

Time Reversal of Ultrasonic Fields—Part I: Basic Principles

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Abstract—Time reversal of ultrasonic fields represents a way to focus through an inhomogeneous medium. This may be accomplished by a time-reversal mirror (TRM) made from an array of transmit-receive transducers that respond linearly and allow the incident acoustic pressure to be sampled. The pressure field is then time-reversed and re-emitted. This process can be used to focus through inhomogeneous media on a reflective target that behaves as an acoustic source after being insonified. The time-reversal approach is introduced in a discussion of the classical techniques used for focusing pulsed waves through inhomogeneous media (adaptive time-delay techniques). Pulsed wave time-reversal focusing is shown using reciprocity valid in inhomogeneous medium to be optimal in the sense that it realizes the spatial-temporal matched filter to the inhomogeneous propagation transfer function between the array and the target. Time-reversal focusing technique provides the optimal inputs to the transducer elements by maximizing the pressure at the target location. The research on time-reversed wave fields has also led to the development of new concepts that are described in the paper: time-reversal cavity that extends the concept of the TRM; iterative time-reversal processing for automatic sorting of targets according to their reflectivity and resonating of extended targets. In the last part of the paper the fundamental differences between a TRM and a phase-conjugated mirror are described.

I. INTRODUCTION

FOCUSING a wave on a target (moving or not) through an inhomogeneous medium is an important problem to solve in acoustics. The use of a time-reversal mirror (TRM) represents an original solution to this problem. We have developed such a mirror in the ultrasonic domain by utilizing piezoelectric transducer arrays [1].

In the time-reversal process, we take advantage of the properties of piezoelectric transducers, i.e., their transmit and receive capabilities, their linearity, and the capability of instantaneous measurement of the temporal pressure waveforms. The pressure field $p(\mathbf{r}_i, t)$ detected with a set of transducer elements located at positions \mathbf{r}_i is digitized and stored during a time interval T . The pressure field is then retransmitted by the same transducers in a reversed temporal chronology (last in, first out). This is equivalent to the transmission of $p(\mathbf{r}_i, T - t)$.

Such a time-reversal procedure allows one to convert a divergent wave issued from an acoustic source into a convergent wave focusing on the source. Unlike an ordinary mirror that produces the virtual image of an acoustic object, the TRM produces a real acoustic image of the initial source. This

process can be used to focus on a reflective target that behaves as an acoustic source after being insonified. The method works even if there is an inhomogeneous medium between the target and the mirror. The TRM consists of a one-or-two dimensional transducer array. Each transducer is connected to its own electronic circuitry that consists of a receiving amplifier, an A/D converter, a storage memory and most importantly a programmable transmitter able to synthesize a time-reversed version of the stored signal.

The research described here was first aimed at overcoming lithotripsy limitations. In state-of-the-art lithotripsy, the determination of stone position is achieved with either an ultrasonic scanner or an X-ray imaging unit. Although the stone position may be accurately known with X-ray systems, the focusing of the destructive ultrasonic wave, through inhomogeneous tissue remains difficult. The sound speed inhomogeneities can distort and redirect the ultrasonic beam. An even more crucial problem is related to stone motion due to breathing. The movement amplitude can be as large as 2 cm and stone tracking is needed for efficient therapy. The time-reversal process can solve these problems. The goal is to locate a given reflecting target among others, as for example, a stone in its surroundings (other stones and organs walls). The region of interest is insonified by the transducer array. The reflected field is sensed on the whole array, time reversed and retransmitted. As this process is iterated, the ultrasonic beam will select the target with the highest reflectivity. If the target is spatially extended the process will converge on one spot whose dimension depends only on the TRM geometry and on the wavelength. A high amplification of the last iteration can be used in stone destruction. This paper introduces the time-reversal approach through a discussion of the different techniques used for focusing in inhomogeneous media. In contrast to time-delay focusing techniques that can only correct distortions produced by a thin aberrator located close to the transducer array, TRM focusing is shown to compensate for distortion wherever the aberrator position.

It is shown that the TRM focusing is optimal in the sense that it realizes the spatial-temporal matched filter to the propagation transfer function through inhomogeneous media. It is a self-adaptive technique that compensates for any geometrical distortions of the array structure as well as for distortions due to the propagation through inhomogeneous media. This research led to the development of new concepts: a matched-filter description of time-reversal focusing; a time-reversal cavity that extends the concept of the TRM; iterative time-reversal processing for automatic sorting of targets

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according to their reflectivity; and resonating of extended targets.

The TRM is a generalization of an optical phase-conjugated mirror (PCM) in the sense that the TRM applies to pulsed broadband signals rather than to monochromatic ones. A comparison of these concepts shows that, in contrast to PCM that works in continuous mode, TRM processing permits choice of any temporal window to be time reversed. This allows operation in an iterative mode.

In a companion paper in this issue [2], the experimental results obtained with a 64-channel time-reversal mirror are described. That paper demonstrates the possibility of robust focusing through an inhomogeneous medium, as well as the performance of the iterative mode for focusing on the most reflective target.

II. ULTRASONIC FOCUSING IN INHOMOGENEOUS MEDIA

A. Adaptive Time-Delay Focusing Techniques

Time-delay focusing techniques are based on the use of transducer arrays. The pressure signals coming from an acoustic source are sensed by each transducer element and then digitized. A cross-correlation algorithm is next used to estimate the time-delay between signals from neighboring array elements. These delays determine the optimal time-delay characteristic required to focus on the source. The source can be passive, for example, a target reflecting an incident wave. In Fig. 1, we first show the principle of focusing in a homogeneous medium based on a set of delay lines (thick bars) that exactly compensate the spherical curvature of the signals sensed on the array. The echographic signals are thus brought into phase before their summation. In the lower part of that figure, we show the influence of an aberrating medium. The signals are no longer aligned by the spherical delay characteristic. We need to superimpose a compensating time delay. The proper time delays are determined by the time shift corresponding to the peak of the cross-correlation between signals from neighboring transducers [3], [4]. Another technique consists in the maximization of the energy of the summed signals with respect to the time-delay law [5]. These two techniques are similar in the sense that the energy maximization can be viewed as the maximization of the cross-correlation between all pairs of signals at the same time [6]. These two techniques are currently being investigated to determine the proper time delay for focusing in medical ultrasonic imaging where the inhomogeneous medium is the tissue path.

However, these techniques rarely give rise to optimal focusing. The first problem comes from the nature of the reflective target on which we want to focus. The ideal target is point-like and behaves as the source of a spherical wave that is distorted in its propagation through the aberrating medium. In practice, medical ultrasound scanners work in the near field and the sources may not be point-like. For example, a kidney stone or a bone has a finite size. Then elementary signals coming from such targets can be very different from one transducer to the other and cross-correlation results will thus be inaccurate.

In more frequent situations, the region of interest does not contain any highly reflective target. This is, for example, the

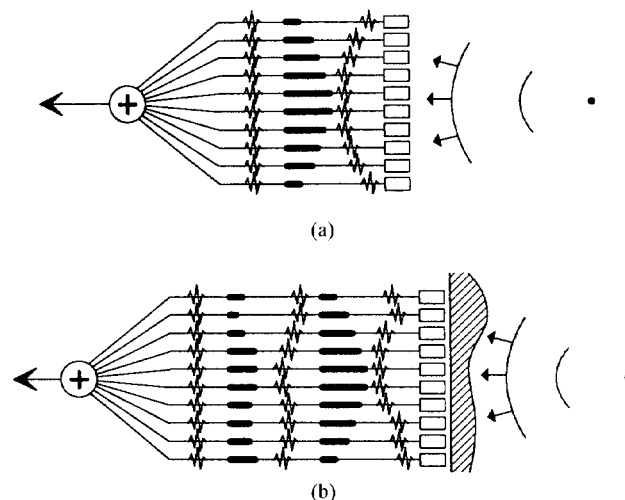


Fig. 1. (a) Cylindrical beamforming. The delay lines (thick bars) compensate for the cylindrical curvature of the signals recorded on the array. (b) Influence of an aberrating layer located against the array. The signals no longer align according to a cylindrical law. A second set of delay lines have to be superposed to the cylindrical compensation to brought the signals in phase before their summation.

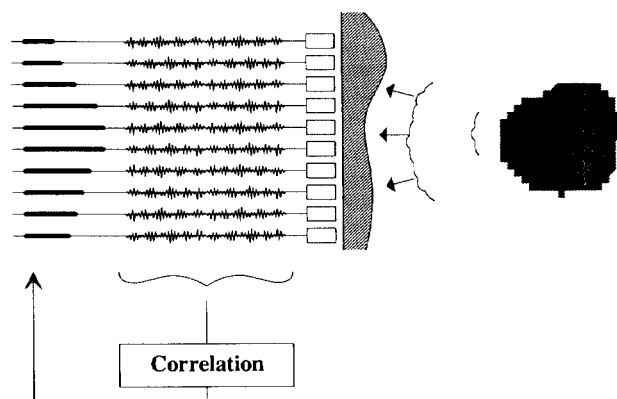


Fig. 2. Echographic speckle noises recorded from a scattering medium. Neighboring transducers sense signals that are correlated. The adequate time delays can be estimated either by cross correlation or by maximization of the energy of the summed signals.

case in imaging of the abdomen where sound speed aberrations in the body can degrade the focusing characteristics of the ultrasonic beam at frequencies in the low megahertz range. Here, the single reflective target is replaced by the many scatterers that comprise the region being imaged. The adequate focusing time-delay law can be estimated for such scattering media with a cross-correlation technique (Fig. 2) [3]–[6]. This possibility results from a remarkable property of the scattered pressure field that results from the summation of the individual echoes reflected on each scatterer. The field propagates with a spatial correlation function width that increases proportionally to the propagation distance. This property, which is described by the Van Cittert Zernike theorem [7] means that, neighboring transducers, as long as they are sufficiently far from the scattering region, will sense echographic signals that are highly correlated [7], [8]. An aberrating layer will shift these signals and the cross-correlation technique will indeed find the proper delay.

Although the problem of the target nature seems to be under control, the nature of the inhomogeneous medium seems to be more critical. An underlying assumption of the cross-correlation technique is that all the effects of the inhomogeneity on the spherical wavefront sum up as a simple distortion of the wave front shape, i.e., the aberrator only modifies the propagation delay between the source and the elementary transducers. Then, knowledge of the proper time-delay characteristic is sufficient for accurate focusing in the receive or transmit modes. However, this hypothesis is very restrictive; it is valid only when the aberrator is thin and located very close to the array (See Fig. 1(b)). In optical astronomy, this hypothesis is valid as long as the atmospheric turbulence is confined close to the telescope and far from the stars. In most medical applications this hypothesis is false: we always work in the near field of the transducers and the inhomogeneities are distributed over the whole volume. A wave propagating in such an inhomogeneous medium is not only delayed, but its spatial and temporal shape is also distorted through refraction, diffraction, and multiscattering. The adaptive delay-line focusing technique no longer works when the array probe-aberrator distance increases [6]. In these situations a more general approach is needed. It consists of a time-reversal process that takes into account all of the information recorded from the medium (time-delay characteristic and shape variation).

B. Time-Reversal of Ultrasonic Fields: Basic Principles

Here we discuss time-reversal mirrors in the framework of acoustic wave propagation in inhomogeneous media. We consider the propagation equation valid in a lossless fluid medium with compressibility $\kappa(\mathbf{r})$ and density $\rho(\mathbf{r})$ that vary with space. Introducing the local sound velocity as $c(\mathbf{r}) = (\rho(\mathbf{r})\kappa(\mathbf{r}))^{-1/2}$, we obtain for the propagation equation of an acoustic pressure field $p(\mathbf{r}, t)$ in the transient regime [10]:

$$\vec{\nabla} \cdot \left(\frac{\vec{\nabla} p}{\rho} \right) - \frac{1}{\rho c^2} \frac{\partial^2 p}{\partial t^2} = 0. \quad (1)$$

Looking at this propagation equation, we note that it has a special behavior with respect to the temporal variable t ; indeed, it contains only a second-order time-derivative operator. This property is the starting point of the time-reversal principle, valid only for a lossless propagation medium. As an immediate consequence, if $p(\mathbf{r}, t)$ is a pressure field solution of the propagation equation, then $p(\mathbf{r}, -t)$ is another solution of the problem. This property is specific to the invariance under a time-reversal operation. If the propagation medium has a frequency-dependant attenuation [9], the propagation equation may contain odd order time-derivative operator and therefore, the invariance under time-reversal is lost. Note that in biological media, if the attenuation coefficient is weak enough in the frequency range used in the experiment, the time-reversal invariance remains valid.

The special property of time reversal has been observed by Stokes [11] in the framework of the classical experiment of reflection and transmission of a plane wave along the interface separating two media of different sound velocity.

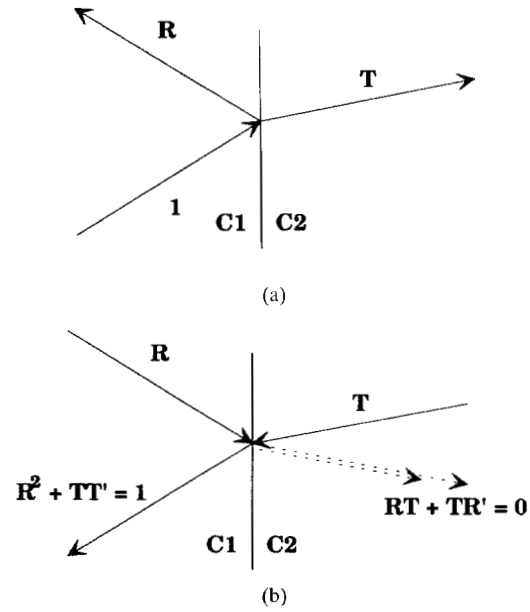


Fig. 3. (a) Reflection and transmission of a plane wave along the interface separating two media of different sound velocities. (b) Time reversal of Fig. 3(a).

Considering a plane incident wave of amplitude 1 propagating from medium 1 to medium 2, we can observe a reflected plane wave of amplitude R and a transmitted wave of amplitude T (Fig. 3(a)). Starting from this configuration in which the pressure field $p(\mathbf{r}, t)$ results from these three plane waves, Stokes tried to see if this experiment could be time-reversed or not. He used the plane wave property that time reversal can be accomplished by reversing the wave vector direction. Thus, the time reversed solution $p(\mathbf{r}, -t)$ can be described by a new set of three waves: two incident waves of amplitude R incident from 1 to 2 and of amplitude T incident from 2 to 1, followed by a transmitted wave of amplitude 1 propagating in medium 1 (See Fig. 3(b)). It can be easily verified that this new solution $p(\mathbf{r}, -t)$ is also a solution of the wave equation. Indeed, defining R' and T' as the reflection and transmission coefficients for an incident wave coming from medium 2, we see from the superposition principle that the two incident waves lead to the generation of four plane waves, two of them propagating in medium 1 with a resulting amplitude $R^2 + TT'$, and the other two propagating in medium 2 with a resulting amplitude $RT + TR'$. An elementary computation of the reflection and transmission coefficients R , T , R' , and T' allows us to verify the following relations:

$$R^2 + TT' = 1 \quad (2a)$$

$$R + R' = 0. \quad (2b)$$

Thus this example shows that the wave equation can be directly interpreted as the time-reversal of the previous situation.

In fact, these arguments can be generalized for different kinds of incident acoustic fields and other geometries of inhomogeneities. It is important to note that the two relations written above are only valid, if the reflected and transmitted plane waves have a real wavenumber (propagative). In a more general situation, the incident field contains evanescent components. Evanescent waves can be created

by incidence at specific angles or, for example, when an incident ultrasonic beam is scattered by a medium whose compressibility $\kappa(\mathbf{r})$ contains spatial-frequency components of linear dimensions that are, roughly speaking, smaller than the wavelength. Evanescent waves cannot be time reversed since their direction of propagation are undefined [12]. The superposition of propagative and evanescent waves leads to a limitation of the time-reversal process, and the basic theory must be analyzed carefully. Due to the finite bandwidth of the incident field, some information is lost in the time-reversal process.

The Time-Reversal Cavity: In any propagation experiment, initial conditions (the acoustic sources and the boundary conditions) determine a unique solution $p(\mathbf{r}, t)$ to the wave equation. Our goal, in time-reversal experiments, is to modify the initial conditions in order to generate the dual solution $p(\mathbf{r}, -t)$. However, due to causality requirements, $p(\mathbf{r}, -t)$ is not an experimentally valid solution. Therefore, we will limit ourselves to the generation of $p(\mathbf{r}, T - t)$.

One solution to the difficult problem of generating the time reversed solution consists in measuring during a time interval T the pressure $p(\mathbf{r}, t)$ in the whole three-dimensional (3-D) volume (T is long enough so that the pressure field vanishes for $t > T$), and then retransmit in all the volume $p(\mathbf{r}, T - t)$. This solution is unrealistic since it requires sampling the whole volume with transmit-receive probes.

A more realistic solution take advantage of the Huygens principle: the wavefield in any point of a volume can be predicted by the knowledge of the field and its normal derivative on a closed surface surrounding the volume. Therefore, the time-reversal operation is reduced from a 3-D volume to a 2-D surface. Indeed, the knowledge of the pressure field and its normal derivative at any point of a closed surface is sufficient to predict the pressure field inside of this surface [13]. Starting from this point of view, the focusing on a target in an inhomogeneous medium can be treated in the following way:

A point-like source located in an inhomogeneous medium creates a spherical wavefront that is distorted after propagation in the medium. We consider a closed surface surrounding the object source and the inhomogeneities, and we assume that we are able to measure the pressure field and its normal derivative at any point on the closed surface (Fig. 4(a)). In a second step, we assume that we are able to create secondary sources (monopole and dipole sources) on the surface that correspond to the time-reversal of the signals measured during the first step. As a result of the secondary sources created on the surface a time-reversed pressure field back-propagates inside the surface and is distorted by the interaction with the inhomogeneities.

Under such conditions, it can be shown [14], [15] that the time-reversed pressure field is focused on the initial source position (Fig. 4(b)). It is important to note that the time-reversal technique provides a better focusing than the correlation technique; in particular, it is not necessary to assume that the inhomogeneities are only located in the neighborhood of the transducer array, and to assume that the effects of inhomogeneities reduce to a simple time-delay varying from one transducer element to another.

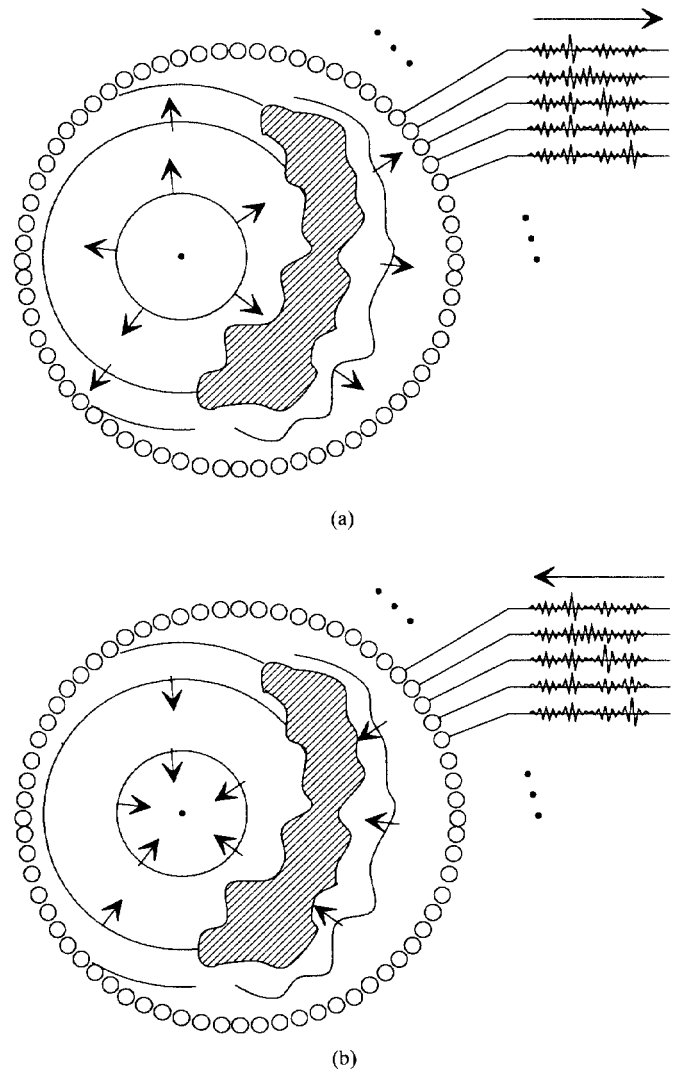


Fig. 4. Time-reversal cavity. (a) Recording step: A closed cavity is filled with transducer elements. A point like source generates a wave front that is distorted by heterogeneities. The distorted pressure field is recorded on the cavity elements. (b) Time-reversed or reconstruction step: The recorded signals are time-reversed and reemitted by the cavity elements. The time-reversed pressure field back-propagates and refocuses exactly on the initial source.

The efficiency of time-reversal techniques obtained from closed cavities is analyzed for weakly and strongly inhomogeneous media in a following paper in same issue [15], and it is shown that the time-reversed pressure field is focused on the initial source position. However, the finite spectral bandwidth of ultrasonic signals restricts resolution because the spatial scales of inhomogeneities that are smaller than the minimum wavelength, are blurred. Therefore, the generation of $p(\mathbf{r}, T - t)$ in the whole volume is not perfect [15].

The Time-Reversal Mirror: The time-reversal cavity is, of course, an ideal concept for focusing through inhomogeneous media. Unfortunately, such a cavity is difficult to realize in practice. The strongest limitation is linked to the difficulty of surrounding the focal region by a set of transducers. In medical applications, as well as, in nondestructive testing, we usually work in pulse-echo mode and the probe is located only on one side of the region of interest. This mode of operation is more practical and allows focusing from an array of transducers. In

this case, the time-reversal cavity may be replaced by a time-reversal mirror. Such a mirror can be plane or prefocused, one-dimensional (1-D) or two-dimensional (2-D). However, its ability to focus through aberrating media may be compared with that of closed cavity.

Other limitations of TRM, even in homogeneous media, are the same as those observed in classical focusing with delay-line techniques.

- 1) Diffraction effects act as low-pass filter on the spatial frequency spectrum of any wavefield. The resulting image of a point is a spot with dimension that depend on the wavelength.
- 2) The limited dimension of the TRM induces a point spread function whose width is related, as in classical focusing techniques, to the angular aperture of the mirror observed from the focal point.
- 3) The spatial sampling of the TRM by a set of transducers introduces grating lobes. These lobes can be avoided by using an array pitch of the order of $\lambda/2$, where λ is the central wavelength of the pressure field. However, such a fine sampling is not necessary if the TRM is prefocused on the region of interest (cylindrical or spherical mirror).
- 4) The temporal sampling of the data recorded and transmitted by the TRM has to be comparable to that of the time-delay law in classical time-delay focusing. A maximum rate of $T/8$ (T central period) is needed to avoid secondary lobes [16].

III. FOCUSING WITH A TRM

As illustrated in Fig. 5, time-reversal focusing on a target through inhomogeneous media requires three steps. The first step consists of transmitting a wavefront, through the inhomogeneous medium, from the array to the target. The target generates a scattered pressure field that propagates through the inhomogeneous medium and is distorted. The second step is a recording step; the scattered pressure field is recorded by the transducer array. In the last step, the transducer array synthesizes the time reversed field. This pressure field propagates through the aberrating medium and focuses on the target.

In the case of a weak inhomogeneous medium that satisfies the first Born approximation (single scattering process), the TRM is able to compensate exactly for the wavefront distortion whatever the aberrator position in the ultrasonic beam [17]–[20]. This is demonstrated by the experimental results presented in [2].

However, a limitation of the TRM compared to the closed cavity appears in the case of strong inhomogeneities. In such a case, multiple scattering processes limit the TRM efficiency. First, it is necessary to measure the pressure field on a very long time interval to take into account the multiple scattered waves that decay slowly; Secondly, the scattered field is radiated in all directions and a mirror cannot measure the complete information needed to optimize the true reversal. In comparison, in a weakly inhomogeneous medium that induces only single scattering the measurement of the distorted pressure field requires a shorter time duration. Moreover,

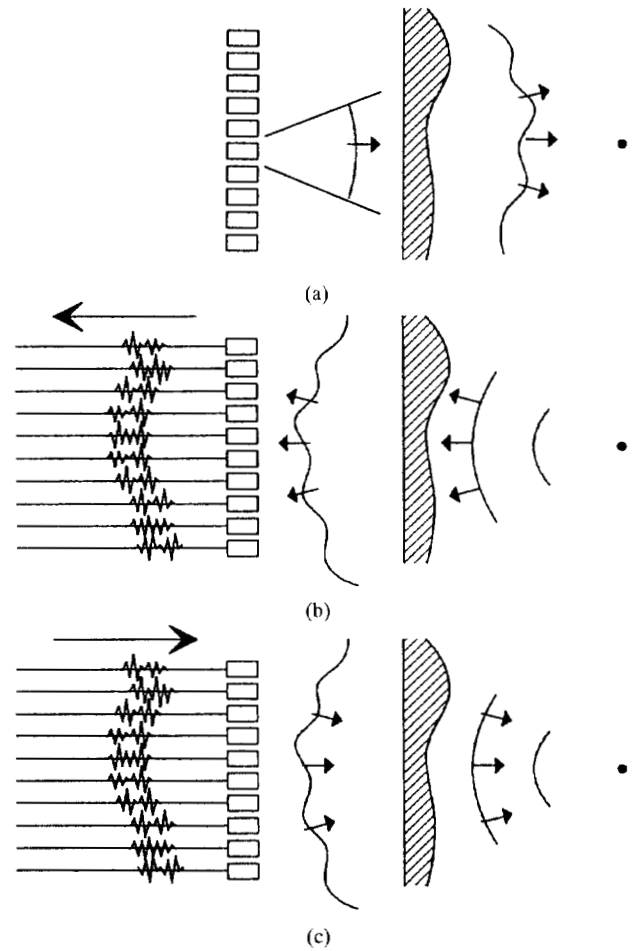


Fig. 5. TRM focusing through inhomogeneous media requires three steps. The first step (a) consists in transmitting a wavefront through the inhomogeneous medium from the array to the target. The target generates a backscattered pressure field that propagates through the inhomogeneous medium and is distorted. (b) The second step is the recording step; the backscattered pressure field is recorded by the transducer array. (c) In the last step the transducer array generates on its surface the time reversed field. This pressure field propagates through the aberrating medium, and focuses on the target.

the time reversal of the field recorded on only one plane is sufficient to achieve optimal focusing on the target.

IV. THE TIME REVERSAL FOCUSING MATCHED-FILTER APPROACH

Time-reversal focusing in inhomogeneous media is a very robust technique, compared to time-delay focusing techniques. In the following we will show that, for any set of transducer elements, time-reversal focusing of pulsed waves is the optimal solution for focusing through inhomogeneous media, in the sense that it realizes the spatial-temporal matched filter to the inhomogeneous propagation transfer function between the elements and the target.

The time-reversal focusing approach is closely related to the matched-filter principle used in signal processing. This well known principle is that the output of a given linear system whose impulse response is $h(t)$ is maximized with an input signal with the form $h(-t)$ [21]. The response results from the convolution $h(t) \otimes h(-t)$. It is an even function whose

maximum is reached at $t = 0$ and is given by the energy of the input signal.

Our goal is to show that the TRM focusing technique provides inputs to the transducer elements that maximize the pressure at the target location. Our analysis is based on the reciprocity theorem and is, therefore, valid in any lossless inhomogeneous media for any set of transducers used in the transmit-receive mode (size, sampling pitch, geometry). It can be formulated as follows.

We wish to find the optimal transient signals transmitted from a set of transducer elements E_i located in r_i in order to focus on a point located in \mathbf{r}_0 . To compute the pressure field produced by the TRM in \mathbf{r}_0 , we first calculate the individual pressure fields produced by each transducer element E_i . The method we propose is based on an approach in the transient regime and it uses diffraction impulse theory.

A. Acoustic Impulse Responses

When a single transducer element E_i transmits an impulsive normal velocity excitation ($v_n(t) = \delta(t)$), the field at point \mathbf{r}_0 can be represented by an acoustic velocity potential $\phi(\mathbf{r}_0, t) = h_i^t(\mathbf{r}_0, t)$ where $h_i^t(\mathbf{r}_0, t)$ is the so-called diffraction impulse response [22]. The acoustic pressure is then obtained through a time derivative of $h_i^t(\mathbf{r}_0, t)$ as $p_0 = (\partial h_i^t(\mathbf{r}_0, t))/\partial t$ [22]. Due to the finite size of the transducer, the diffraction impulse response of element E_i at location \mathbf{r}_0 in the transmit mode is not simply a delayed dirac pulse, it is a signal of nonzero duration [22], [23]. This duration is related to the different transit times of the impulses arriving in \mathbf{r}_0 from the Huygens point sources filling the transducer aperture. A similar impulse response $h_i^r(\mathbf{r}_0, t)$ can be defined in the receive mode as the acoustic force acting on the transducer due to an impulsive pressure source located in \mathbf{r}_0 [22].

Analytic solutions of the diffraction impulse responses have been derived in homogeneous media for some transducer geometries [22], [23]. However, in inhomogeneous media, the evaluation of the diffraction impulse response is generally very complicated. Fortunately, it is not necessary for our purpose to calculate these impulse responses in inhomogeneous media. The only important point is to show the diffraction impulse responses in the transmit mode and in the receive mode are identical even for an inhomogeneous media. For this purpose we use the reciprocity theorem that indicates the position of a point source and an observer can be reversed without altering the observed acoustic pressure [10], [24]. Note the reciprocity theorem is valid in homogeneous as well as in lossless inhomogeneous media [24].

Introducing the so-called Green function G , where $G(\mathbf{r}_0, t_0|\mathbf{r}, t)$ is the field produced in \mathbf{r} at time t by an impulsive source located in \mathbf{r}_0 and excited at time t_0 , the reciprocity theorem can be written as

$$G(\mathbf{r}_0, t_0|\mathbf{r}, t) = G(\mathbf{r}, t_0|\mathbf{r}_0, t). \quad (3)$$

Since $h_i^t(\mathbf{r}_0, t)$ is the integral over the transducer surface S_i of $G(\mathbf{r}, t_0|\mathbf{r}_0, t)$:

$$h_i^t(\mathbf{r}_0, t) = \int_{S_i} G(\mathbf{r}, t_0|\mathbf{r}_0, t) dr \quad (4)$$

and $h_i^r(\mathbf{r}_0, t)$ is the integral over the transducer surface S_i of $G(\mathbf{r}_0, t_0|\mathbf{r}, t)$

$$h_i^r(\mathbf{r}_0, t) = \int_{S_i} G(\mathbf{r}_0, t_0|\mathbf{r}, t) dr. \quad (5)$$

The reciprocity theorem tells us that the two impulse responses are identical. In the following they will be written as $h_i(\mathbf{r}_0, t)$. Note that this assertion is obvious in homogeneous medium since the Green function $G(\mathbf{r}_0, t_0|\mathbf{r}, t)$ reduces to $\delta(t - t_0 - |\mathbf{r} - \mathbf{r}_0|c_0)/|\mathbf{r} - \mathbf{r}_0|$, which is invariant under an exchange of \mathbf{r} and \mathbf{r}_0 . Next, we take into account acousto-electric responses of the transducers. In the transmit mode, $h_i^{tae}(t)$ relates the electrical input to the normal velocity output. In the receive mode, $h_i^{rae}(t)$ relates the acoustic force input to the electrical output. In general, these two impulse responses are identical and are written $h_i^{ae}(t)$ [25], [26].

B. Time-Reversal Focusing

As illustrated in Fig. 5, the TRM focusing process consists in three steps. After the first step (target illumination), a point target located in \mathbf{r}_0 behaves as an acoustic source. Within a convolution with a constant waveform, the electrical signal observed (output signal) at element E_i is given by

$$h_i^{ae}(t) \otimes h_i(\mathbf{r}_0, t). \quad (6)$$

These signals are recorded in step 2 for each transducer element. This step is illustrated for an homogeneous medium in Fig. 6(a) and for an inhomogeneous medium in Fig. 7(a). Note that in this last case the signals recorded by each transducer element can have very different shapes. In step 3, these signals are time-reversed and we obtain for the new electrical input:

$$h_i^{ae}(T - t) \otimes h_i(\mathbf{r}_0, T - t). \quad (7)$$

These signals are then retransmitted through the same transducer element. In order to compute the total acoustic field at \mathbf{r}_0 due to the TRM, we first consider the individual field produced by a single transducer element E_i . This field results from the convolution product of the electrical input signal ($h_i^{ae}(T - t) \otimes h_i(\mathbf{r}_0, T - t)$) with the transmit acousto-electric and diffraction impulse responses ($h_i^{ae}(t) \otimes h_i(\mathbf{r}_0, t)$) and can be written as

$$h_i^{ae}(T - t) \otimes h_i(\mathbf{r}_0, T - t) \otimes h_i(\mathbf{r}_0, t) \otimes h_i^{ae}(t). \quad (8)$$

The electrical input signal (the time-reversed signal) is the optimal input to the linear system defined by the two consecutive impulse responses $h_i^{ae}(t)$ and $h_i(\mathbf{r}_0, t)$. The maximum of the field in \mathbf{r}_0 is reached at time T independent of the element E_i location (Figs. 6(b) and 7(b) right). The total acoustic field produced by the TRM is obtained by the simultaneous transmission of the individual time-reversed pressure fields from all the elements. It leads at \mathbf{r}_0 to the summation of all the individual transducer contributions:

$$\sum_i h_i^{ae}(T - t) \otimes h_i(\mathbf{r}_0, T - t) \otimes h_i^{ae}(t) \otimes h_i(\mathbf{r}_0, t). \quad (9)$$

All the individual signals reach their maxima at the same time T , resulting in a constructive interference. Such a matched-filter process produces a maximum of the field in \mathbf{r}_0 for a

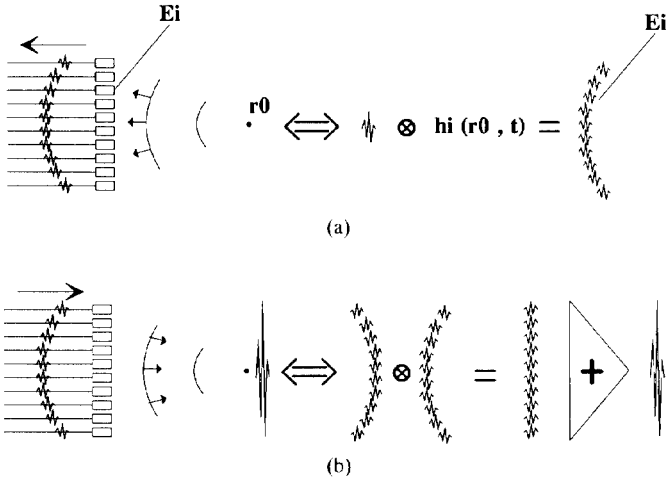


Fig. 6. Matched-filter approach of TRM focusing in homogeneous medium. (a) On the left part of the figure, the spherical wave front originating from a point source is recorded on the array. On the right part, the individual waveforms recorded on the array elements correspond to (6). They are presented as the convolution product of the acoustoelectric response with the individual diffraction impulse responses $h_i(\mathbf{r}_0, t)$. (b) The left part of the figure represents the time-reversed propagation toward the source. The right part of the figure shows the mathematical construction of the individual pressure waves sensed at the source location. They correspond to the convolution product of the two wave fronts as written in (8). All the individual signals reach their maxima at the same time. The simultaneous emission of all the transducers results in a constructive summation of all these individual waves.

specified input energy. The process is illustrated in Figs. 6(b) and 7(b). Note that this process is valid for any kind of array geometry. For example, a large array may be initially distorted geometrically. TRM focusing is self-adaptive and allows an automatic compensation of these effects. Besides, this analysis is based on the reciprocity theorem only. Therefore, it is also valid in any inhomogeneous medium. In this case, the individual acoustic fields can have very different shapes, however, all their maxima occur at the same time T . The time-reversal focusing is the best choice in inhomogeneous media to focus since all the individual time-reversed waves will reach their maxima at position \mathbf{r}_0 at the same time T .

Although the TRM techniques maximize the pressure at the target location, the axial resolution is degraded due to the different convolution of the signal by the acoustoelectric impulse response of the transducers. The experimental results presented in the companion paper [2] confirm the validity of this model.

V. THE ITERATIVE TIME REVERSAL MODE

A. Automatic Target Selection

One major advantage of the TRM is the ability to choose the origin and the duration of the signals to be time reversed. This is done through the definition of the temporal window that selects the data to be time reversed. When the medium of interest contains several reflectors, the time-reversal technique cannot directly be focused on one point. Indeed, if the medium contains two targets of different reflectivity illuminated by a short pulse, the time reversal of the echoes reflected from these targets will generate two

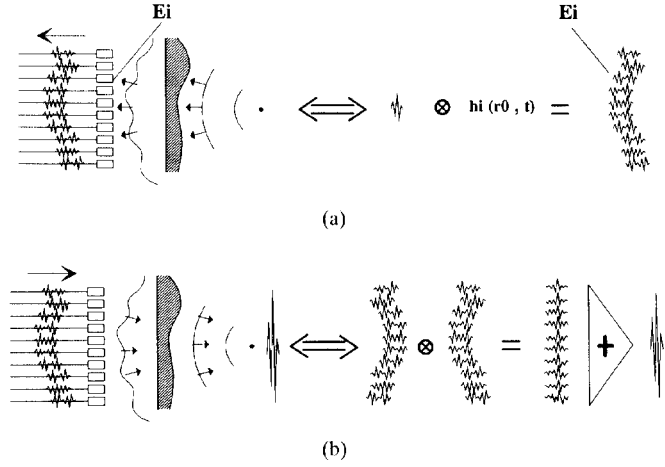


Fig. 7. Matched-filter approach of TRM focusing in inhomogeneous medium. (a) and (b) are similar to (Fig. 6(a)) and (Fig. 6(b)) except that the aberrator distorted the spherical wavefront originating from the point source (Fig. 6(a)). The diffraction impulse responses are very complicated, and the individual acoustic waveforms recorded on the array can have very different shapes. In (b), due to the convolution product of (8) all the individual waves reach their maxima at the same time at the source location.

wavefronts refocused on each target. The mirror produces the real acoustic images of the two reflectors on themselves. The highest amplitude wavefront illuminates the most reflective target, while the weakest wavefront illuminates the second target. However, what we have described is, in fact, only valid if we neglect the multiple scattering processes that can occur between the two targets. In order to avoid these multiple scattered waves, we can select echoes within a particular time-reversal window. In this case, the time-reversal process can be iterated [1], [19]. After the first time-reversed illumination, the weakest target will be illuminated more weakly and will reflect a wavefront much fainter than the one coming from the strongest target (Fig. 8). After some iterations, the process will converge and produce a wavefront focused on the most reflective target. This process converges if the target separation is sufficient to avoid the illumination of one target by the real acoustic image of the other target [19], [20].

Although this simple presentation implies the iterative mode is very attractive, one can argue a contradiction exists between the concept of iteration and the physical principle of time-reversal invariance. Indeed, the complete time reversal of an acoustic "scene" results in the time-reversed scene. Therefore, the iteration of the time-reversal operation gives stationary results, a contradiction with wave field modification after each iteration. In fact, the contradiction is only apparent. A complete time-reversal operation requires a closed time-reversal cavity surrounding the acoustic scene and a recording time T long enough to take into account all the multiple scattered waves. In our technique, we utilize only a finite spatial aperture and short temporal windows. Therefore, some information is lost. This information loss gives the iterative mode its target selection capabilities.

Experimental results show the efficiency of the process on single wire targets [2]. In lithotripsy applications the iterative mode allows the automatic selection of one of the kidney stones [2].

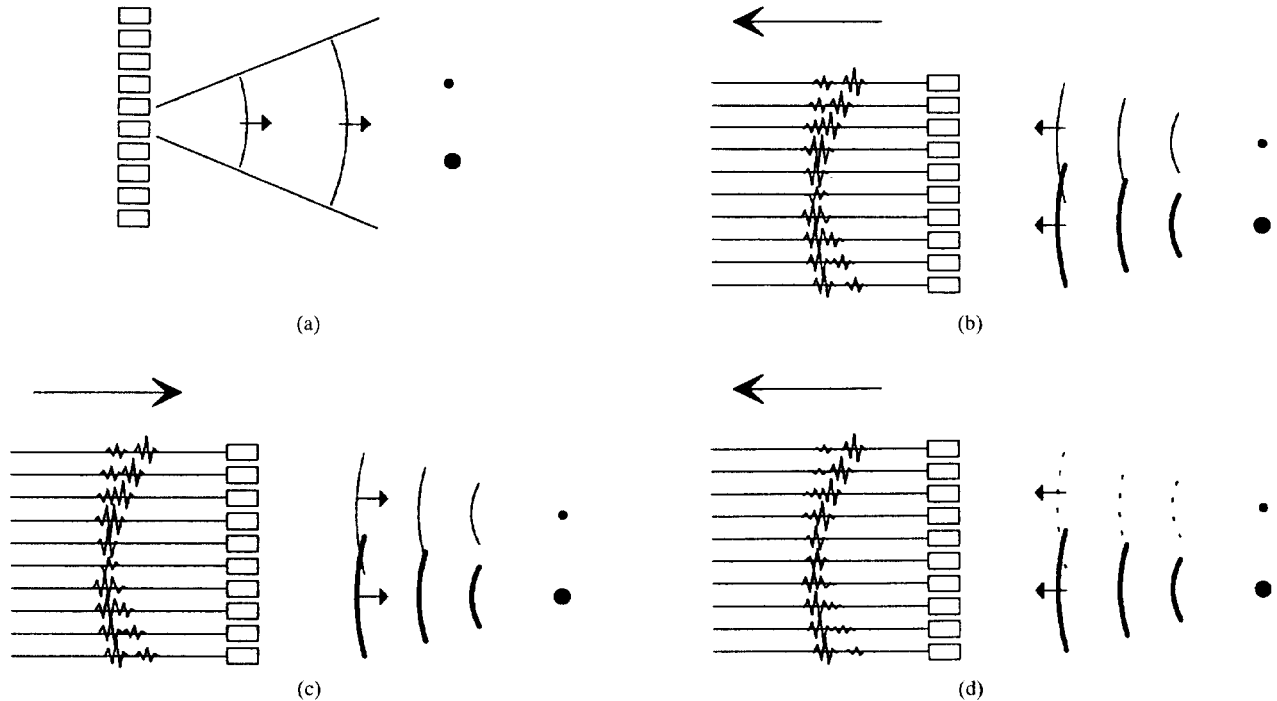


Fig. 8. Principle of the iterative time-reversal mode. (a) A first transmitted wave illuminates a sector containing the two targets. (b) The reflected waves are recorded on the array. (c) The data are time-reversed and reemitted. The time-reversed wave fronts refocus on the two targets. The highest amplitude wavefront illuminates the most reflective target, while the weakest wavefront illuminates the second target. (d) The new reflected wave fronts are recorded before another time reversal. Note the weakest target has been illuminated more weakly and reflect a wavefront much fainter than the one coming from the strongest target. After some iterations the process will converge and produce a wavefront focused on the most reflective target.

B. Target Resonance

In lithotripsy applications, the iterative process is complicated by a stone being an extended target. An incident beam can induce acoustical resonances of the stone and the wave reflected by the stone may contain different wavefronts resulting from these resonances [2]. In such cases, the time-reversal mirror is also efficient. It allows some matching of the new transmit waveform in order to produce stone resonances through the iterative time-reversed transmission.

A simple explanation of this phenomenon can be given in a 1-D experiment conducted with a plane transducer illuminating at normal incidence a parallel face plate medium. The pulse-echo signal $e(t)$ reflected by the plate contains the multiple reflected waves generated in the resonant plate. For an ideal transducer excited by an electrical pulse $\delta(t)$, the signal $e(t)$ represents the impulse response of the resonator. The time-reversal process leads to a transmission of $e(-t)$. This is the optimal signal needed to excite the faceplate resonance and corresponds to the faceplate matched-filter that gives the maximum system response (Fig. 9).

The same concepts may be extended with care to 3-D vibrating structures (the extended target) illuminated by 2-D time-reversal mirrors. Although, the theoretical modeling of this process is complex, and we present simple physical ideas that serve as guidelines for this study. The vibration of a complex 3-D structure is a linear combination of the different natural Fourier modes of the problem. Each mode is characterized by a wave vector K_n . For example, in the case of the parallel face plate there is only one mode with

a wave vector orthogonal to the faces and with a modulus that depends on the plate thickness. In the case of a complex structure, the TRM can intercept the plane waves radiated by these modes if the corresponding wave vector directions are inside the angular aperture of the TRM. Besides, due to the array finite bandwidth, the phase conjugation of these modes will work only for wave vectors whose modulus correspond to frequency included in the transducer bandwidth. These ideas suggest the possibility of an automatic selection and excitation of some vibration modes of an object.

VI. COMPARISON BETWEEN TRM's AND PHASE-CONJUGATED MIRRORS

The basic principle of a time-reversal mirror is an extension of an optical phase-conjugated mirror. If $P(r, \omega)$ is the temporal Fourier transform of $p(r, t)$, then the temporal Fourier transform of $p(r, -t)$ is $P^*(r, \omega)$. Therefore, the time reversal of pulsed signals is equivalent to the phase conjugation of monochromatic waves. However, this equivalence is only valid mathematically, there are some fundamental differences between these two techniques.

A. Linearity Versus Nonlinearity

Since the time responses of optical detectors are very long compared to the period of optical waves, an optical phase-conjugated mirror (PCM) requires a nonlinear effects to measure and to conjugate the phase information on monochromatic wavefields [27]. The optical phase conjugation can be understood as a real time dynamic holographic method.

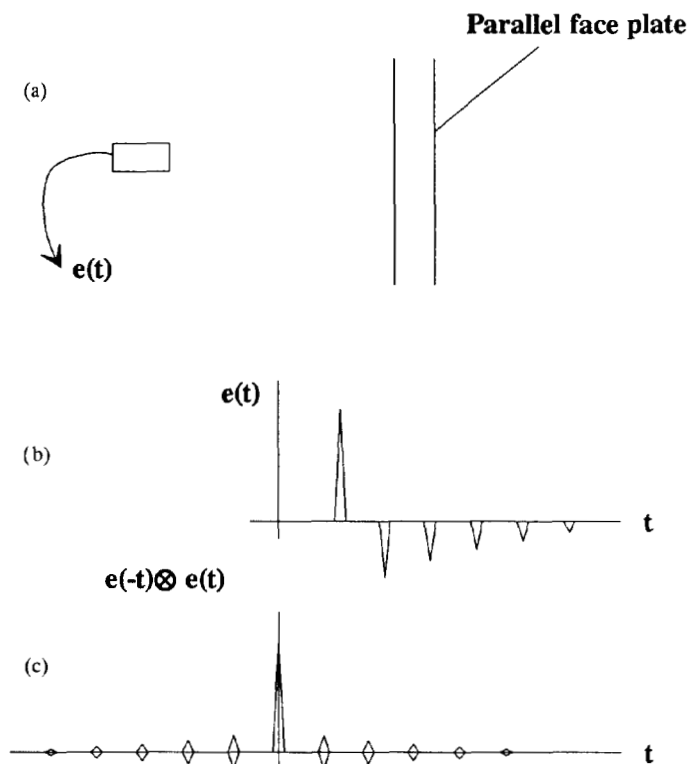


Fig. 9. (a) Parallel faceplate resonating the pulse-echo signal $e(t)$ reflected by the plate contains the multiple reflected waves generated in the resonant plate (b). The time-reversal process leads to a transmission of $e(-t)$. This is the optimal signal needed to excite the faceplate resonance. It corresponds to the faceplate matched-filter (c) and it gives the maximum system response $e(-t) \otimes e(t)$.

Indeed, it is well known that the holographic technique allows the generation of the complex conjugate of an incident monochromatic wave.

The basic principle of holography consists in the superposition of the incident monochromatic wavefield with a reference wave, and in the measurement of the resulting interference fringes on the holographic plate. The interference of two waves transforms the phase information of the incident wave, that is necessary but not otherwise available, into intensity information that is experimentally available. This is a nonlinear process.

After development of the holographic plate, a photographic transparency is obtained with the information of the incident wave and its complex conjugate. An appropriate illumination of this photographic transparency (e.g., the complex conjugate of the reference wave) leads to the generation of the complex conjugate of the incident wave. This technique involves two reference waves, one during the recording step and a second one during the reading step (Fig. 10).

In the case of a PCM, the holographic plate is replaced for example by a nonlinear photorefractive medium (LiNbO_3 , BSO, ...) that spatially changes its photorefractive index (with a time response of a few tens of milliseconds) in reaction to the interference of the incident wave with one of the reference waves. The refractive index changes are due to electrooptic effects. Interference between the incident

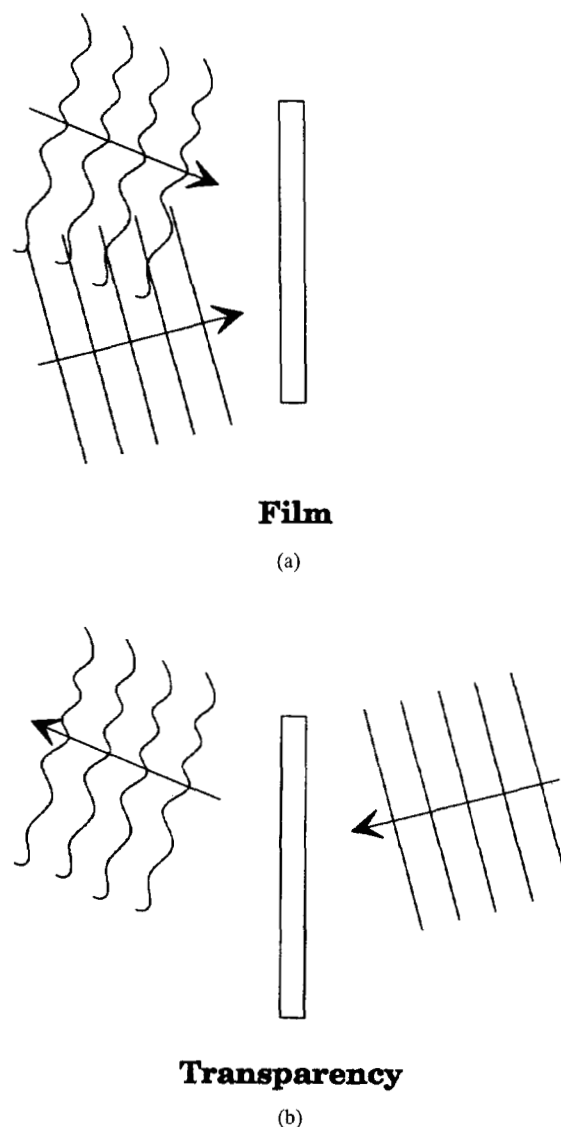


Fig. 10. Principle of monochromatic phase-conjugated holography. The first step (a) consists in the recording on the holographic plate of the interference fringes between the incident monochromatic wavefield and a reference plane wave. After development of the holographic plate, a photographic transparency is obtained. The illumination of this transparency (b) by the complex conjugate of the plane reference wave leads to the generation of the complex conjugate of the incident wave.

beam and the reference beam results in phase volume gratings in the photorefractive media. The second reference wave is then diffracted by this resulting grating, thus generating the complex conjugate of the incident wave. Several similar configurations can be realized either with this kind of wave mixing (four wave-mixing) or with other nonlinear techniques using the stimulated Brillouin scattering effect [27]. In all these techniques a wave incident on the PCM produces a phase-conjugated wave in a nonlinear medium.

In the time-reversal technique, wave mixing is not required and nonlinear effects are not necessary. They are replaced by the use of linear reversible transducers linked to a read-write memory. All these steps are linear. Additionally, the phase conjugation occurs in the whole volume of the photorefractive medium, while the time reversal occurs only on a 2-D surface area.

B. Continuous Versus Transient Regime

In optics, once the photorefractive medium has modified its photorefractive index, a continuous state is reached. That means that the concept of time disappears from the process. All the continuous monochromatic waves generated in the aberrating media (single- or multiscattering) and in the photorefractive medium will interfere. This requires laser coherence length to be much longer than dimensions of the optical circuit. However, due to causality requirements, there is a transient period during which the different waves generated cannot interfere. In the time-reversal experiments, the ultrasonic signals are brief and all the processes occur in the transient regime.

In the continuous regime, the interference between the incident wave and the any subsequent multiply reflected waves between the PCM and the aberrator play a very particular role. It can be shown that the resulting interference between all these waves allows the exact compensation of the distortion due to the aberrating medium. This compensation is achieved for weakly inhomogeneous media (first Born approximation) as well as for strongly heterogeneous media containing discontinuities [28]–[30]. The following experiment illustrates the particular role of the continuous regime in the case of a medium containing discontinuities [29], [30].

Recall that a PCM works in a continuous mode and phase conjugates an incident wave, for example, a plane wave of amplitude 1 propagating in medium 1 of sound velocity c_1 (Fig. 11(a)) and consider the effect of inserting a new medium 2 of sound velocity c_2 between the source of the incident wave and the PCM (See Fig. 11). An initial reflected wave of amplitude R and a transmitted wave of amplitude T result. The transmitted wave T is then phase conjugated by the PCM. This phase-conjugated wave is reflected and transmitted through the interface 2 to 1. A new reflected wave comes back to the PCM with a new incidence and is phase conjugated. The process repeats itself periodically. The main point is that the set of waves, generated in medium 1 from all the phase-conjugated waves, produces a resulting plane wave of amplitude 1 in the reverse direction of the incident wave and complete destructive interference in the direction of the initial reflected wave. Such a result means that, for an observer located in medium 1, the insertion of medium 2 between medium 1 and the PCM does not change anything. The observer see only the phase-conjugated wave of amplitude 1 with a reverse wavevector. This important result is linked to the property of time invariance of R , T , R' , and T' . The result is only obtained in the continuous mode [29], [30] in which all the generated waves interfere to destroy the initial reflected wave.

In time-reversal mirrors, such effects cannot be used. We work with pulsed waves in the transient regime. On the one hand, the impulse waves generated along the interface cannot interfere. On the other hand, the recording step does not overlap with the transmitted step and the different time reversed waves cannot interfere with the incident waves. Therefore, a single TRM located in medium 2 can neither cancel the wave of amplitude R reflected on the 1 to 2 interface, nor generate a time reversed wave of amplitude 1 in

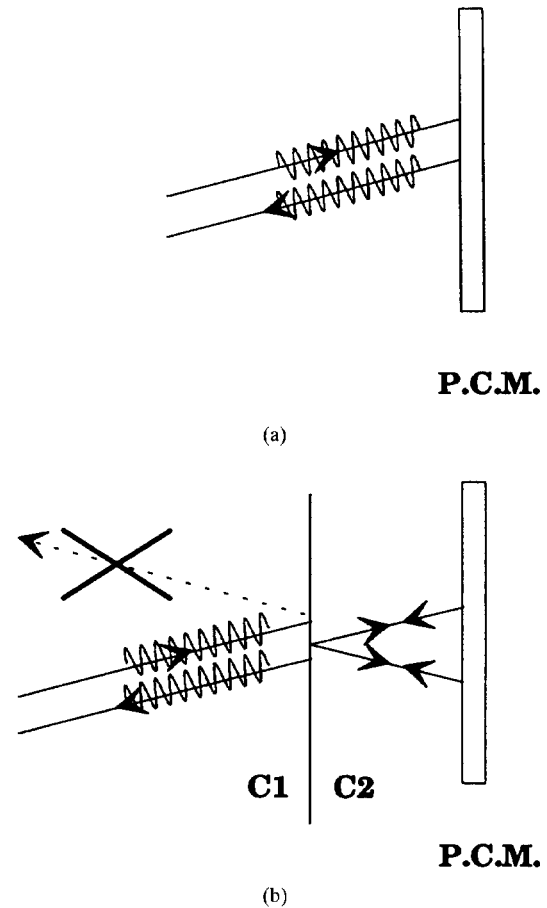


Fig. 11. (a) Principle of the PCM. An incident monochromatic plane wave is reflected and phase conjugated. The wave vector of the phase-conjugated wave is reversed compared to the incident wave vector. (b) The effect of PCM on phase velocity discontinuities. An incident plane wave induces along the interface the generation of a complicated set of waves. Due to the multiple reflections between the interface and the PCM, the resulting field in medium 1 reduces to the phase conjugated of the incident wave. There is a complete destructive interference in the direction of the initial reflected wave.

medium 1. In order to obtain this time reversed wave, a second TRM has to be placed in medium 1 behind the incident wave source [20]. This set of two TRM's is equivalent to a TR cavity.

C. Ultrasonic Phase-Conjugated Mirrors

Although our time-reversal approach developed for an ultrasonic field is quite different from the phase-conjugation approach, different authors have developed ultrasonic phase-conjugated mirrors based on the use of monochromatic signals and nonlinear effects [31].

The basic method used to produce phase-conjugated wavefronts is to mix the incident wavefront in space with a wavefield oscillating with a frequency twice that of the incident radiation. This approach requires an elastically nonlinear medium within which the mixing can be carried out. Piezoelectric or ferromagnetic media can be used to process the non linear mixing. Ohno [32] has studied such an effect with LiNbO₃ piezoelectric crystal working at 50 MHz. The incident wavefront propagating in the crystal is mixed with an electric field oscillating on the second harmonic. Brysev [33] has de-

veloped such a parametric interaction in Ni-ferrite polycrystal by mixing the incident wavefront with an oscillating magnetic field that modulates the sound velocity in the ferrite. Sato [34], [35] has used suspended microparticules as the nonlinear medium.

Another original technique has been developed by Nikoonahad [36]. The PCM is based, as in the TRM approach, on a transducer array that samples an incident monochromatic wavefront. The signals collected from each array element are electronically mixed with a second harmonic signal, using an array of mixers. The difference frequency signal so obtained has a phase that is the conjugate of that of the incident wavefront. The transducer array then retransmits the conjugated signals. One limitation of the technique is the continuous wave mode operation. Also, in principle, the phase reversed signals have to be fed back to the same array element at which the wavefront was sampled and this results in a short circuit across the two ports of the mixer. To overcome this difficulty, Nikoonahad feeds the conjugated signals to the element immediately adjacent to where the wavefront was sampled. The transducer array is then made of two sets or interlaced elements: those that sample the wavefront and those that transmit the conjugated wavefront. In order to avoid phase shift due the spatial offset, the array pitch has to be $\lambda/4$.

VII. CONCLUSION

Time reversal is a new approach for focusing an ultrasonic field. In this paper, focusing with a TRM through inhomogeneous media is shown to be optimal in the sense that it realizes the spatial-temporal matched filter to the propagation. However, TRM focusing has only been described in the transmit mode. The extension of this matched filter focusing technique in the receive mode requires further study.

The time-reversal cavity is a new concept that extends the TRM approach. It can be realized with two TRM's surrounding the region of interest and can solve different problems in nondestructive testing where liquid-solid interfaces strongly distort the incident beam.

Time-reversal techniques also represent an original solution to target selection in an iterative mode that allows selective focusing on the most reflective target in a multitarget medium. The important case of spatially extended targets also requires further study as does the study of time reversal in a scattering region that does not contain any highly reflective target.

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