Time-Reversed Waves : Subwavelength Focusing

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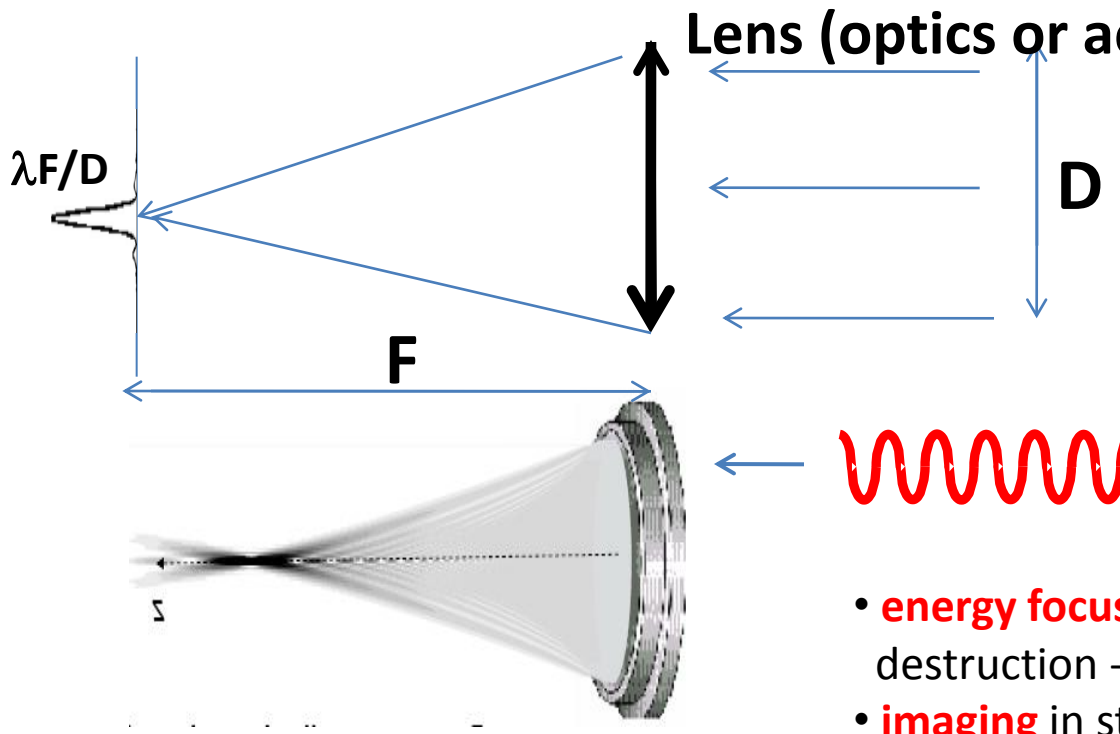
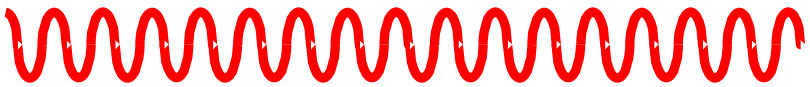
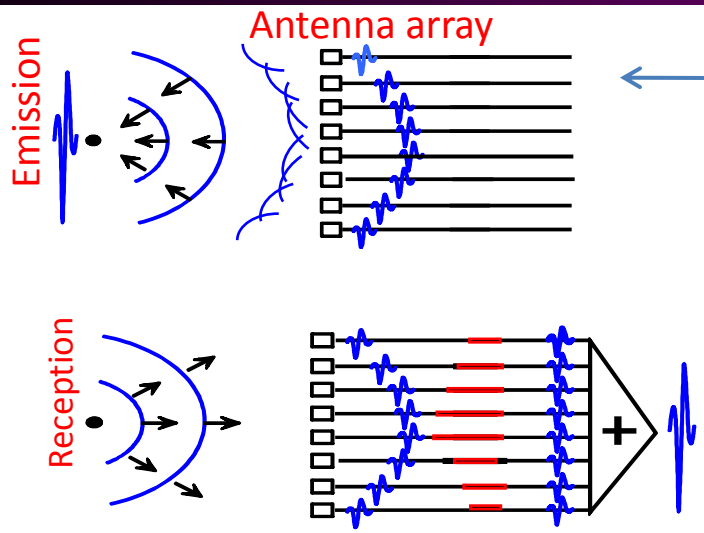
Focusing a Wave in an Homogeneous Medium



Lenses work both for stationary waves (**monochromatic** or incoherent) and for **pulsed** waves



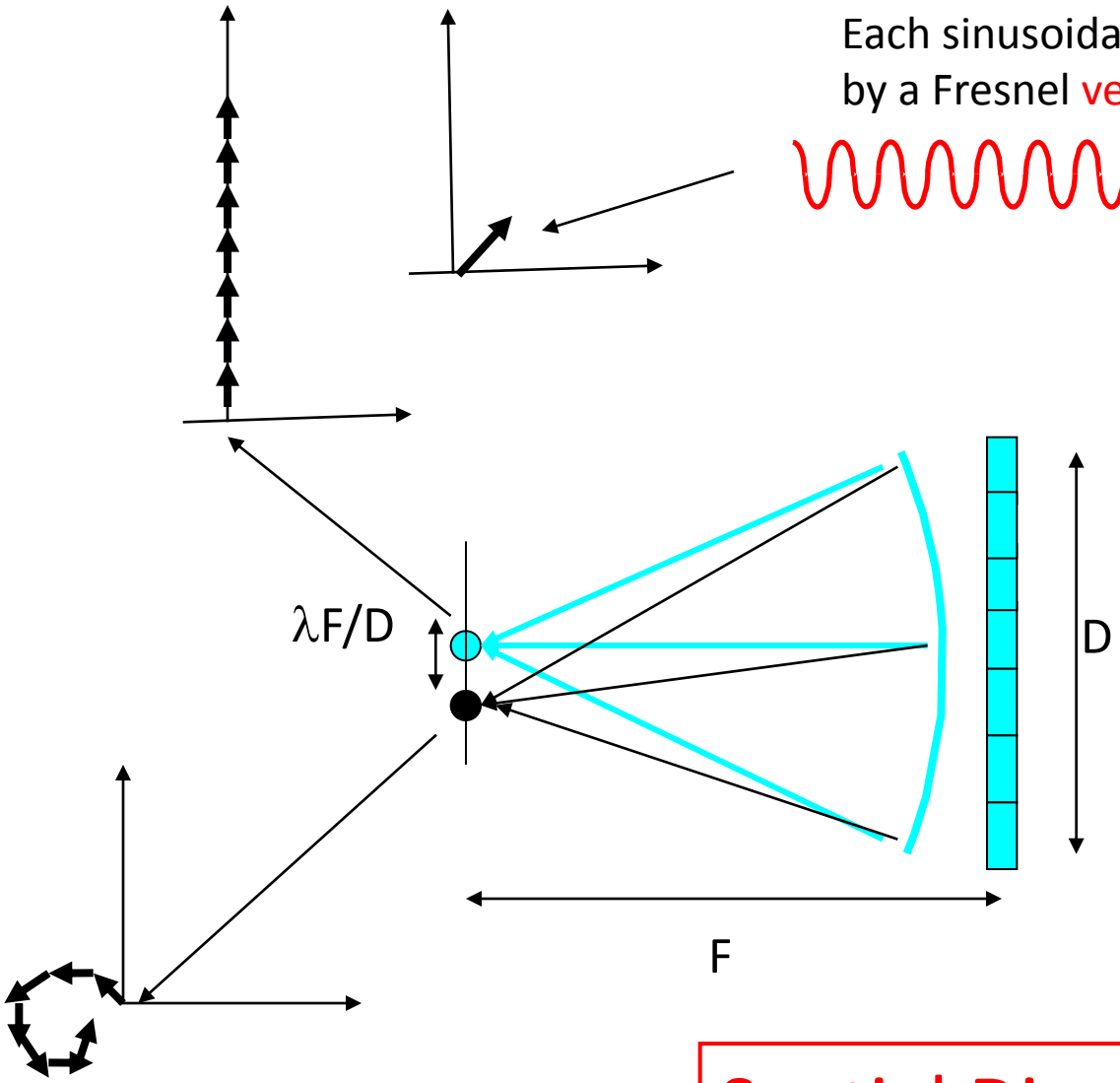
- **energy focusing** (therapy by thermal destruction - hyperthermia)
- **imaging** in stationary mode



- **imaging** in pulsed mode (sonar, radar, lidar)
- **shock wave focusing** (therapy by mechanical destruction)
- **telecommunications** discrete broadband telecommunications

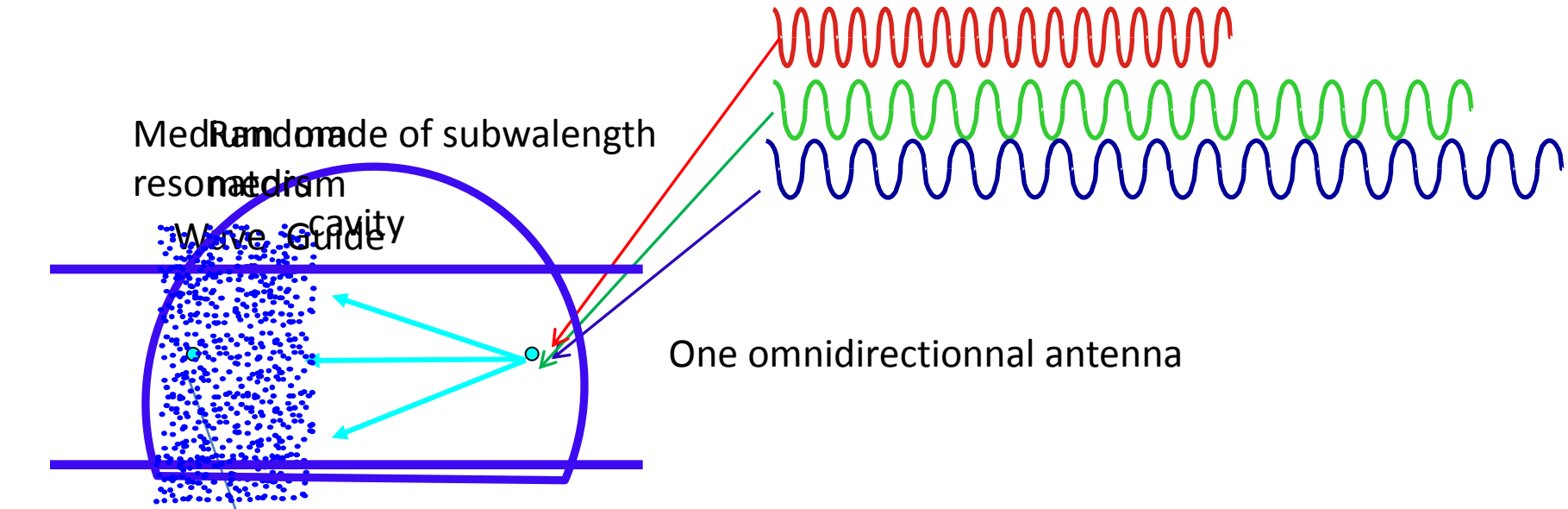
Spatial Focusing in monochromatic mode

Each sinusoidal wavelet is represented by a Fresnel vector (a complex number)

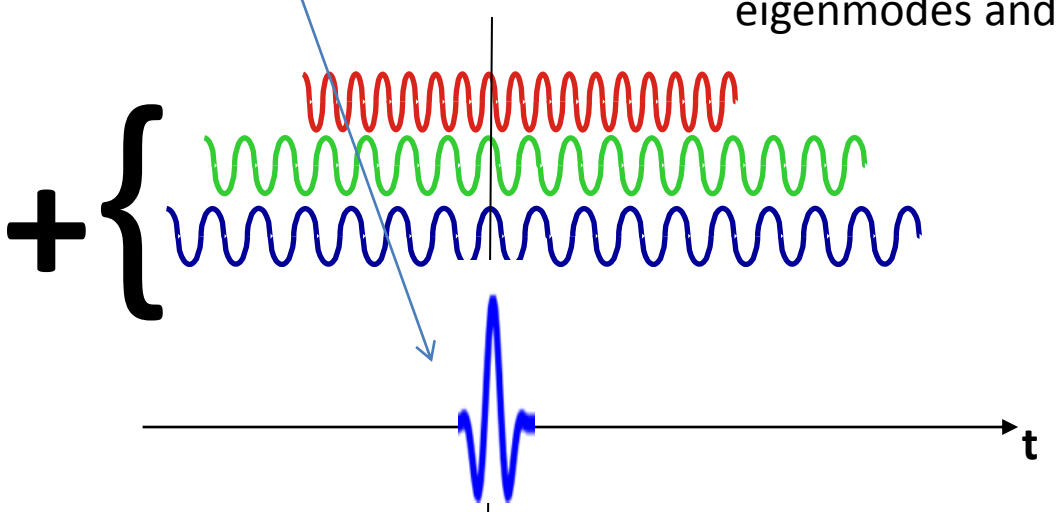


Spatial Diversity

Spatio-Temporal Focusing for broadband signals in an heterogeneous medium or in bounded medium



Dispersive medium described by
eigenmodes and eigenfrequencies distribution



Frequency Diversity

Focusing a broadband wave in an heterogeneous medium

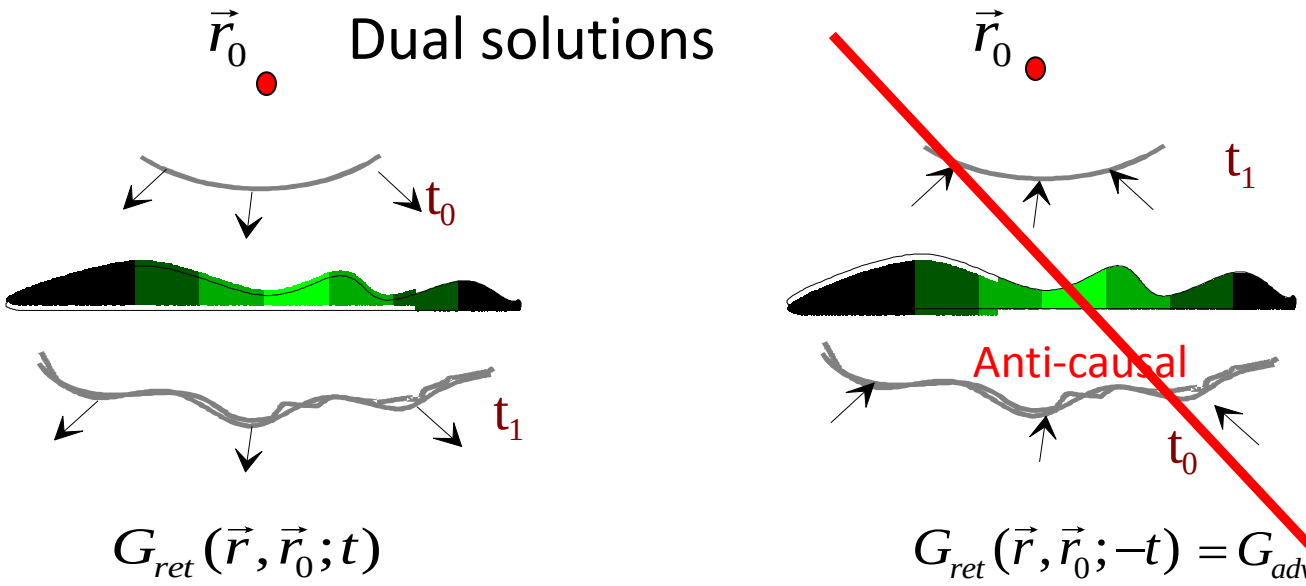
A solution : building a time-reversed wave (causality Pb)

The acoustic case : non dissipative **heterogeneous** domain with a ponctual source

$$\left\{\Delta - \frac{1}{c^2(\vec{r})}\frac{\partial^2}{\partial t^2}\right\} G(\vec{r},\vec{r}_0;t) = -\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(t\right)$$

Green's function

Dual solutions



$G_{ret}(\vec{r},\vec{r}_0;t)$

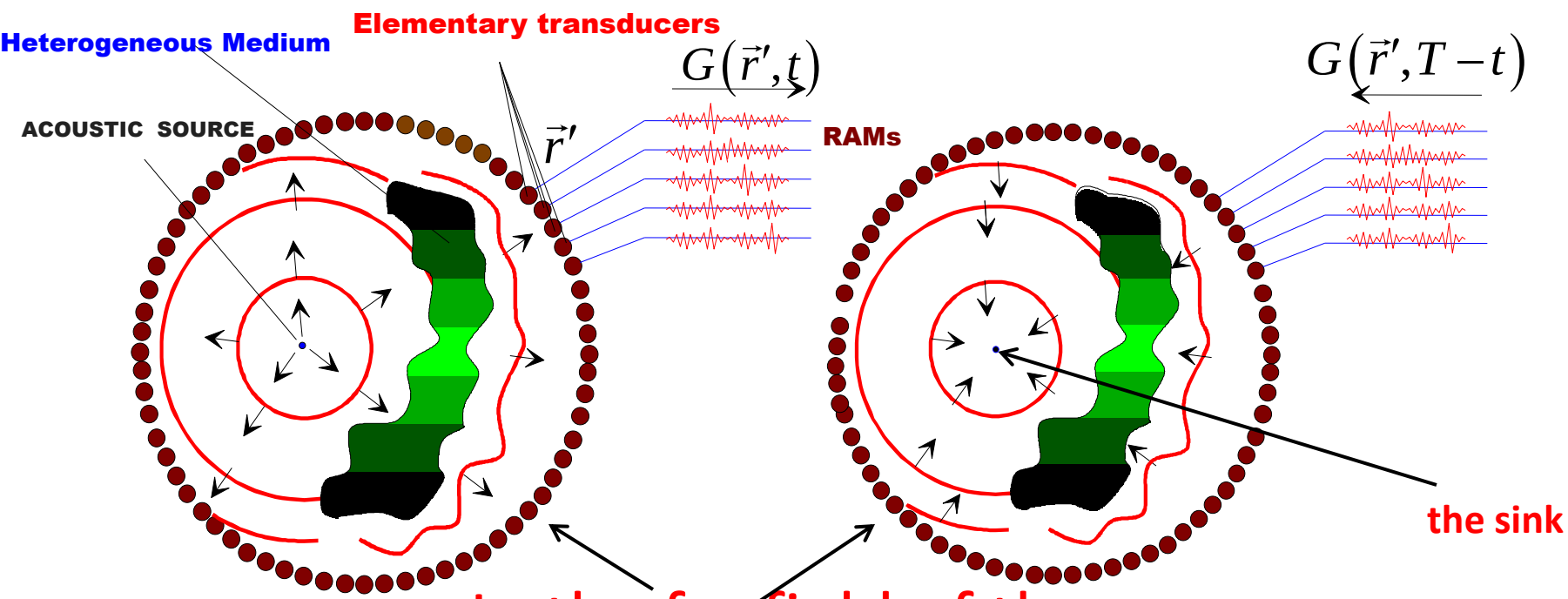
$G_{ret}(\vec{r},\vec{r}_0;-t) = G_{adv}(\vec{r},\vec{r}_0;t)$

To build the TR field : Time reversed the causal field on the boundary : the time reversal cavity

TR on the boundary : the TR Cavity

- record on the boundary $G(\vec{r}',\vec{r}_0;t);\partial_n G(\vec{r}',\vec{r}_0;t)$
- transmit from the boundary $G(\vec{r}',\vec{r}_0;T-t);\partial_n G(\vec{r}',\vec{r}_0;T-t)$

Forward Step ***Time-reversed Step***

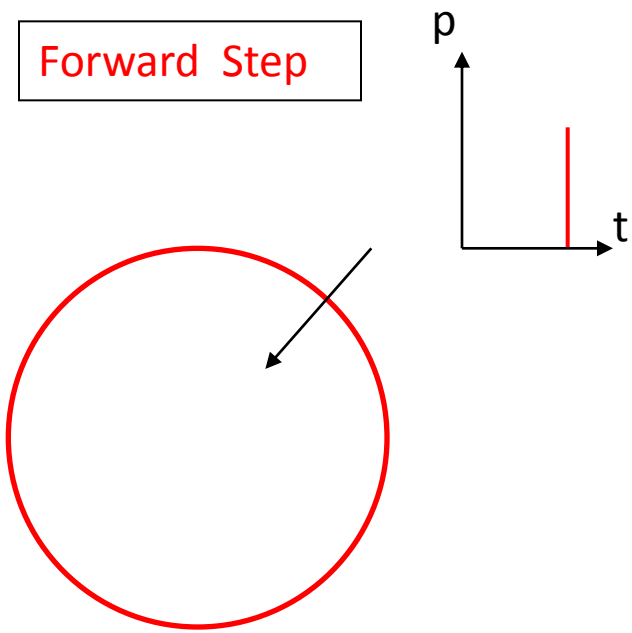


In the far field of the source

Origin of Diffraction Limits in Wave Physics

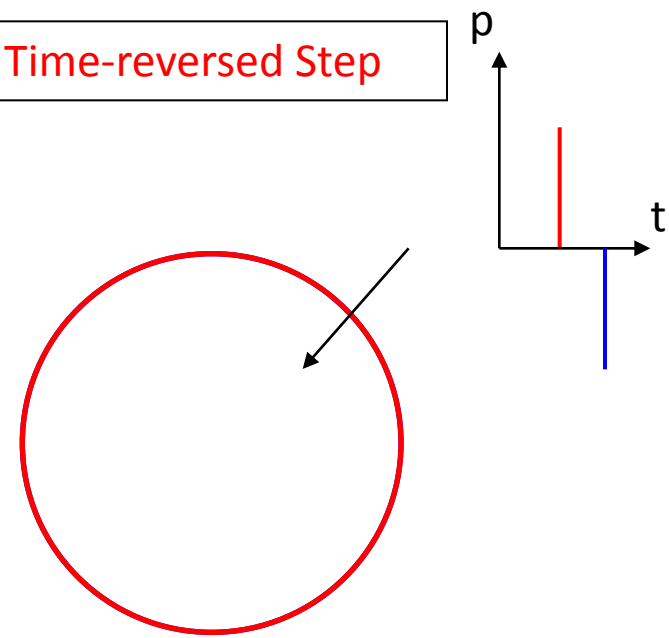
Pulsed mode – the homogeneous medium

Forward Step



$$G^0_{ret}(R,t)$$

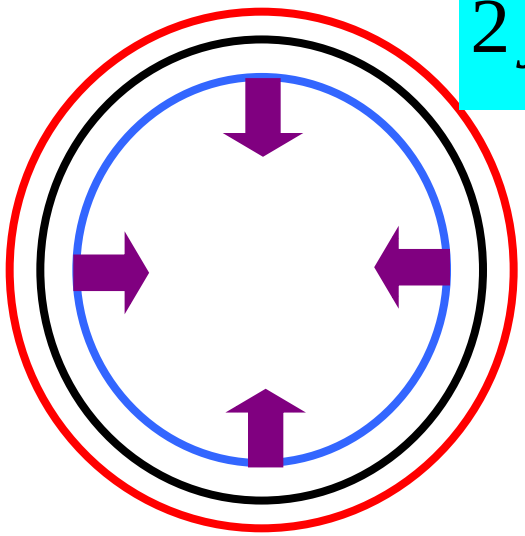
Time-reversed Step



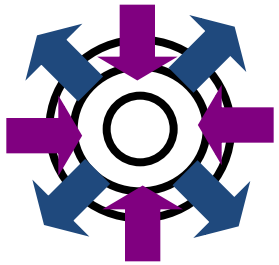
$$G^0_{adv}(R\ ,t)-G^0_{ret}(R,t)$$

The time-reversed step for monochromatic waves : origin of the diffraction limits

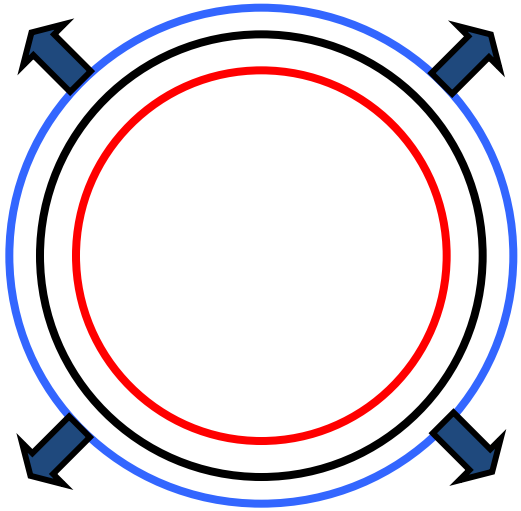
$$2j\,\mathrm{Im}\left\{\hat{G}^0_{ret}\left(R,\omega\right)\right\}$$



Converging only



Both converging and diverging waves interfere



Diverging only

$$\hat{G}^0_{adv}(R,\omega)=\frac{\exp\left\{j\left(-kR-\omega t\right)\right\}}{R}$$

with a singularity

$$\boxed{\frac{\sin\{kR\}}{R}\exp\left(-j\omega t\right)}$$

without singularity

Diffraction limit (λ/2)

$$-\hat{G}^0_{ret}=-\frac{\exp\left\{j\left(kR-\omega t\right)\right\}}{R}$$

with a singularity

J. de Rosny, M. Fink

Heterogeneous medium

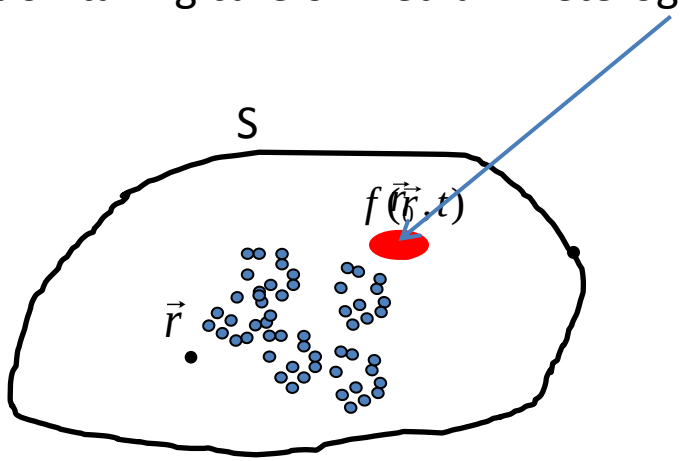
Phase 1 –Transmission by an acoustic source

$$\left\{\Delta - \frac{1}{c^2(\vec{r})}\frac{\partial^2}{\partial t^2}\right\}\varphi(\vec{r},t) = -\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(t\right)$$

For a point like source at time 0 located at $\vec{r}_0$

$$\varphi\left(\vec{r},t\right) = G\left(\vec{r},\vec{r}_0;t\right)$$

Causal Green's function taking care of medium heterogenitises

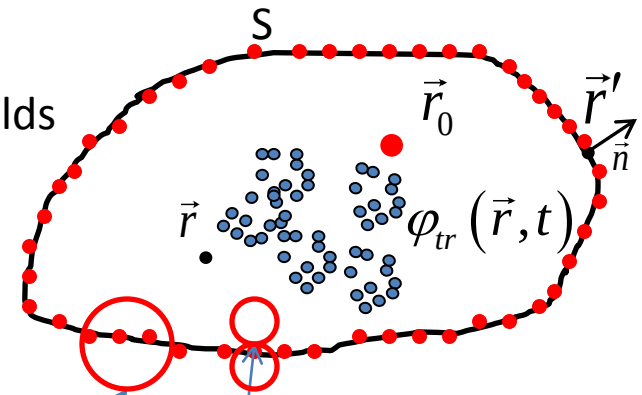


Phase 2 –Time-Reversed Wavefield

One records on every point $\vec{r}'$ the incident field and its normal derivative along surface S

And after a delay T, one transmits the time-reversed wavefields

$\varphi(\vec{r}',t') = G(\vec{r}',\vec{r}_0;T-t')$ and $\partial_n\varphi(\vec{r}',t') = \partial_n G(\vec{r}',\vec{r}_0;T-t')$



The diffraction Integral theorem gives the wavefield inside the volume at points $\vec{r}$ from its value and its normal derivative on points $\vec{r}'$ along surface S

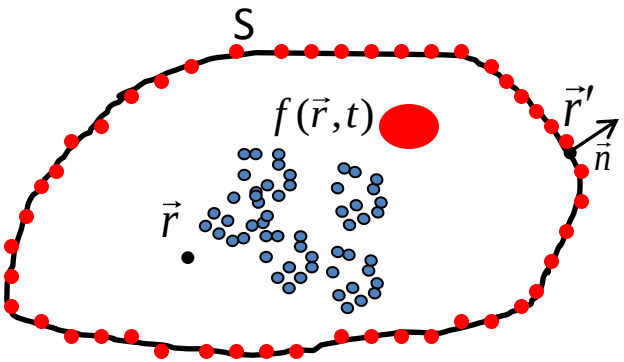
$$\boxed{\varphi_{tr}(\vec{r},t) = \int_{-\infty}^{+\infty} dt' \iint_S \left[G(\vec{r},\vec{r}';t-t')\,\partial_n\varphi(\vec{r}',t') - \partial_n G(\vec{r},\vec{r}';t-t')\,\varphi(\vec{r}',t') \right] \mathrm{d}^2\vec{r}' = ?}$$

monopole

dipole

Time-reversed Source : the Sink

$$\left\{\Delta - \frac{1}{c^2(\vec{r})}\frac{\partial^2}{\partial t^2}\right\}\varphi(\vec{r},t) = f(\vec{r},t)$$



One applies t $\rightarrow$ -t

$$\left\{\Delta - \frac{1}{c^2(\vec{r})}\frac{\partial^2}{\partial t^2}\right\}\varphi(\vec{r},-t) = f(\vec{r},-t)$$

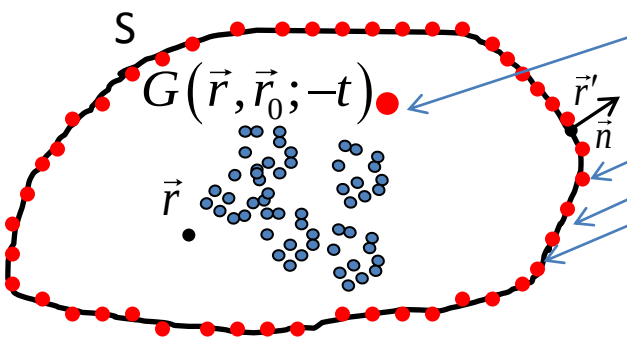
To obtain $\varphi(\vec{r},-t)$ we need also to use another source term $f(\vec{r},-t)$
This is the concept of an acoustic sink

For a ponctual source $f(\vec{r},t) = -\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(t\right)$

then $f(\vec{r},-t) = -\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(-t\right) = -\delta\left(\vec{r}-\vec{r}_0\right)\delta\left(t\right)$

To get $G(\vec{r},\vec{r}_0;-t)$ one needs both : 1. a time-reversed source $\boxed{f(\vec{r},-t)=-\delta(\vec{r}-\vec{r}_0)\delta(t)}$

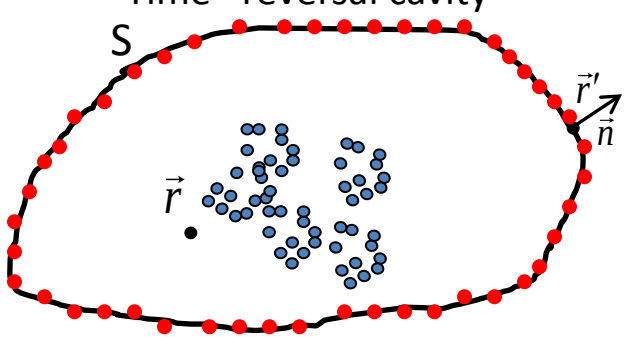
: 2. the time-reversed wavefield generated by surface sources (dipoles and monopoles) located along S



$$\boxed{\varphi_{tr}(\vec{r},t)=G(\vec{r},\vec{r}_0;-t)-G(\vec{r},\vec{r}_0;t)}$$

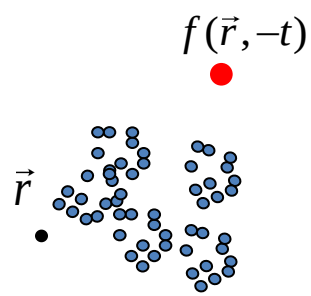
$G(\vec{r},\vec{r}_0;-t)$

Time –reversal cavity



+

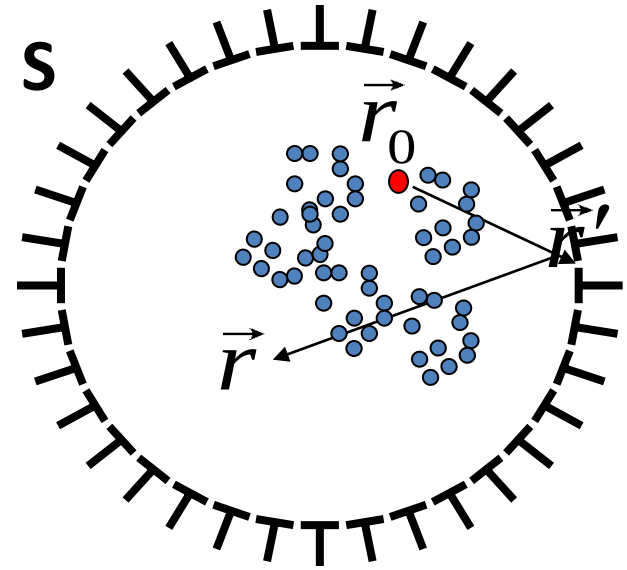
Time-reversed source



$$\varphi_{tr}(\vec{r},t) \quad + \quad G(\vec{r},\vec{r}_0;t) \quad = \quad G(\vec{r},\vec{r}_0;-t)$$

Theoretical description of an ideal time TR Cavity.

For any heterogeneous medium



For a Dirac excitation

$$\varphi_{tr}(\vec{r},t) = G_{ret}(\vec{r},\vec{r}_0;-t) - G_{ret}(\vec{r},\vec{r}_0;t)$$

For a monochromatic signal

$$\widehat{\Phi}_{tr}(\vec{r};\omega) = -2j\,\mathrm{Im}\,\widehat{G}(\vec{r},\vec{r}_0;\omega)$$

For a limited bandwidth signal

$$\varphi_{tr}(\vec{r},t) = -2j\int_{\Delta\omega}\mathrm{Im}\,\widehat{G}(\vec{r},\vec{r}_0;\omega)e^{j\omega t}d\omega$$

$$\varphi_{tr}(\vec{r},t=0) = -2j\int_{\Delta\omega}\mathrm{Im}\,\widehat{G}(\vec{r},\vec{r}_0;\omega)d\omega$$

Field at the focal time. at $\vec{r} = \vec{r}_0$ related to **LDOS** (local density of states)

D. Cassereau, M. Fink, Sept 1992, IEEE UFFC

Electromagnetic Time-Reversed Waves

$$\nabla'_r \times \nabla'_r \times \overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,t) - \frac{\varepsilon(\vec{r},\omega)}{c_0^{\,2}} \frac{\partial^2}{\partial t^2} \overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,t) = -\delta(\vec{r}-\vec{r}_0)\,\delta(t)\,\overleftrightarrow{\mathbf{I}}$$

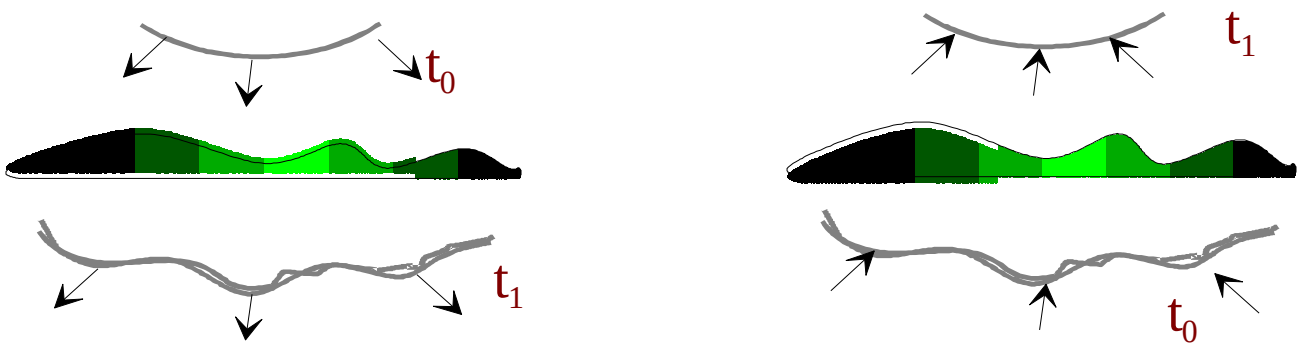
Dipole source

Dyadic Green's function

$$\mathbf{E}(\vec{r},t) = \mu_0 \frac{\partial^2}{\partial t^2} \overleftrightarrow{\mathbf{G}}(\vec{r},\vec{r}_0,t) \otimes \mathbf{p}(t)$$

$\vec{r}_0$

Dual solutions



$$\overleftrightarrow{\mathbf{G}}_{ret}(\vec{r},\vec{r}_0,t)$$

For a monochromatic signal

$$\mathbf{E}_{tr}(\vec{r},\omega) = -2\,j\,\mu_0\,\omega^2\,\mathrm{Im}\left[\hat{\overleftrightarrow{\mathbf{G}}}(\vec{r},\vec{r}_0,\omega)\right]\mathbf{p}^*$$

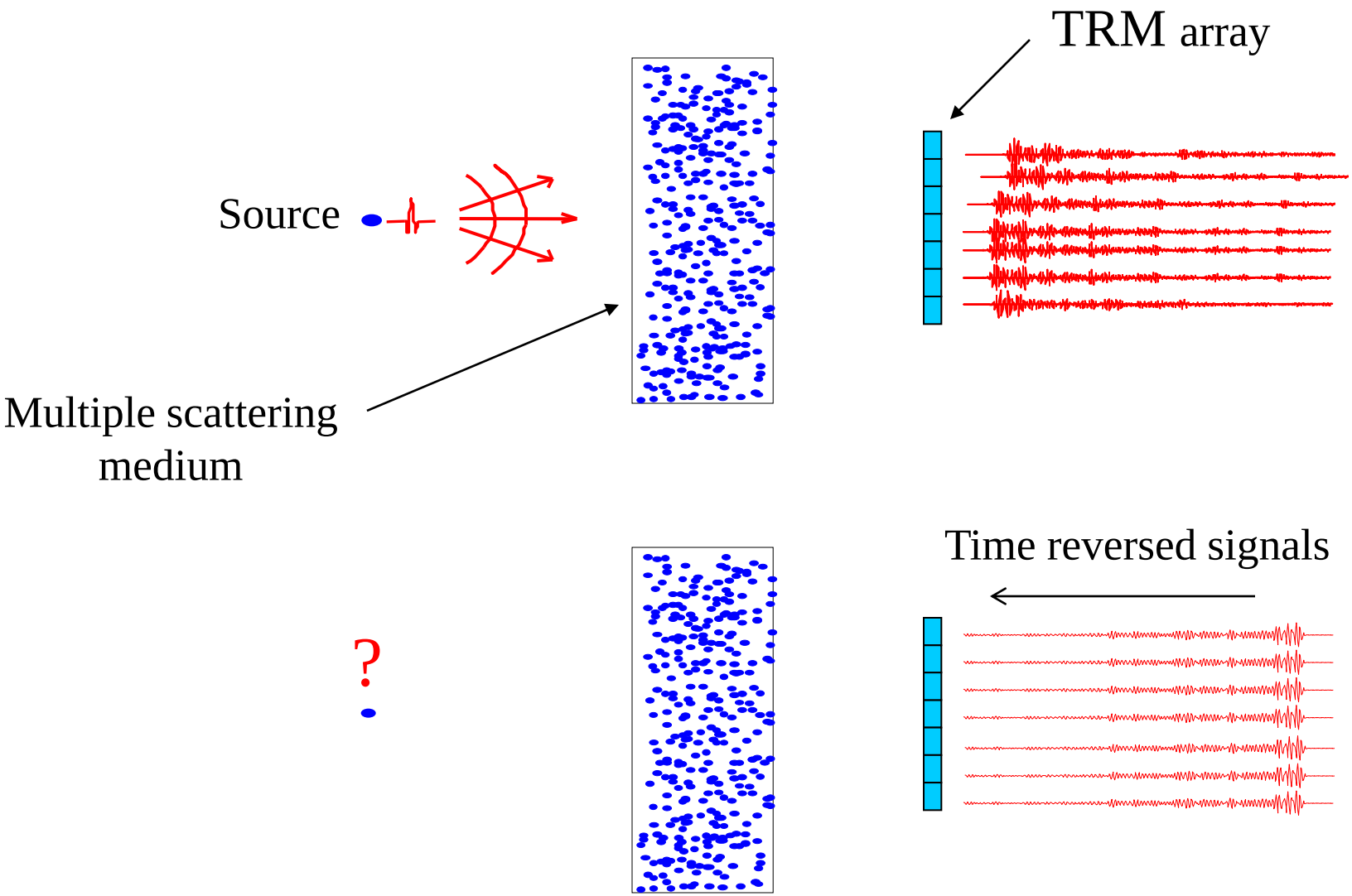
R. Carminati, R. Pierrat,
J de Rosny, G. Lerosey, M. Fink

How to decrease the number of antenna in a time-reversal mirror ?

- Introducing a diffusive medium between the source and the TRM
- Experiment inside a reverberating cavity
- a source embedded in a smart material made of subwavelength resonators

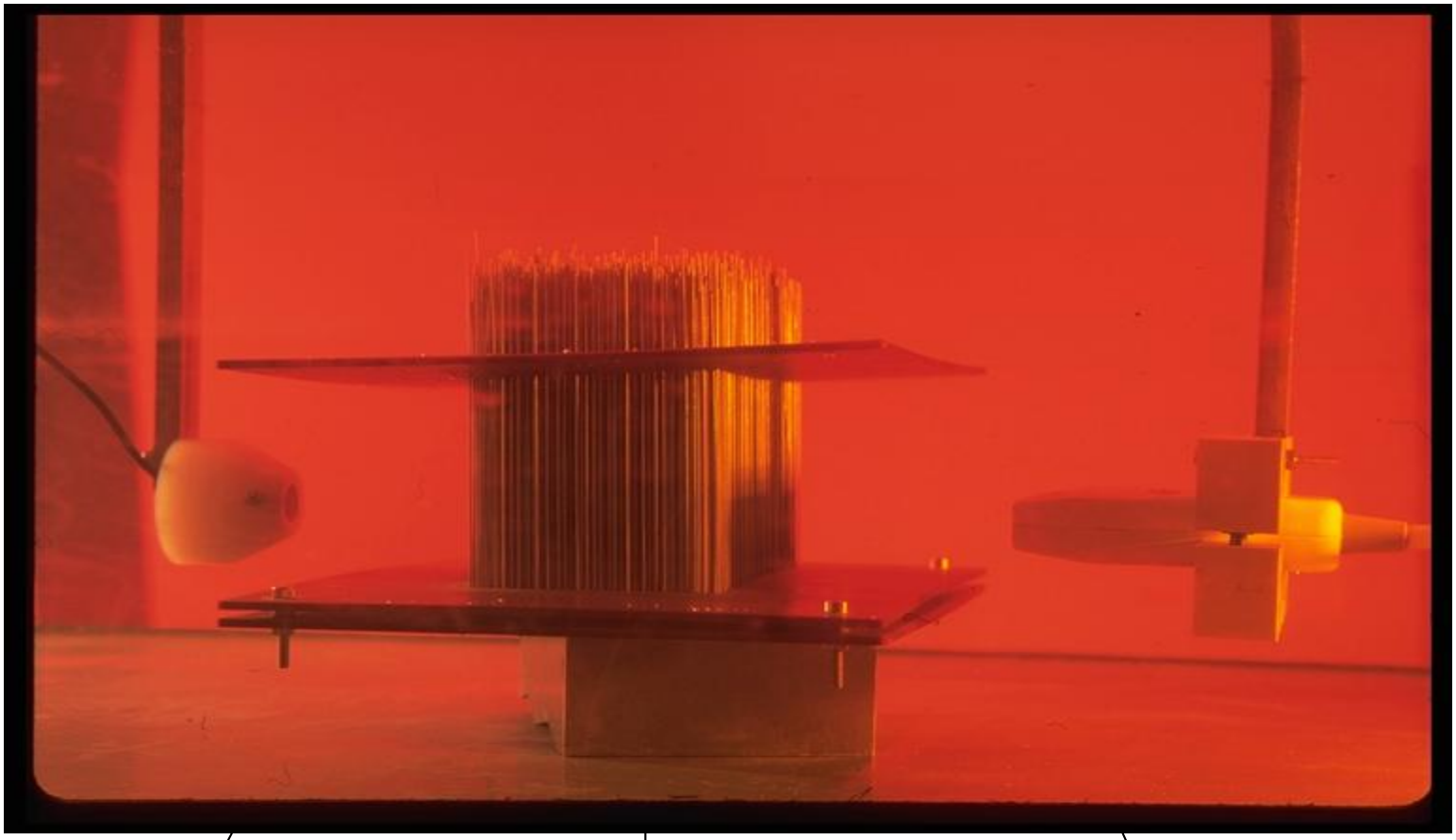
- **Broadband signals**

Case 1 . Time Reversal through a diffusive medium



A.Derode, A. Tourin, P. Roux, M. Fink, Phys Rev Let , 75, 23, p 4206-4209, 1995

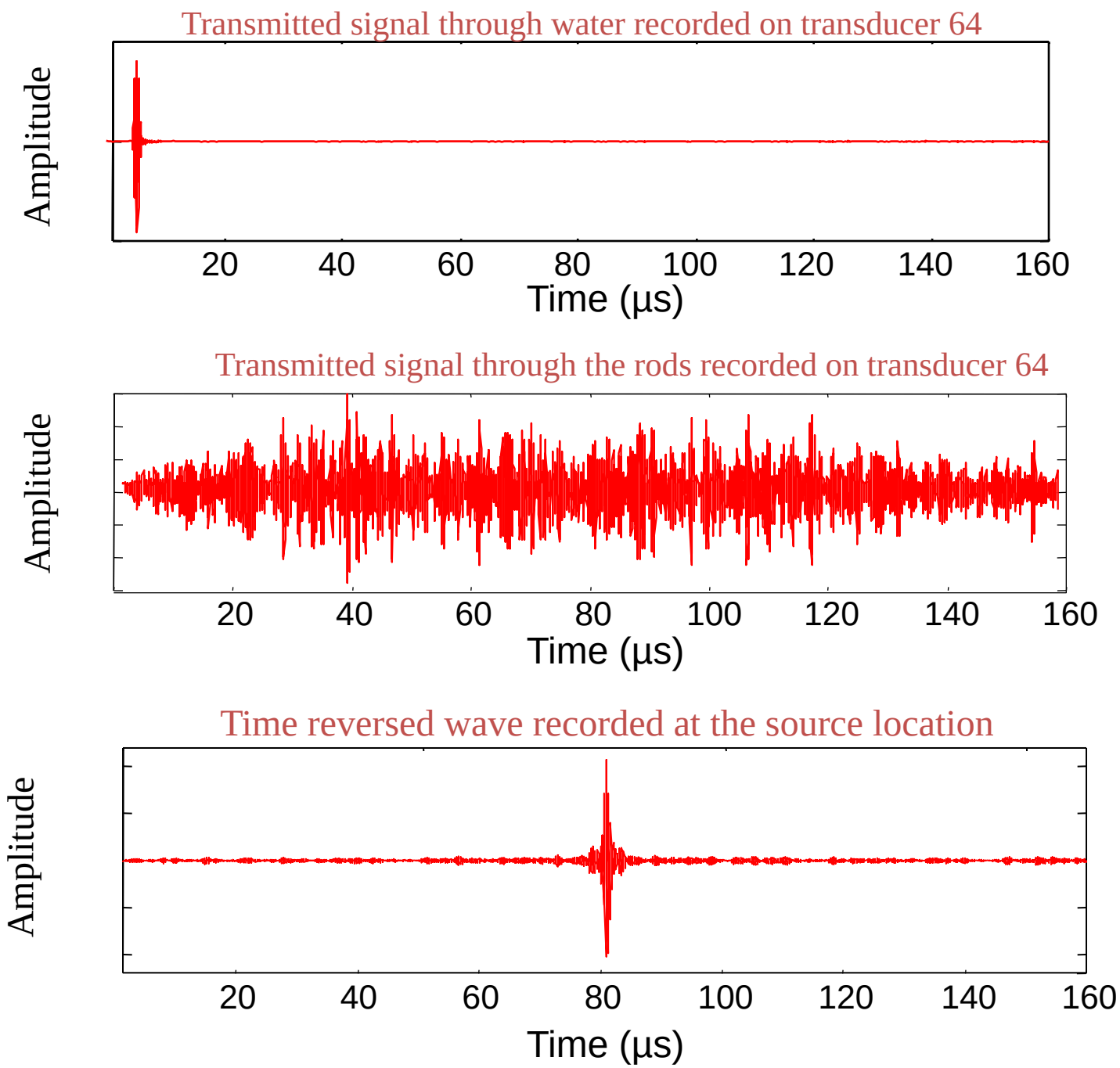
The experimental setup



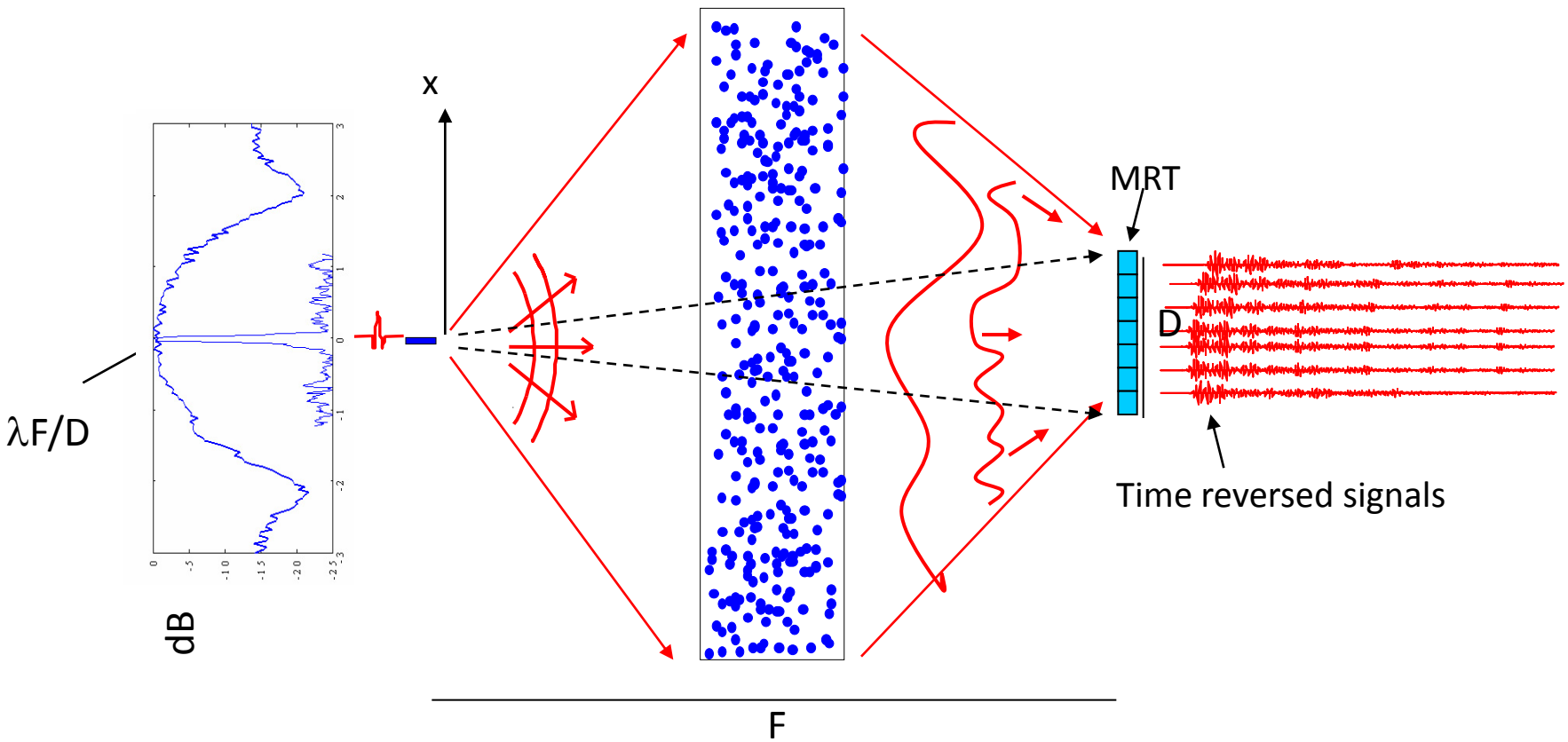
Acoustic source
ν=3 MHz, λ=0.5 mm

Steel rods forest

Linear array, 128 transducers
Element size ¾λ


Transmitted signal through water recorded on transducer 64
Amplitude
20
40
60
80
100
120
140
160
Time (µs)
Transmitted signal through the rods recorded on transducer 64
Amplitude
20
40
60
80
100
120
140
160
Time (µs)
Time reversed wave recorded at the source location
Amplitude
20
40
60
80
100
120
140
160
Time (µs)

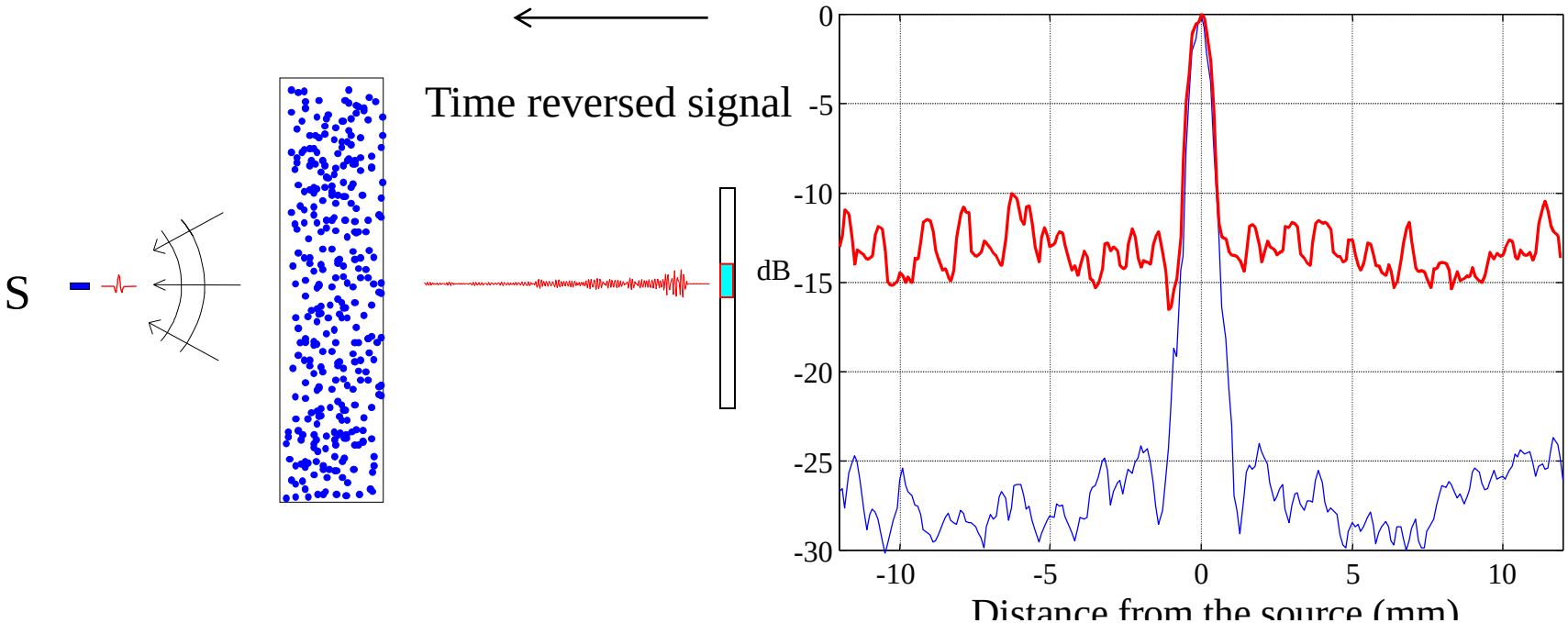
Spatial Focusing



Focal spot : beamwidth at -6 dB : **35 mm / 1 mm**

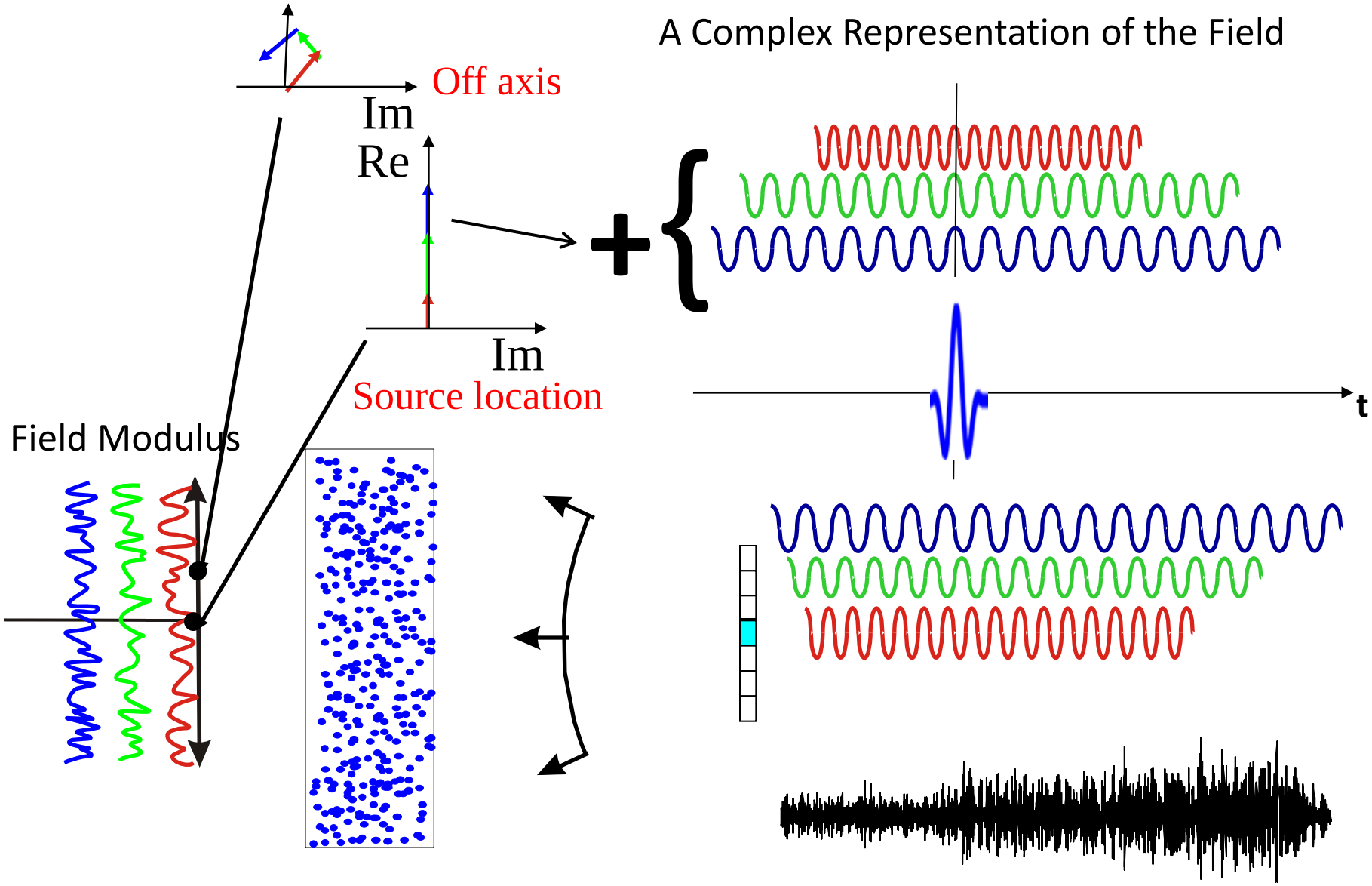
Spatial resolution does not depend of the array aperture !!!

One channel time reversal mirror

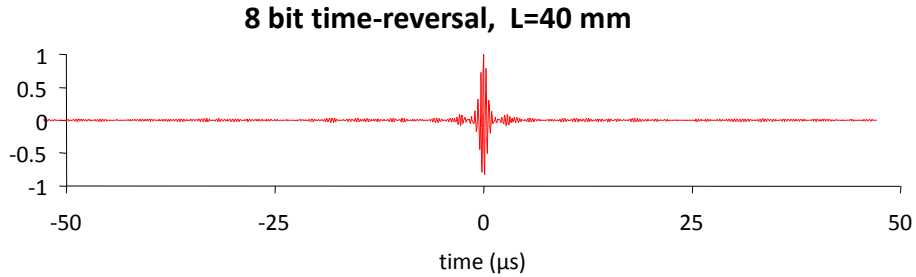


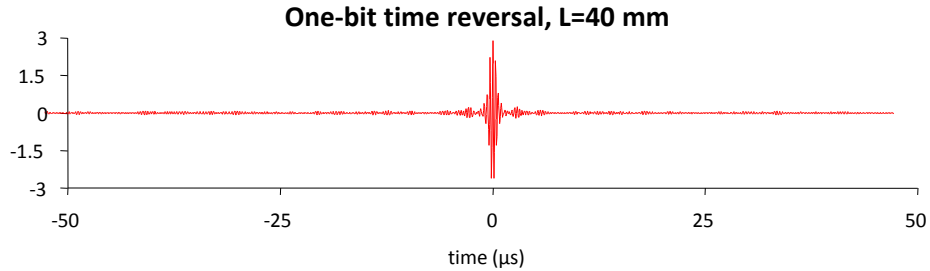
Directivity patterns of the time-reversed waves around the source position with 128 transducers (blue line) and 1 transducer (red line).

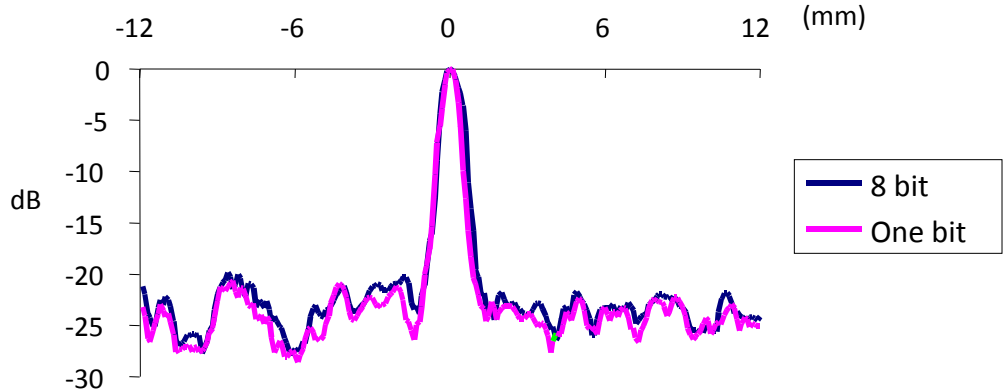
Polychromatic Focusing



One-bit versus 8-bit time reversal

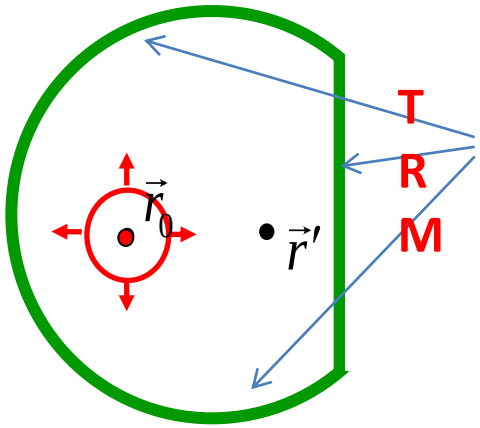






A one channel time-reversal mirror (TRM)

- Case 2



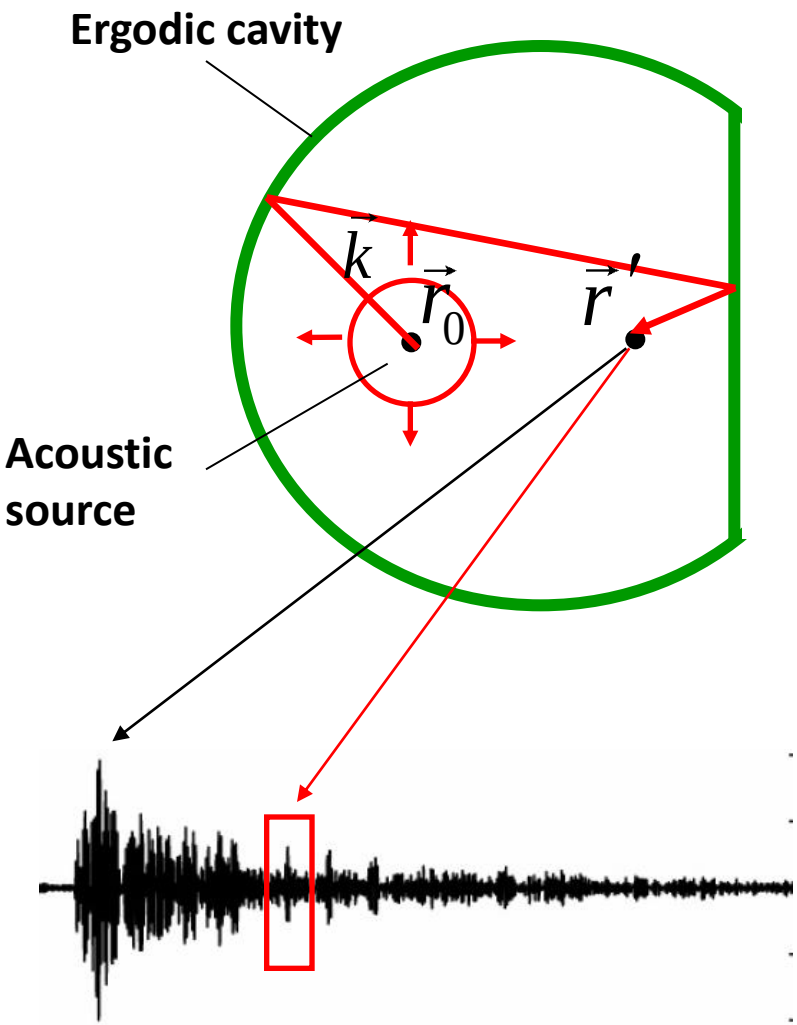
Reflecting boundaries in the far field redirect the Incident field

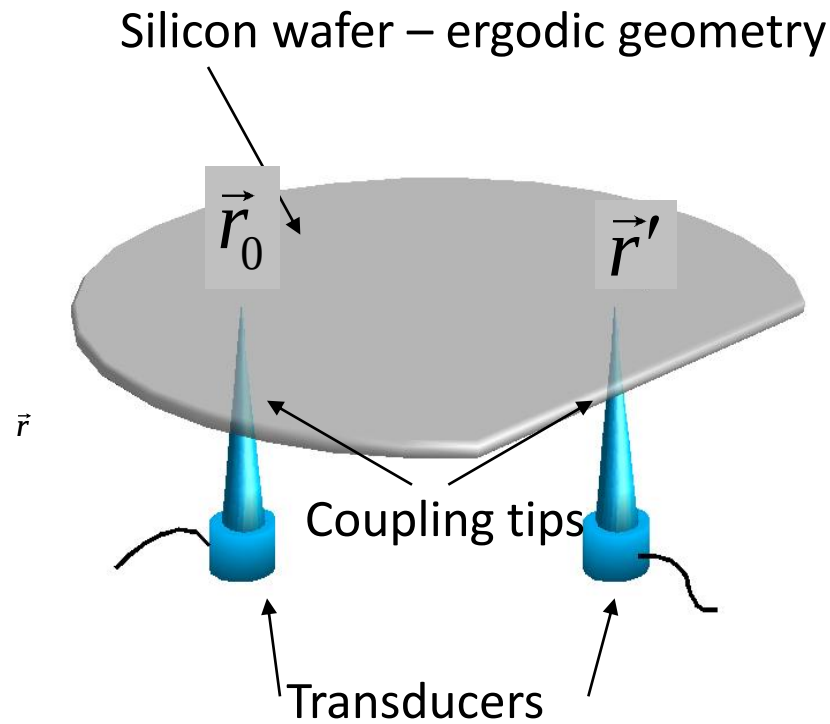
- Case 3

in open space

Resonating sub-wavelength scattererers in the near field of the source

Case 2 . Time-Reversal in a chaotic cavity with a one channel TRM





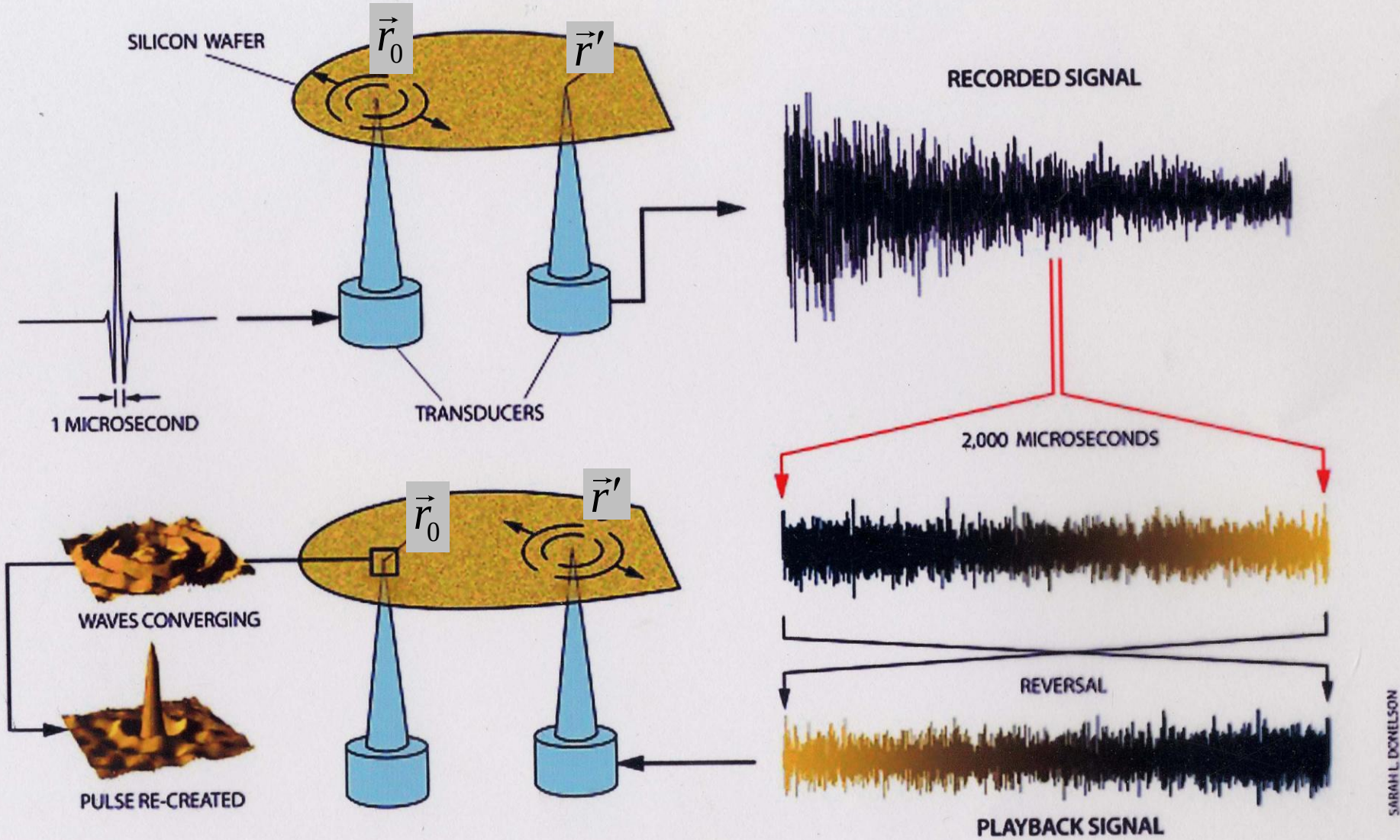
Space – time coupling

each $\vec{k}$ **is coded in a time t**

$$\Delta k_i . \Delta r_i \Rightarrow \Delta t . \Delta \omega$$

C Draeger, J de Rosny, M. Fink

C. Draeger, M. Fink, Phys Rev Lett **9** (3), p.407-410, 1997

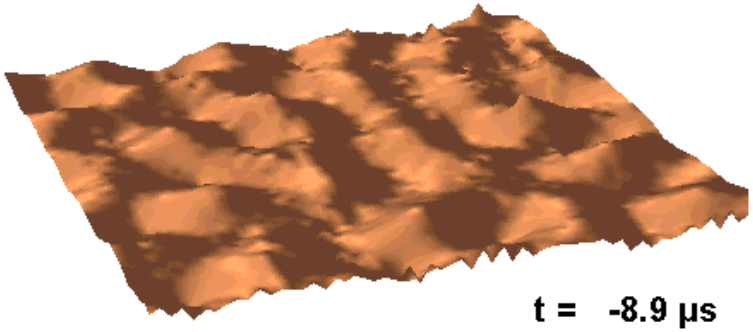


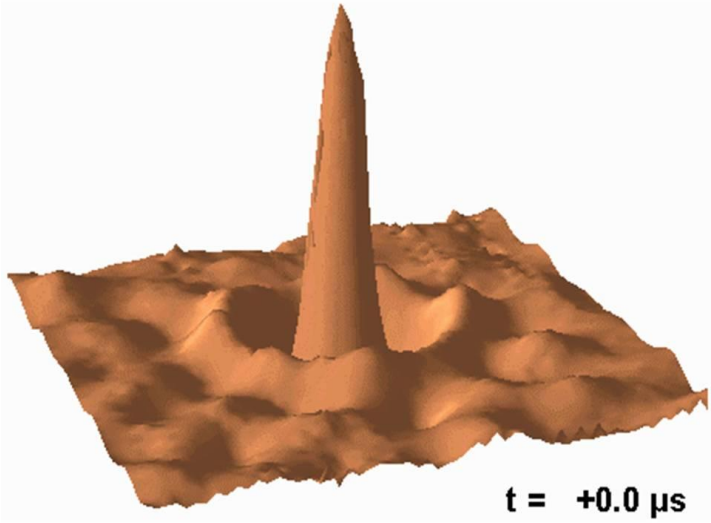
SINGLE TRANSDUCER can time-reverse a wave in an enclosed "cavity." A source transducer emits a pulse at location A on a small silicon wafer (*top*). A transducer at location B records chaotic reverberations of the pulse reflected off the wafer edges hundreds of times. The transducer at B plays back a short segment of that signal in reverse (*bottom*). After many reflections, these recombine to re-create the short pulse focused again at location A, as was revealed by imaging the waves on the wafer near A (*bottom left*).

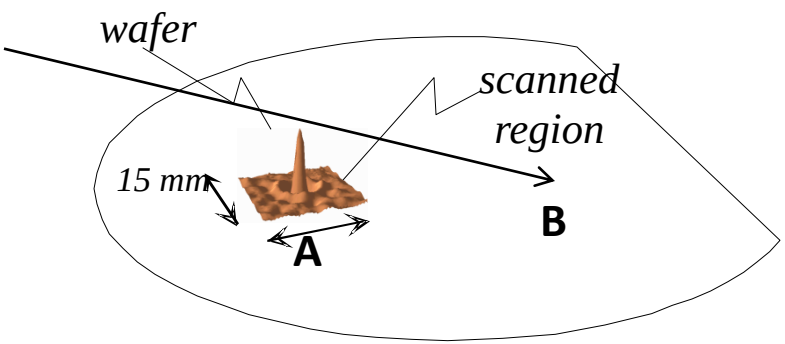
Time-reversed wave observed with optical interferometer

Transmission from point B of a 2 ms temporal **code**

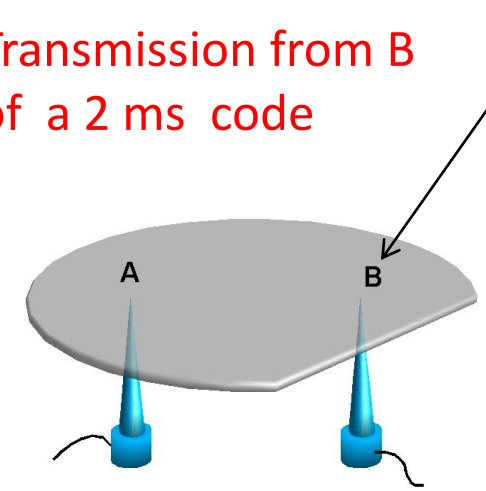
Around point A 15mm x15 mm

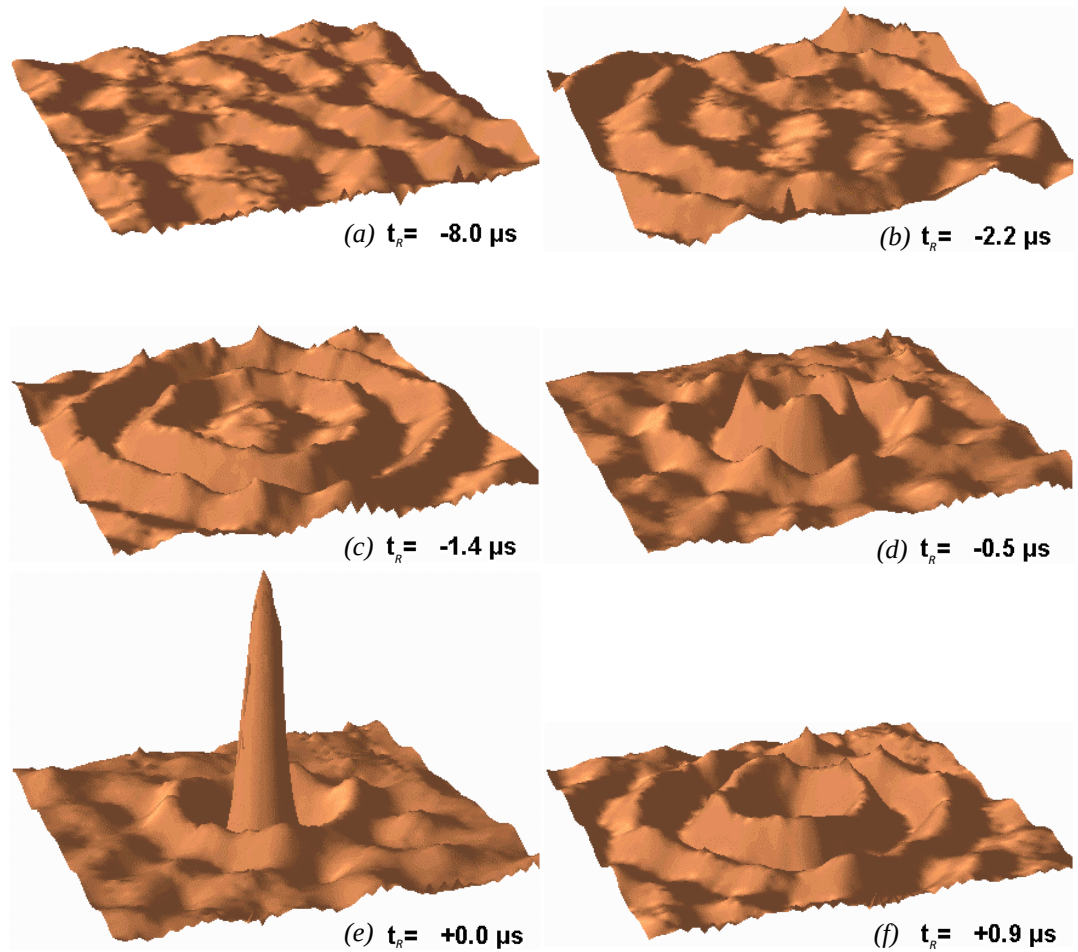






Transmission from B of a 2 ms code

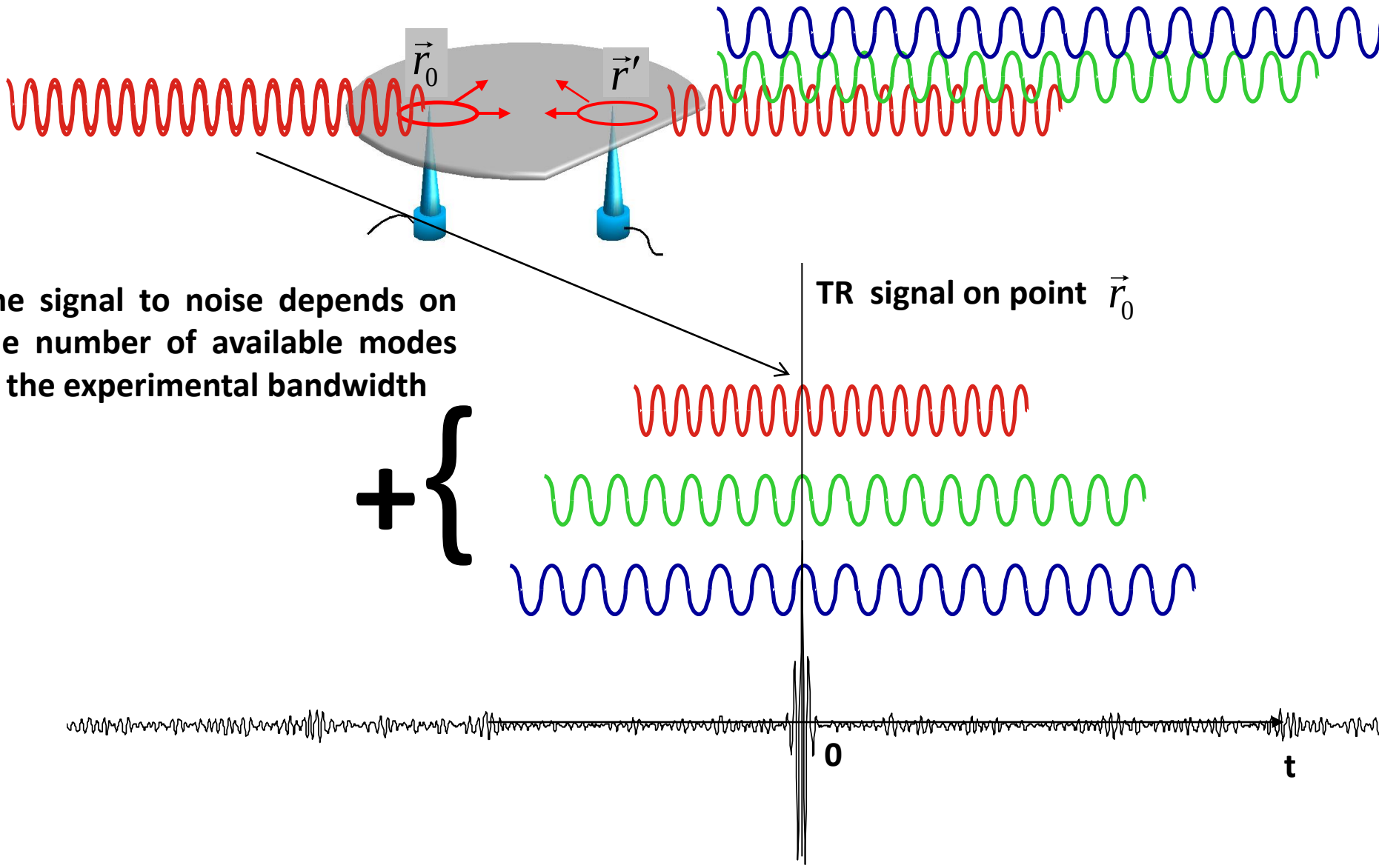




The signal to noise Is proportional to the bandwidth square root $\sqrt{\frac{\Delta\omega}{\delta\omega}}$

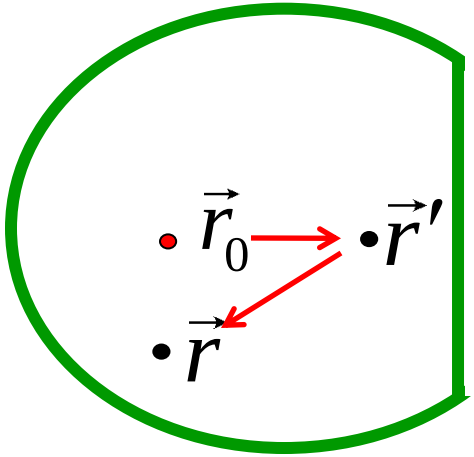
Indeed, a monochromatic source located at B will never focused at A

The one-channel TRM works only for broadband signal



The signal to noise depends on the number of available modes in the experimental bandwidth

Multi-modal Focusing



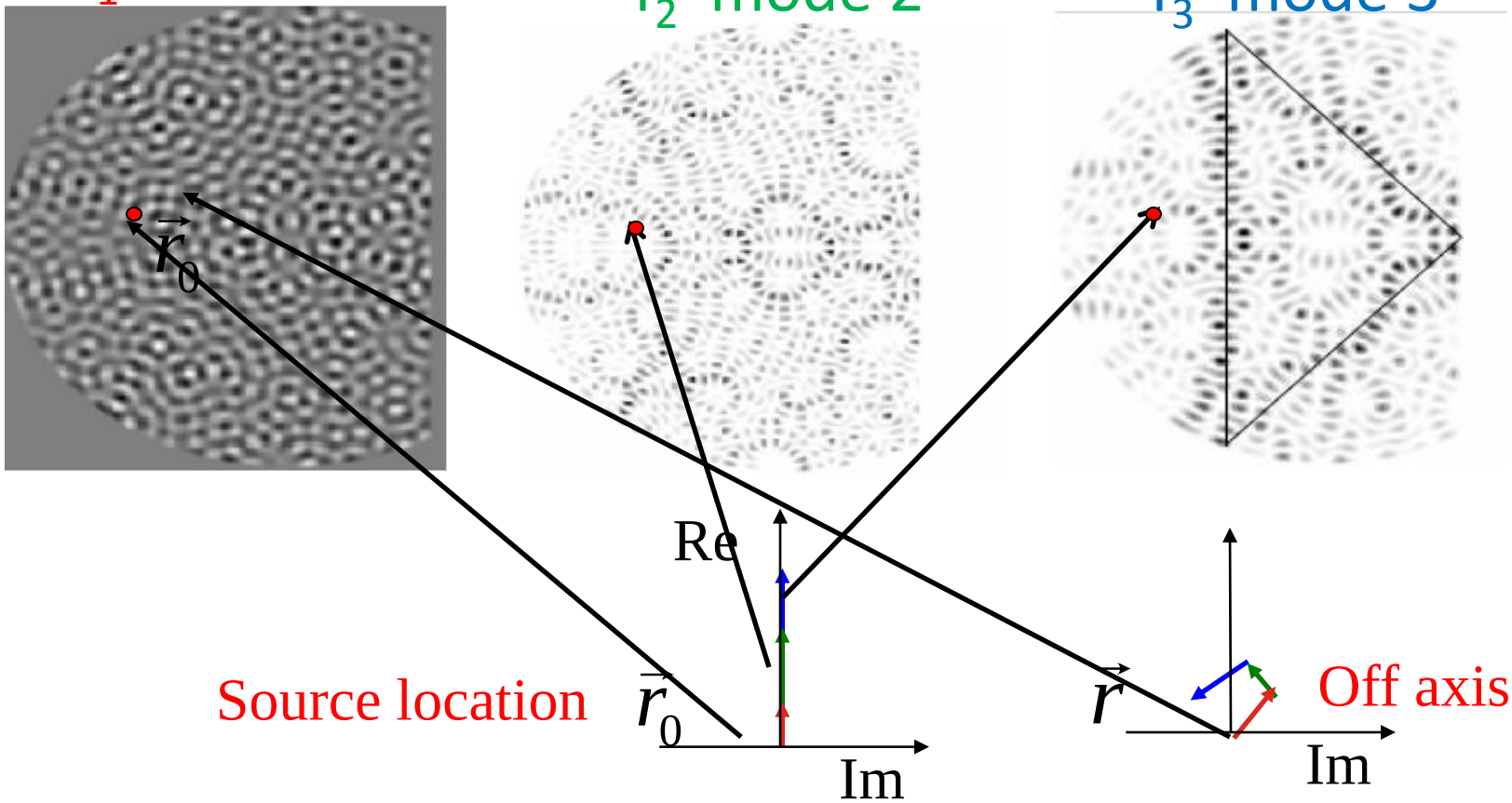
Eigenmodes u_n with $\Delta u_n = -\dfrac{\omega_n^2}{c^2} u_n$

$$G(\vec{r}\,',\vec{r}_0;t) = \sum_n u_n(\vec{r}_0) u_n(\vec{r}\,') \frac{\sin(\omega_n t)}{\omega_n}$$

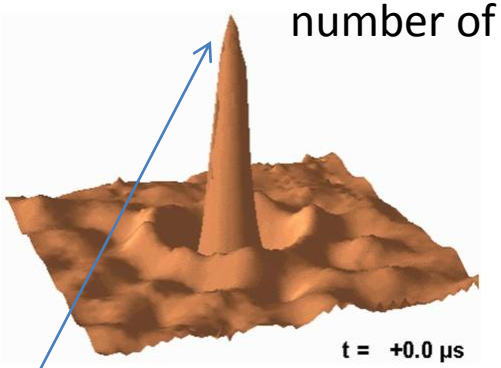
f_1 mode 1

f_2 mode 2

f_3 mode 3

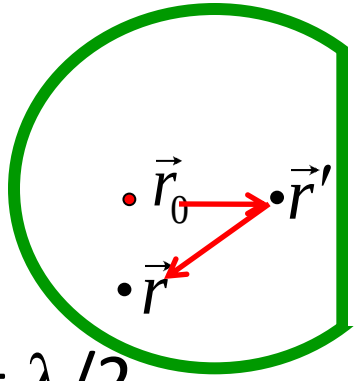


The focal spot



number of modes

$$J_0(2\pi|\vec{r}-\vec{r}_0|/\lambda_n)$$

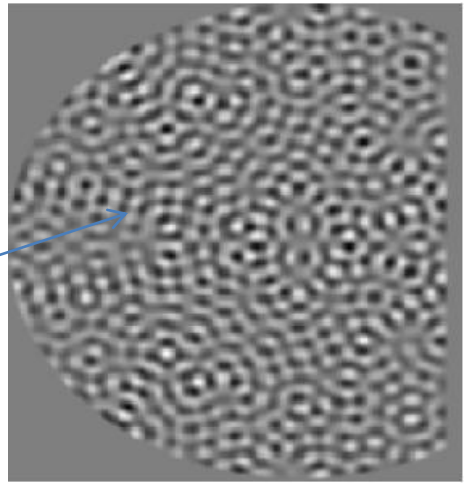


Size of the focal spot = λ/2

$$\varphi_{tr}(\vec{r},t=0) \approx -2j\int_{\Delta\omega} \mathrm{Im}\,\hat{G}(\vec{r},\vec{r}_0;\omega)\,d\omega$$

Here we have 400 uncorellated eigenmodes

The size of the focal spot is of the order of the size of one modal cell

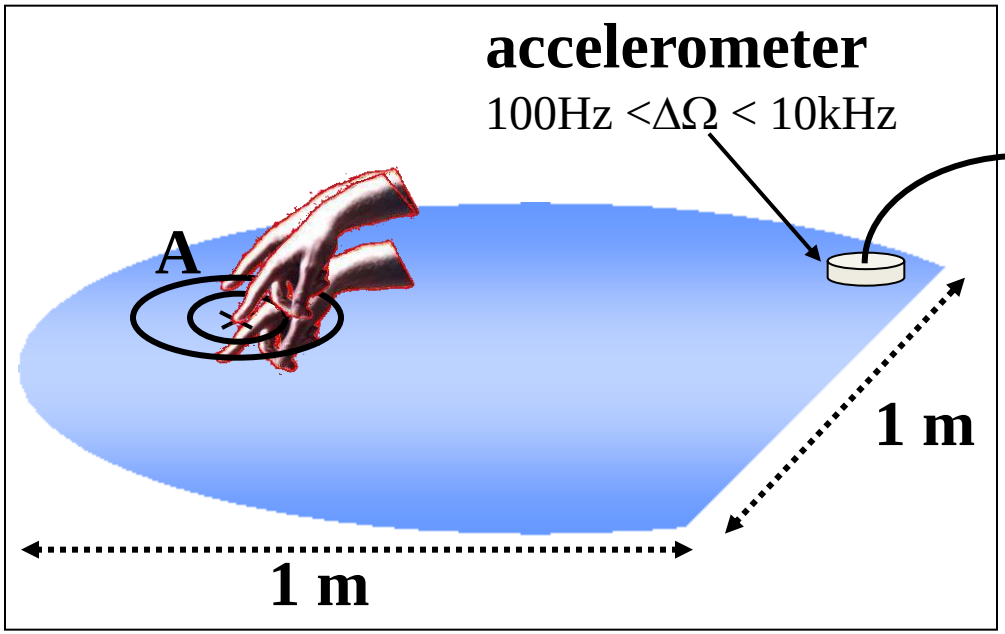


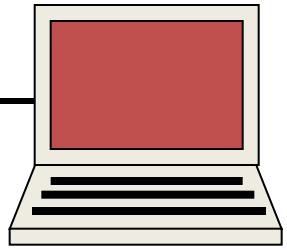
How to get focal spot smaller than the wavelength ?

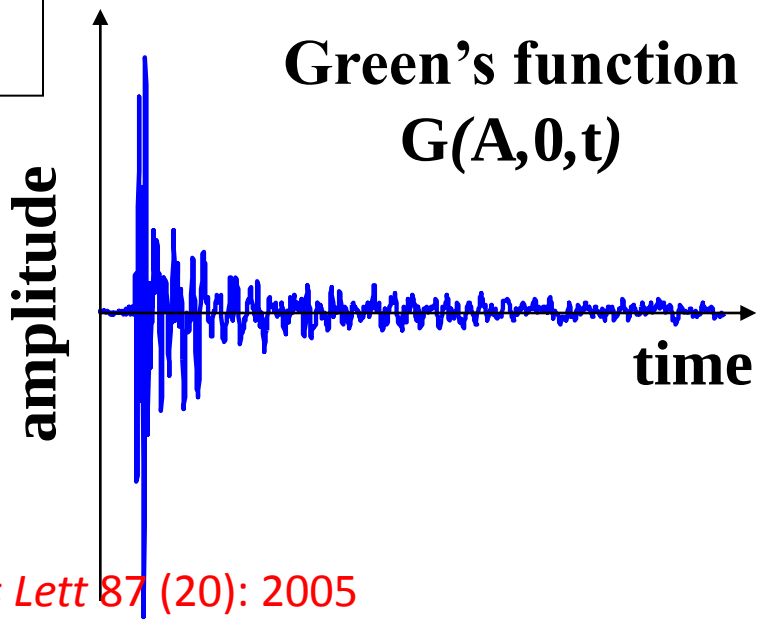
Building materials with sub-wavelength resonators closed to the source

A nice application: Tactile objects

How to transform any solid object in a tactile device (screen) ?







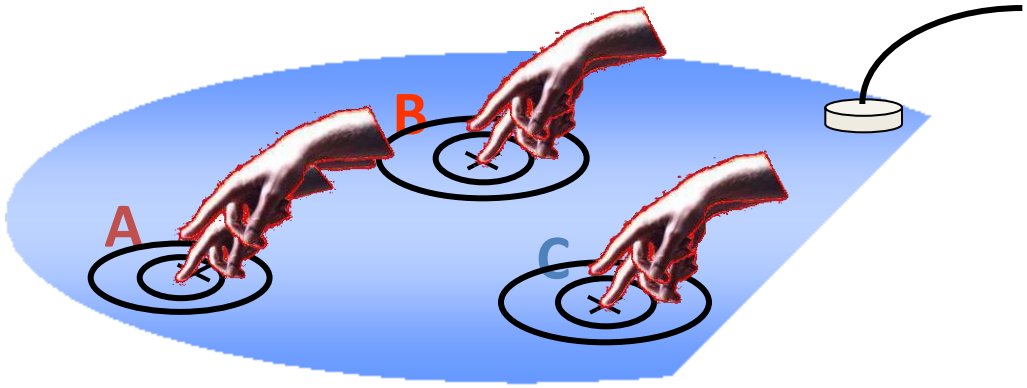
A start-up Sensitive Object founded in 2003 (30 members) sailed in 2010 to Tyco Electronics

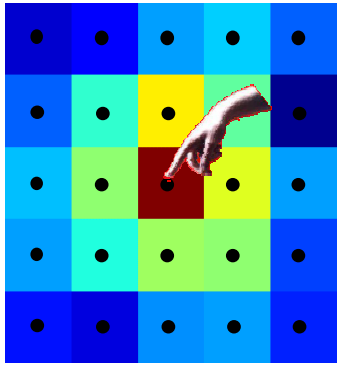
sensitive Object

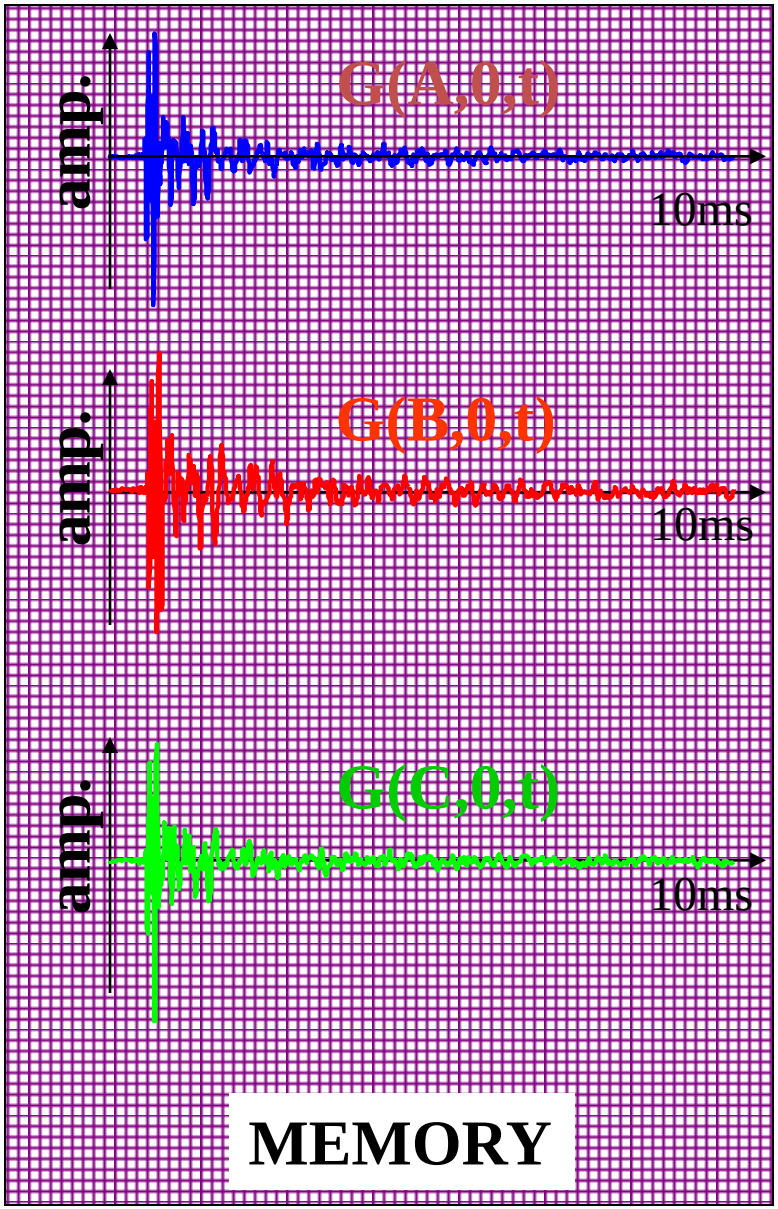
R.K Ing, N. Quieffin, S. Catheline, M. Fink, Appl Phys Lett 87 (20): 2005

Teaching Green's functions

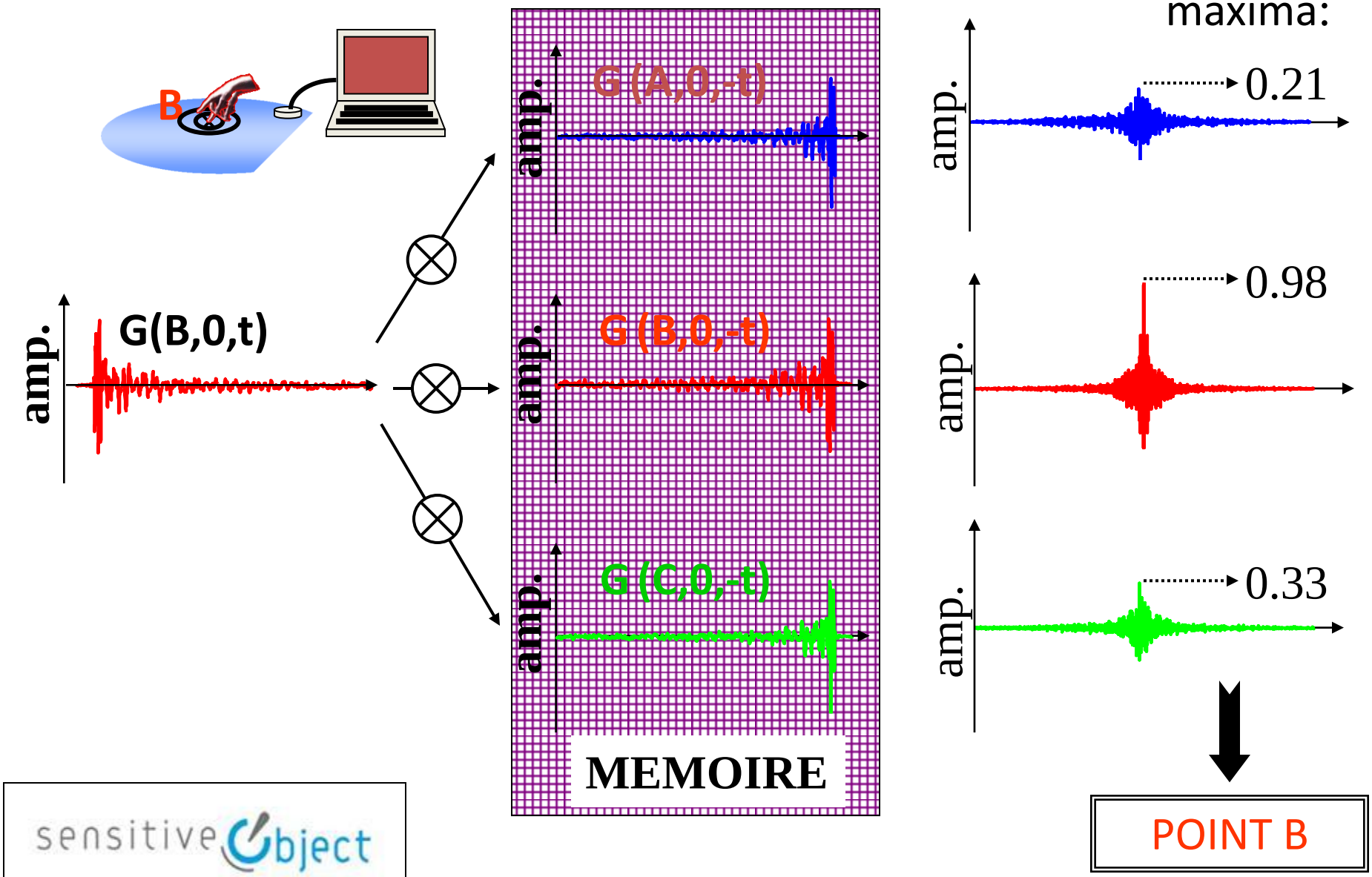
sensitive Object





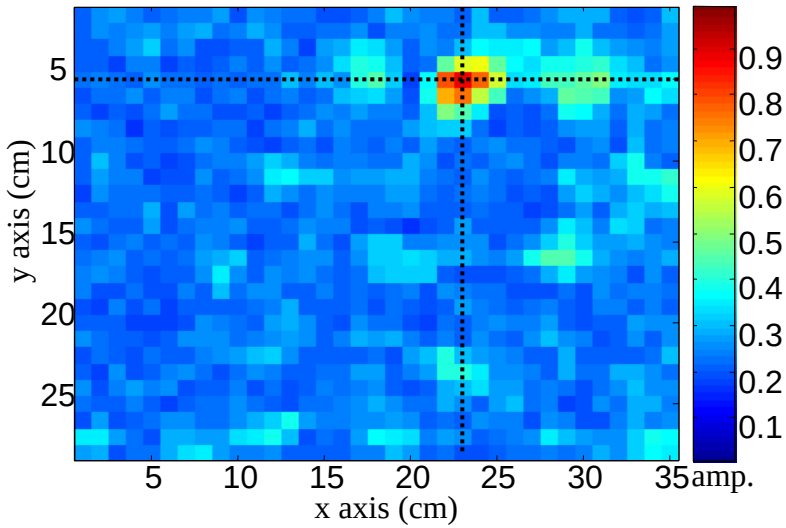


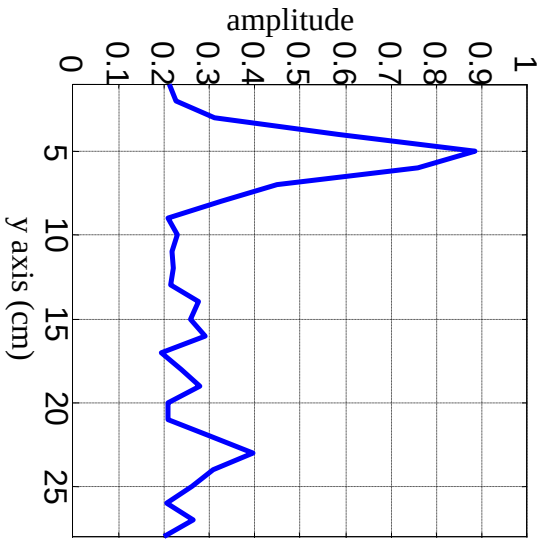
Source localisation by cross-correlattion
Numerical simultation of TR

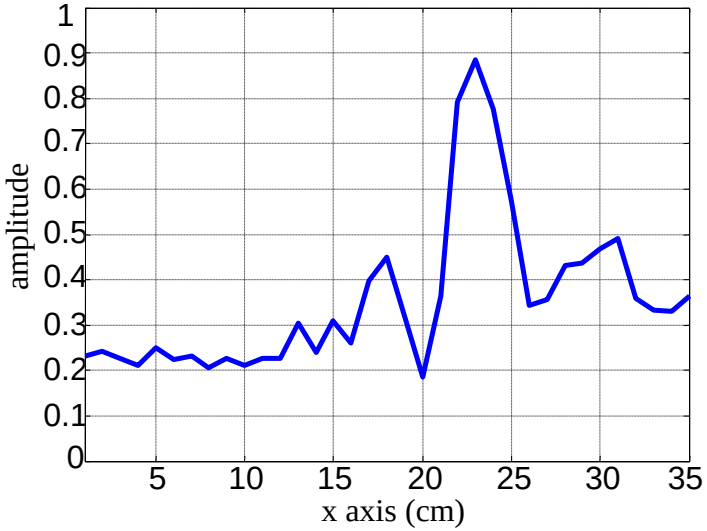


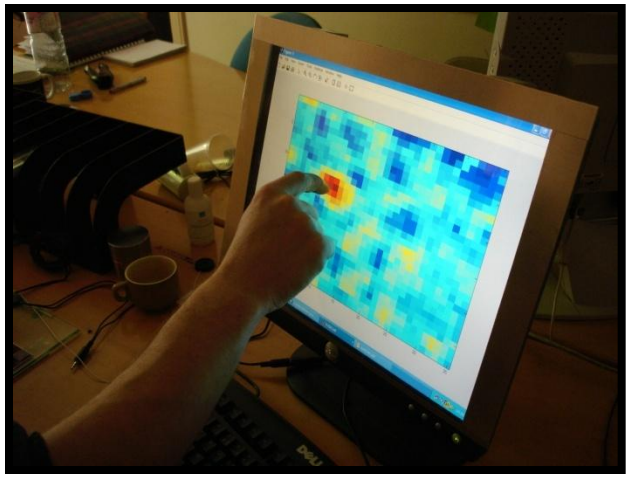
sensitive Object

The focal spot

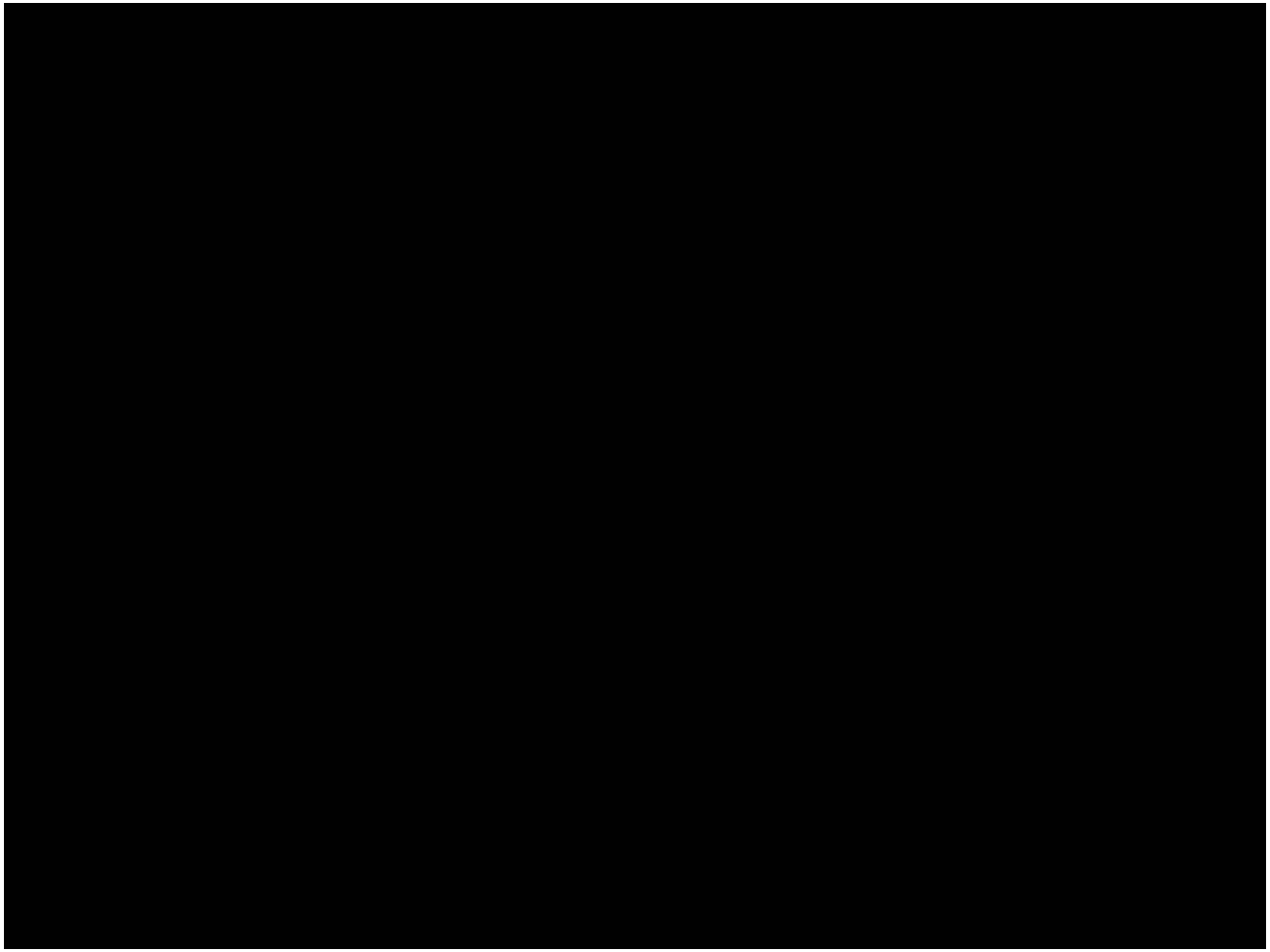






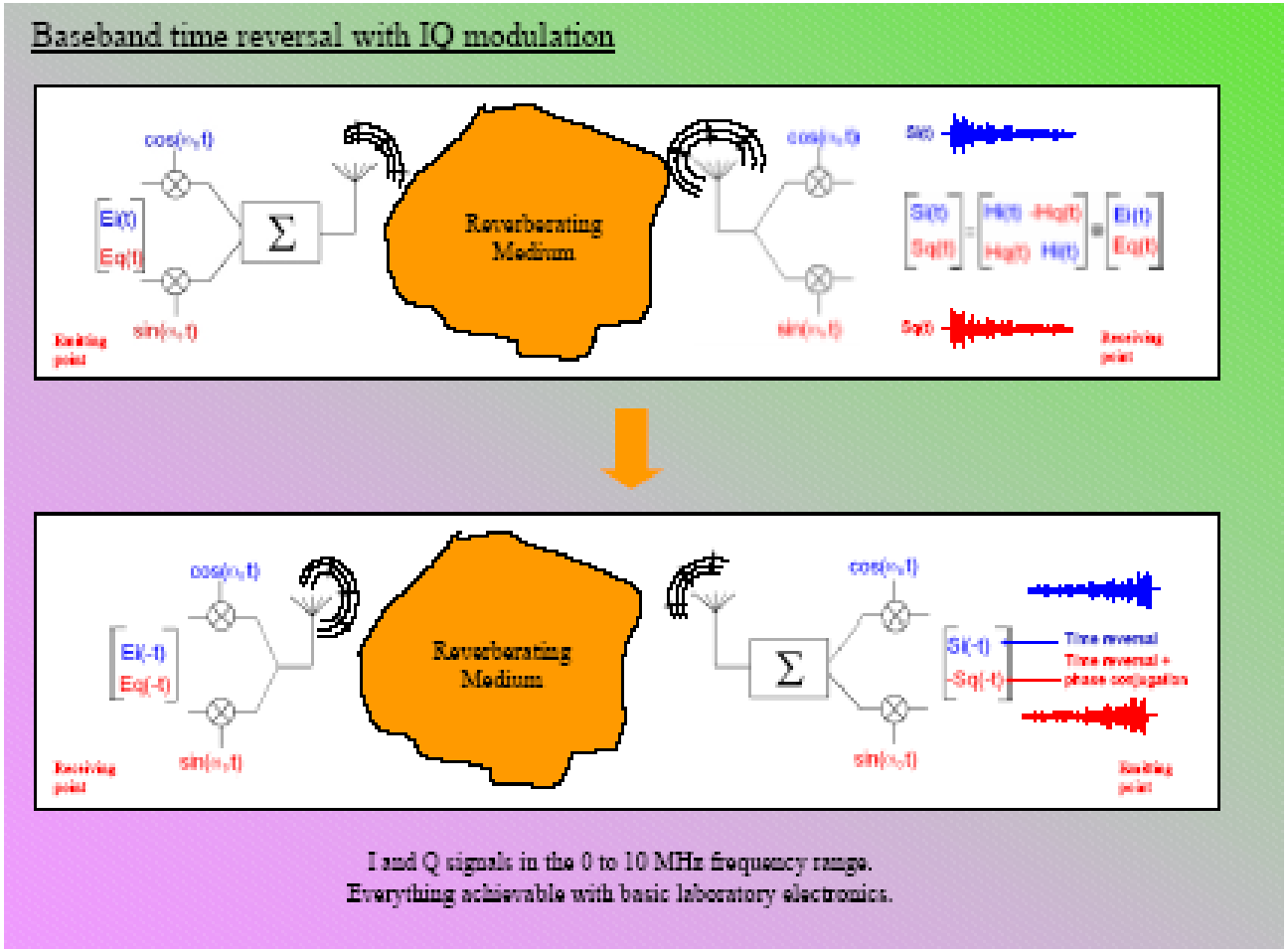


Products



sensitive Object

Time Reversal of Electromagnetic Waves (2.4 GHz)

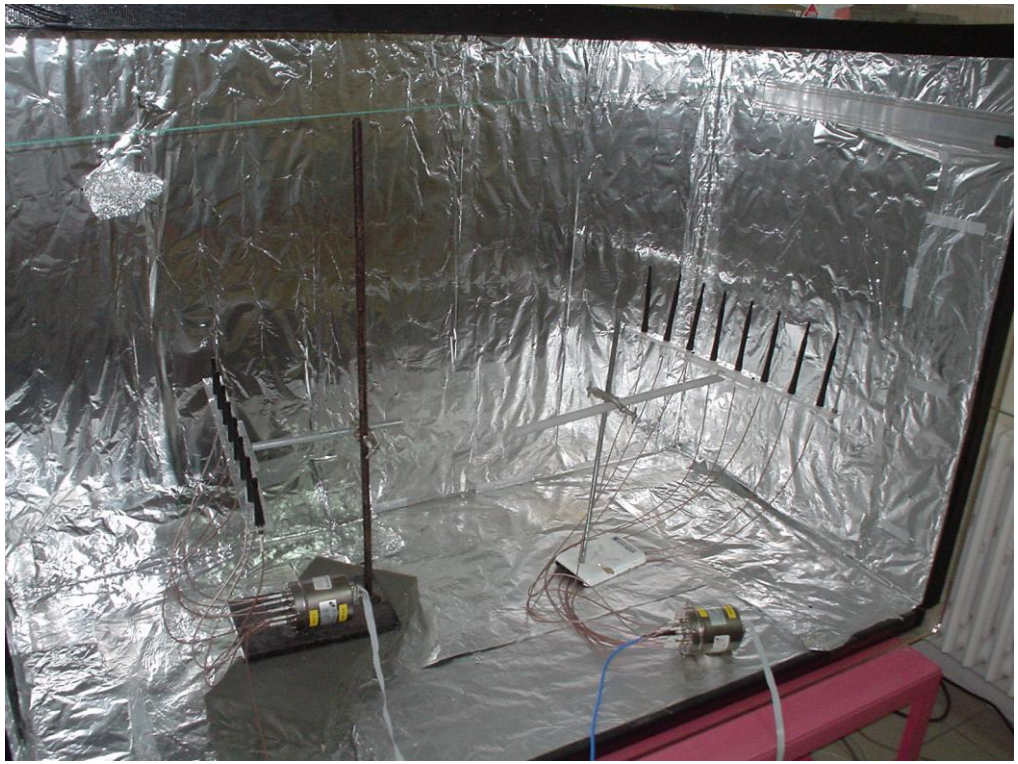


G.Lerosey , J de Rosny, A Tourin, M Fink,

Electromagnetic TRM

• 2 arrays of 8 antennas separated of approx 6.15 cm, i.e. half a wavelength (12.3cm @ 2.44 GHz) inside a reverberating cavity

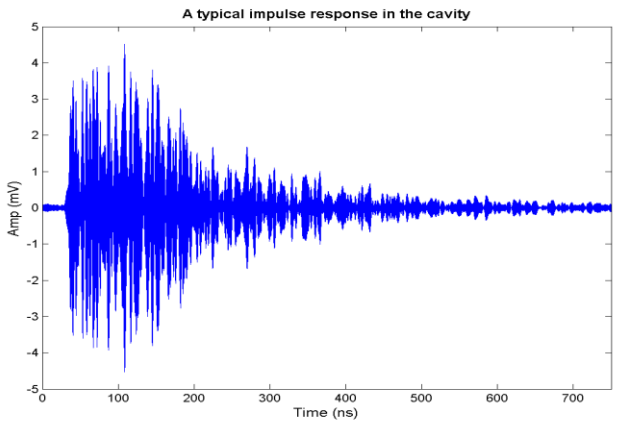
•

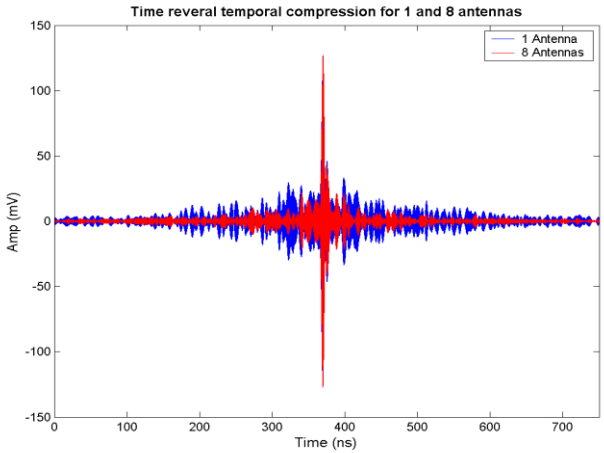


G Lerosey, J. de Rosny, A Tourin, M Fink, Phys Rev Lett **92**, 194301, 2004

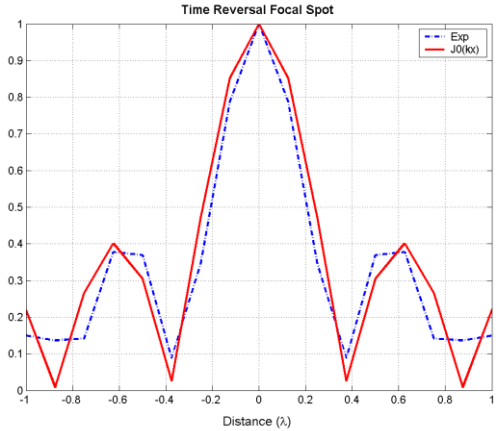
Spatio-temporal focusing for high rate communications

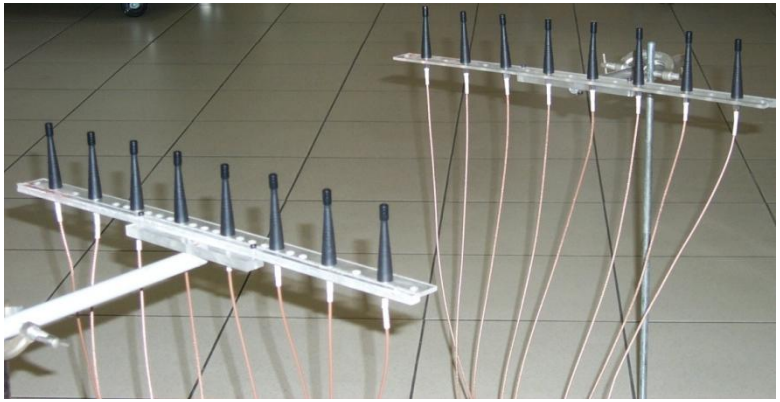
initial pulse 10 ns





Time compression





Spatial focusing $\lambda/2$

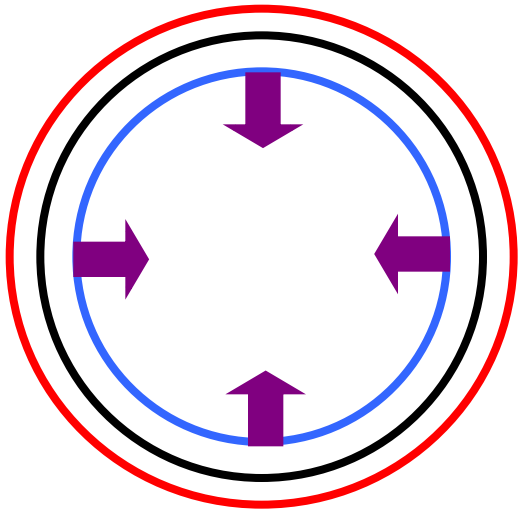
Super Resolution

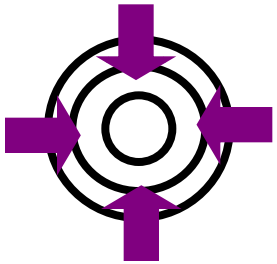
1- The Sink (active or passive)

2- Medium with sophisticated Green's functions

$$\varphi_{tr}\left(\vec{r},t=0\right)=-2j\int_{\Delta\omega}\operatorname{Im}\widehat{G}\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

The Acoustic Sink





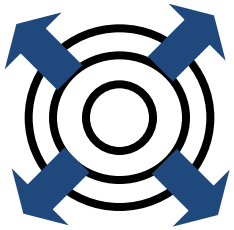
Converging only

$$\hat{G}^{0}_{adv}(R,\omega) \propto \frac{\exp\{j(-kR-\omega t)\}}{R}$$

with a singularity

J. de Rosny, M. Fink, Phys Rev Let 89 (12), p 124301, 2002

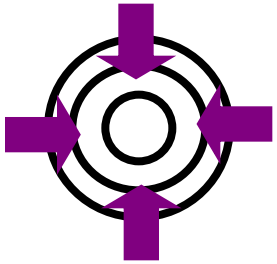
The Acoustic Sink Formalism



$$\left(\Delta - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\varphi(\vec{r},t) = f(t)\delta(\vec{r}-\vec{r}_0)$$

Propagating term

Point-like source



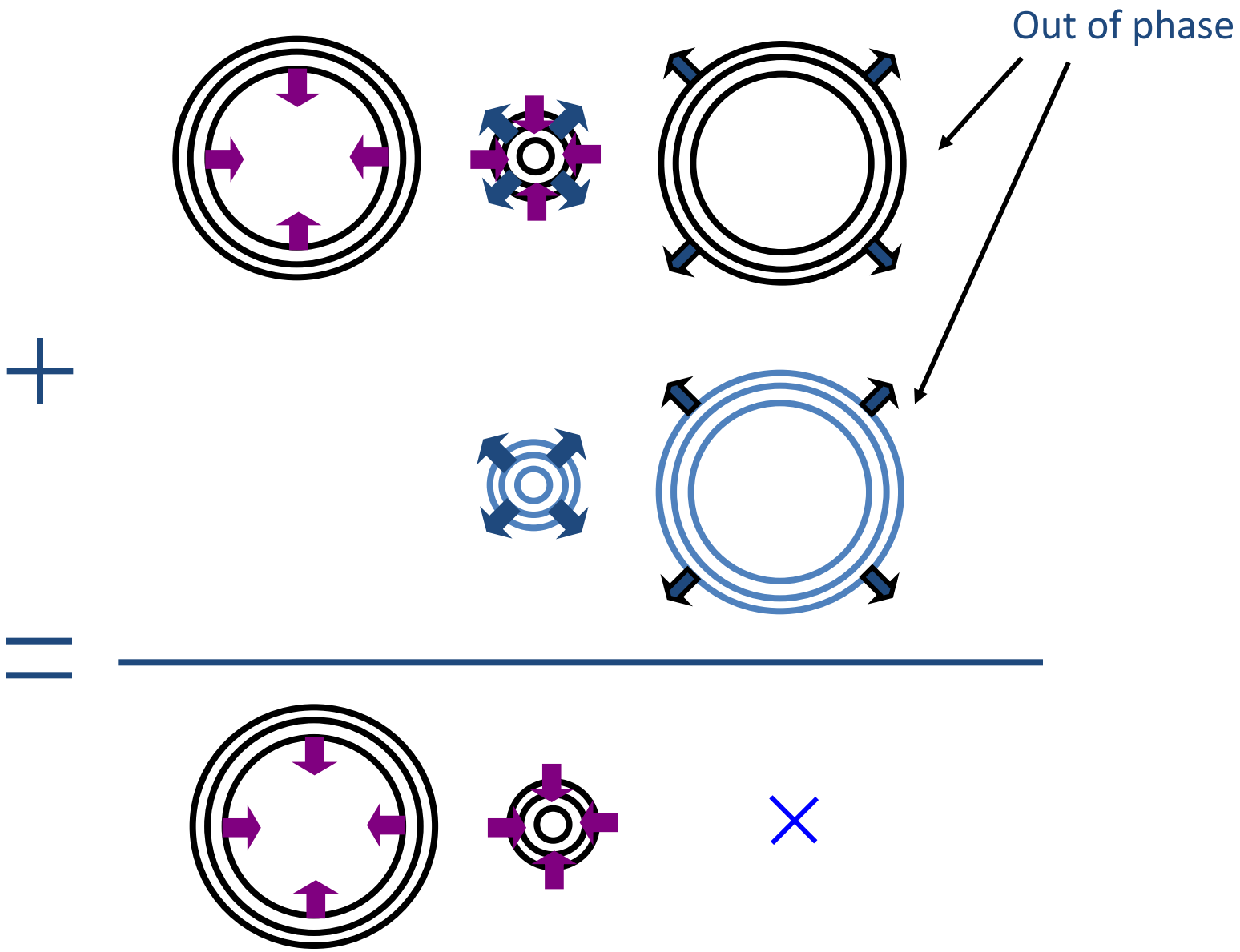
$$\left(\Delta - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\varphi(\vec{r},-t) = f(-t)\delta(\vec{r}-\vec{r}_0)$$

Converging wave

Source at r_0 excited by *f(-t)*

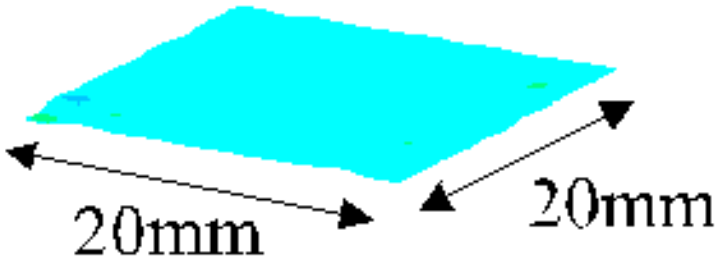
(TR source)

Principle of the acoustic sink

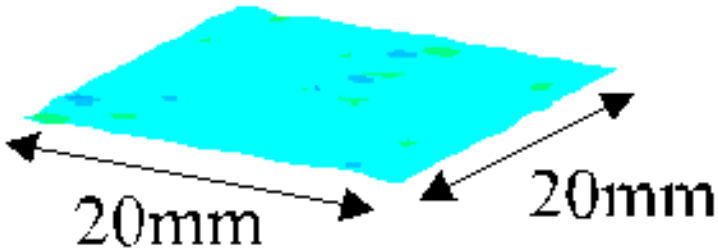


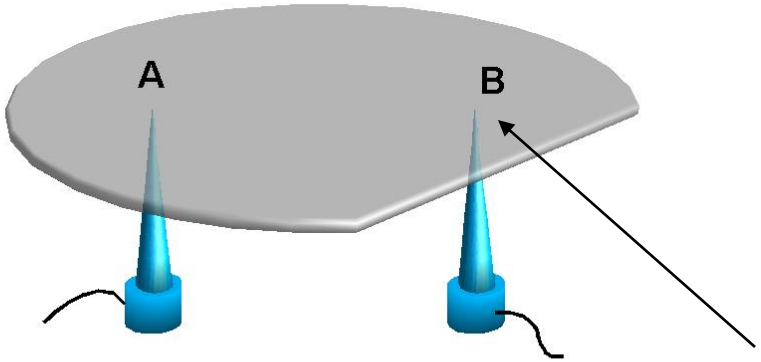
Experimental results

Wavefield Time Reversal



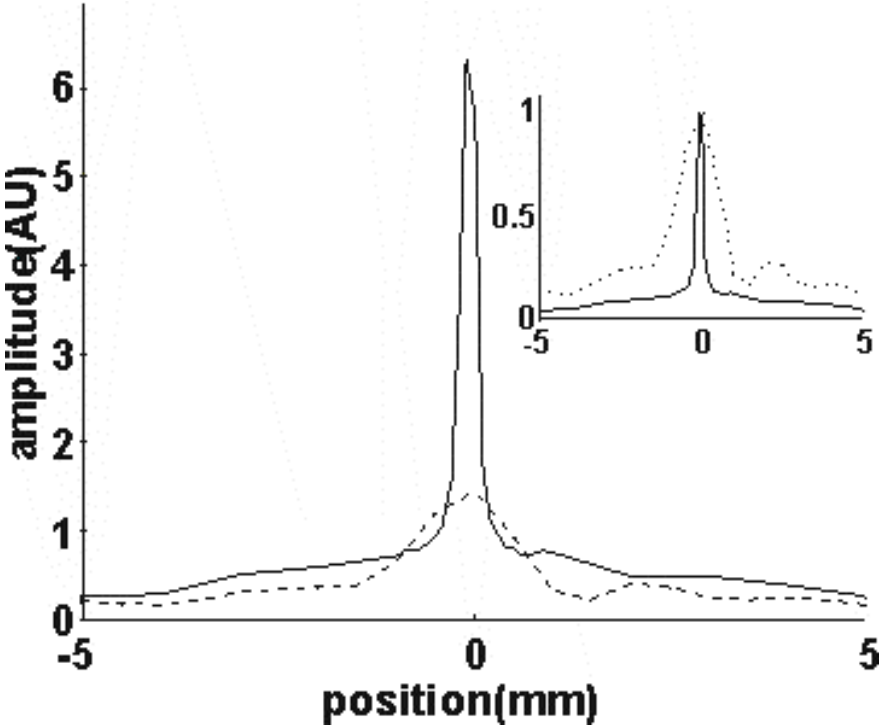
WaveField Time Reversal and Source Time Reversal

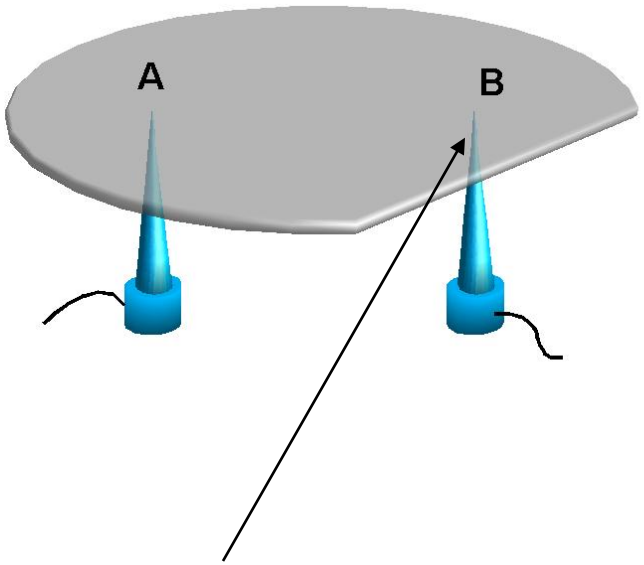




$\lambda/14$ tip

Focal spots with and without an acoustic sink





$\lambda/14$ tip

Is it possible to build a passive Sink ?

The coherent perfect absorber :

the time-reversed Laser : a sink for monochromatic wave only

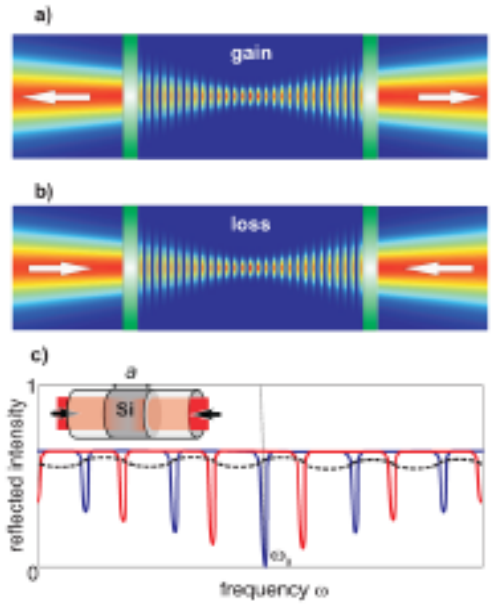


FIG. 1: (a) Schematic of a laser oscillator: a gain medium with complex refractive index $n = n' + in''$, embedded in an optical cavity, emits outgoing coherent waves with defined phase and spatial pattern at frequency ω_0 when the amount of amplification n'' reaches a threshold value. (b) Schematic of the time-reversed laser oscillator that realizes a coherent perfect absorber: when the gain medium is replaced by an absorbing medium with complex refractive index $n^* = n' - in''$, incoming waves with definite phase and spatial pattern [the time-reversed waves of the lasing mode in (a)] are fully absorbed in the medium. (c) Typical behavior of total intensity reflection coefficient versus frequency in a simple two-port photonic device, comprising a slab of lossy material (like Si) of thickness a embedded in a fiber or waveguide (inset), excited at the two input sides. The red and blue lines show the spectral reflection curves for coherent input waves with controlled phase difference $\Delta\theta$ ($\Delta\theta = 0$, blue curve; $\Delta\theta = \pi$, red curve). The dashed black line depicts the spectral reflection curve for incoherent input waves. The peaks in the coherent reflection curves correspond to the odd- and even-symmetric modes sustained by the slab structure.

PRL 105, 053901 (2010) | Selected for a Viewpoint in *Physics* | PHYSICAL REVIEW LETTERS | week ending 30 JULY 2010

Coherent Perfect Absorbers: Time-Reversed Lasers

Y. D. Chong,* Li Ge, Hui Cao, and A. D. Stone

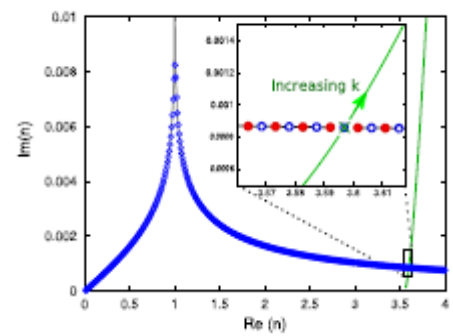


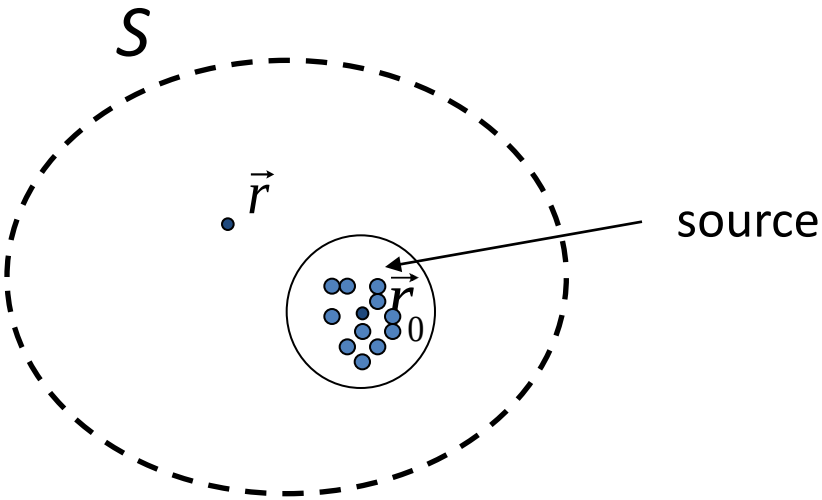
FIG. 2 (color online). Complex refractive indices leading to perfect absorption for the device of Fig. 1, for $ka = 664.7$. Only parity-even solutions are shown (blue circles). The green curve shows the refractive index of Si, $n(k)$. Inset: Parity-even solutions (hollow blue circles) and parity-odd solutions (filled red circles) in the region $3.55 < \mathrm{Re}(n) < 3.62$; the green cross shows the index of Si at $ka = 664.7$, $a = 100\ \mu m$ (i.e., $\lambda = 945.3$ nm).

Is it possible to build a blackbody (broadband) of size smaller than the wavelengths ?

Super-Resolution with metamaterials

Medium with sophisticated Green's functions

Sophisticated Green's functions



- select a medium with $\boxed{\mathrm{Im}\left\{\hat{G}_{ret}\left(\vec{r},\vec{r}_0,\omega\right)\right\}}$ that oscillate faster than the wavelength

Building a medium with a distribution of eigenmodes ocillating faster than the usual wavelength : introducing a new spatial scale between subwavelength resonators

$$\varphi_{tr}\left(\vec{r},t=0\right)=-2j\int_{\Delta\omega}\mathrm{Im}\,\hat{G}\left(\vec{r},\vec{r}_0;\omega\right)d\omega$$

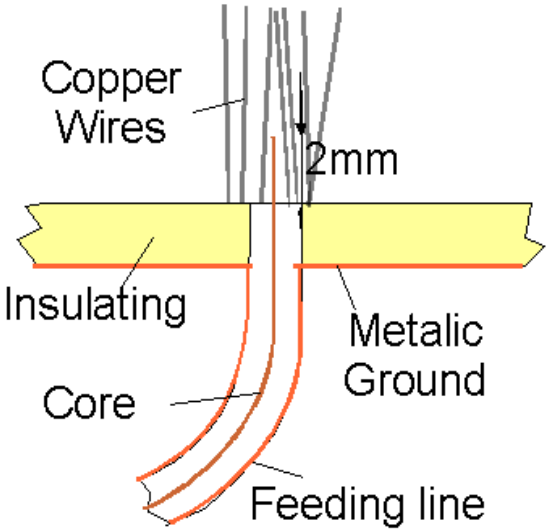
$$\varphi_{tr}\left(\vec{r}=\vec{r}_0,t=0\right)=-2j\int_{\Delta\omega}\mathrm{Im}\,\hat{G}\left(\vec{r}_0,\vec{r}_0;\omega\right)d\omega$$

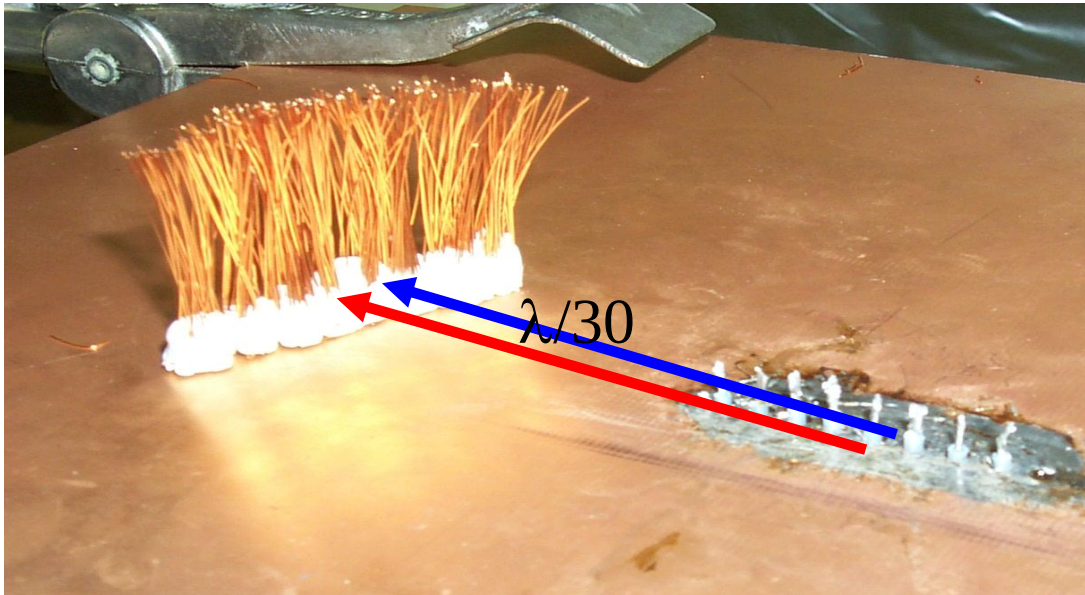
Number of modes

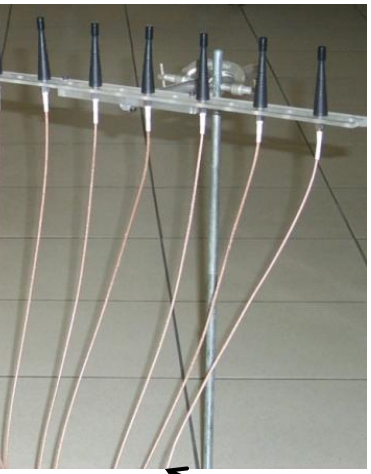
How to create a fast oscillating eigenmodes ?

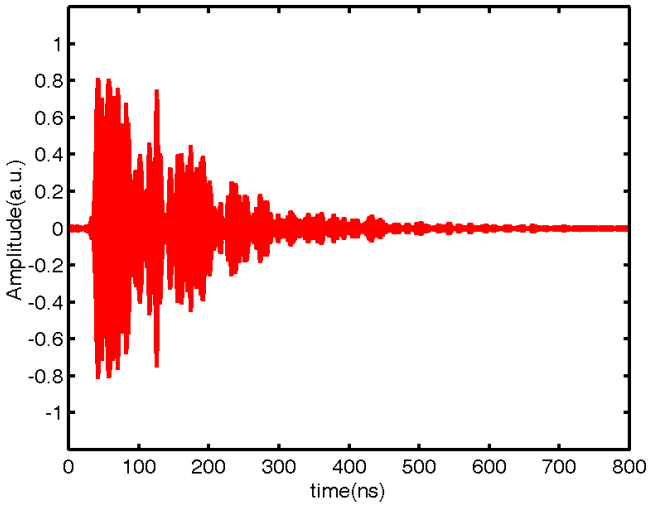
G. Lerosey, J de Rosny, A. Tourin, M. Fink
Science, Vol 315, 1120-1122, Feb 2007

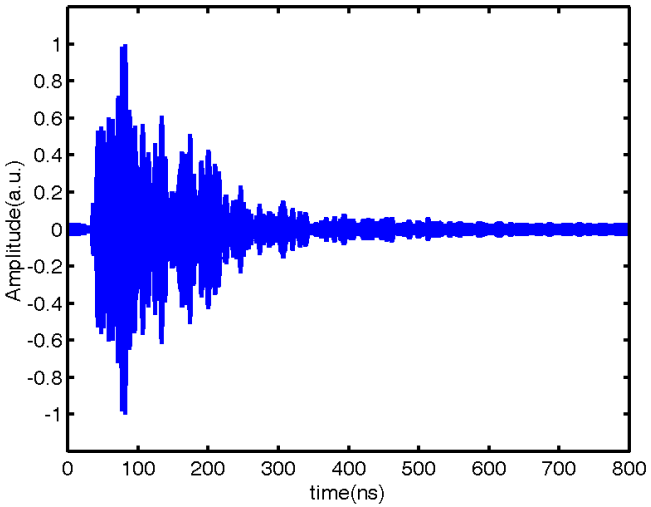
An Electromagnetic Example







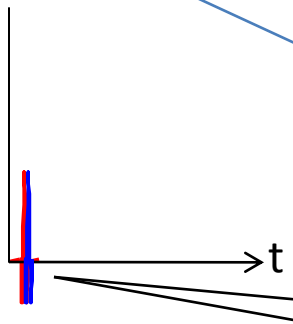




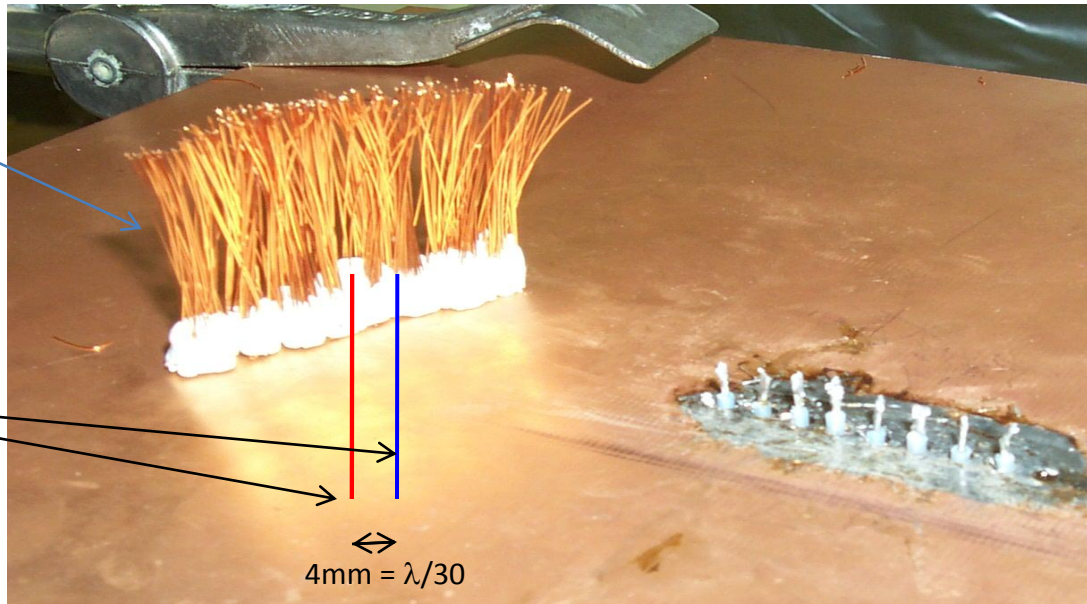
Signal recorded in far field by one antenna of the TRM

Each cupper wire with a length equal to $\lambda/4$, Resonating scatterers Mean distance between resonators $\lambda/120$

First experiment in a reverberating chamber 1 m3

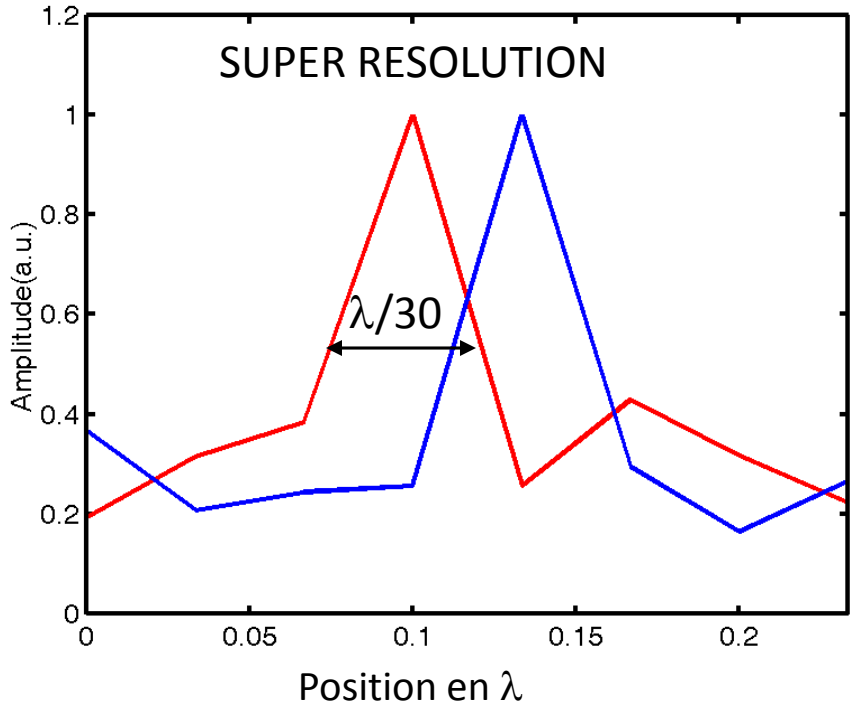


Impulsion 10 ns

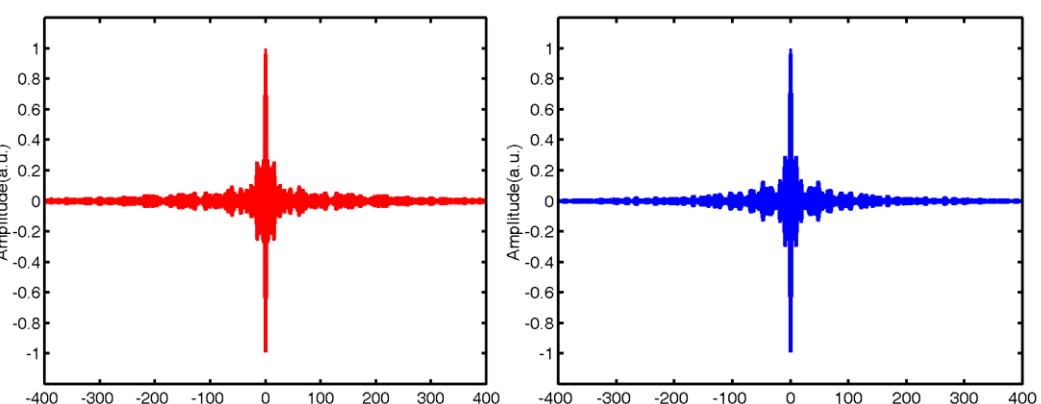


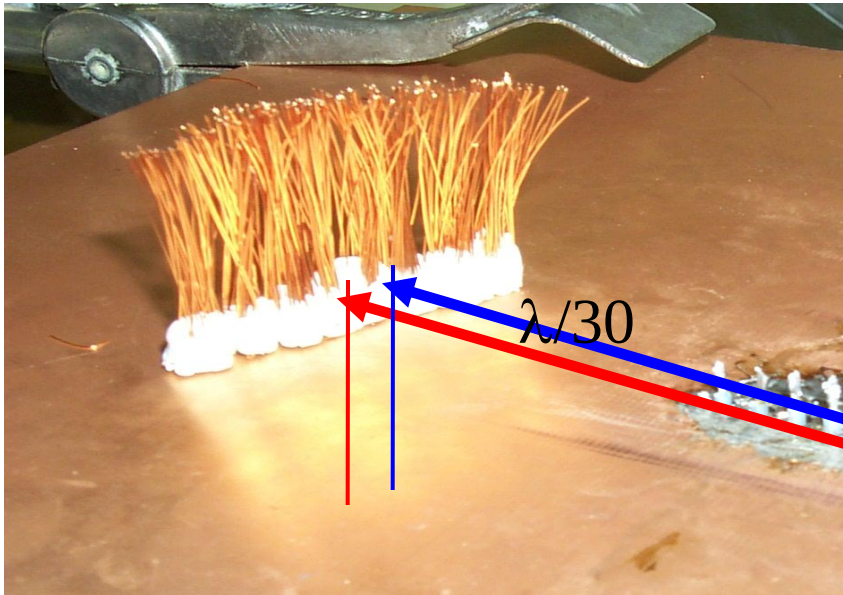
Bandwidth 100 MHz

Sub-wavelength resolution with far field time reversal



Signals observed on wire red or blue from the time-reversal signals



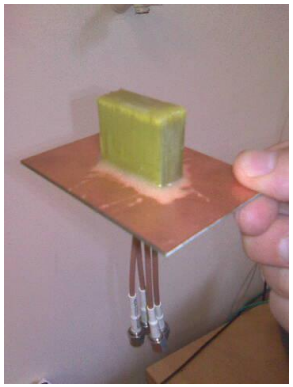


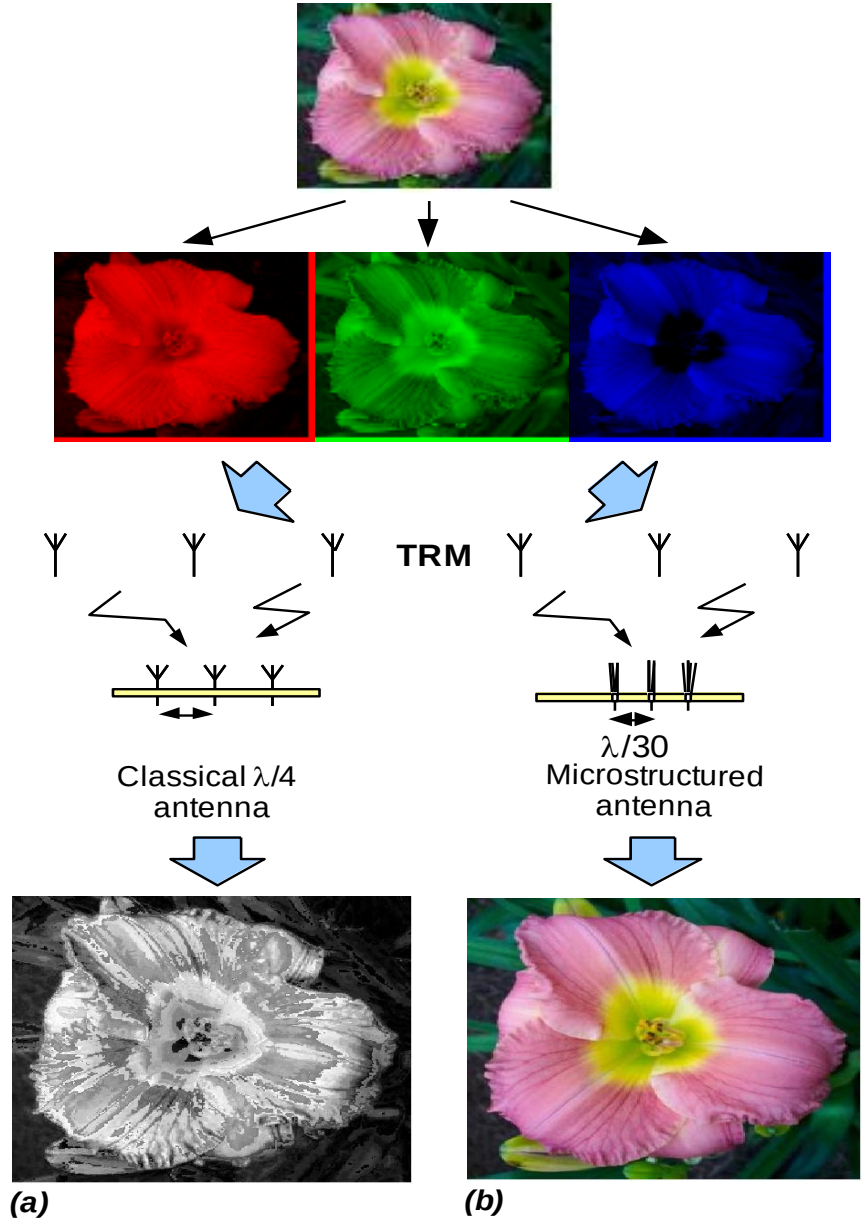
Telecommunications

3 bitstreams (RGB) with a data rate of 50 Mbits/s each.
The intended global data rate is thus 150 Mbits/s.

The TRM is made of 3 antenna
2.45 GHz central frequency
180 MHz bandwidth

New prototype in PCB material



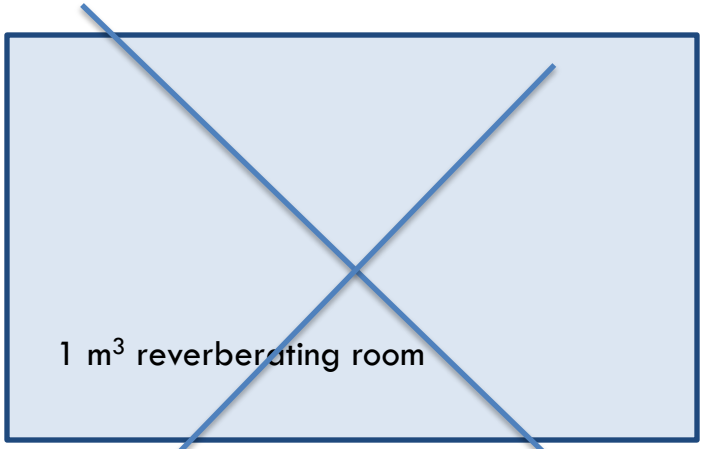


Examples of subwavelength resonators

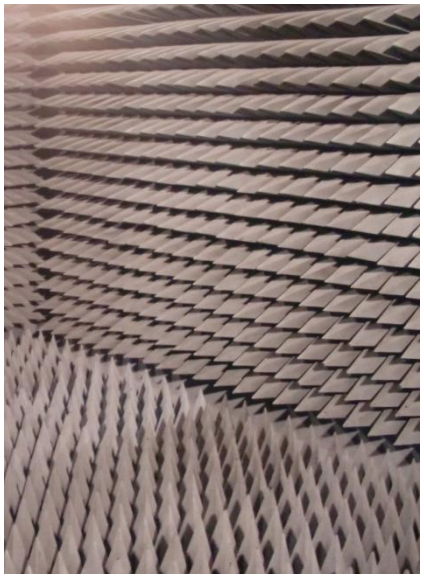
1. Periodic wire media (microwave)
2. Metamaterials with SRRs (microwave)
3. Acoustic (sonic wave)
4. Optics

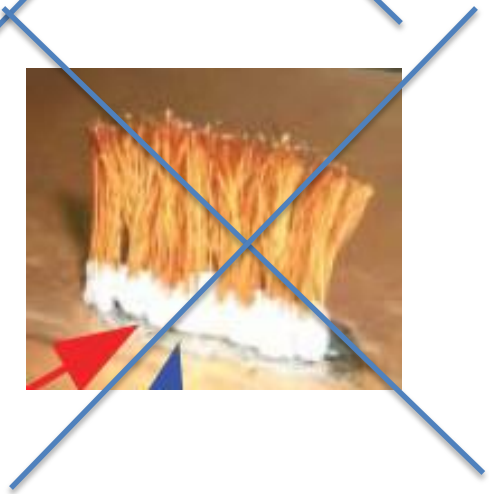
Generalization to Periodic structures in free space

Fabrice Lemoult, Geoffroy Lerosey, Julien de Rosny, Mathias Fink
« Resonant Metalenses for Breaking the Diffraction Barrier »
Phys Rev Lett 104, 203901 (May 2010)



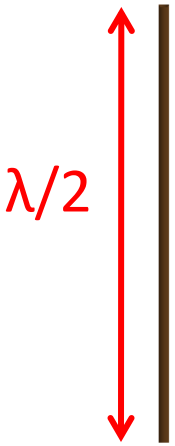
Vacuum or anechoic chamber



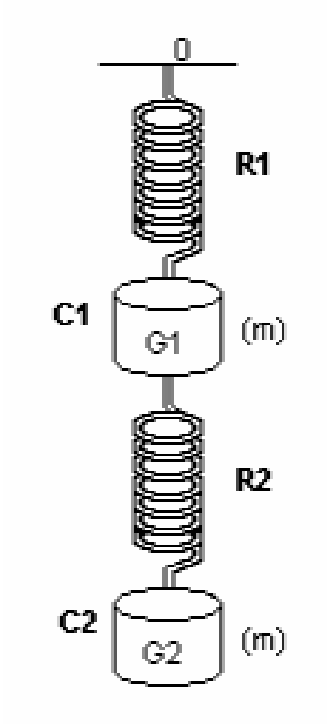


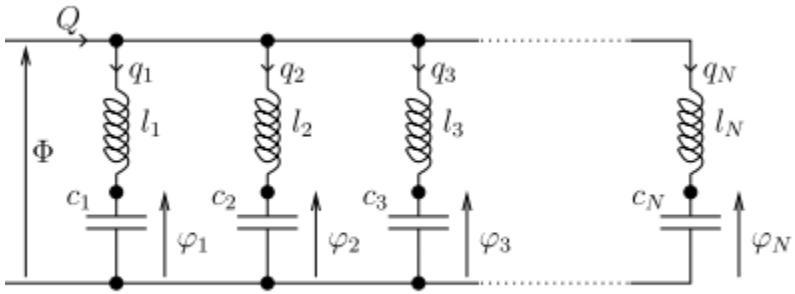
Periodic arrangement of identical wires

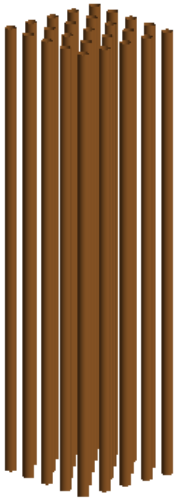
Coupling subwavelength resonators



= resonator (f_0)



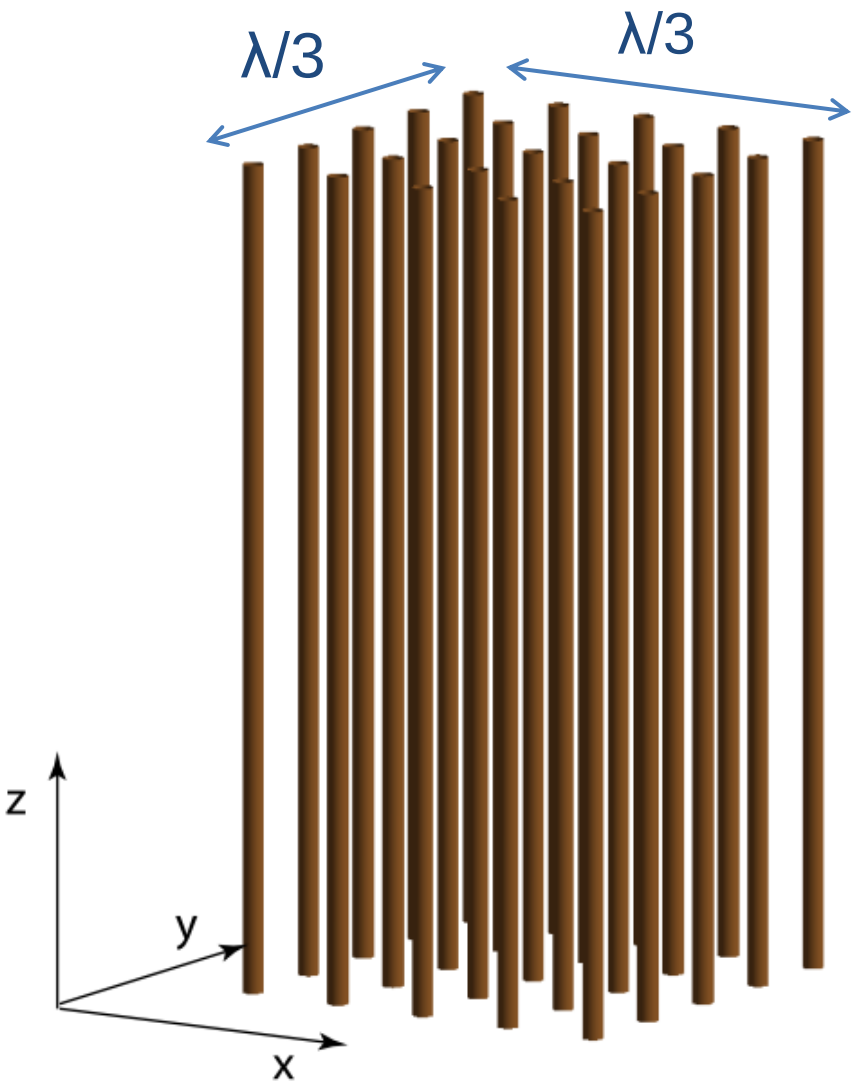




Coupling N resonators :

N resonant fréquencies (f_n)

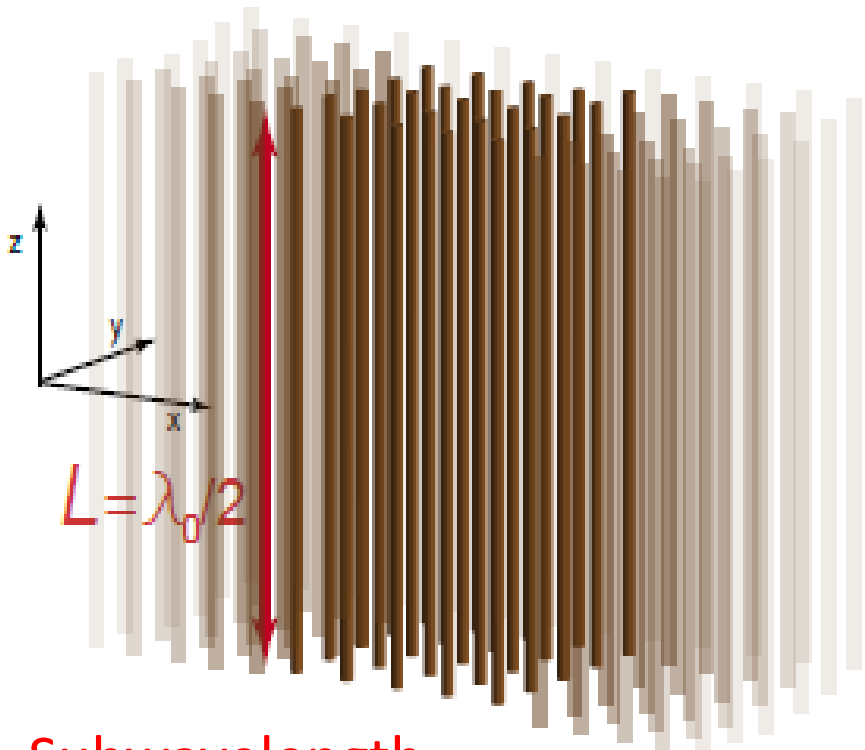
The closely spaced subwavelength resonators approach: « resonant metalens »



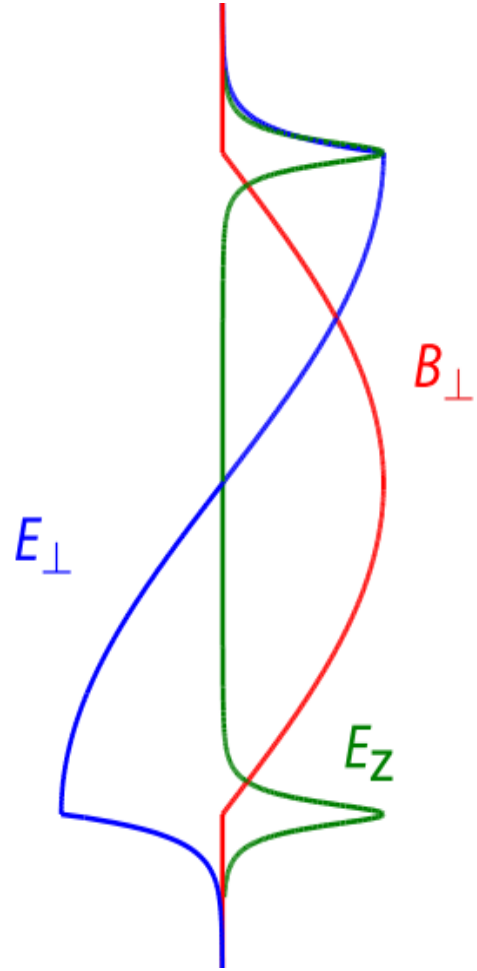
In the transverse plane:
20*20 deeply subwavelength (λ/80) spaced resonators!

(Resonance in the z dimension)

z-Bounded wire medium: introducing dispersion for TEM modes

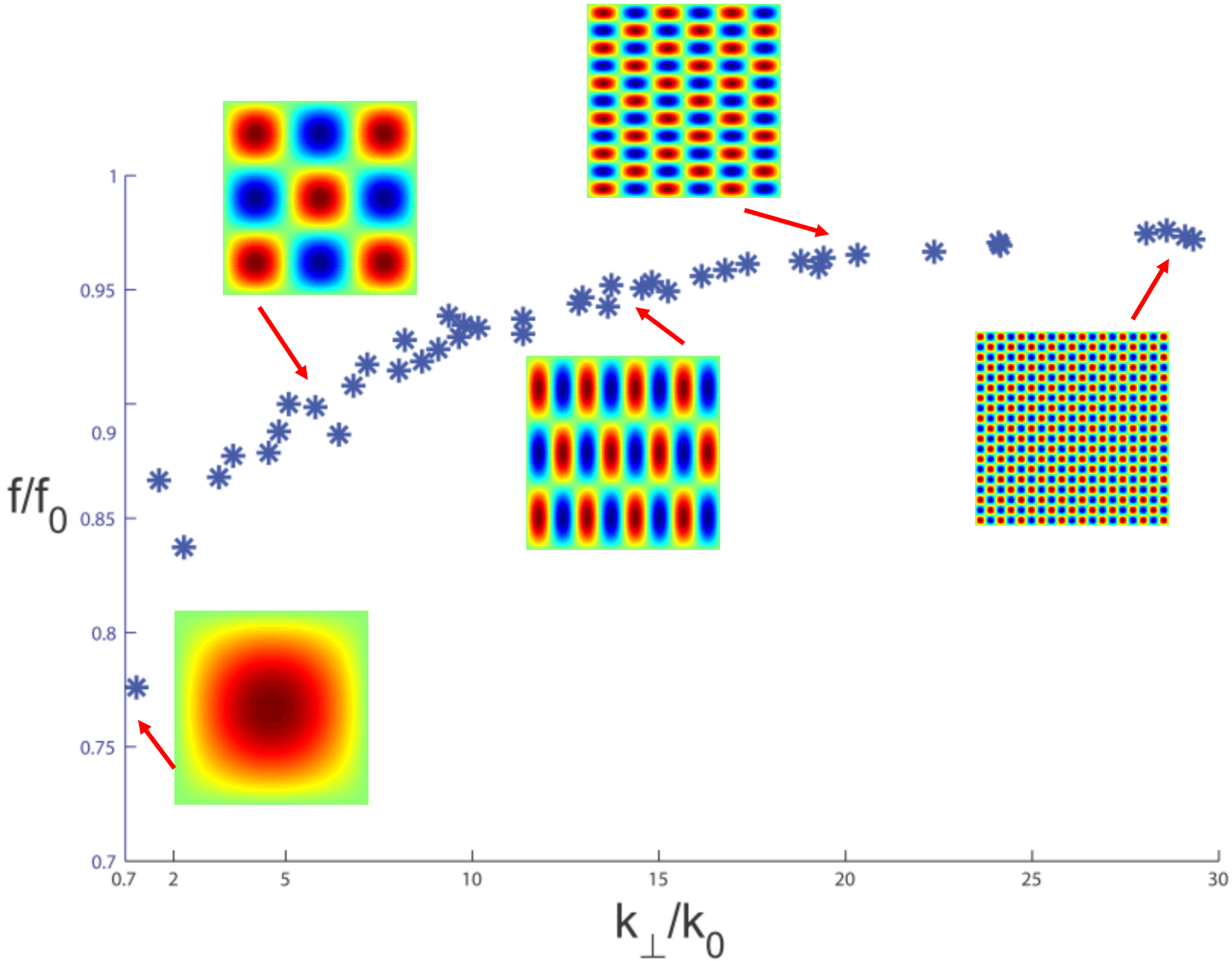


Subwavelength modes evanescent at –L/2 and L/2 interfaces

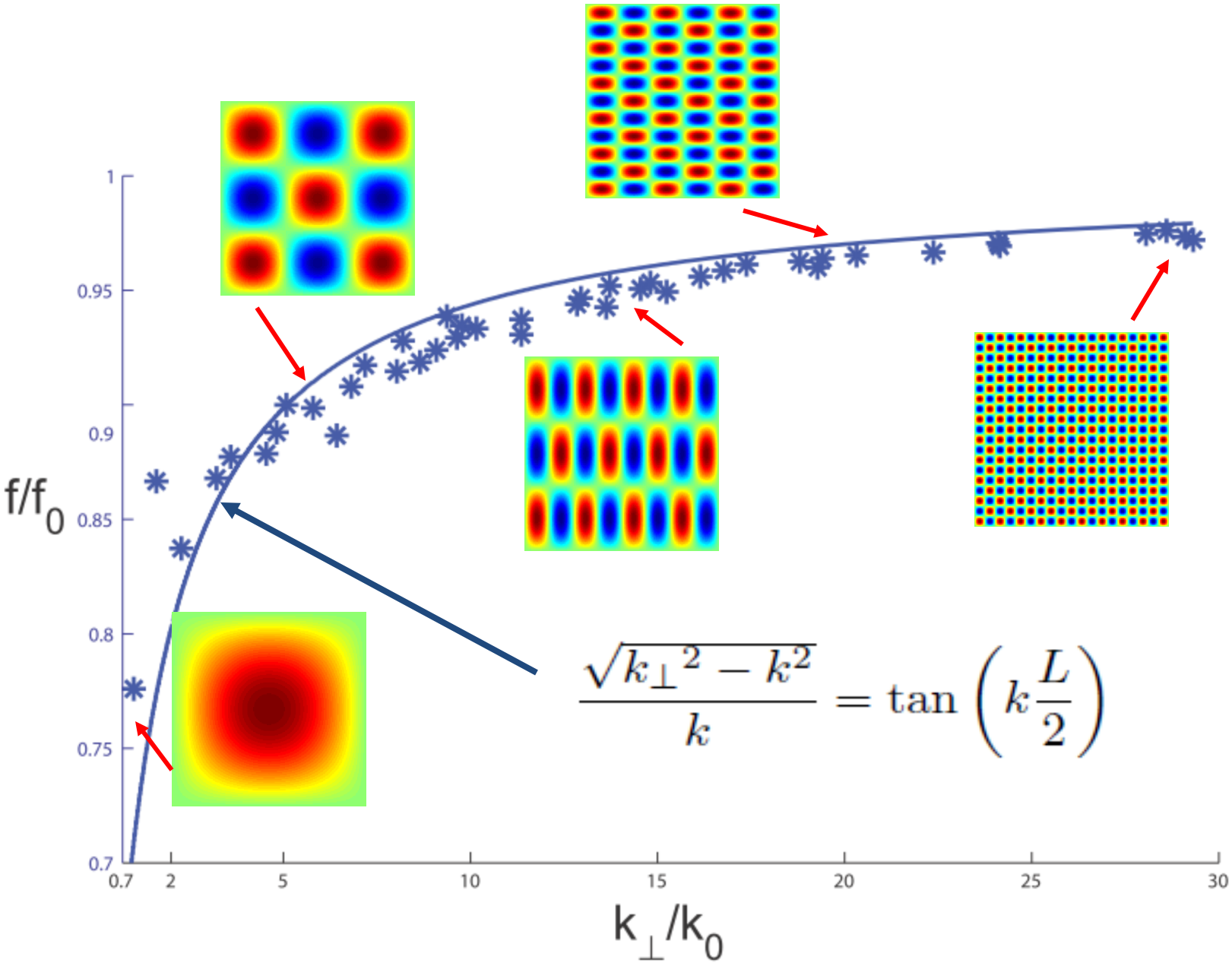


Fabry-Perot like resonance

Dispersion relation (CST Microwave studio)



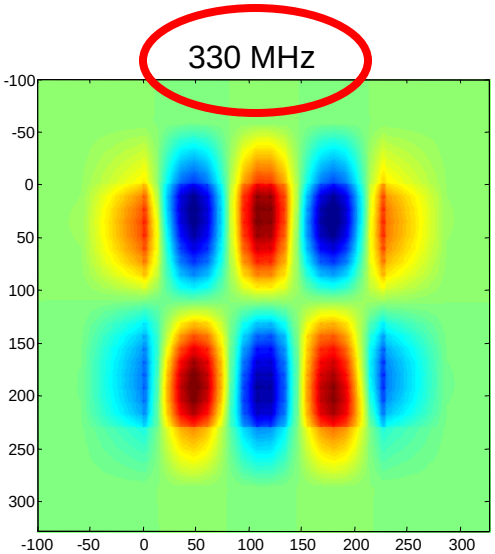
Dispersion relation theoretical derivation

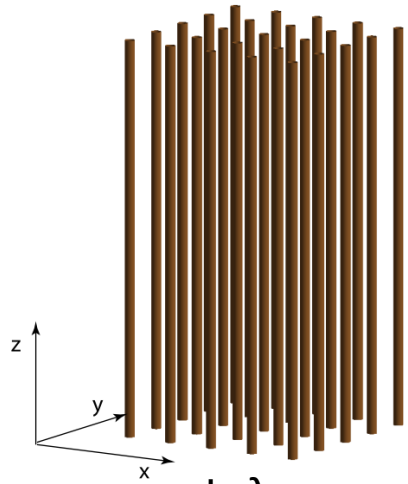


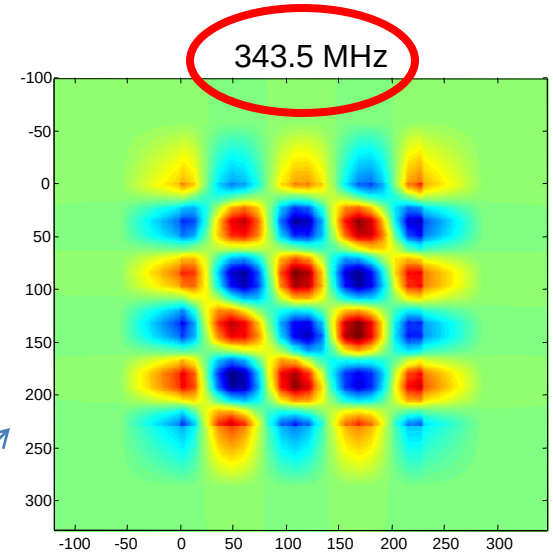
$$\frac{\sqrt{k_\perp{}^2 - k^2}}{k} = \tan\left(k\frac{L}{2}\right)$$

A periodic wire medium with a distribution of fast oscillating eigenmodes

N x N coupled resonators



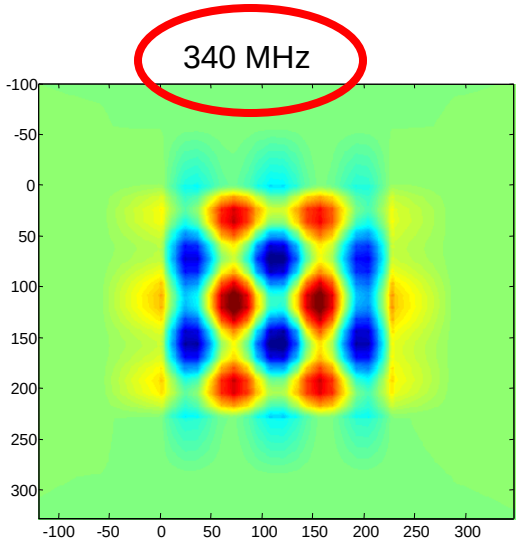


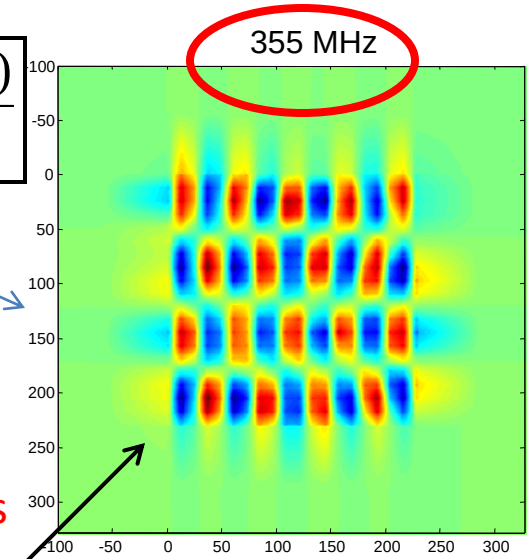


sub-λ source:

$$G(\vec{r}',\vec{r}_0;t)=\sum_n u_n(\vec{r}_0)u_n(\vec{r}')\frac{\sin(\omega_n t)}{\omega_n}$$

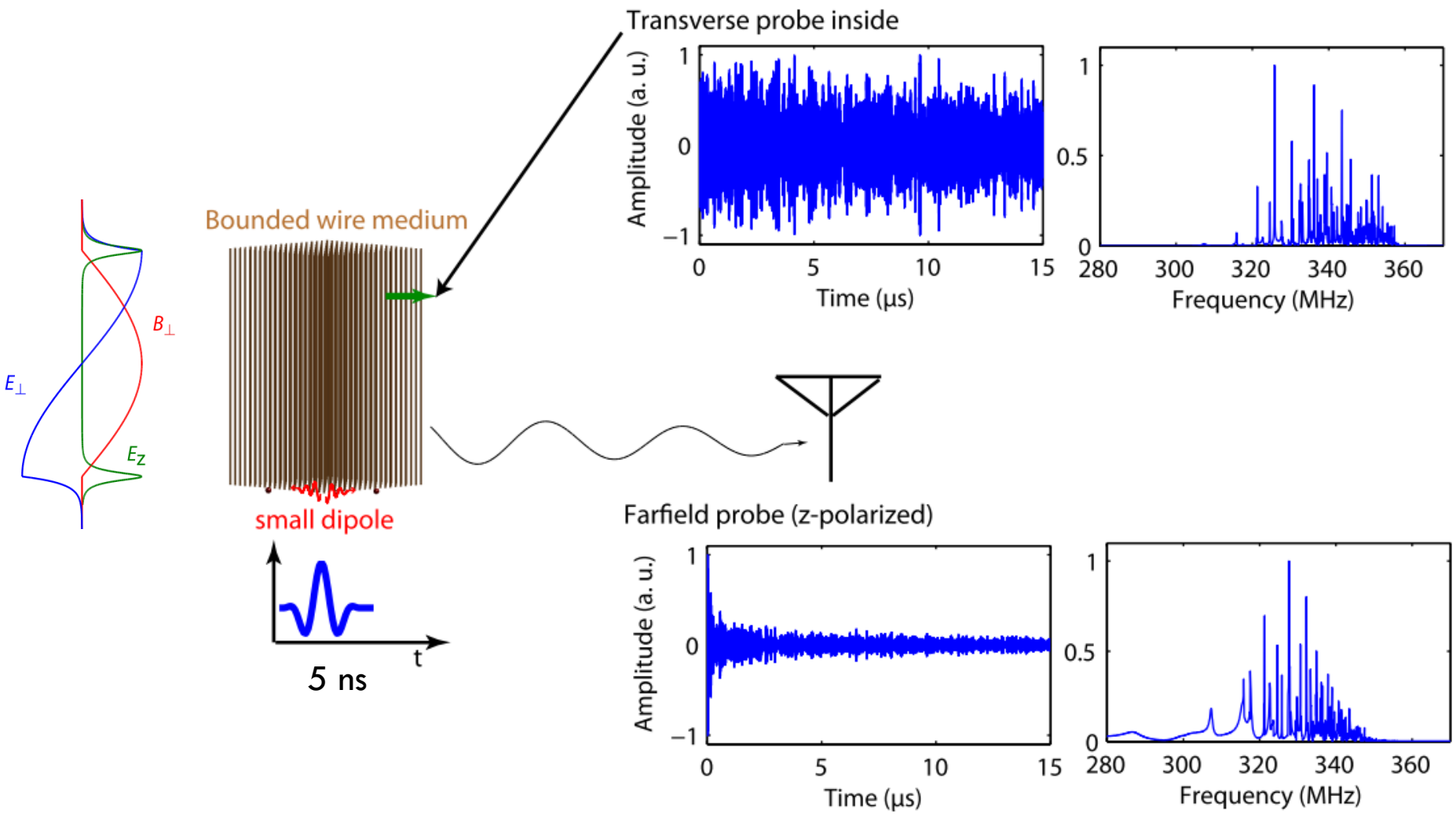
The number of modes is NxN





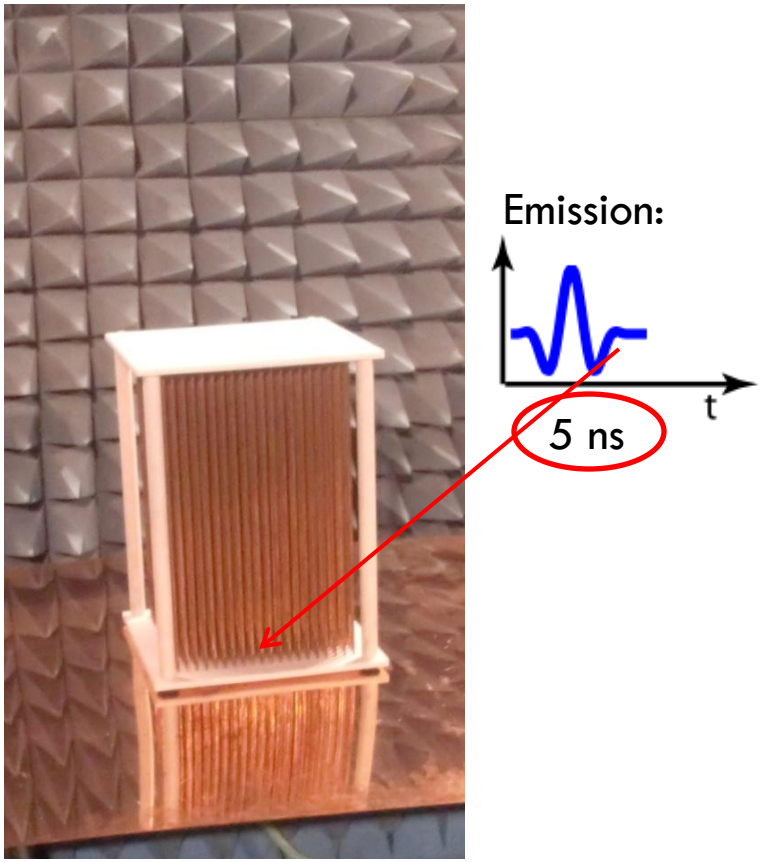
Sub-λ features are stored
In the spectrum.
High spatial frequency corresponds
to highest temporal frequency

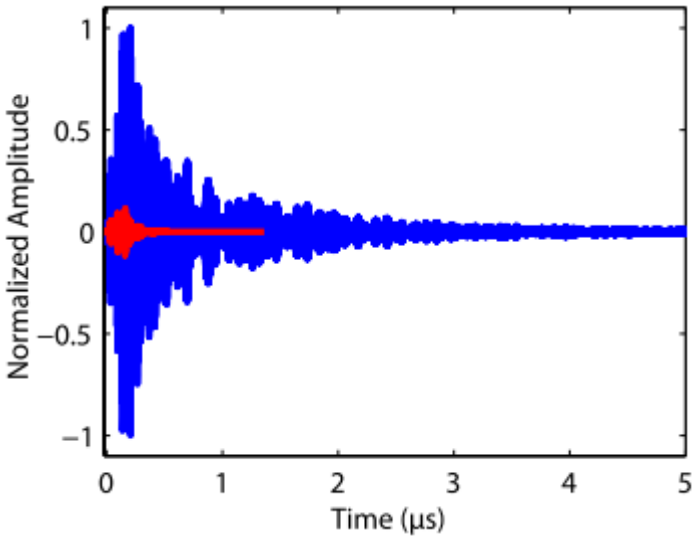
A numerical simulation

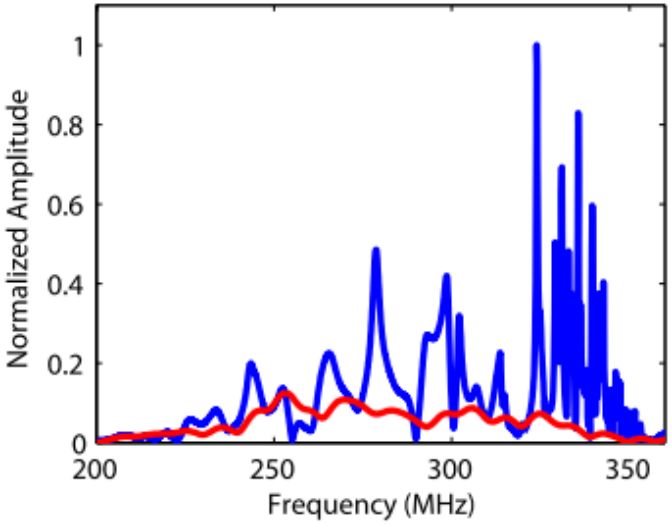


« sound board » equivalent

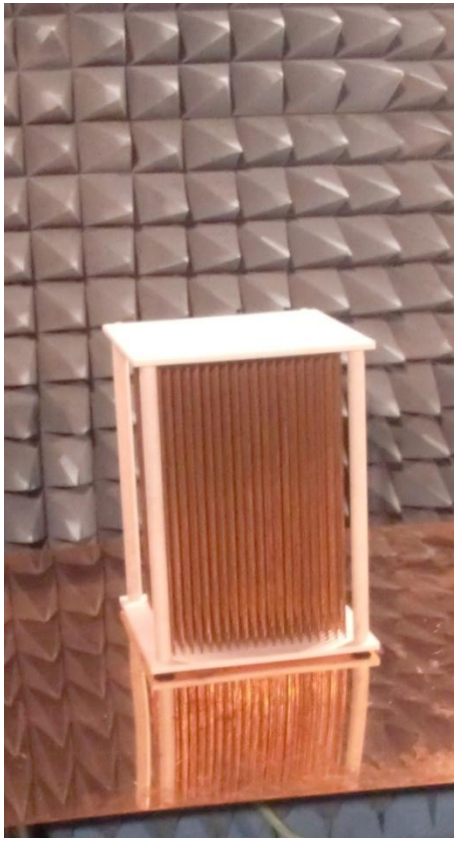
The experiment, 220 to 360 MHz

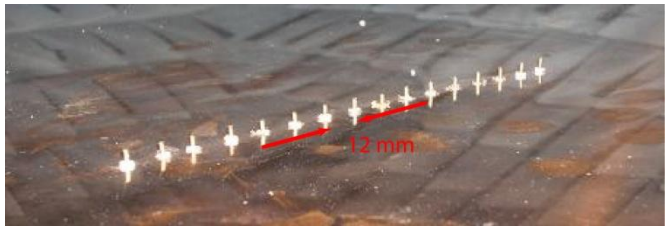


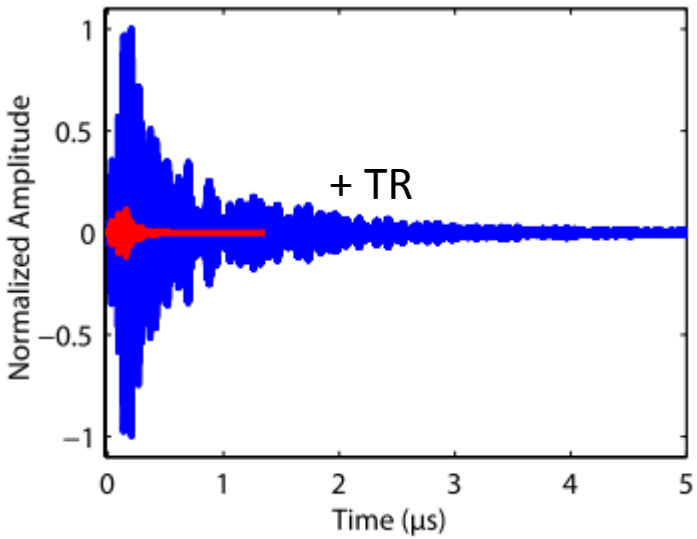


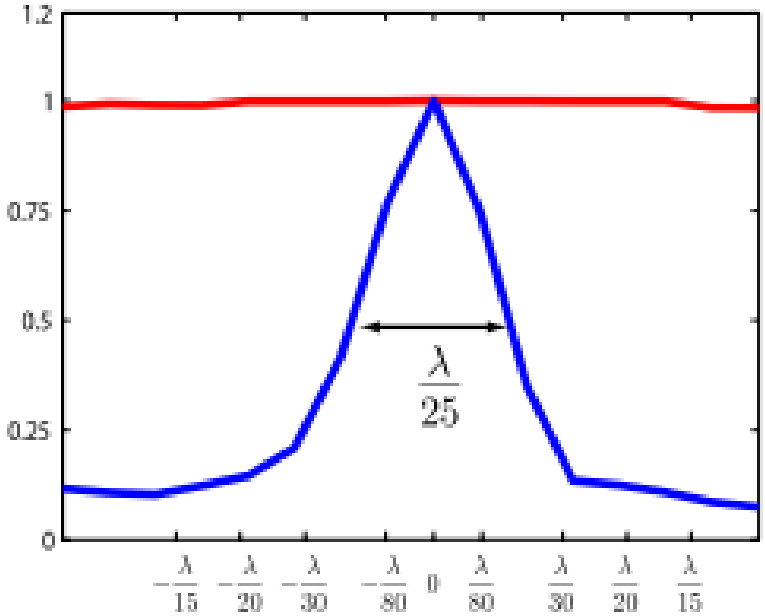


Experiment: sub-λ focusing

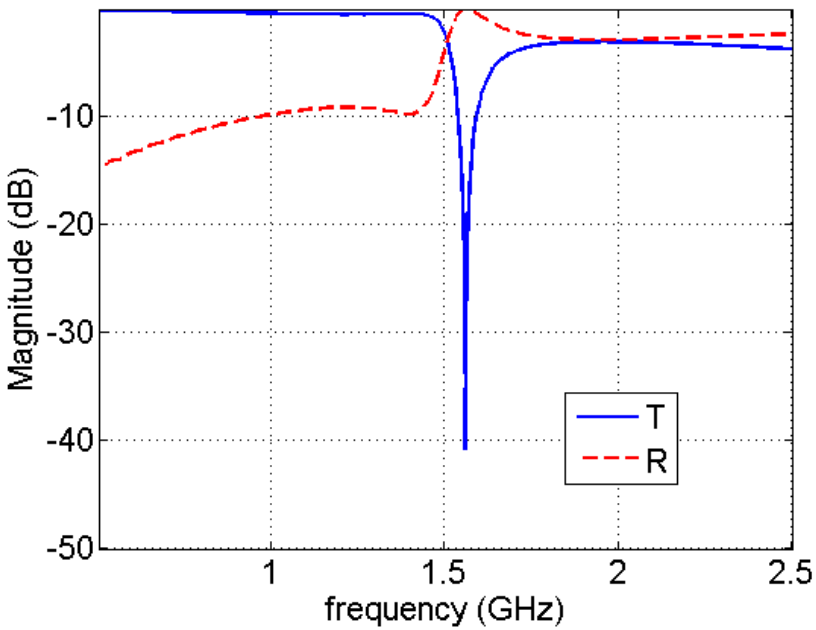


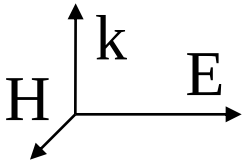


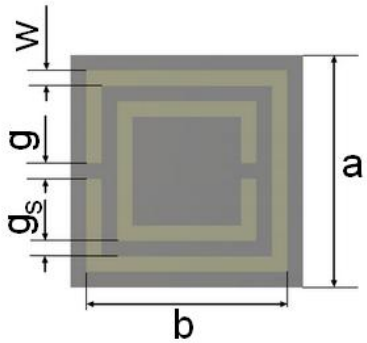


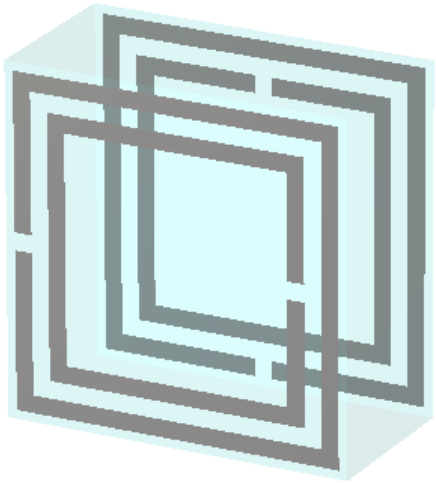


Let's move to other resonators









- Magnetic resonance around 1.6 GHz (~λ/18)

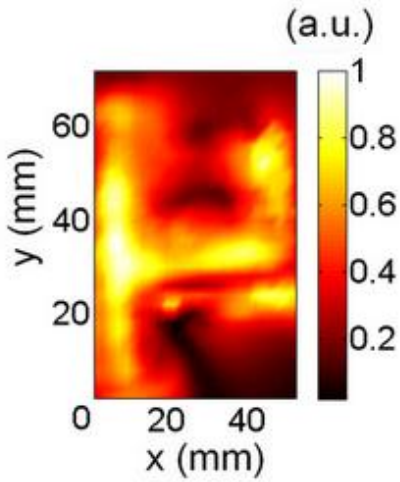
A. Ourir, J. de Rosny

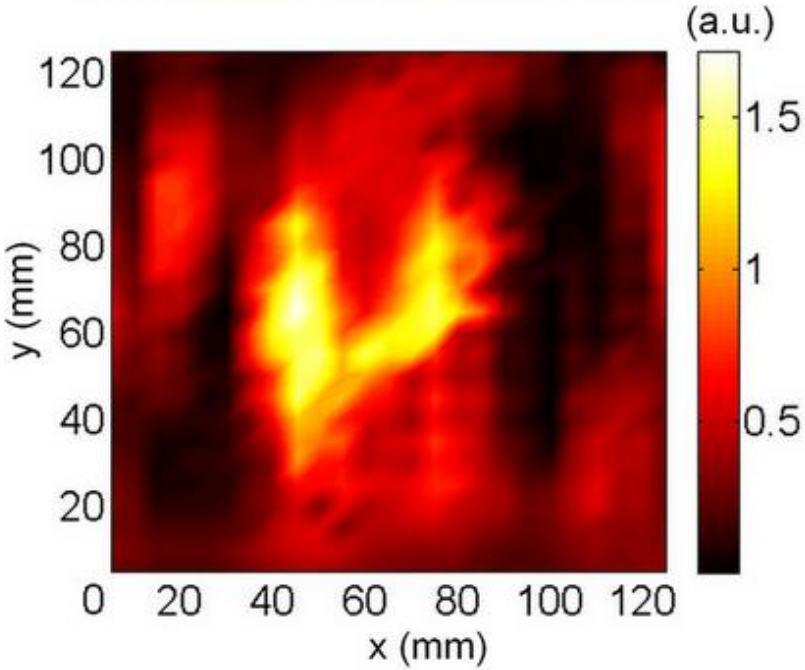
Far field subwavelength imaging using a magnetic resonant metalens

μ-shaped source

Magnetic resonant metalens

Near field image of μ





Far field image of μ, resolution λ/15

A. Ourir, J. de Rosny

How about sound?

Helmholtz resonance in air @ 420 Hz (λ=0.8m)

subwavelength acoustic resonator (diameter 6 cm)

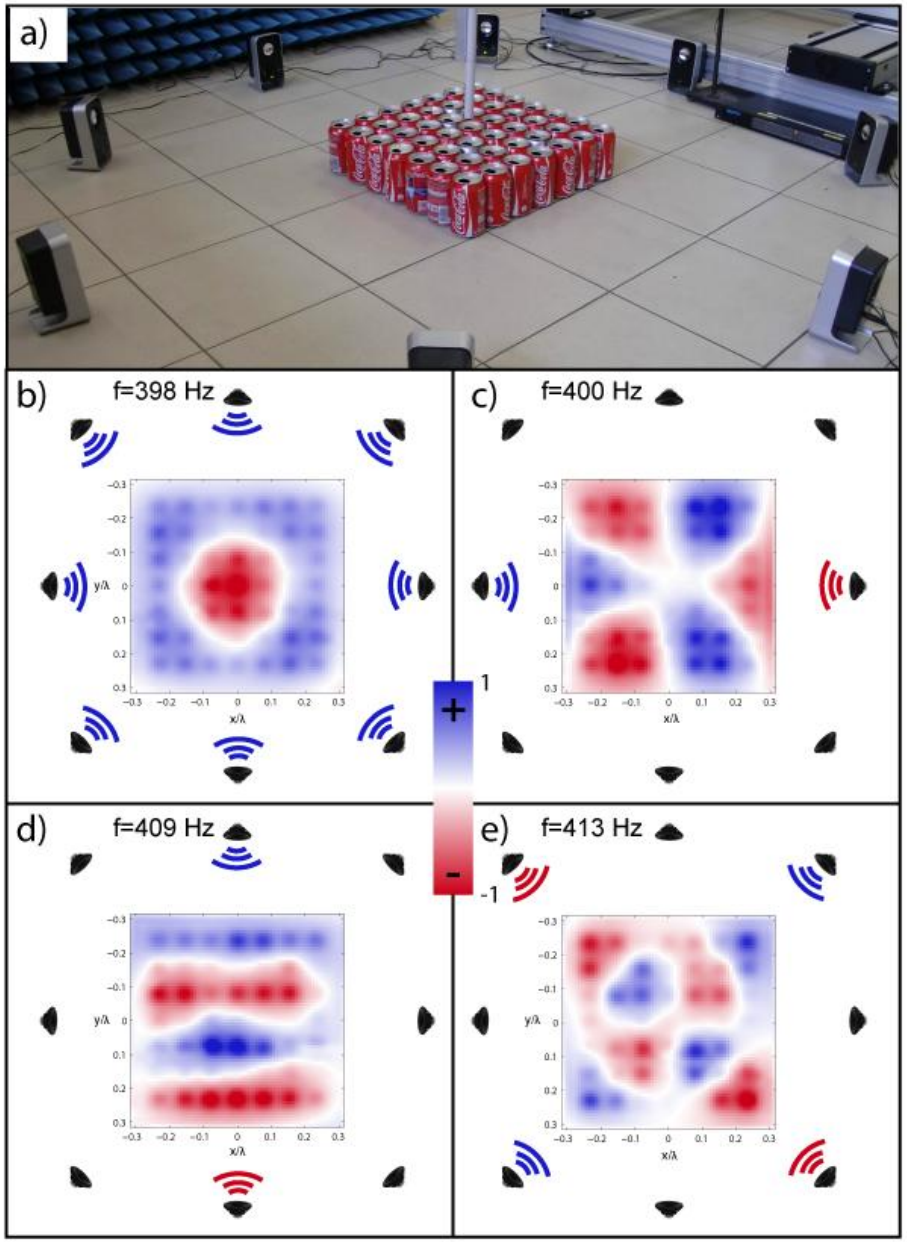
Lemoult F, G Lerosey et al., submitted

A resonant metalens for sound

Lemoult et al., submitted

42 cm, $\lambda/2$ @ resonance

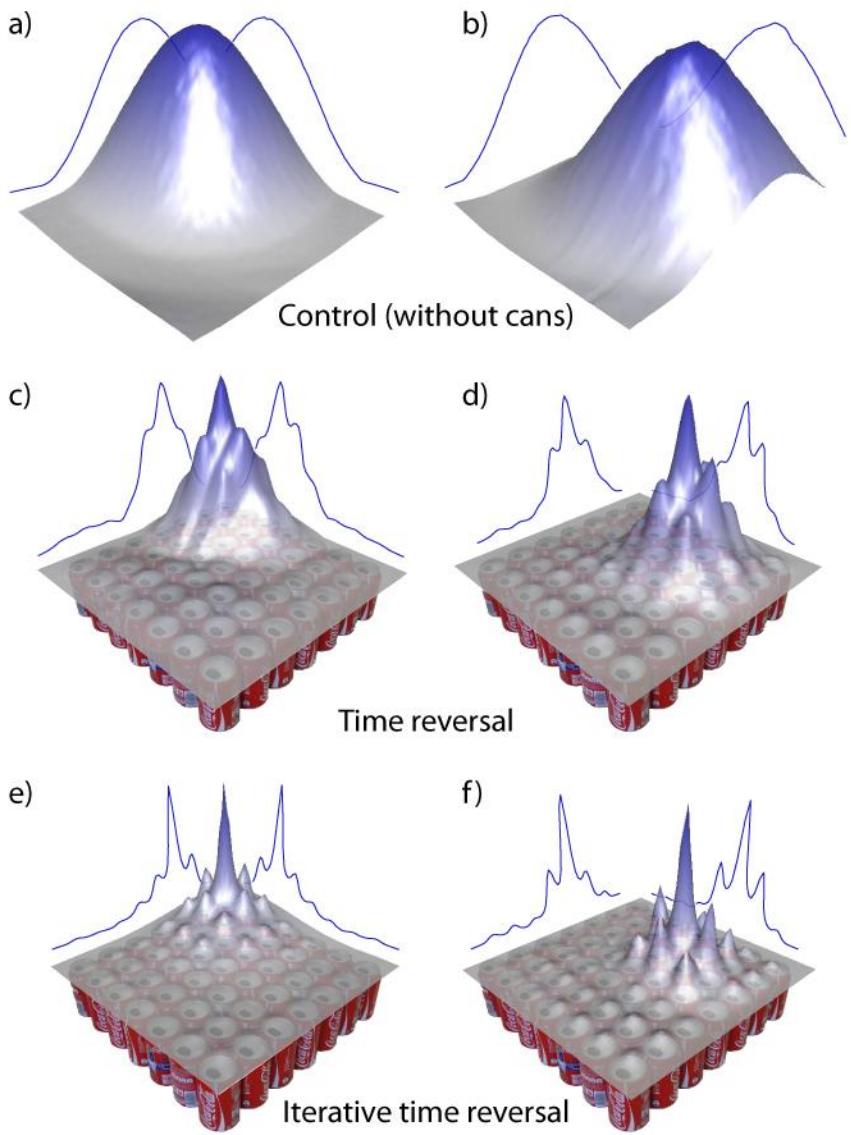
Results



Excitation of subwavelength coherent modes

Lemoult, Lerosey et al., submitted

Far field Subwavelength focusing of sound



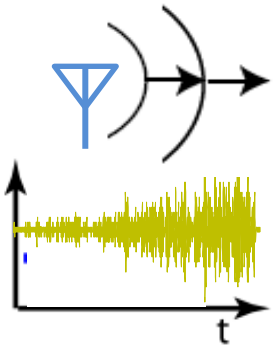
Lemoult et al., submitted

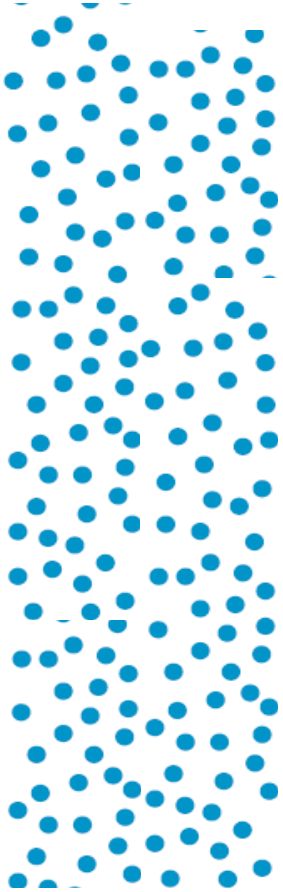
Demonstration of $\lambda/25$ focal spots with audible sound around 400 Hz

When the medium is dissipative,
Time Reversal is not the optimal solution to
Focus. Time reversal is a matched filter
but it is not an inverse filter for propagation.

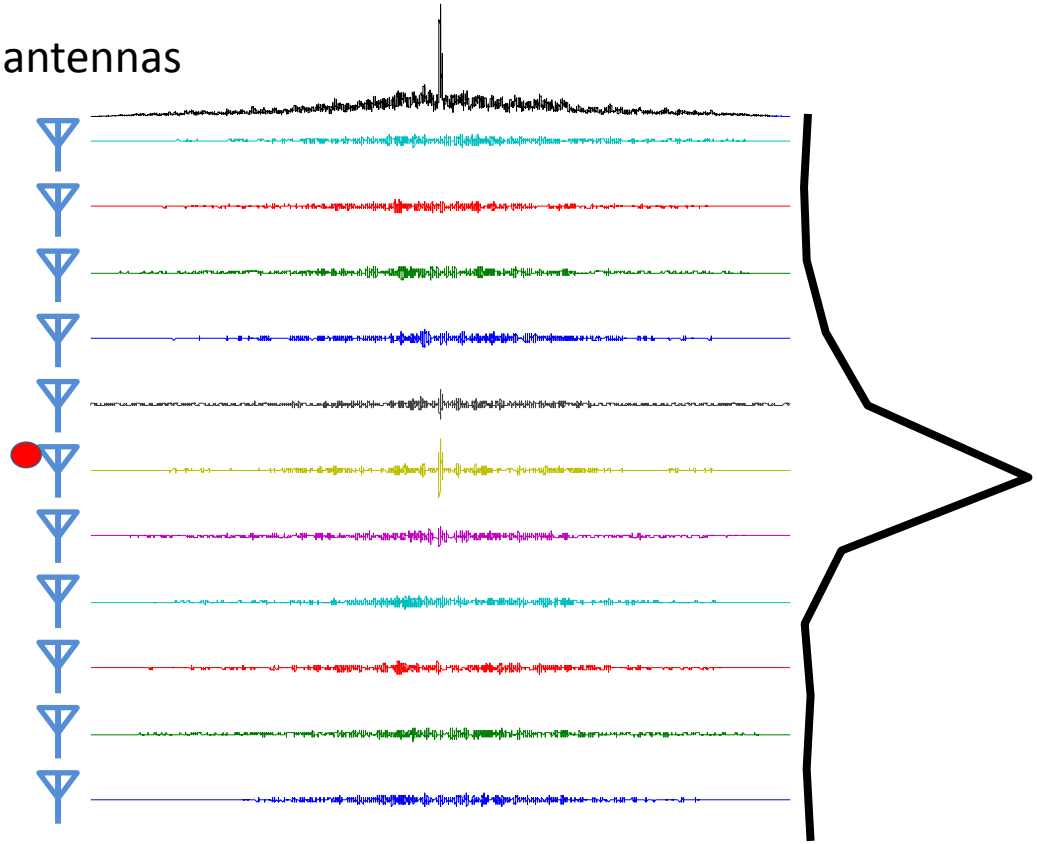
One channel time reversal

Ns = 1 antenna





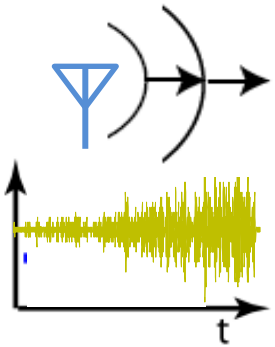
L antennas

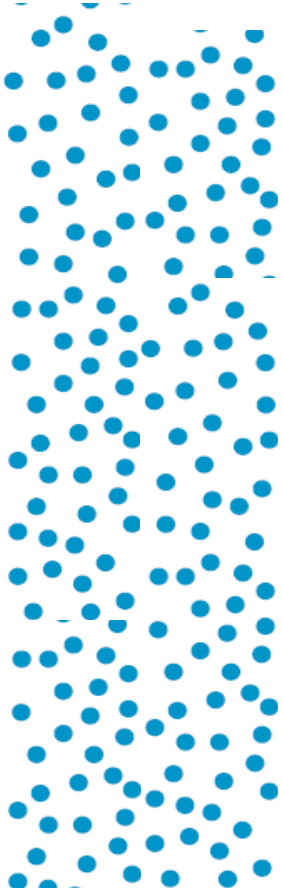


F. LEMOULT, G. LEROSEY, J. DE ROSNY, M. FINK, Phys Rev Lett 103, 173902 (2009)

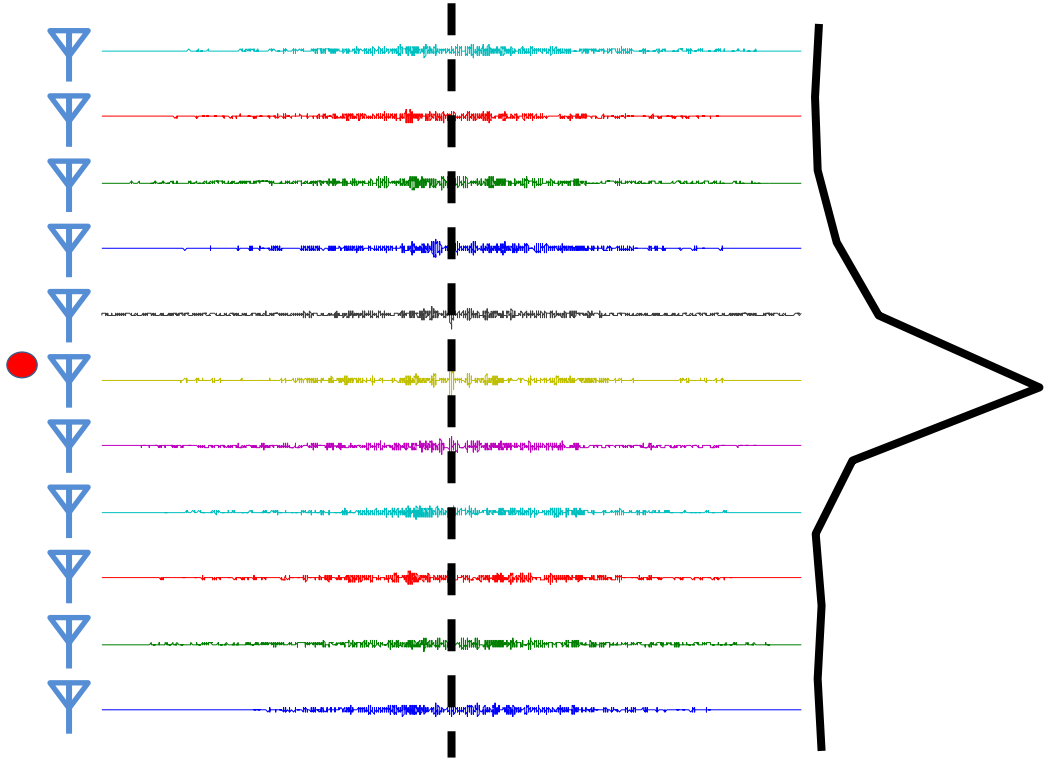
Iterative spatial inverse filter

Ns = 1 antenna

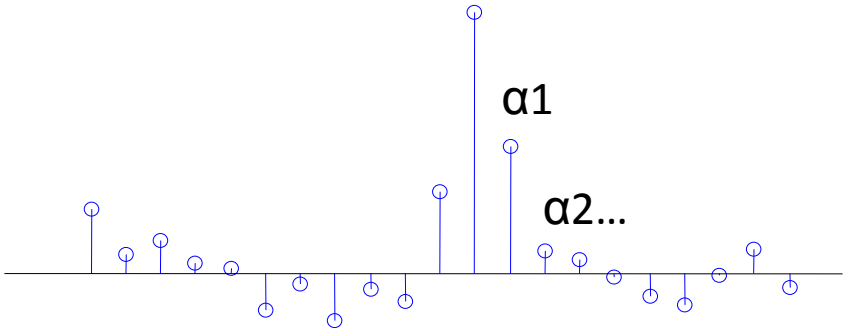




L antennas

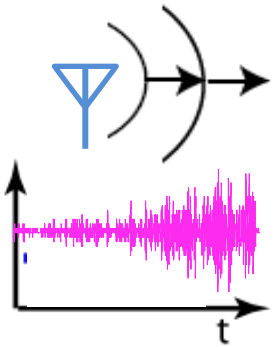


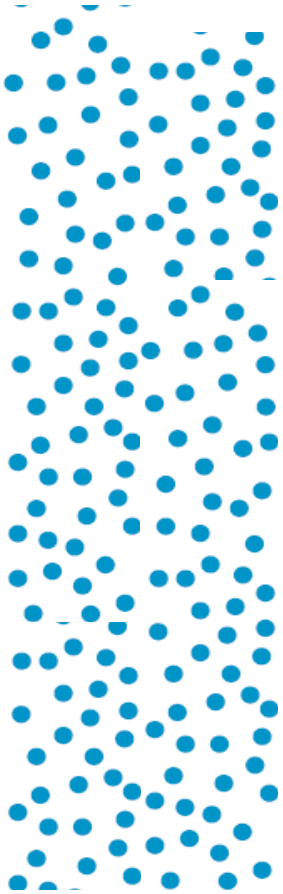
Focal spot at t=0:



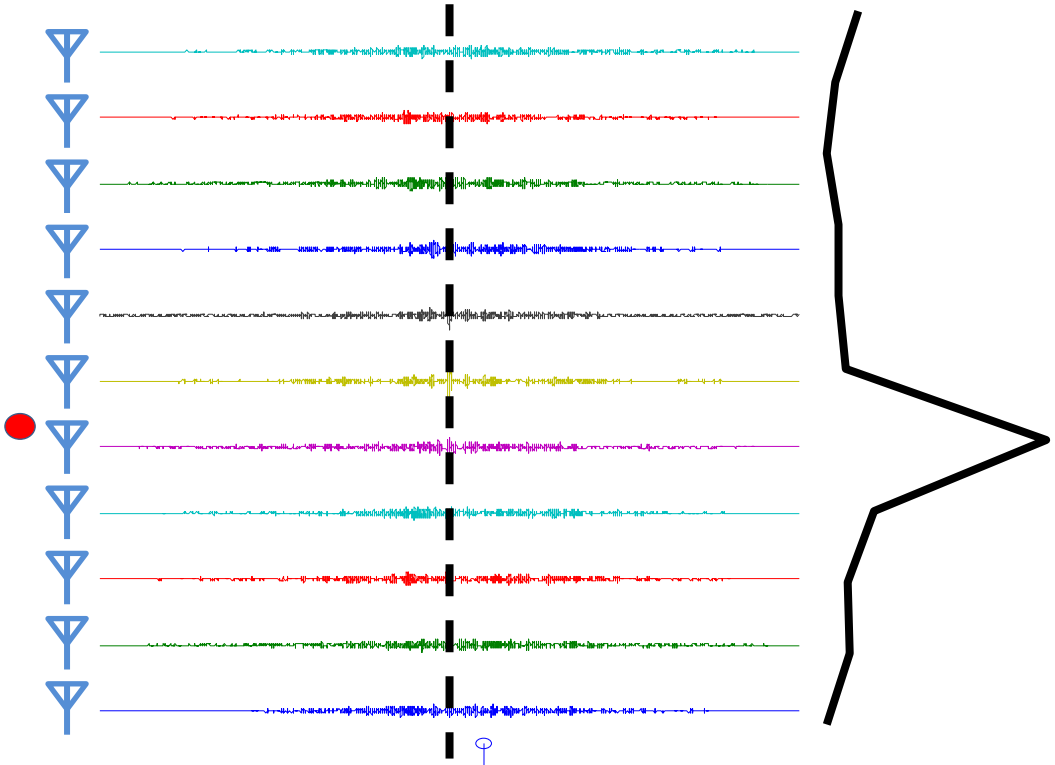
Iterative spatial inverse filter

Ns = 1 antenna

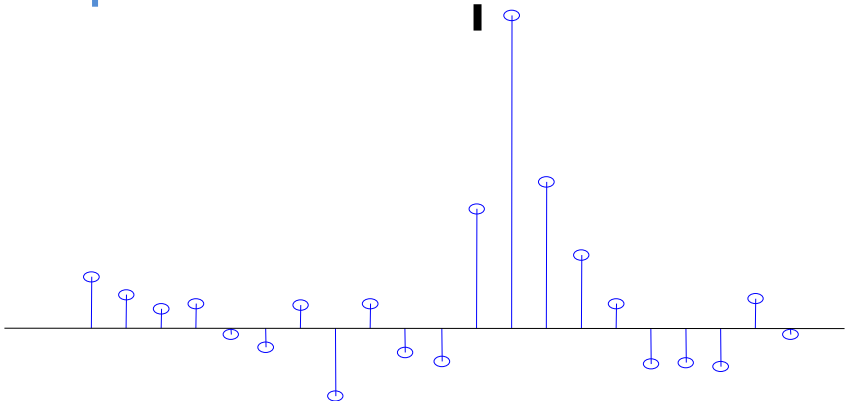




L antennas

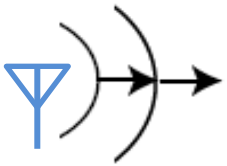


Focal spot at t=0:

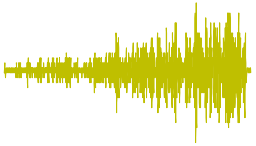


Iterative spatial inverse filter

Ns = 1 antenna

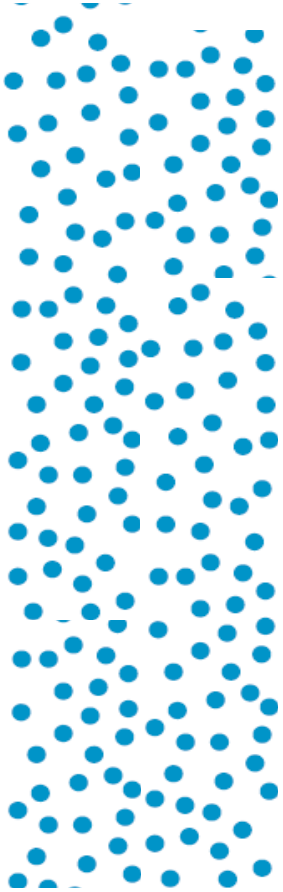


1x

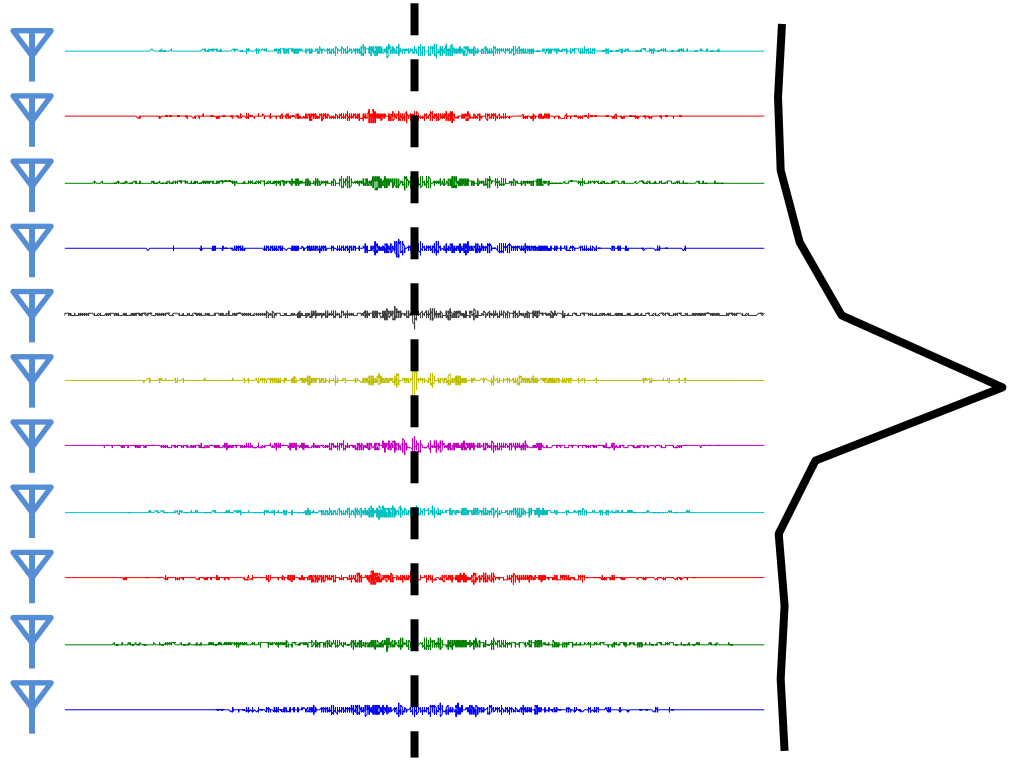


+ (- α2)x

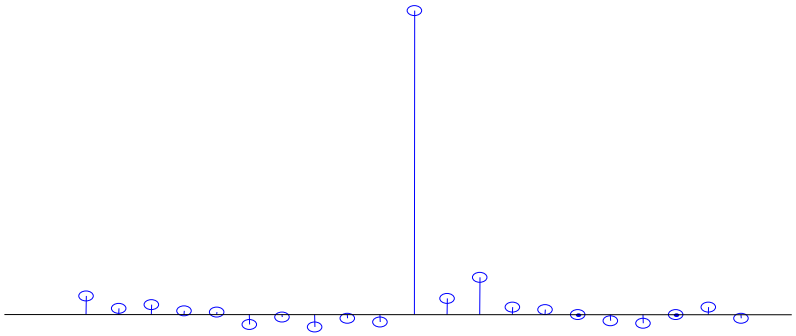
+ (- α3)x…



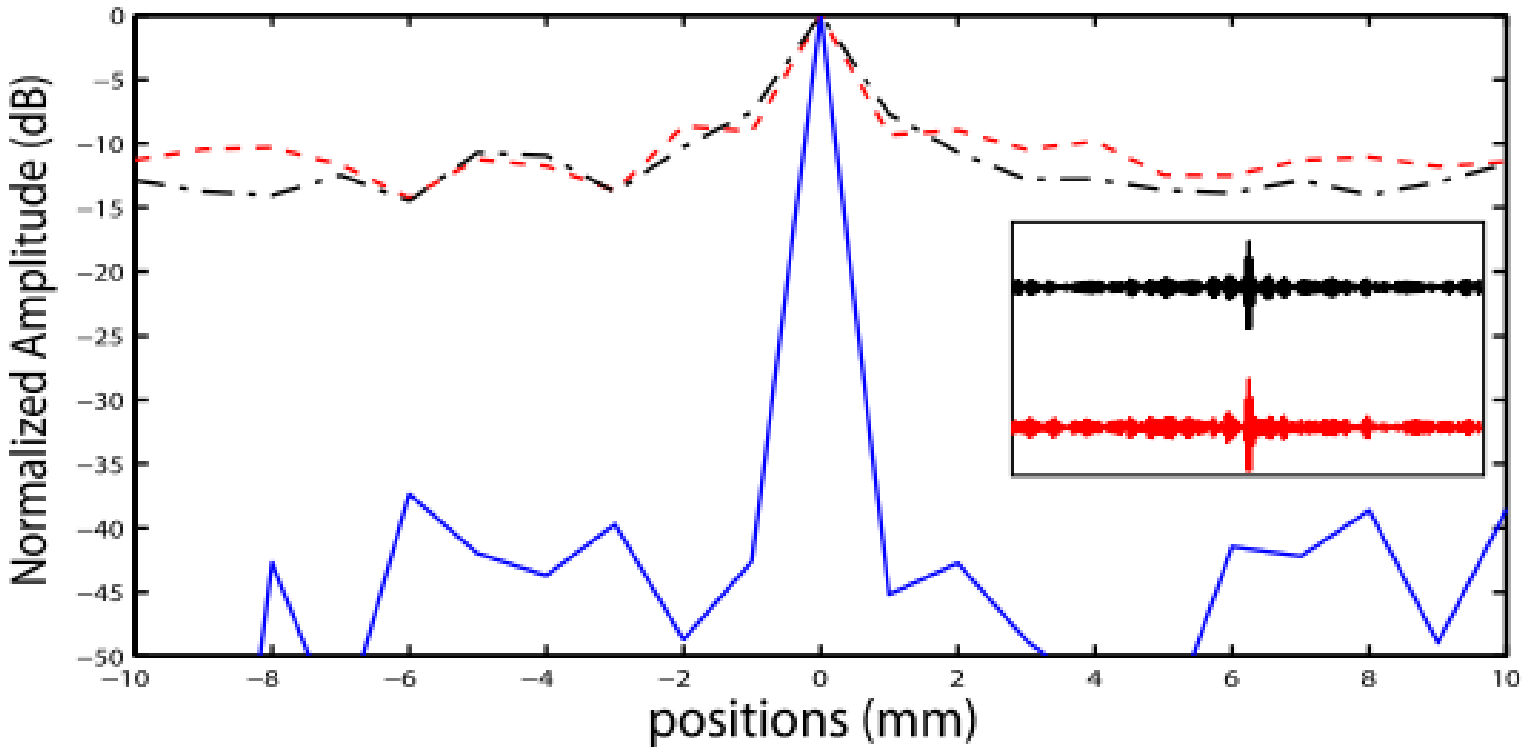
L antennas



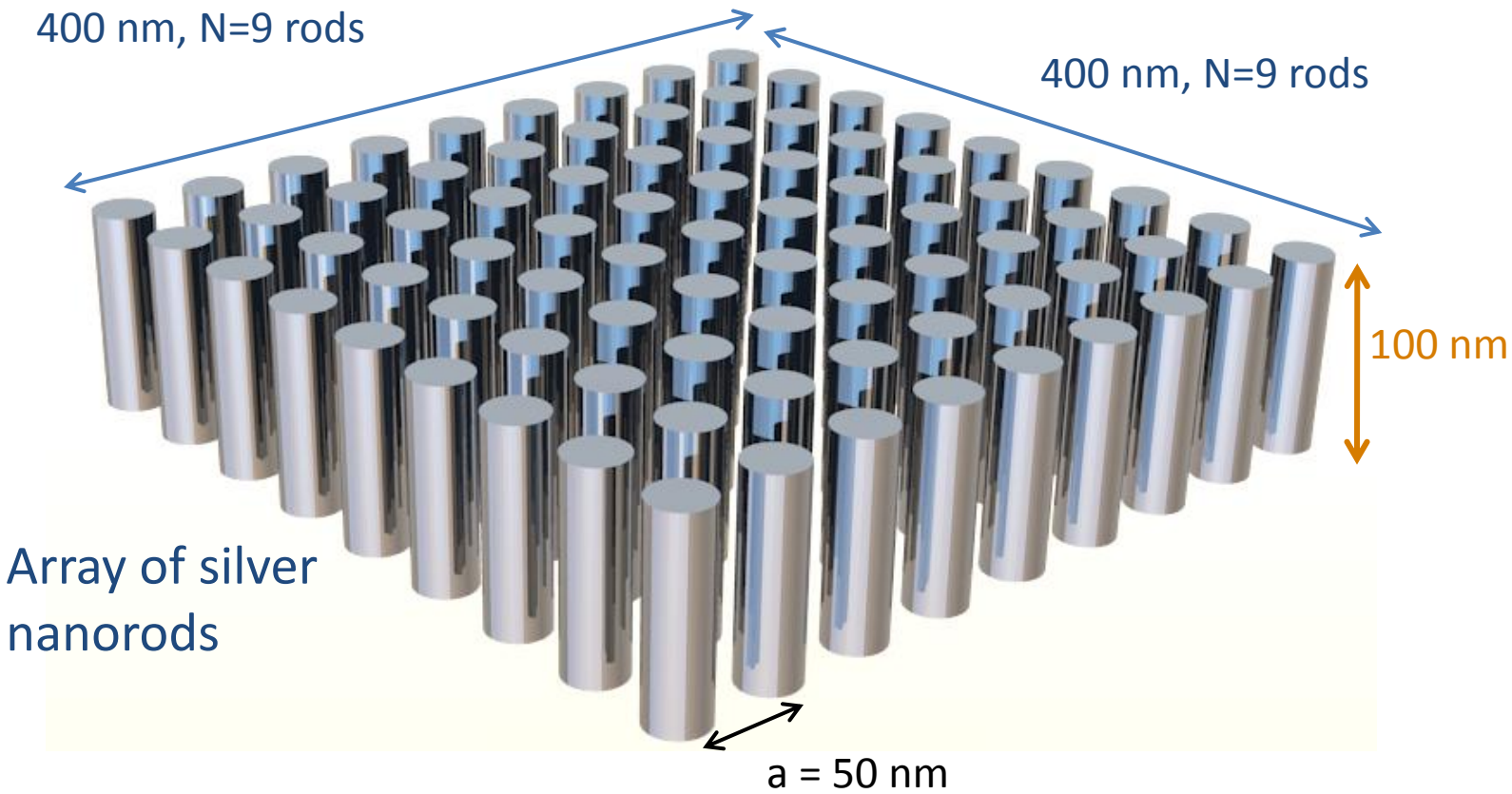
Focal spot at t=0:



Spatial inversion

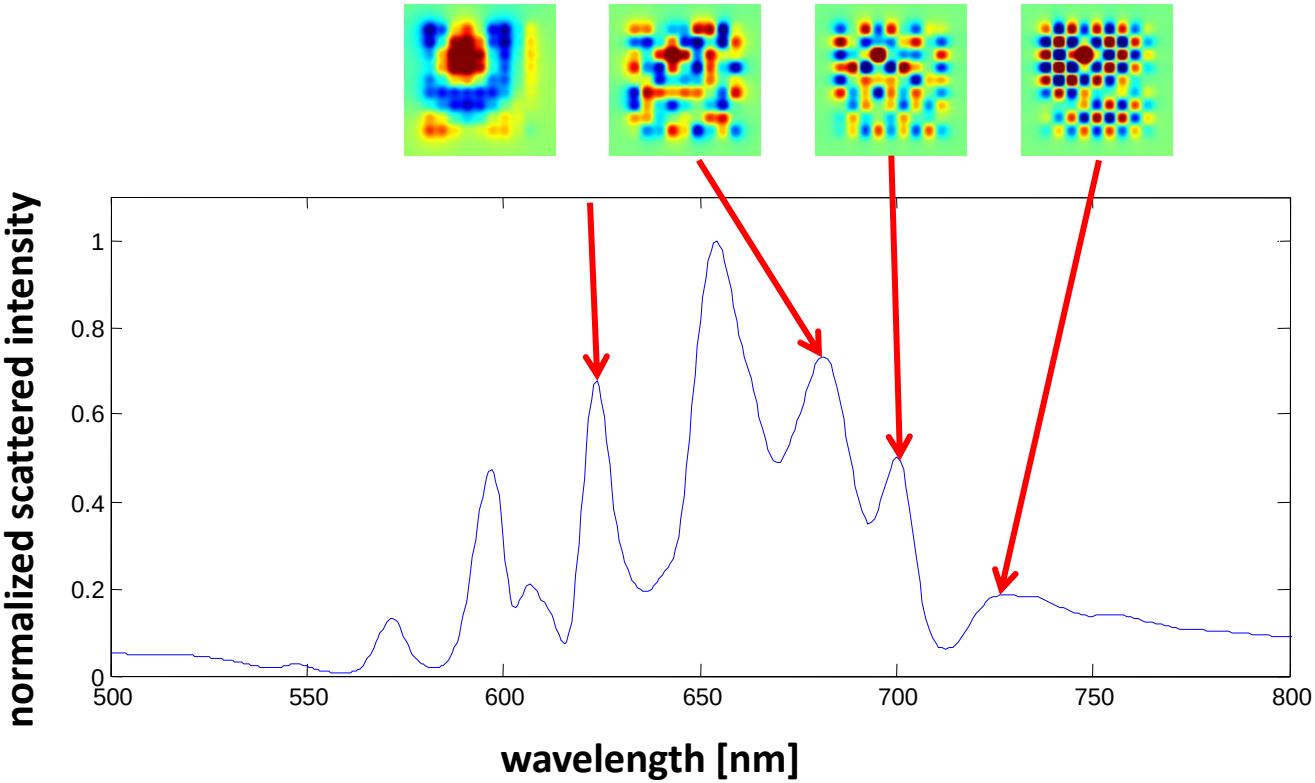


A plasmonic resonant metalens for the visible



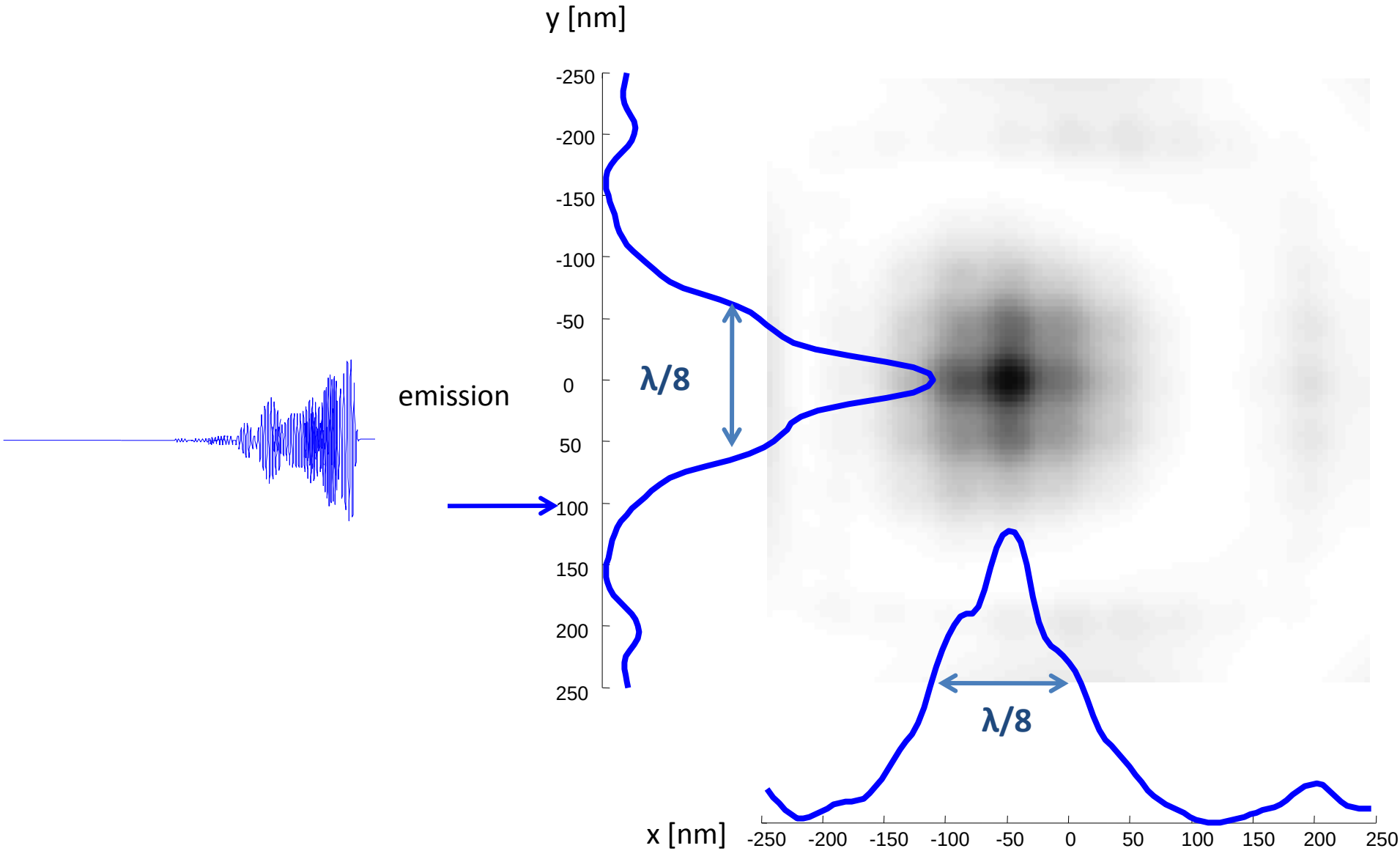
Lemoult F, Lerosey G et al., to be submitted

Subwavelength modes signature in the far field



Lemoult, Lerosey et al., to be submitted

Time reversal of a dipole far field: focusing



Time reversal at optical frequencies?

Control of light transmission through opaque scattering media in space and time

Jochen Aulbach,[1,2,*] Bergin Gjonaj,[1] Patrick M. Johnson,[1] Allard P. Mosk,[3] and Ad Lagendijk[1]

Controlled Spatiotemporal Focusing Through Turbid Media

Ori Katz*, Yaron Bromberg, Eran Small, Yaron Silberberg

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Shaping speckles: spatio-temporal focussing of an ultrafast pulse through a multiply scattering medium

David J. McCabe,[1,*] Ayhan Tajalli,[1] Dane R. Austin,[2] Pierre Bondareff,[3] Ian A. Walmsley,[2] Sylvain Gigan,[3] and Béatrice Chatel[1]

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Conclusion

Spatio-temporal degrees of freedom

PRL **103,** 173902 (2009) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2009

Manipulating Spatiotemporal Degrees of Freedom of Waves in Random Media

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(Received 4 June 2009; revised manuscript received 7 August 2009; published 20 October 2009)

- an array of closely spaced **coupled sub-λ resonators**
- needs **broadband illumination** or frequency scanning
- **codes a subwavelength** wavefield into a **temporal**/freq **signature**
- **radiates efficiently** all subwavelength modes same efficency
- can be used for **subwavelength focusing and imaging** from far field
- **limited** by **period** and of course **losses**