nederlands akoestisch genootschap
NAG
journaal
nr. 174
maart 2005

Time-Reversed Experiments with Acoustics

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Abstract

In many mechanical systems the propagation of elastic and acoustical waves is reversible in time. A time-reversed reconstruction of the original signal can be obtained by recording the evolving signal, than reversing the recorded signal, and sending the reversed signal again through the system. The reconstruction of the time-reversed signal occurs in time as well as in space. The process is very robust and works even in systems with multiple scattering of the waves due to complex boundaries or heterogeneous material properties. Examples of such complex signals are the 'Coda' response in seismology and reverberation in room acoustics. In open systems where the waves can escape the region of interest, multiple transducers at the boundary of the region are required to perform an accurate reconstruction. The array of transducers acts as a time-reversal mirror refocusing the reversed signal to the original location of the source. In cavity like closed spaces a single transducer pair is sufficient to perform the time-reversed reconstruction. Any part of the reverberating source signal in a chaotic cavity reaches the receiving transducer after a while and contains enough information to reconstruct the original signal by time reversion. Braking of the time-reversal symmetry occurs for example by absorption, changes in acoustic velocity and dynamical changes of the structure. Such changes immediately influence the reconstruction quality. Time reversed reconstruction has many applications in non destructive materials testing and medical research. Laboratory experiments in a liquid filled bottle and initial audio experiments in a reverberating room are presented to illustrate some of the concepts.

1. Introduction

The understanding of the fundamental properties of wave propagation in strongly scattering systems stimulated many applications including such diverse topics as medical imaging through turbid media and ultrasonic underwater communication [1]. In strongly scattering environments waves still obey under very general conditions time reversal symmetry. I.e. it is possible to reverse the evolution of the system and evolve backwards in time until its begin conditions. Many fundamental aspects of time reversal properties have been demonstrated in acoustics by Mathias Fink et al. in Paris [2, 3] and lead to a variety of applications in materials testing, medical imaging and therapy, wireless communication and sonar.

In open systems the time-reversal experiments use an array of receiving transducers covering a part of the boundary area where outgoing waves leave the region of interest. The array records the scattered signal from the source. In the reconstruction stage of the experiment, the recorded signals are played through the same array but reversed in time. The signal from the array evolves as if time is reversed and causes a reconstruction of the original signal after a while at the place and time of the source. The time reversal process is very robust and works with a relatively small arrays covering only part of the boundary, limited section of the recorded signal, or even 1-bit digitalization of the recorded signal [3].

Time-reversed reconstruction also works in closed spaces, - cavities -, where the reflecting boundaries of the system prevent the escape of the waves [4]. For time-reversal experiments in a cavity an array with many elements is not required. A single pair of transducers launching and detecting waves anywhere in the cavity can be used to perform the reconstruction. The scattering from the walls cause enough delayed copies of the original to reach the detector and form the basis for the time-reversal reconstruction.

The time-reversal in cavities is easy to perform and requires only basic experimental equipment and can be used as an economic and effective method to probe small changes in the acoustical properties. In the following sections the origin of time-reversed experiments is discussed and some of the features are demonstrated in simple laboratory based experiments.

2. Theoretical considerations

2.1 The wave propagation

The free propagation of acoustical waves in a complex environment is disturbed by scattering, diffraction and absorption [5]. In a strongly inhomogeneous system were the acoustical properties vary on the length scale of the acoustical wavelength or larger the resulting signal is the sum of all the scattered signals that traveled along different paths. The length scale to characterize the scattering is the mean-free-path _ between scattering events. If _ inside the medium is much larger than the region of interest, the waves essentially propagate freely and the signal between transducers is undisturbed. In a closed system the waves propagate until they meet the boundaries and are reflected or absorbed. The reflections cause reverberation on the received signal. This response also occurs if _ is comparable to the distance between transducers. The received signal is than a combination of the direct wave and the delayed scattered waves.

A characteristic wave equation is the propagation of sound in gases and liquids. The pressure $p(\vec{r},t)$ in the gas in a source free region is described by a Helmholtz type second order partial differential equation [5]:

$$\vec{\ }.(\frac{1}{\rho(\vec{r})}\vec{\ }p(\vec{r},t))-\kappa(\vec{r})\frac{f^2 p(\vec{r},t)}{f\,t^2}=0\,, \tag{1}$$

where $\rho(\vec{r})$ is the density and $\kappa(\vec{r})$ the compressibility of the gas. The mean-free-path of waves in the system is determined by the variations in $\rho(\vec{r})$ and $\kappa(\vec{r})$. The full solution of $p(\vec{r},t)$ is obtained by adding a source term and specifying the properties at the boundaries around the region of interest (see figure 1). For an open system the boundary conditions far enough from the region of interest can be taken radiative; i.e. letting all waves pass through without reflection. In closed systems the boundary conditions for the pressure and the normal derivative of the pressure need to be specified to find the solution. For example for a rigid wall the derivative in the direction $\vec{n}$ (figure 1) normal to the surface vanishes. In general the boundary conditions should specify conditions for the pressure and the first derivative of the pressure. The source term can be included in equation (1), or incorporated by specifying the boundary value on a small inclusion in the whole system (small circle in figure 1). Such an inclusion can also be used to model scattering regions in the system. Representing the heterogeneity of the system by inclusions instead of choosing a spatial dependent $\rho(\vec{r})$ and $\kappa(\vec{r})$ has the advantage that the waves propagate in a homogeneous system and can be described by free wave propagation. Furthermore, if $p(\vec{r},t)$ and its derivative is known at the boundary, the solution in the interior is given as an integral over the boundary surface in the form of a Helmholtz integral [5]. Absorption can be included in equation (1) by choosing $\kappa(\vec{r})$ complex. The imaginary part of $\kappa(\vec{r})$ describes the absorption.

The basic acoustical equation (1) without absorption is a generic example of a time-reversible wave equation. This can easily be verified by observing that if $p(\vec{r},t)$ is a solution, than also $p(\vec{r},-t)$ is a solution even for very complicated spatial dependence of $\rho(\vec{r})$ and $\kappa(\vec{r})$. The wave equation (1) also has reciprocity symmetry. I.e. the role of a point-like transmitter and receiver can be interchanged and still give the same signal transfer response. In time reversal experiments both the time-reversal symmetry and the reciprocity are used in the reconstruction.

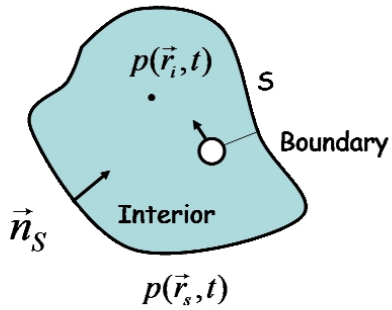

Figure 1: The Helmholtz type equation (1) specifies the pressure $p(\vec{r}_i,t)$ at a point $\vec{r}_i$ in a source free region. From the values at the boundary S the interior solutions can be obtained through the Helmholtz-Kirchhoff integral [5]. The conditions for a fully reflecting boundary require that the pressure derivative in the direction $\vec{n}_S$ normal to the surface vanishes.

Time-reversal reconstruction and reciprocity of the scalar wave equation (1) can be extended to the elastic wave equation and to the propagation of electromagnetic waves describe by the Maxwell equations. In general these wave equations are not scalar but have different wave propagation modes. For example in the elastic case the compression wave and two shear waves still obey reciprocity and time-reversal symmetry in a non-dissipative system. In the coming sections the discussion will be limited to the acoustical case.

2.2 Time-reversed experiment

A time-reversed experiment is using the time-reversal property of the wave propagation described by (1) and the connection between boundary values and the solution in the interior. A typical experiment in an open system uses an array of transducers on the boundary of the region of interest to record the signal from a (small) source $p_0(\vec{r}_0,t)$ inside the system (see figure 2). The heterogeneity of the system will distort the signal and result in a recorded signal $R_i(t)$ on each of the transducers i. Replaying the time reversed recordings $R_i(T-t)$ through the transducers generates the conditions on the boundary associated with the time reversed solution $p(\vec{r},T-t)$ of the recorded signal. The signals evolve as if time is running backwards and a reversed reconstruction of $p_0(\vec{r}_0,T-t)$ occurs at a time $t=T$. The reconstruction occurs in time and at the place $\vec{r}_0$ of the source. In a real experiment usually only a small section of the boundary is covered by the array. Yet, clear and robust time reversal reconstruction can be achieved with such a ‘time-reversal mirror’.

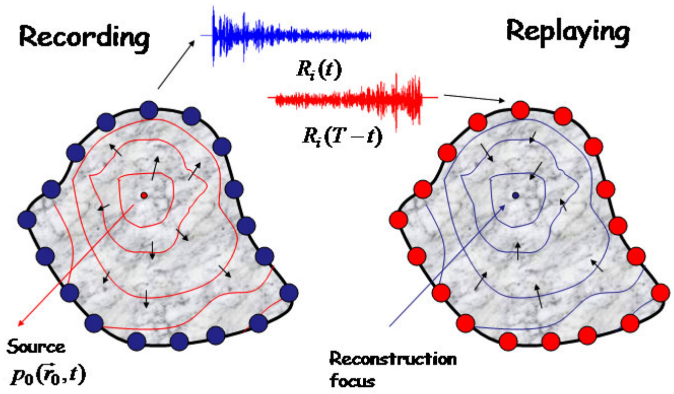

Figure 2: Time-reversed experiment in an open system

After the reconstruction the signals evolves further and becomes itself an apparent source. The interference of the signal still arriving and the signal already leaving limits the achieved spatial focusing to a diffraction limit that is in the order of the wavelength. The original source may have much smaller dimensions. To regain focusing on the length scale of the original source a local active source is required to suppress the reemission of the reconstructed wave [6].

The time-reversal process is very robust. Systematic studies of the spatial and temporal quality of the reconstruction of pulsed signals showed the influence of the duration, the used bandwidth and the digitalization quality of the recorded signals. In general the temporal quality is determined by the spectral content of the signals. In short, the longer the recorded sections the better the reconstruction. Better amplitude and time sampling of the signal enhances the reconstruction quality. The spatial quality is to a good approximation determined by the diffraction limit of a source with an effective aperture given by the size of the time-reversal mirror and the spreading of the signal in the medium. The size of the mirror may be small, but if the medium is scattering enough to distribute the wave over a large volume, the effective aperture can be enhanced considerably.

In an open system multiple transducer positions are required to be able to reconstruct the signal. This is different in a closed system, - cavity -, or a semi-open system such as a waveguide. The boundaries reflect the signal and disperse the waves throughout the enclosed region. In particular for a cavity system with chaotic properties the mode structure is so complex that waves are able to propagate between any two points inside the cavity. The chaotic nature can be induced by the complexity of the boundaries of by the heterogeneity of the medium. Under such conditions time-reversal experiments can be performed with a single transducer pair [4] and an experiment can easily be performed without the need for a complicated array set up.

In the following sections some of the concepts of time-reversal reconstruction discussed so far will be demonstrated in some basic laboratory experiments. As examples of relatively simple experiments in an enclosed space the time reversal reconstruction in a system based on a simple liquid-filled bottle and time-reversed reconstruction in a reverberating space will be discussed here. Of particular interest is how the time-reversal symmetry in the system can be disturbed.

Example 1: Ultrasonic experiments in a bottle

The basic experiment in a cavity is illustrated in figure 3. In a 25 ml glass bottle two needle tranducers (Valpey Fisher Pinducer VP93) operating in the 0.1 – 5 MHz ultrasonic range are used as a transmitter and a receiver. The source is an arbitrary wave generator (Agilent 33120) under computer control that can produce different pulse shapes but also arbitrary signal shapes downloaded from the computer. The receiver is connected to a pre-amplifier (Femto DHVPA-100) and recorded on a deep memory digital oscilloscope (Tektronix TDS620) also under computer control. The wave generator and the oscilloscope are controlled using Labview (National Instruments) and the results are further analyzed with Matlab (Mathworks inc.).

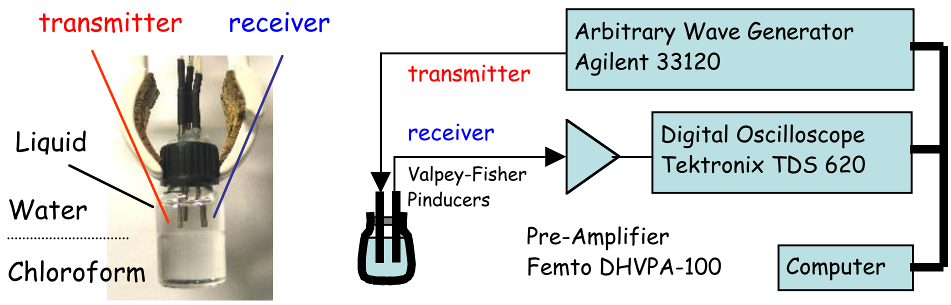

Figure 3: Time-reversed experiment in closed system consisting of a bottle filled with a liquid and using broad-band pin-shaped ultrasonic transducers at a frequency near 1 MHz. The set up is based on a computer controlled arbitrary wave generator and a digital oscilloscope.

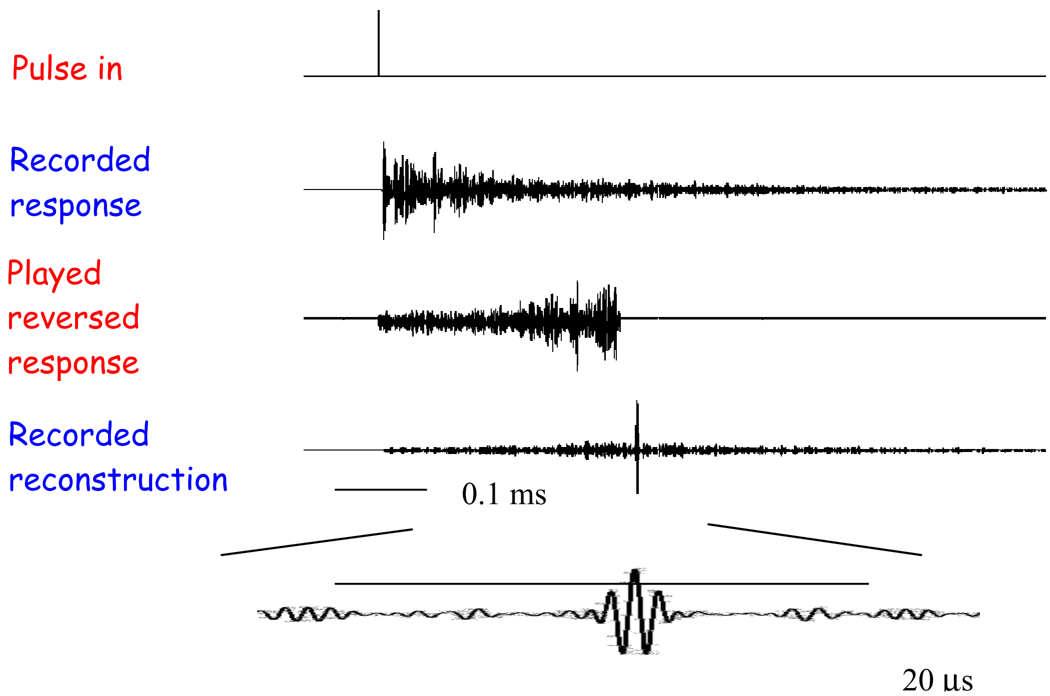

Figure 4: Time-reversed reconstruction in a water filled bottle immersed in a thermostatic bath using the set up illustrated in figure 3. The lowest trace shows a detail of the reconstructed pulse.

An example of a time-reversal reconstruction with this set up is shown in figure 4. The recorded impulse response is reversed in time and sent through the same transducer as the original pulse. I.e. the reciprocity in the system is used to avoid switching of transmitter and receiver in the experiment. The response of the reconstructed signal shows a clear peak illustrating the time reversal reconstruction of the signal. Before and after the reconstruction a lower reproducible signal is observed where the contributions to the full reconstruction start to build up and afterwards decay. The reconstructed signal is symmetric around the maximum, as can be seen in the bottom trace in figure 4. In fact, the reconstruction signal can be represented as a cosine series based on the relative contributions of the resonant modes in the cavity at the places of the transducers [4].

The reconstructed signal is very stable and remains undisturbed for days as long as the position of the transducers and the liquid level remains the same. As such it is a sensitive probe of changes in the acoustical properties in the system. Changes in detector positions or the liquid level on a sub-wavelength scale does destroy the reconstruction.

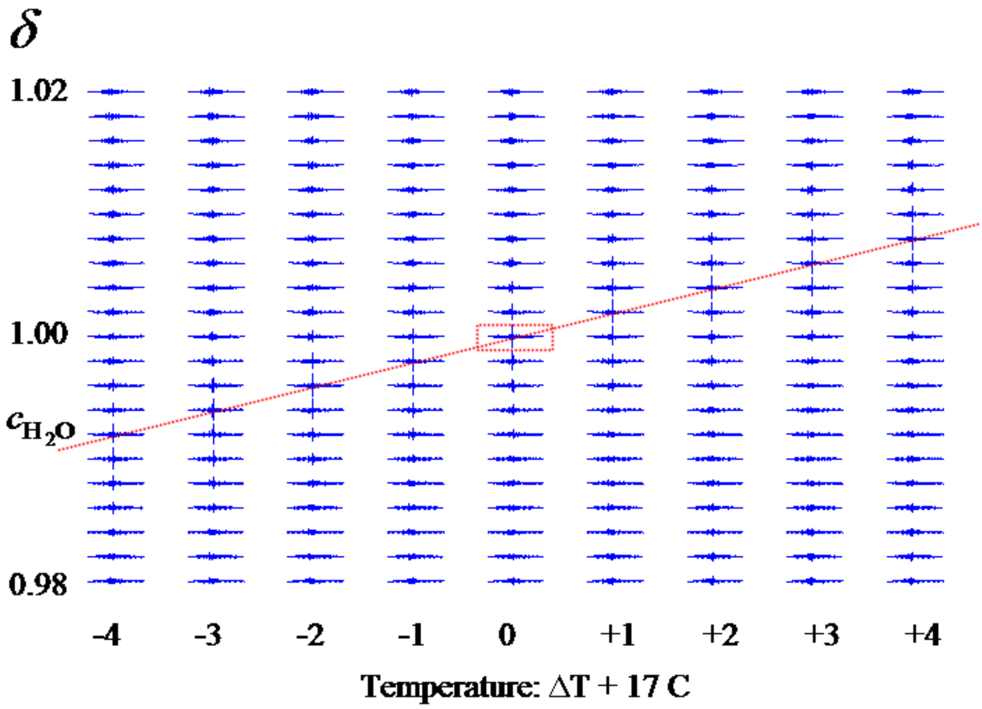

Figure 5: Recovery of time reversal reconstruction in a H_2O sample as function of temperature by time dilation. Each trace represents a reconstruction response using the conditions with $\Delta T=0$ and $\delta=1$. The dotted line represents the relative temperature dependence of the sound velocity in water: $c_{H20}\ (17 + \Delta T\ ^0C) = 1480\ m\ s^{-1} + 2.4\ m\ (s\ ^0C)^{-1}\ \Delta T\ ^0C$.

The reconstruction is in particular sensitive to changes in the replay rate of the arbitrary wave generator. A change in the replay rate effectively compresses or stretches the time axis of the replayed signal and acts as a time dilation for the wave propagation. Since the reconstruction is the result of the interference of waves that travelled for a long distance in the system, a small change in the replay frequency has a drastic influence on the relative phases of the components of the reconstruction. In [7] is demonstrated that the decay of the peak-peak voltage V_{pp} scales with: $V_{pp} \cup 1 - \frac{1}{6}(2\pi f_0 \Delta)^2 \delta^2$, where f_0 is the central frequency in the signal, Δ the length of the time section used for the reconstruction and δ the dilation of the time axis: $\delta = \frac{f_{rep}}{f_{ref}} - 1$, with f_{rep} the used replay frequency and f_{ref} the original replay frequency. Thus a dilation δ=0.001 is already sufficient to suppress the reconstruction when working at f_0 = 1MHz and Δ = 1ms.

Changing the temperature of the liquid will change the sound velocity and the reconstruction [8]. The effect on the reconstruction is similar to the time-dilation effect. A lower velocity leads to longer paths length and is equivalent to choosing $|\delta| > 1$. The combined effect of temperature and dilation are illustrated in figure 4. Each of the traces depicts a reconstruction trace for different temperature and replay rate. The middle trace is the original optimal condition. Changing temperature or δ only destroys the reconstruction. The reconstruction is recovered when both temperature and replay rate are changed simultaneously. The dotted line in figure 4 indicates the temperature dependence of water and is reflected in the recovery of the time-reversed reconstruction. The strict compensation only works for homogeneous liquids. When the bottle is filled with two non-mixing liquids such as water and chloroform the compensation by dilation does not work because the temperature dependence of the sound velocity is different for each liquid [9].

The time-reversal experiments in a cavity type sample holder are currently used in our laboratory to monitor slow changes in complex liquids and soft condensed matter such as liquid mixtures, colloidal suspensions, gels and glasses. Apart from the temperature effect, also changes in the acoustical properties due to phase transitions and dynamic changes in the heterogeneous material influences the time reversal reconstruction. Of interest are slow processes of defect formation, creep and aging. The influence of absorption can usually be compensated by amplifying the reverse signal before reemitting through the system.

Example 2: Time-reversed audio reconstruction in reverberating rooms

In the audio range time-reversed techniques are also feasible. In open systems the use of arrays of speakers and microphones can be used to construct a time-reversal mirror and allow for complex wave field generation in general [10, 11]. In a strongly reverberating room a situation similar to the cavity situation discussed in the previous section is valid.

To demonstrate some of the features of time-reversed reconstruction a basic audio setup based on personal computer audio equipment was used. With the modern sound cards available at a moderate price high quality digitalization and replay up to 24 bit resolution and 200 kHz rates are easily realized. The used setup is based on the firewire linked sound system (Edirol FA-101) in combination with digital speaker set (Edirol MA10D) and two measuring microphones (Behringer EMC8000).

The typical reverberation impulse response to a 35 ms tone burst at a center frequency of 1750 Hz is illustrated in figure 7. The reconstruction is clearly visible in the bottom trace even though the staircase is not providing a very long and well mixed signal. Part of the reconstruction is the direct signal that has not been excluded from the reversed and replayed sound.

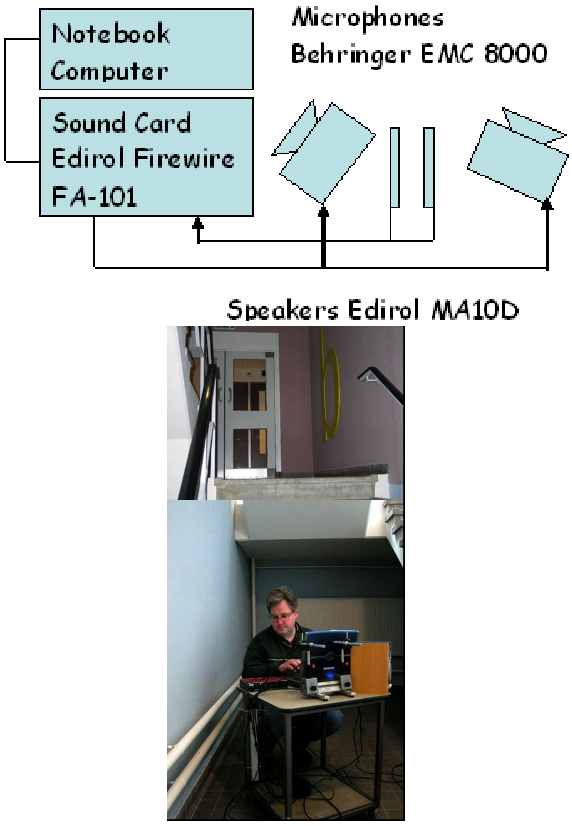

Figure 6: Audio time-reversed experiments in a 5 story high stair case at the van der Waals-Zeeman Institute. Left: photo of the stair case. Right: schematic audio set up based on standard personal computer sound equipment (see text).

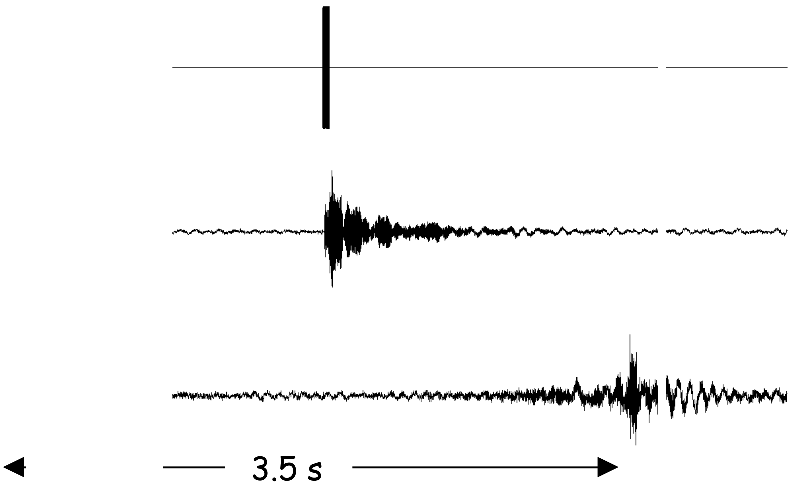

Figure 7: Audio time-reversed reconstruction. Top: input pulse with a center frequency of 1750 Hz. Middle: the recorded response of the reverberating stair case with a reverberation time of approximately 1s. Bottom: time-reversed construction

That indeed the signal is composed of many delayed and constructively added signals can be verified by using the time dilation method outlined in the previous section. By changing the sampling frequency in the replayed signal, the reconstruction of the long paths is suppressed and will not contribute to the reconstruction. The effect of time dilation is illustrated in figure

8. The reconstruction is destroyed when a sampling frequency of 11125Hz instead of 11025 Hz is used to replay the reversed signal.

Also the spatial dependence of the reconstruction can easily be demonstrated. Figure 9 illustrates a recorded train of responses and the reconstruction. The used signal was actually transposed by a factor three to obtain a signal and response at a 3x higher frequency. By listening to the signals the effect of the reconstruction can be directly appreciated. When the speaker is moved by only 20 cm, about the wavelength of the tone burst, the reconstruction is destroyed. This illustrates that the reconstruction is also spatially localized on a lengthscale of the wavelength of the sound.

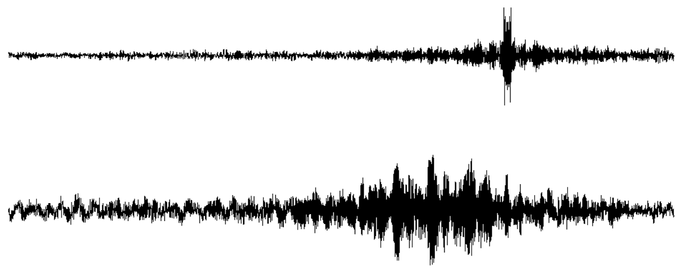

Figure 8: Audio time-reversed reconstruction with the same pulse as in figure 3 using different replay rate of the reversed recorded pulse response. Top: with the same replay frequency as the recording rate $F_{rec} = F_{rep} = 11025$ Hz, Bottom: $F_{rep} = 11125$ Hz. The signals are scaled to the same maximum amplitude.

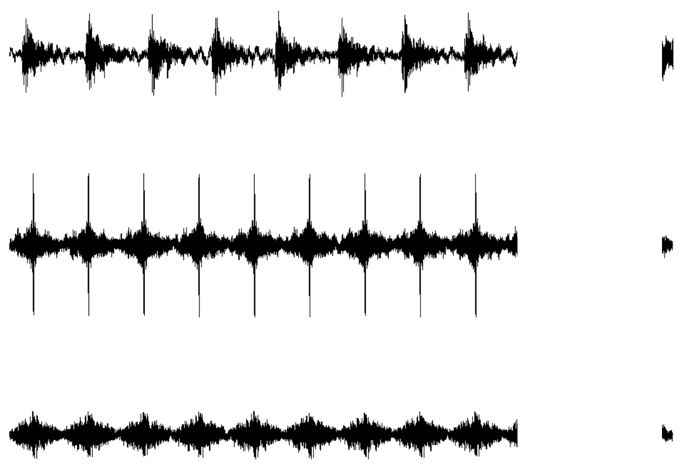

Figure 9: Time-reversed reconstruction of a train of pulses separated by a time longer than the reverberation time. Top: the recorded response, Middle: time-reversed reconstruction. Bottom: time-reversed reconstruction after moving the speaker by 20 cm. The two reconstruction signals have the same scale.

Conclusions

Wave synthesis and signal reconstruction using time-reversed techniques is a powerful approach in elastics and acoustics to control multiple scattering of waves in complex systems. Already many applications using the concepts of time-reversal reconstruction in open and closed systems have been developed in acoustic and elastic media and many will probably follow in acoustics as well as other areas where wave propagation is influenced by scattering. For example the effect of multiple scattering on wireless communication and the use of time-reversal techniques to optimize data transfer between antenna arrays has been demonstrated recently [12]. Also the intricate connection between time-reversal symmetry, wave imaging by correlation of ambient noise and wave localization has emerged as a new direction [13, 14]. Many of the basic principles can be explored by basic laboratory experiments with ultrasonic, acoustic, and RF waves.

Acknowledgements

This work is partly supported by the research program of the 'Stichting voor Fundamenteel Onderzoek der Materie (FOM)', which is financially supported by the 'Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO)'.

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