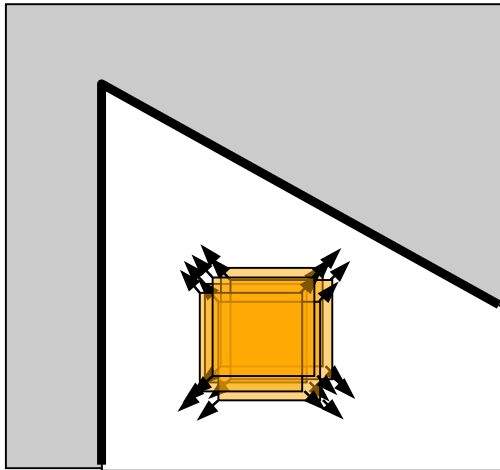
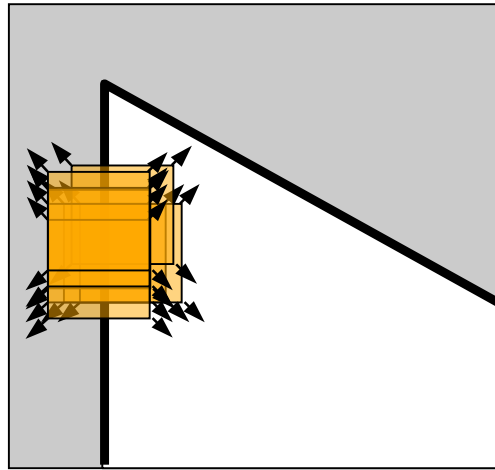


## Corner detection: the basic idea

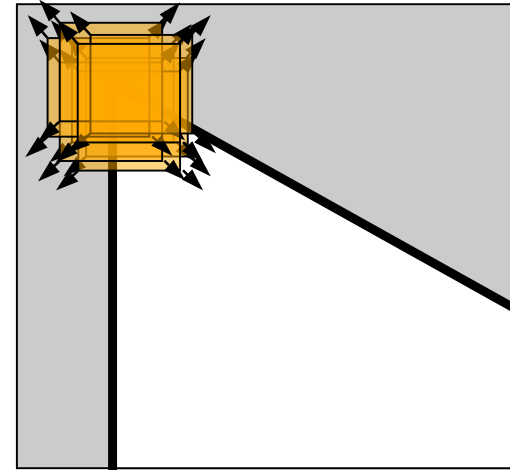
- At a corner, shifting a window in any direction should give a large change in intensity



“flat” region:  
no change in  
all directions



“edge”:  
no change along  
the edge direction



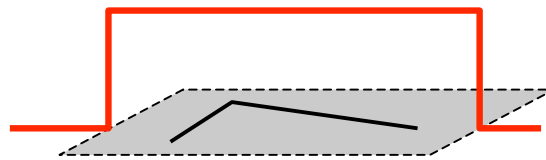
“corner”:  
significant change  
in all directions

# A simple corner detector

- Define the sum squared difference (SSD) between an image patch and a patch shifted by offset  $(x,y)$ :

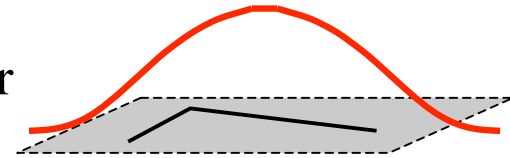
$$S(x,y) = \sum_u \sum_v w(u,v) (I(u,v) - I(u-x, v-y))^2$$

where  $w(u,v) =$



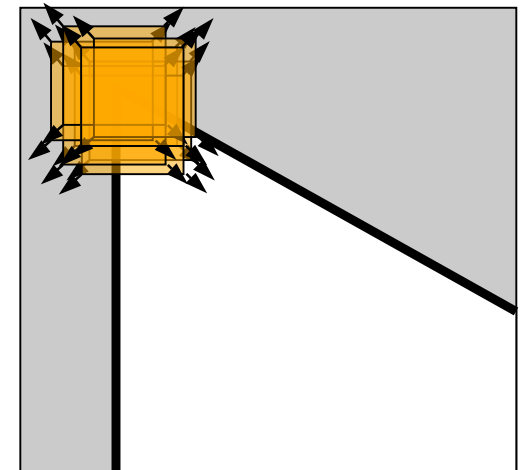
1 in window, 0 outside

or



Gaussian

- If  $s(x,y)$  is high for shifts in all 8 directions, declare a corner.
  - Problem: not isotropic



# Harris corner detector derivation

- First-order Taylor series approximation:

$$S(x, y) = \sum_u \sum_v w(u, v) (I(u, v) - I(u - x, v - y))^2$$

$$S(x, y) \approx \begin{bmatrix} x & y \end{bmatrix} A \begin{pmatrix} x \\ y \end{pmatrix}$$

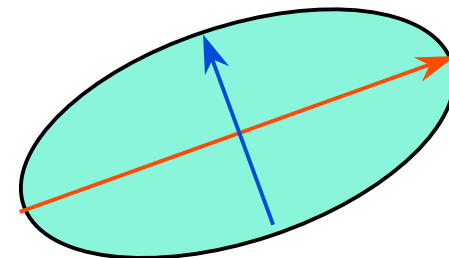
$w(u, v) [x \delta I / \delta x + y \delta I / \delta y]^2$   
 $u = x \quad v = y$  in partial derivatives

- where A is defined in terms of partial derivatives  $I_x = \partial I / \partial x$  and  $I_y = \partial I / \partial y$  summed over (u, v):

$$A = \sum_u \sum_v w(u, v) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

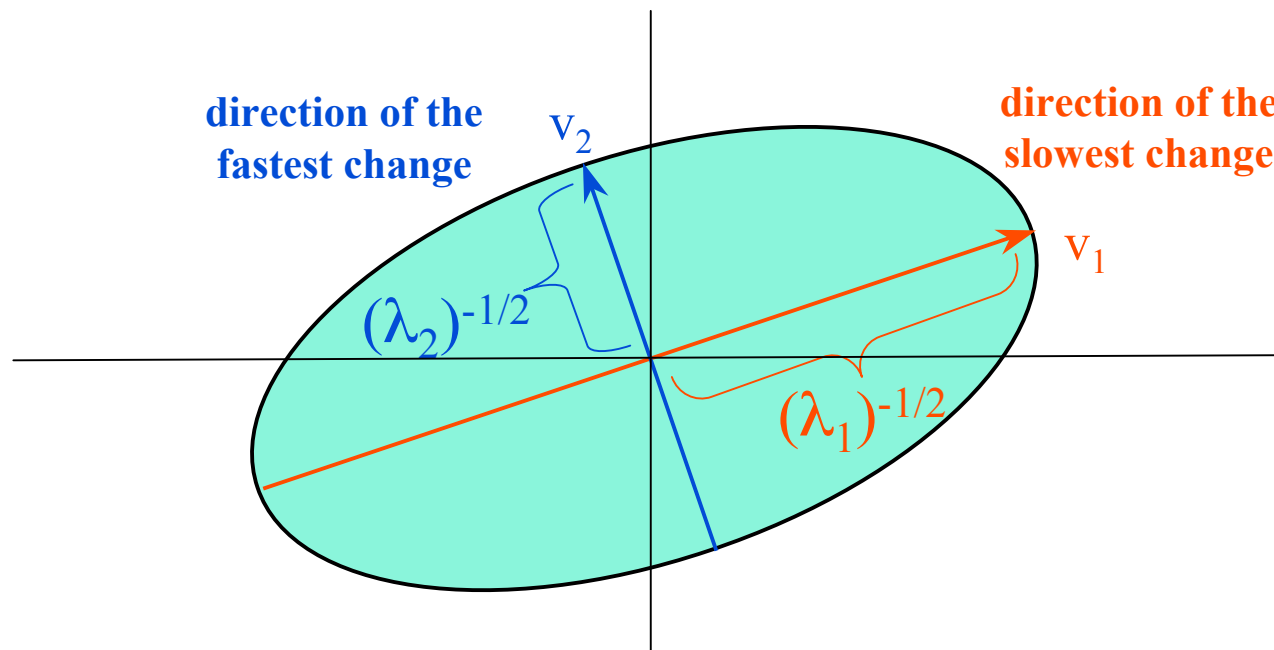
- For constant t,  $S(x, y) < t$  is an ellipse

ellipse is the shape, the noise is not necessarily Gaussian



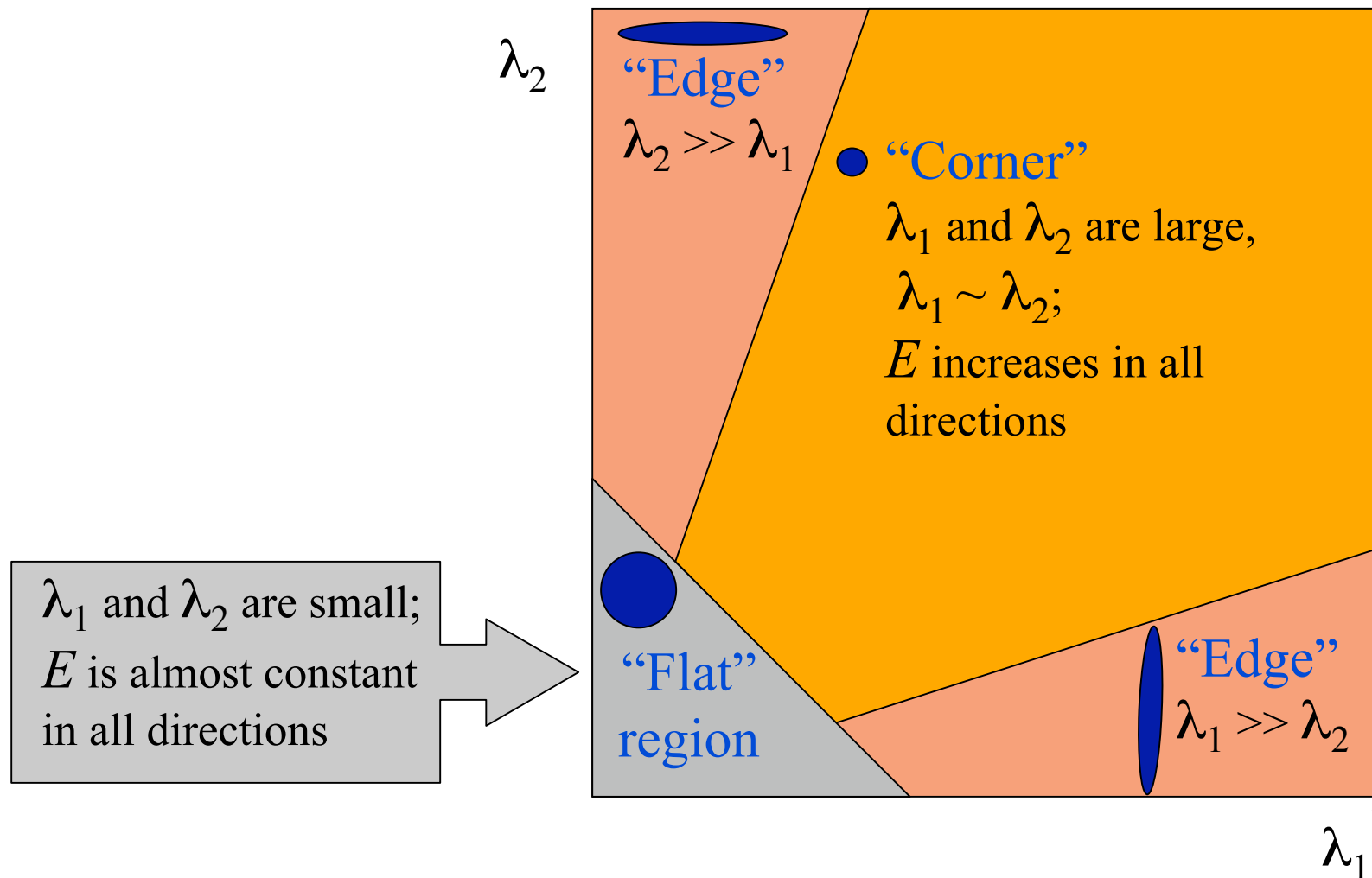
# Eigenvector analysis

- The eigenvectors  $v_1, v_2$  of  $A$  give an orthogonal basis for the ellipse
  - I.e. directions of fastest and slowest change
  - for  $\lambda_2 > \lambda_1$ ,  $v_2$  is the direction of fastest change (minor axis of ellipse) and  $v_1$  is the direction of slowest change (major axis)



# Classify points based on eigenvalues

- Classification of image points using eigenvalues of  $\boxed{A}$ :



# Harris corner detection

- But square roots are expensive
  - Approximate corner response function that avoids square roots:

$$R = \lambda_1 \lambda_2 - k (\lambda_1 + \lambda_2)^2$$

$k = 0.04 \text{ -- } 0.06$

$R = \text{determinant} - k * \text{trace}^2$

with  $k$  is set empirically

- After thresholding, keep only local maxima of  $R$  as corners
  - prevents multiple detections of the same corner

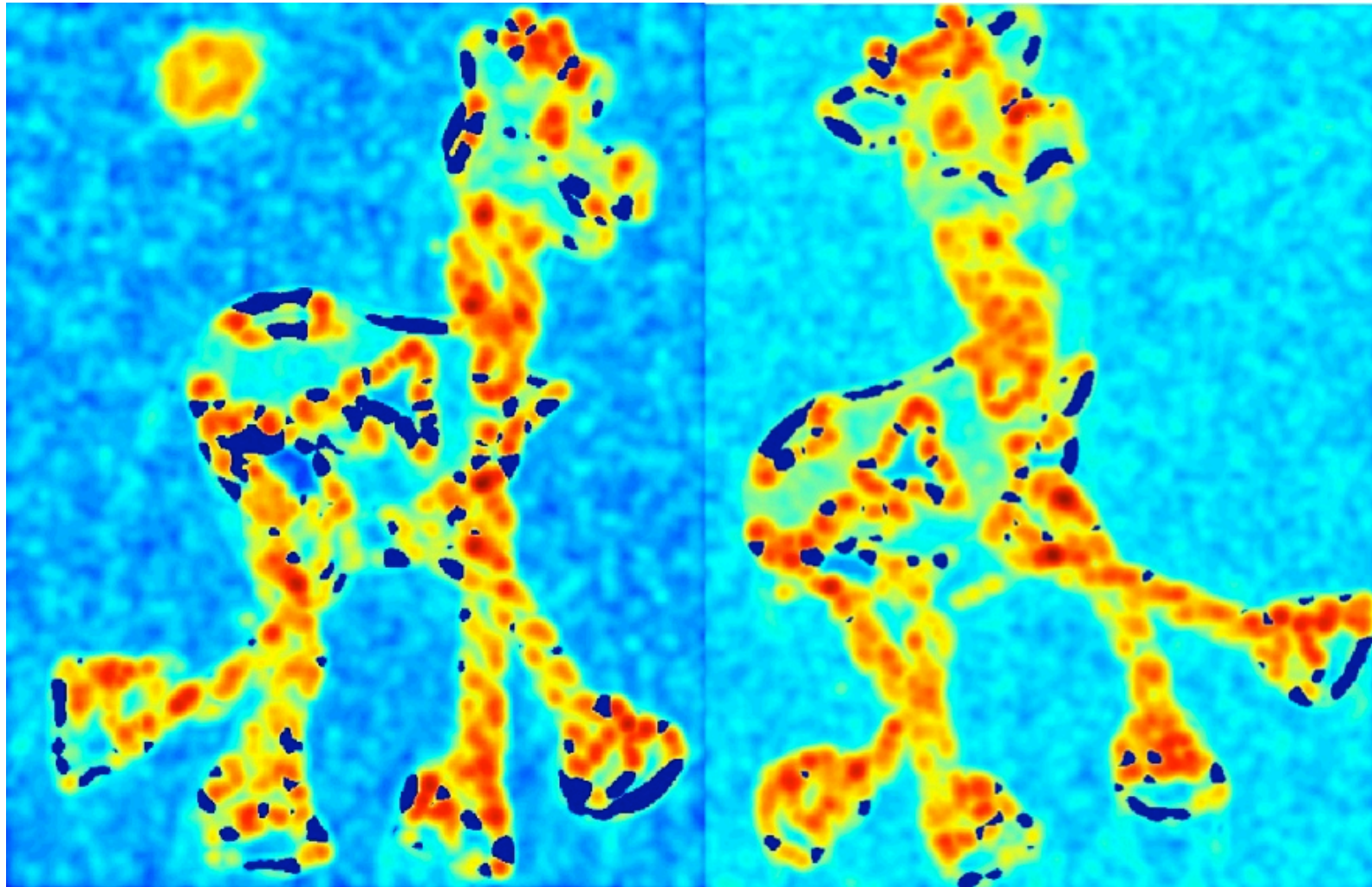
## Harris detector, step-by-step





# Harris detector, step-by-step

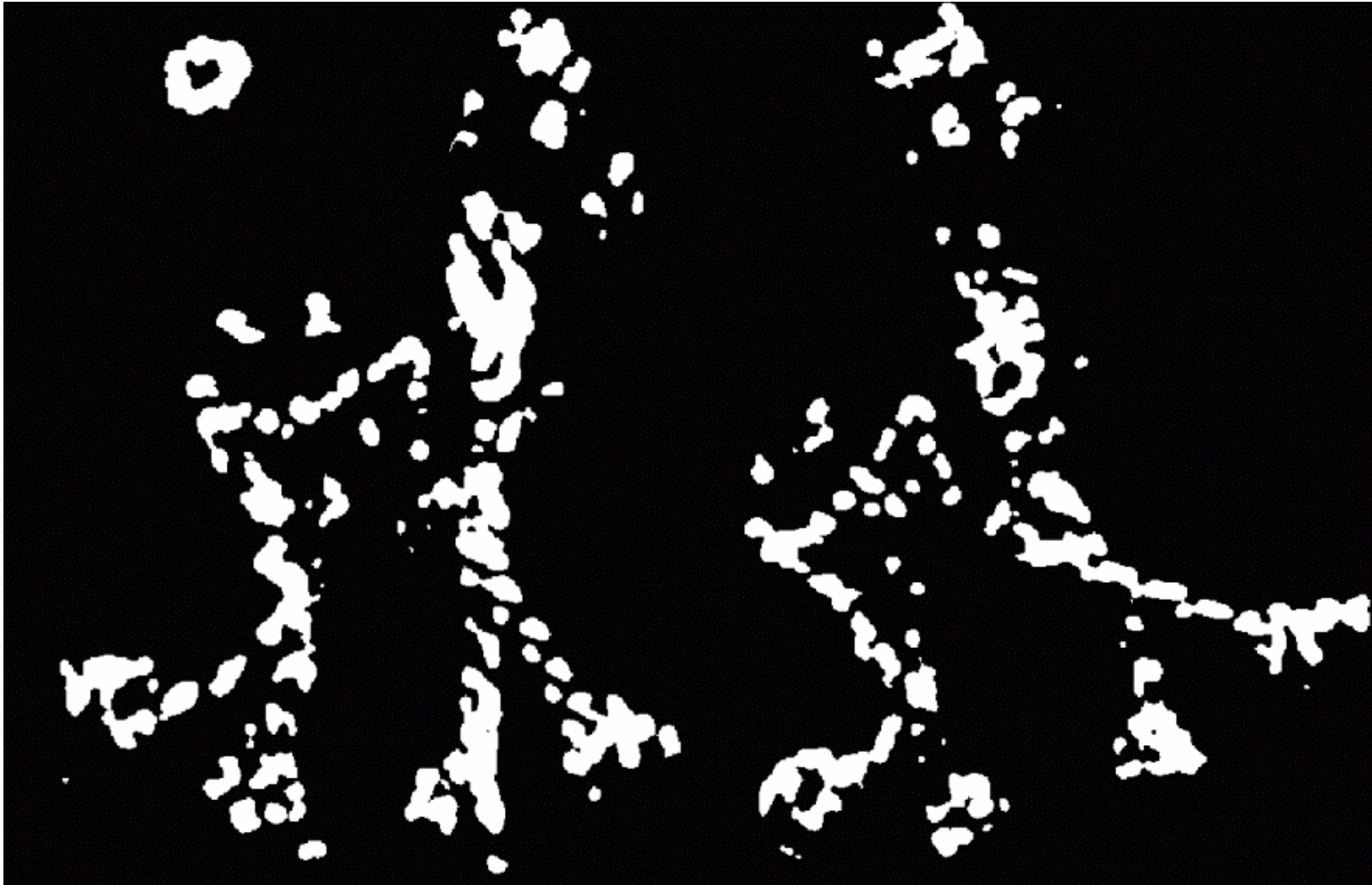
- Compute corner response  $R$





## Harris detector, step-by-step

- Threshold on corner response  $R$



## Harris detector, step-by-step

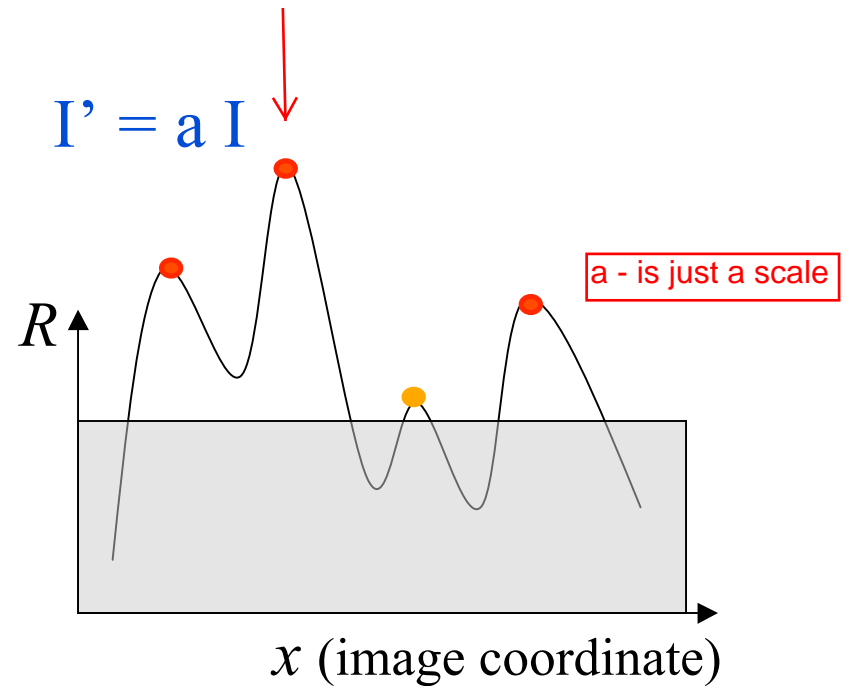
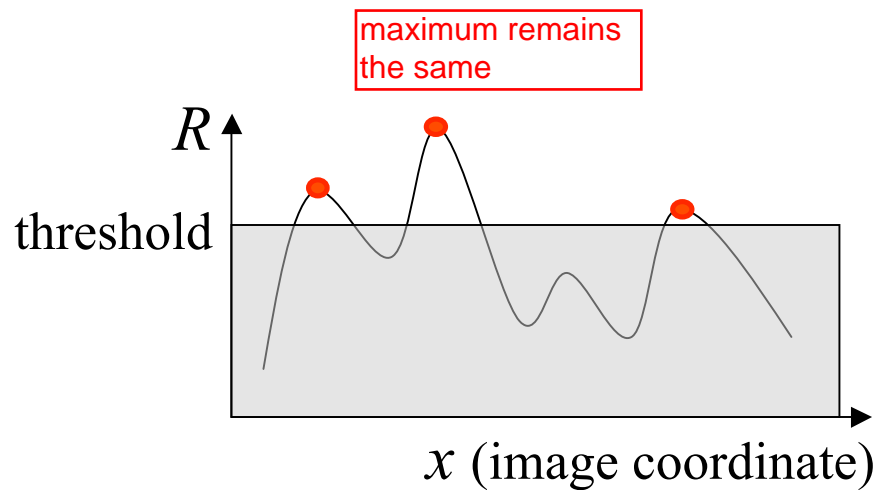
- Take only local maxima of  $R$



# Harris detector properties

- Invariant to intensity shift:  $I' = I + b$ 
  - only derivatives are used, not original intensity values

- Insensitive to intensity scaling:  $I' = a I$

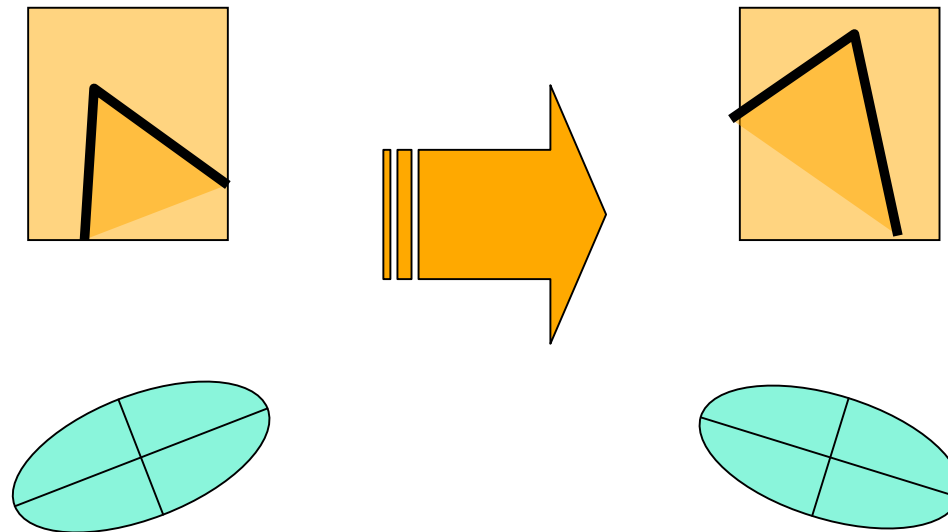


- So Harris is insensitive to affine intensity changes
  - I.e. linear scaling plus a constant offset,  $I' = a I + b$

# Harris detector properties

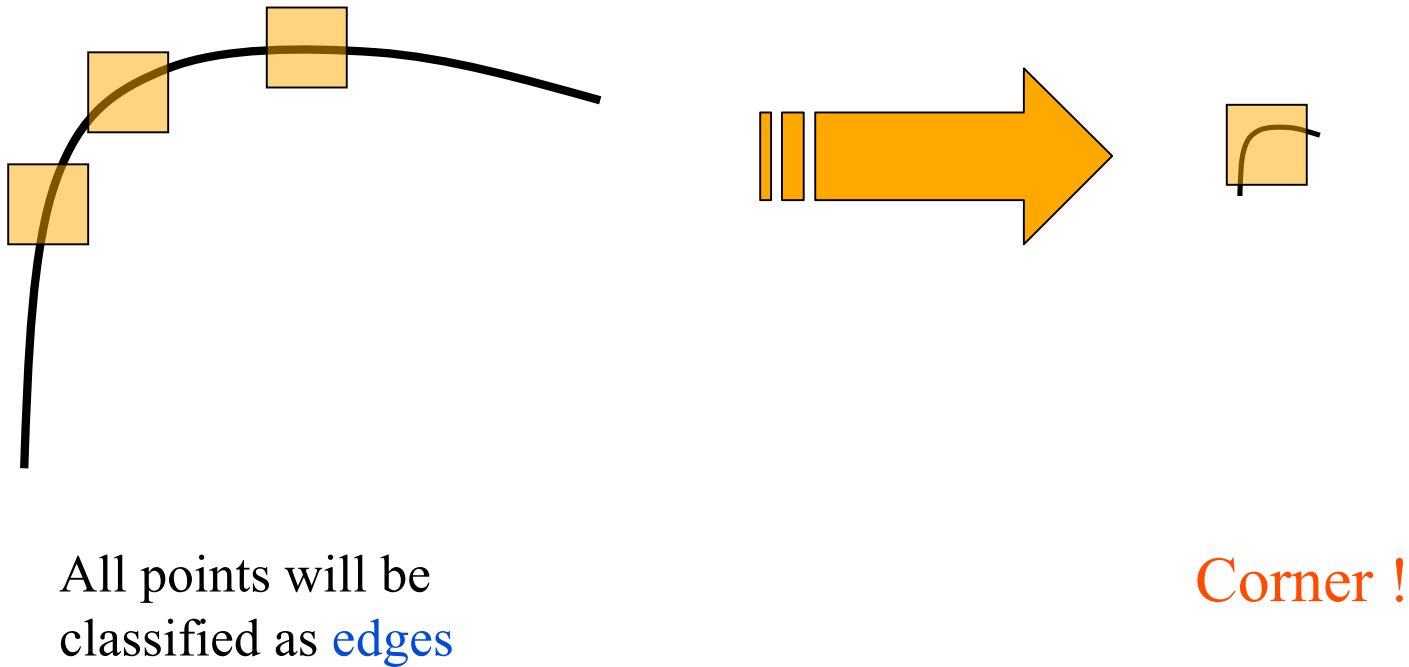
- Rotation invariance

- Ellipse (eigenvectors) rotate but shape (eigenvalues) remain the same
- Corner response  $R$  is invariant to image rotation upto sampling



# Harris detector properties

- But Harris is *not* invariant to image scale

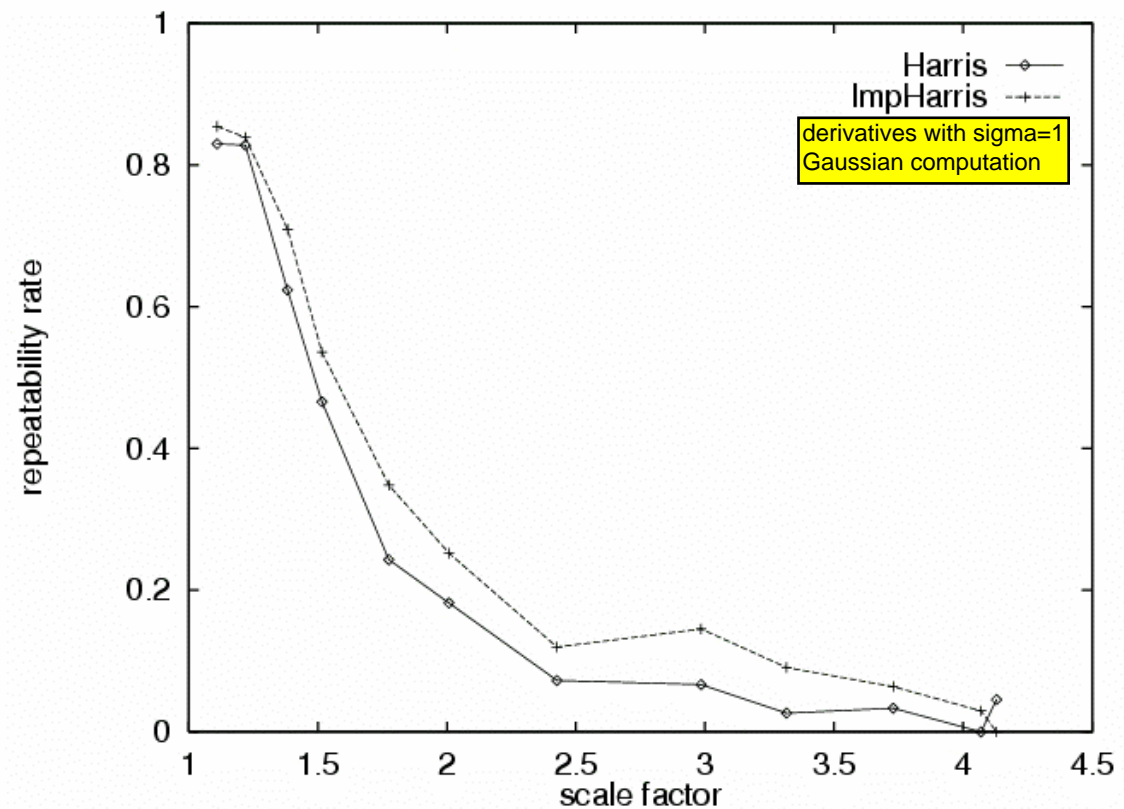
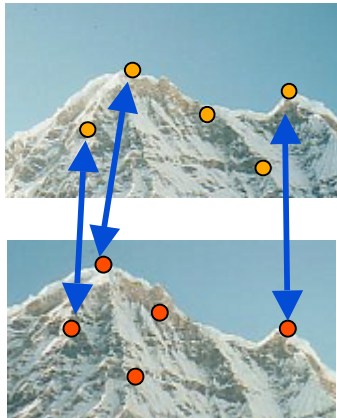


# Experimental evaluation

- Quality of Harris detector for different scale changes

Repeatability rate:

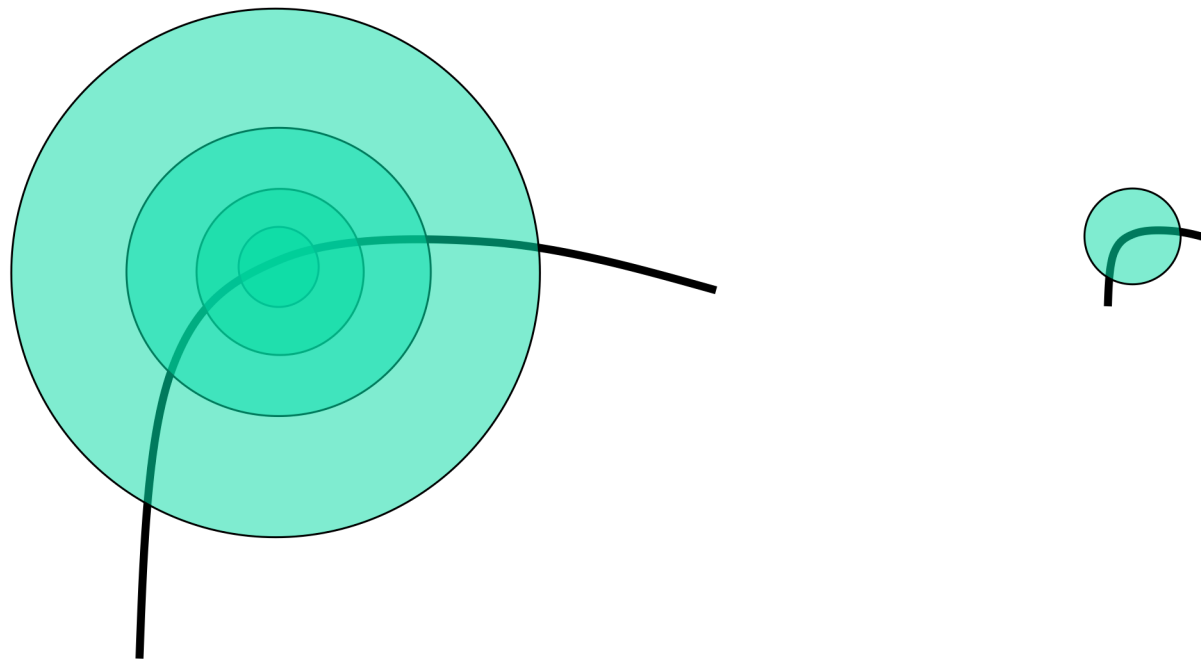
$$\frac{\# \text{ correspondences}}{\# \text{ possible correspondences}}$$



the image is Van Gogh's sower painting

# Scale invariant interest point detection

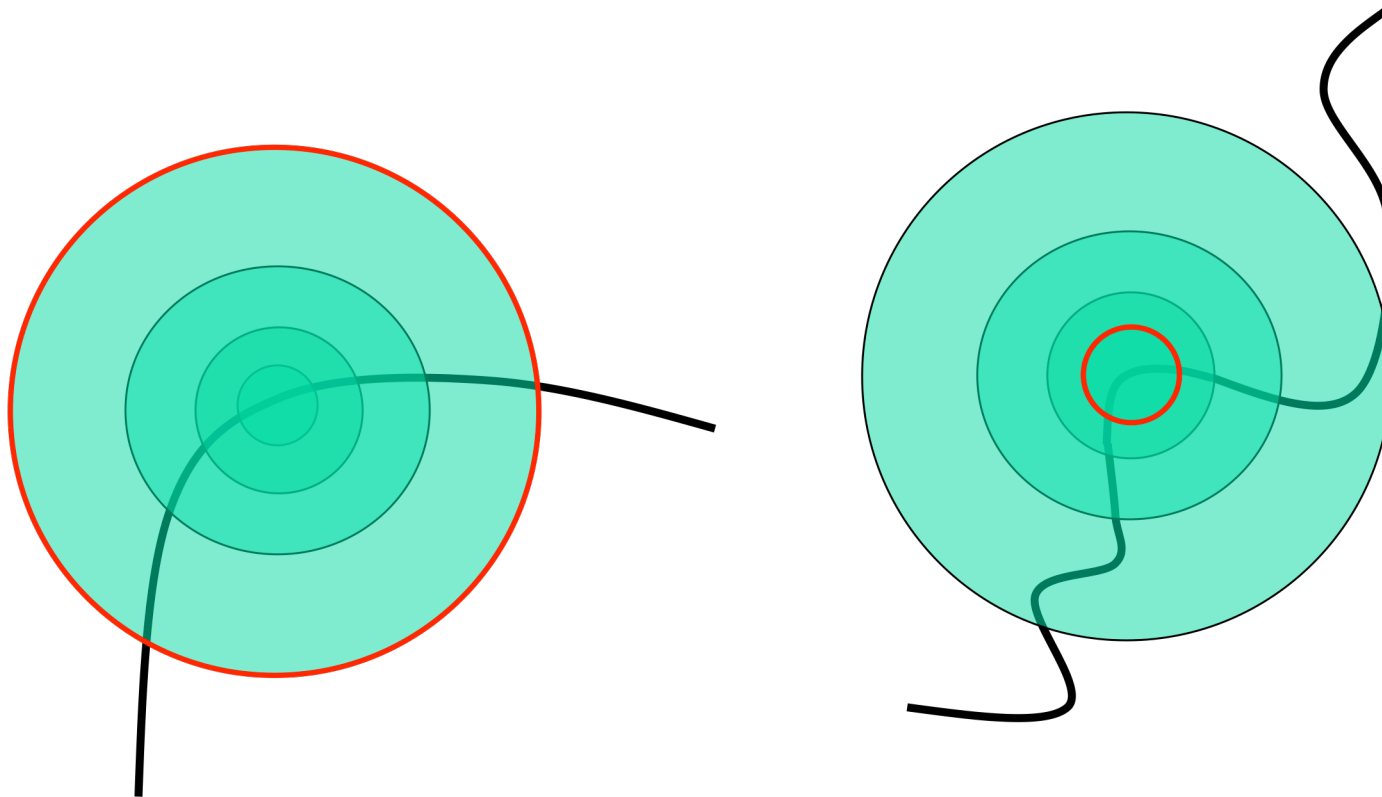
- Consider regions (e.g. circles) of different sizes around a point
- Regions of corresponding sizes will look the same in both images





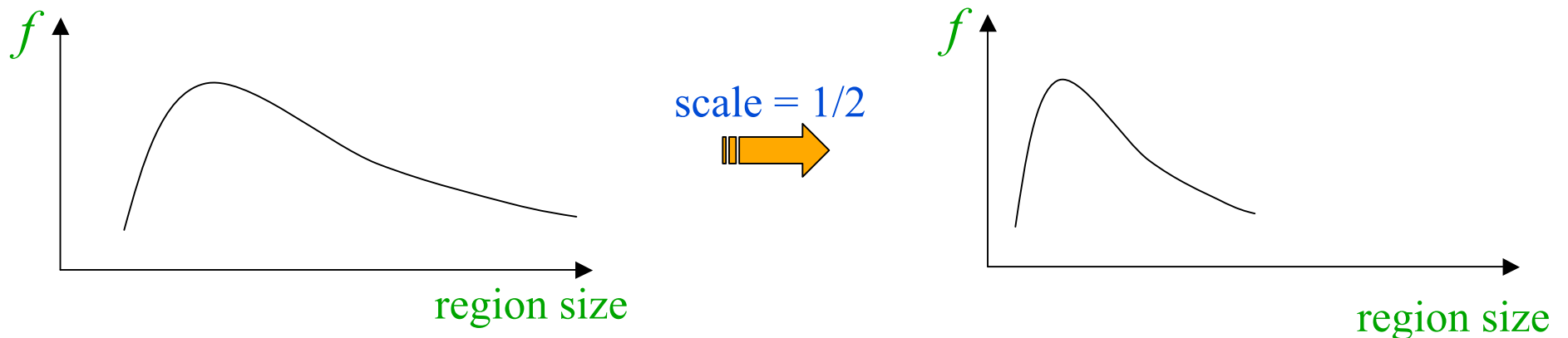
# Scale invariant detection

- The problem: how do we choose corresponding circles *independently* in each image?



## A solution

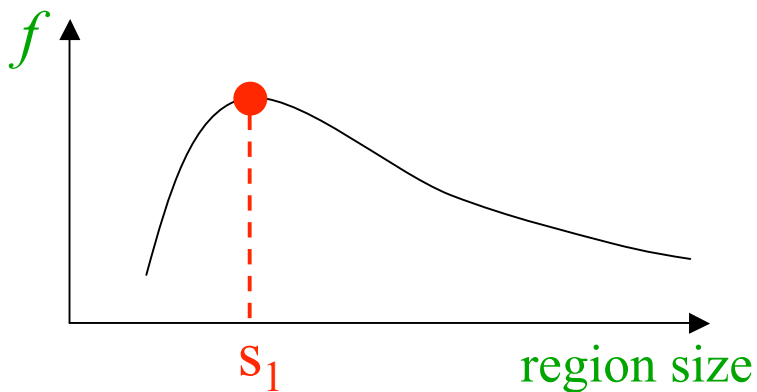
- Design a function which is “scale invariant”
  - I.e. value is the same for two corresponding regions, even if they are at different scales
  - Example: average intensity is the same for corresponding regions, even of different sizes
- For a given point in an image, consider the value of  $f$  as a function of region size (circle radius)



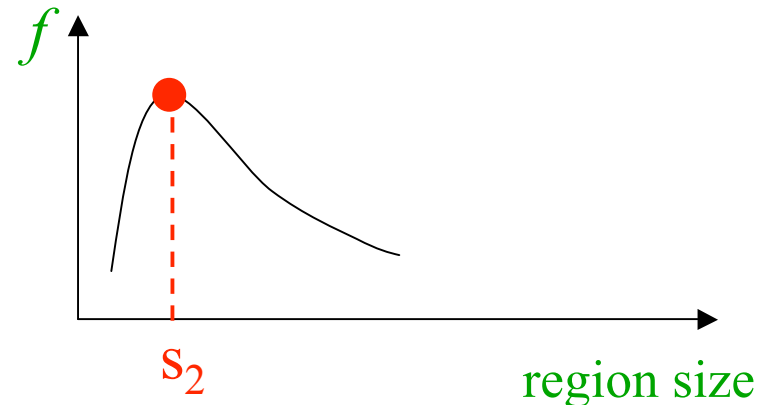
# A solution

- Take a local maximum of this function
  - The region size at which maximum is achieved should be invariant to image scale
- This scale invariant region size is determined independently in each image

in reality you don't have a good theoretically valid solution just a generally good approximation

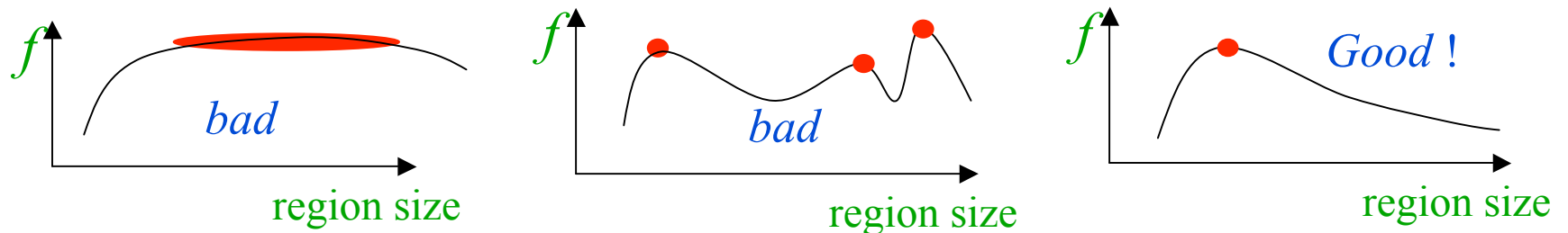


scale = 1/2  
→



# Choosing a function

- A good function for scale detection has one sharp peak



- A function that responds to image contrast is a good choice
  - e.g. convolve with a kernel like the Laplacian or the Difference of Gaussians (almost the same)