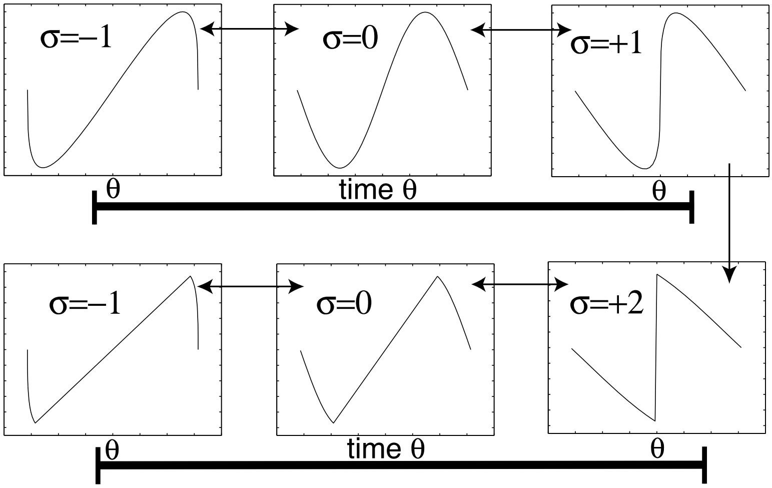

FIGURE 4. The wave can be propagated within the shock distance $\sigma = \pm 1$ without losing information (top row). When the wave has been propagated past shock to $\sigma = 2$, the wave can be propagated within [-1: 2] without losing *more* of its information (bottom row).

the back-propagated wave V_B (dotted line) coincide with V_0 in the left and right parts of the interval. The shock has eaten the middle part, there has been a nonlinear attenuation. The dotted straight line actually does not contain any points, it is only the line connecting the rightmost point on the left part with the leftmost point on the right part.

Before shock, a wave can be backpropagated and the original wave will be obtained.

A Fourier series approach is made with n harmonics. With $n = 10$, Figure 5 a, the Fourier series adapts to all the relevant parts as can be seen in the close-up in the next frame (Figure 5 b). Still, in the middle interval the curve rises to levels which are far above one. An important point is that this curve, given only the wave at $\sigma = 1.5$, may very well have been the original one as there is an infinite selection of possible original signals for which V would be the same. As many original signals are possible solutions we must choose criteria that will help us decide among the ones we think are correct. This can be made as a choice of the allowed function. In the third frame in Figure 5 c, a five harmonic Fourier series ($n = 5$) is chosen and the nearly correct solution is obtained.

Increasing the propagation distance to $\sigma = 5$, the last frame in Figure 5 d, even smaller parts of the back-propagated wave V_B contains information. Not even $n = 2$ gives a decent result. Trying $n = 1$ seems unnecessary as we know it would conform perfectly.

The distance $\sigma = 5$ is past the limit for the numerical accuracy used in our examples. We need to be fairly certain that the solution is correct for when the original signal is not known.

This shows that there is a practical limit but in theory there will always be traces of the original wave in the parts of the wave which has not been absorbed by shocks which before the back-propagation is all of the wave except the shocks themselves. It is only

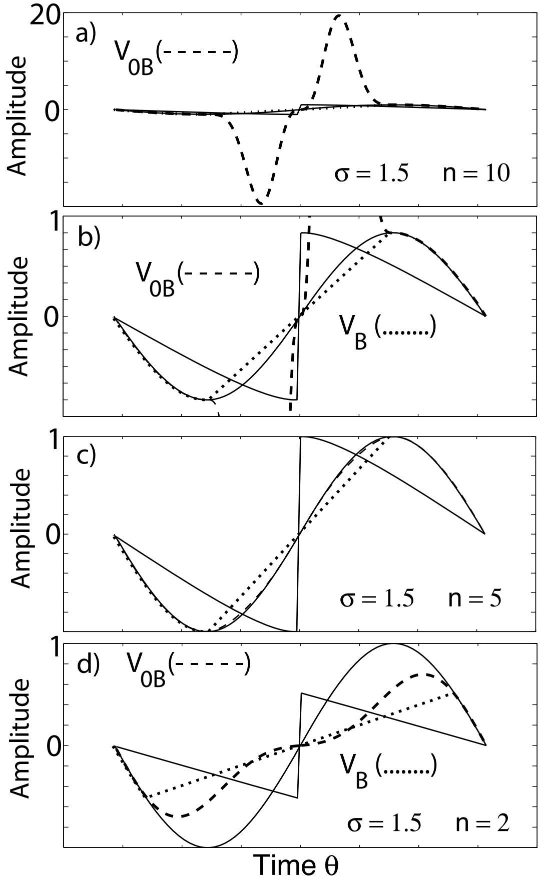

FIGURE 5. The back propagation method of a sine wave for distance $\sigma = 1.5$ (a,b,c) and $\sigma = 5$ (d). After shock formation information is lost in the shocks.

after the back-propagation that these parts shrink into the regions which coincide with parts of the original signal. An interesting fact is that a wave V at a distance with fully developed shock structure, can be described accurately by the position and amplitude of the shocks [20] while all the information from the original wave V_0 is found only in the slopes in between the shocks.

The boundary condition for the example of multi-harmonic waves is chosen to be $V_0 = \sin 4t + \sin 5t$. In Figure 6 is seen how the method works. With 12 terms in the Fourier series the result is not very good, even though the remaining parts of the slopes coincides with the function. For 7 terms the result is better - the exact solution is not found but it is qualitatively right. It should be noted that the distance $\sigma = 0.2$ is approximately 20 shock distances.

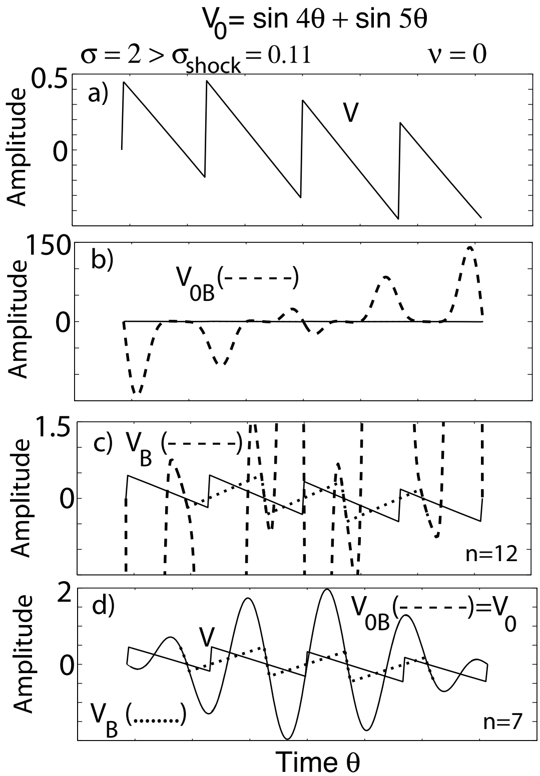

FIGURE 6. Back propagation results for a bi-frequency source.

DISSIPATIVE NONLINEAR WAVES

In this section the size of the dissipation over nonlinearity ratio is always $\nu = 0.05$.

In Figure 2, a wave propagated the distance $\sigma = 100$ was compared with a simple sine wave. They have almost the same shape. We will here show that it is possible to see that the propagated wave has been shocked and estimate the propagated distance. This is done through comparing the ratios of the Fourier coefficients of the harmonics.

The ratios of the Fourier coefficients with dissipation makes it possible to determine the distance travelled. They all start at zero, soon reaches a maximum and thereafter decreases monotonically. There are two possible distances for every possible ratio, therefore two ratios are needed to decide if the signal has passed through the shock phase or not. For sinus waves the analytical Fay solution [21] (good when the distance $\sigma > 3$) or the Fubini solution [22] (good when the distance $\sigma < 1$) are used. The choice between these two solutions is made by comparing the ratios b_n/b_1. An example of this is shown in Figure 7.

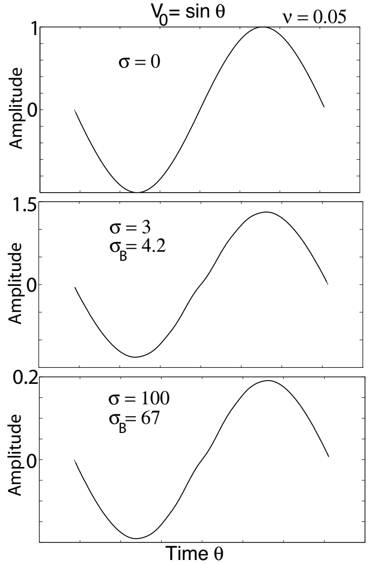

FIGURE 7. Both the propagation distance and the original amplitude is estimated. The dissipation parameter is $\nu = 0.05$

For a wave propagated to $\sigma = 3$, the estimated back-propagation distance is $\sigma_B = 4.2$ which gives an estimated amplitude of 1.3. For a wave propagated all the way to $\sigma = 100$, the estimated back-propagation distance is $\sigma_B = 67$ which gives an estimated amplitude of 0.19.

For original signals with multi-harmonic content many shocks per period are usually present. The way the back-propagation is done is to look at every half-period one at a time, extend it into a whole period, and do single frequency back-propagation separately. Then the back-propagated wave V_B is created through the usual Fourier algoritm. Multi-frequency waves turn out to be possible to determine more easily, i.e. for larger distances, than for a single frequency. It should be noted that in the examples the distance parameter σ should be compared to the shock formation distance rather than the numerical values as σ is a dimensionless parameter. For a single frequency wave with amplitude one the shock formation distance is $\sigma_{shock} = 1$ but for multiple frequencies smaller. For example, $V_0 = \sin 4\theta + \sin 5\theta$ has its shock formation at approximately $\sigma_{shock} = 0.11$.

Heavy noise contaminating the signal might in some cases disturb to the extent that the outcome will be wrong. However, noise with small amplitudes will not have a decisive influence [8]. The methods used are described in Section 2 above. A known original signal V_0 is propagated, usually past shock formation. In practical situations the input V would be a measured signal.

This method expects sine waves and is therefore more stable than the non-dissipative equivalent, which is why better results are obtained for the same distances here.

CONCLUSIONS

Methods has been presented for the back-propagation of signals in nonlinear and dissipative media. The time reversal invariance which makes linear time reversal appealing exist only for dissipation free propagation in the region before shock formation. Still in theory there are always parts of the wave which retains information about the source signal. In the methods were shown how the nonlinearity and the dissipation may be detected and compensated for. For larger distances a high numerical accuracy is required. Multi-frequency waves contain more information than single frequency and therefore it is more easy to identify the original signals from them.

The accuracy of the calculations in this article may of course be improved and give the desired result for larger distances. But there is a limit to how well non-accurately measured signals may be analyzed in order to determine source signals and nonlinearities.

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