Time Reversed Focusing in Finite-Length Rods with Defects

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This paper continues our studies on using time reversed acoustic energy to accelerate the rate of recovery in a self healing material. The project is divided into two main areas; acoustic time reversal and epoxy curing. Recent epoxy test results have proven very promising in respect to using acoustic energy for enhancing the curing rate of the epoxy. Our time reversal methods have also been expanded to allow for easy focusing of acoustic energy at a cracked location without any actual knowledge of the crack's position.

I. Introduction

There are many instances when minor damage affects the functionality of difficult to access space structures. Often the damage is caused by collisions with space debris and is nearly impossible to repair when employing the traditional method of reaching the damaged site manually. It is therefore important to consider materials and structures with built-in mechanisms for self-repair.[1] Research on self-healing materials has seen a rapid growth in recent years, though the emphasis thus far appears to have been on developments, including increasing the rate of recovery, at the materials level.[2,3,4,5,6,7,8] Many applications have relied on heating/cooling to both initiate and speed the recovery process.[9,10,11,12,13,14] Some research has also been done on the effect of introducing ultra-violet light during the recovery stage.[15] The rate of healing is crucial from a structural point of view, given that the healing process frequently must compete with the damage-inducing processes in an application. Our study considers the problem from a structural point of view, while recognizing that healing occurs at the molecular/materials-level. In particular, the current emphasis is on investigating whether crack healing to the point of full mechanical recovery can be accelerated using focused acoustic energy.

An appealing method for focusing energy at a damaged point is one that realizes a defect has occurred, but does not need actual knowledge of where that defect is located. One method that has been investigated is the time-reversal method. There are many different applications and methods of time-reversal that have been researched.[16,17,18,19,20] One particular method of time-reversal involves an emitting source and receiving sources. The emitter sends a signal through a medium and the receivers record that signal. If a defect is present, it will affect the way in which the signal travels through the medium. This provides a way to detect if damage has occurred and to begin focusing acoustic energy at that point without actually knowing where the defect is. If the receivers time reverse and play back the signals they read in, those played back signals will combine together and focus on the damage point. Our work is based on the use of this technique for the purpose of focusing acoustic energy at a damage point in order to accelerate the recovery rate of the self-healing material at that point.[16,21]

This paper details our most recent studies on epoxy curing and time reversal. The testing method for the epoxy curing portion of the project has been improved to allow for more reliable, consistent, and conclusive results. For the time reversal portion of the project, we looked at the potential problem of ringing in the system caused by natural frequency oscillations. We have also furthered our time reversal testing by focusing energy at a "defect" location within a rod.

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II. Experimental Implications

In order to further the studies on accelerated crack healing, we have tried to answer a few fundamental questions: (i) what effect, if any, does acoustic excitation have on the healing rate of the material, (ii) how does ringing affect the system, and (iii) what is a viable method for focusing acoustic energy. To answer the first question, we look at epoxy curing as an experiment that is analogous to the process of self healing. The epoxy curing and self healing processes both proceed through the following five stages: (1) surface rearrangement, (2) surface approach, (3) wetting, (4) diffusion, (5) equilibrium and randomization.[21] For the second question, we introduce acoustic energy into a system and see how quickly that energy dissipates. The last item is addressed by performing tests using a form of acoustic time reversal to determine if we are able to focus acoustic energy at a crack location in a system.

II.A. Epoxy Curing Studies

Testing methods and practices used in quantifying the curing process of epoxy systems continue to be refined and variables such as air inclusion have been realized and eliminated. Epoxy-filled half-inch diameter brass tubes remain in use for this study and frequency responses as well as the internal temperature of the tube-epoxy system during its 24 hour cure are recorded every 1-5 minutes during a 28 hour test. Testing materials and procedures remain largely the same, but slight modifications have been made to the marble elevator used, degassing and epoxy injection systems, and plans to better control the ambient air temperature surrounding the testing apparatus have been created.[22]

The experiments that we performed compared the curing processes of epoxy with and without acoustic excitation. For the acoustic excitation experiments, we utilized an overhead speaker in the system. By adjusting the speaker frequencies and the distances within the apparatus we ensured that the brass tube was within an antinode. Ensuring that the brass tube was located within an antinode allowed for maximum energy transfer.

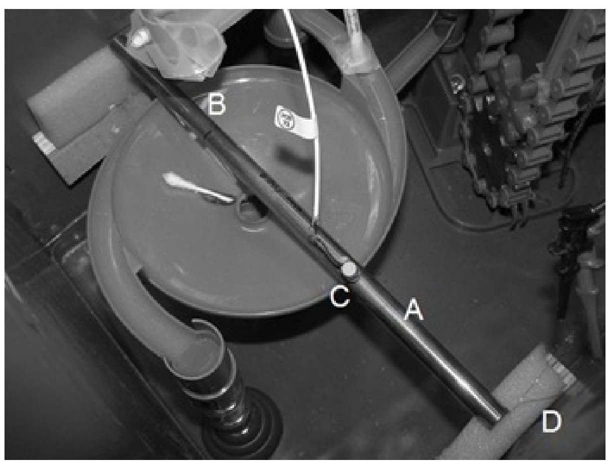

Figure 1. An overhead view of the testing apparatus. Seen diagonally across the picture is the epoxy-filled tube (A) supported by foam supports at each end. Marbles are dropped by the elevator onto the mark approximately 1/3 down from the far end of the tube (B) and are collected by the round dish directly beneath the tube. Vibrations are received by the accelerometer seen in the center of the picture (C) and also visible are thermistor leads emerging from the near end of the tube (D).

II.B. Time Reversal Testing

There were three separate experiments that were performed which dealt with the time reversal aspect of this project. The goal of the first experiment was to determine how quickly the amplitude of the ringing in the system would diminish. The second set of tests, acoustic focusing, are an extension to the time reversal tests that were presented in our last paper in which we now use multiple transducers to focus energy at a point in the rod that lies between them .[23] The third test is similar to the second, except that we look at the effects of iteratively playing back the reversed signals in an attempt to combine their energy with the energy of the pulses that remain in the system between iterations.

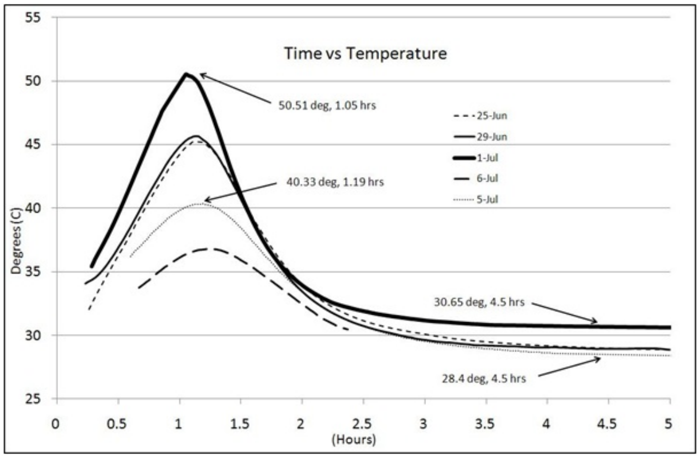

Figure 2. Seen here are 5 graphs, each from different tests, showing the internal temperature of the tube-epoxy system as time progresses. Near the right-hand side of the graph it can be seen that the tube approaches the ambient air temperature of the laboratory in which the experiments were conducted. Of note here is the pattern that higher and earlier peak temperatures seem to occur when the ambient air temperature is highest, and vice-versa. Future testing will monitor and control the surrounding air temperature in an effort to reduce these disparities so that more accurate test results can be achieved.

II.B.1. Ringing Test

A setup identical to the one used for our previous time reversal testing was used for the ringing tests. This setup uses a steel rod as the material through which the acoustic waves are propagated. A ceramic piezoelectric-transducer stack (PZT) is placed on each end of the rod. The PZTs are connected to a custom, multi-channel, voltage/current amplifier. This amplifier is controlled by an FPGA data acquisition card which is programmed remotely by a desktop computer. Figure 3 shows the setup used for this test.

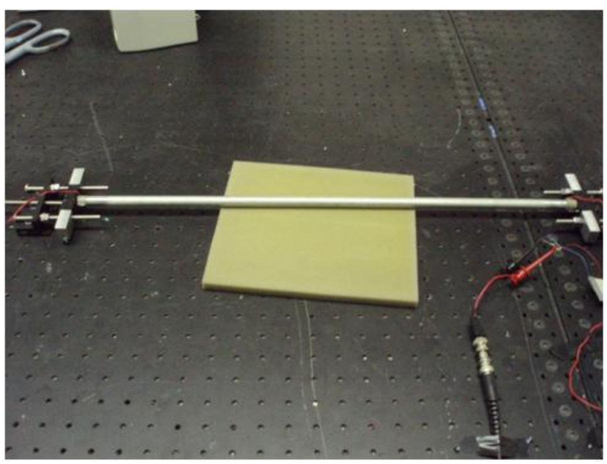

Figure 3. Setup for the ringing tests

The test is performed as follows:

1. An acoustic signal is played out from one of the transducers. This is known as PZT0. The PZT on the opposite side of the rod is known as PZT1.
2. 25,000 samples are then read in from both PZT0 and PZT1.
3. The samples are analyzed to determine how quickly the ringing disappears.

Two separate types of acoustic signals were played out. For one set of the ringing tests, we used a single energy pulse of max amplitude. By this we mean that PZT0 was fed max voltage for one single data point.

The other set of tests used the same multi-tone pulse that we have used in previous experiments for our time reversal testing.[23] This multi-tone pulse is just multiple sine waves of different frequencies that are concatenated. The center sine wave has a frequency of $130KHz$. The waves directly to the left and right of the center wave have a frequency of $110KHz$. The sine waves on the ends have a frequency of $90KHz$. These signals are generated via LabVIEW. The ringing effects from both the single pulse and multi-tone pulse were analyzed.

II.B.2. Acoustic Focusing Test

The setup used for the acoustic focusing test is similar to our previous test setup with one major difference; we add a second steel rod segment to the system. This is essentially just added on to the previous setup. The rod is placed so that one of the PZTs becomes sandwiched between the two rods. This PZT is now the "defect" in the system. This transducer creates a reflection point in the system which gives us a location to attempt to focus energy and, at the same time, allows us to determine the response amplitude at that location. This is more analogous to crack focusing than our previous experiment in which we "focused" energy at a transducer on the end of the rod. A third PZT is then placed on the open end of the second rod segment. This system is placed under compression as before. The same custom amplifier and FPGA data acquisition card were used for this testing. Figure 4 shows the setup used for the acoustic focusing test. Notice that the right hand rod segment is shorter than the left. The "defect" location has been placed into an arbitrary position in the system (i.e., the size of the second steel segment was nearly random). Figure 5 gives a close up view of the right hand side of the system. Figure 6 is a zoomed in picture so that you can get a better look at the PZT that is between the two rods.

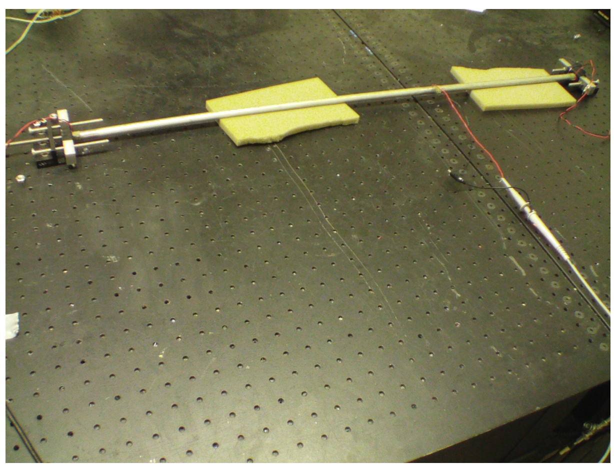

Figure 4. Setup for the acoustic focusing tests

We will refer to the end transducers as PZT0 and PZT1. The middle transducer will be known as the defect PZT. The idea is that PZT0 sends an initial multi-tone pulse. This pulse will travel through the steel rod. When the pulse reaches the defect PZT, part of its energy will be reflected back towards PZT0 and the rest of the energy will continue propagating through the system and towards PZT1 (the opposite end PZT). You could consider this from the point of view that the defect PZT has actually just emitted a weaker version of the signal that PZT0 originally emitted. We want for PZT0 and PZT1 to capture the first instance of this weak signal emitted by the defect PZT as well as the time delay that it took for the signal to arrive at the PZTs. The end PZT0 and PZT1 then time reverse the signal that they each captured and play it back along with the time delay. Reversing and playing back the signals in this manner will cause them to arrive at the defect PZT simultaneously. This means that their amplitudes will combine at the defect PZT and a focusing of their energy occurs.

One of the real advantages of this algorithm is the fact that we are able to focus energy at a crack location without ever having any actual knowledge of that crack's position. This makes the algorithm very robust in terms of implementation. It also has implications that this could be extended to focus energy on multiple crack locations. Another great thing about this method is that the focusing occurs only at a

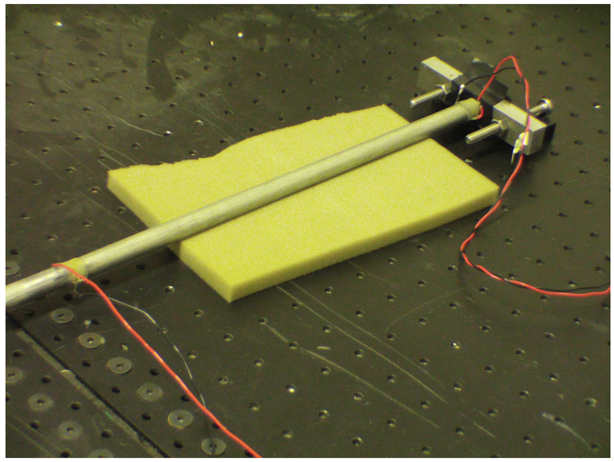

Figure 5. Close up of right hand side of the test setup

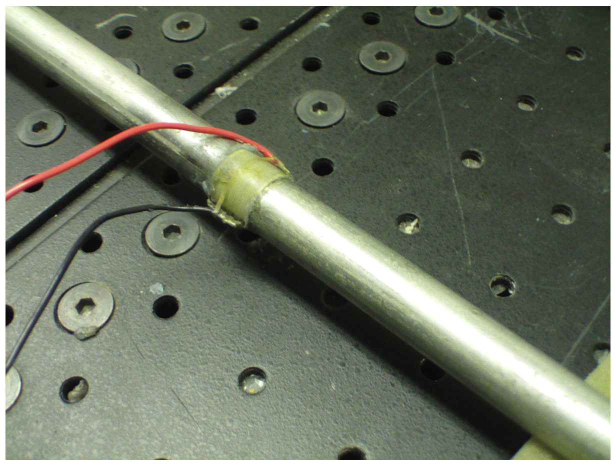

Figure 6. Close up of the "defect" PZT

point in the system which not only reduces chances of unwanted damage, it also can potentially increase the overall efficiency of the system by requiring lower amounts of energy to be put into it.

The test is performed as follows:

1. A multi-tone acoustic pulse is played from one of the end transducers (PZT0).
2. 1,000 samples are read in from all three PZTs, and the maximum amplitude reached at the defect PZT is recorded.
3. A normalized correlation filter is applied to the signals read from PZT0 and PZT1 in order to extract the pulses of interest and reject unwanted noise.
4. The filtered signals are scaled to the maximum output amplitude.
5. The program waits for 2.5 seconds to let any ringing die down.
6. The filtered, scaled signals are then played out on their respective channels.
7. 1,000 samples are again read in from all three PZTs and the maximum amplitude reached at the defect PZT is recorded.
8. The amplitude at the defect during the initial stage is compared to that of the time reversal stage.

Variations on this algorithm were implemented to determine if a focusing was actually occurring. The variations were having only PZT0 playback during the time reversal phase and, the opposite, having only PZT1 playback during the time reversal phase. This allowed us to determine the amplitude of the individual waves and to see if the amplitude at the defect PZT during the time reversal phase is approximately the sum of these waves.

II.B.3. Iterative Acoustic Focusing Test

The setup used for the iterative focusing tests is the same as that described in the previous section. However, in these tests we made use of only one of the end PZTs instead of both. The purpose of this test is to see the effects of iteratively adding energy into the system. This energy is added such that it is in phase with the pulse propagating in the system which resulted from the previous iteration. The idea was to look at the amplitude growth of the desired pulse and the amplitude growth of the unwanted vibrations. We consider the unwanted vibrations to be any recorded response that is not directly part of the propagated signal. It is assumed that this is representative of the ambient vibrations in the whole system. We want the vibrations at any location on which we are not attempting to focus energy to be minimal.

The concept of this method is that the end PZT0 used will send an initial pulse as before. As the pulse strikes the defect PZT in the middle, part of its energy is reflected back towards PZT0 and the rest continues to propagate through the system and towards the other end. Again, we have PZT0 capture this first instance of the pulse reflected back from the defect PZT. The time delay that it took for the pulse to travel to the defect PZT and back is recorded. So far, this is all almost identical to the tests described in the previous section. However, now we will continually play back the time reversed signal from PZT0 as well as the time delay that was recorded. What happens when doing this is that pulse played out in the first iteration will travel down the rod, strike the defect PZT, then partially reflect back towards PZT0. During this time, PZT0 is actually still playing out the time delay portion of the signal which was the total time for the wave to travel to the defect PZT and back to PZT0. So at the *exact* moment that the reflection wave from iteration 1 reaches PZT0, iteration 2 will begin and PZT0 will play the next wave which will be in phase with the reflection wave from iteration 1. The amplitude of these two waves will combine together and travel down the rod towards the defect PZT. This wave will again strike the defect PZT and reflect back, although this time it was larger in amplitude than the previous iteration. This process continues, each time the wave on the current iteration adds to the energy of the wave from the previous iteration.

This algorithm is as follows:

1. A multi-tone acoustic pulse is played from one of the end transducers. This is known as PZT0. The PZT in the middle of the system is again referred to as the defect PZT. The PZT on the opposite end is not used in this test.
2. 1,000 samples are read in from PZT0 and the defect PZT.
3. A normalized correlation filter is applied to the signals so that the transmitted pulse can be located within those signals.
4. The Voltage RMS for the both the desired signal and the unwanted noise seen at the defect are recorded.
5. PZT0's filtered signal is rescaled to be maximum amplitude and time reversed with the time delay in place (i.e., time it took for pulse to travel to defect PZT and back)
6. PZT0 continually plays back this time reversed signal.
7. At each iteration, the defect PZT reads in 1,000 samples and records the voltage RMS for both the desired signal and the unwanted noise that it sees.

II.C. Results

In order to have a properly functioning crack healing system, there must be two working components; a method to detect cracks within a system and focus energy at that location, and a system of material which has been characterized. In this study, we have utilized time reversal testing and epoxy curing studies as separate experiments. This paper focuses more heavily on the time reversal portion of the system and the results pertaining to those experiments will be more detailed.

II.C.1. *Epoxy Curing Results*

Our epoxy curing tests have shown that the curing process is accelerated when acoustic energy is introduced into the system. Figures 7 and 8 show the FFT of the response recorded by the accelerometer that is placed on the brass tube. Recall that it is the marble dropping onto the brass tube that causes this vibrational response. The FFT of the epoxy test without acoustic excitation is shown in Figure 7 and the test with acoustic excitation is shown in Figure 8. The idea is that as the epoxy cures, its vibrational response will shift from what is seen in the beginning (the red FFT) to what is seen in the end (the blue FFT). You can see from the graphs that after two hours the epoxy that has the acoustic excitation is much closer to its final FFT position than the epoxy that does not have acoustic excitation.

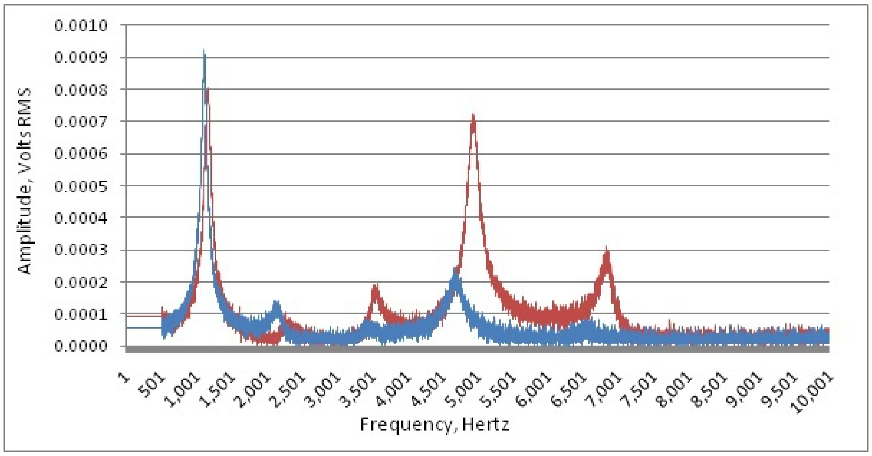

Figure 7. Fast Fourier transforms taken at 2 hours (blue) and 30 hours (red) of the unexcited test.

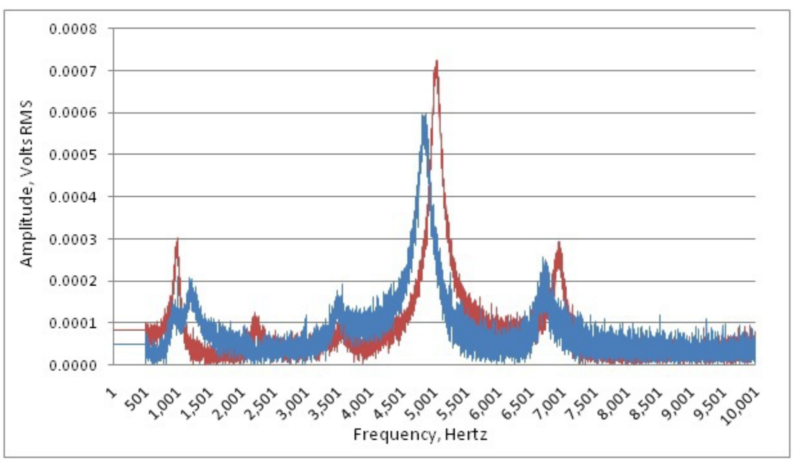

Figure 8. Two hour (blue) and 30 hour (red) FFTs taken of the acoustic test..

II.C.2. *Ringing Results*

The graphs obtained from these tests show that the ringing in the system diminishes rather quickly. In the graphs for both PZT0 and PZT1, you are able to see the initial pulse that strikes the transducer and all the subsequent reflection pulses that strike it. The amplitude of each pulse is significantly less than the pulse that was read before it. This matches well with the analytical model that was used to predict the effects of ringing in the system. This is the same for both the single pulse and for the multi-tone signal. Figures 9 and 10 show the graphs of the signals read by each PZT for the single pulse test. Figure 11 shows the multi-tone signal that is used for the second set of ringing tests. This is the same signal used in the acoustic focusing testing that is also presented in this paper. Figures 12 and 13 show the graphs of the signals read by each PZT for the multi-tone signal ringing test.

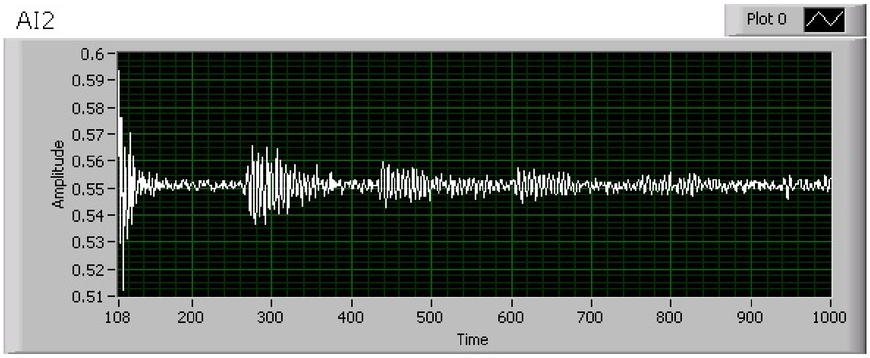

Figure 9. Graph of signal read by PZT0 after sending single pulse

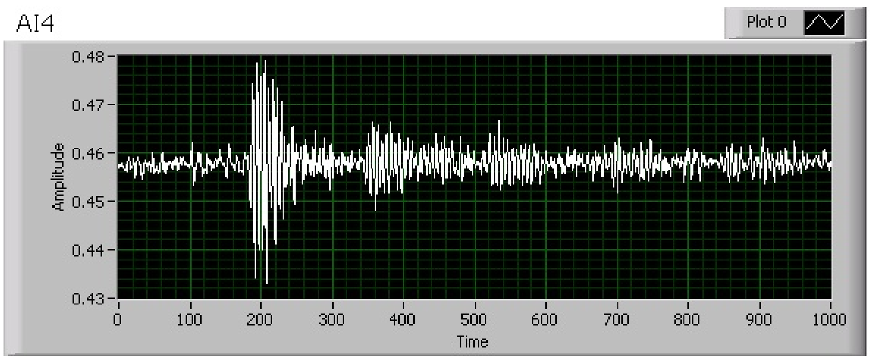

Figure 10. Graph of signal read by PZT1 after single pulse is sent by PZT0

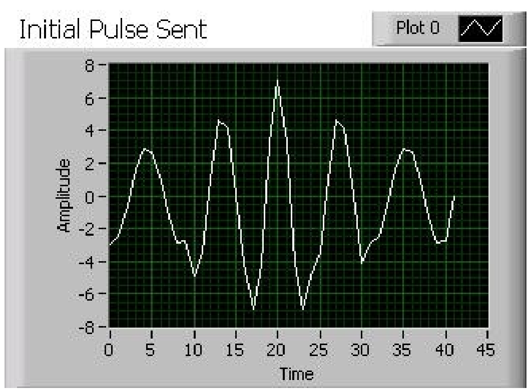

Figure 11. Graph of multi-tone signal sent by PZT0

II.C.3. Acoustic Focusing Results

The tests showed that the amplitude recorded at the defect PZT during the time reversal phase was almost double the amplitude recorded during the initial phase. The variations of the test algorithm (only playing back from one PZT during the time reversal phase, as described in the testing section) were implemented to see the response at the defect PZT from the individual waves sent from the end PZTs. The sum of the amplitude of these responses is approximately that of the amplitude recorded when both end PZTs are used

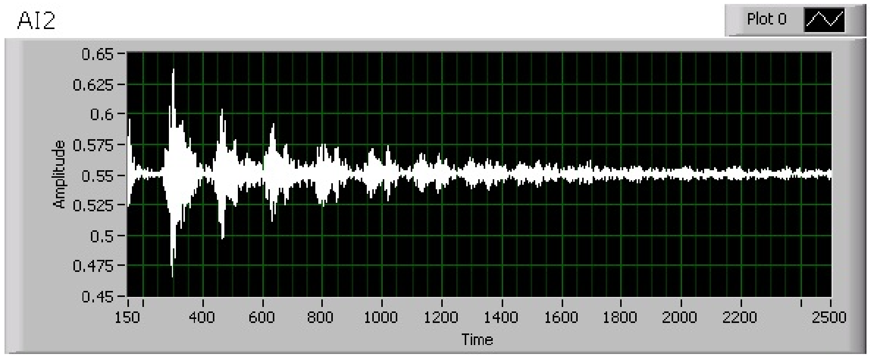

Figure 12. Graph of signal read by PZT0 after sending multi-tone signal

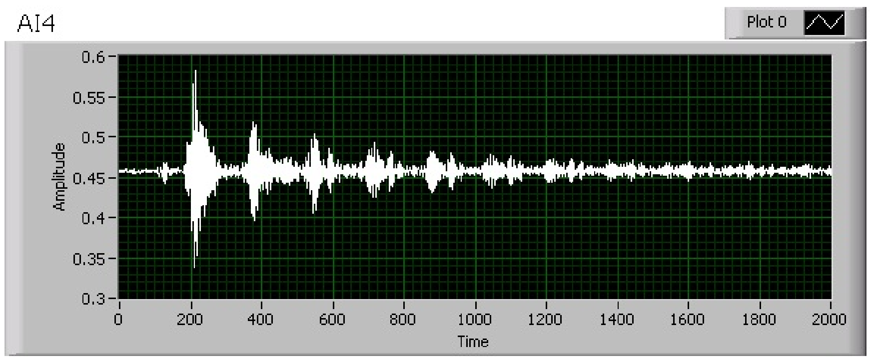

Figure 13. Graph of signal read by PZT1 after the multi-tone signal is sent by PZT0

during the time reversal phase. This strongly suggests that the waves are combining at the defect PZT and creating a focusing of acoustic energy at that specific location. Figure 14 shows the signals read by the end PZTs during the initial phase after PZT0 sends the multi-tone signal. Figure 15 shows the signals that are played back during the time reversal phase. These signals are obtained by applying a normalized correlation filter, along with a few other minor adjustments, to the noisy signals read in by each PZT. Figure 16 shows the response at the defect PZT during both the initial phase and the time reversal phase. You can clearly see that the amplitude of the response at the defect is much greater during the time-reversal phase. Chart 17 shows the amplitude of response at the defect transducer for the initial phase, normal time reversal phase, time reversal phase with only PZT1 playing back, and the time reversal phase with only PZT0 playing back. The amplitudes are an average of five test runs.

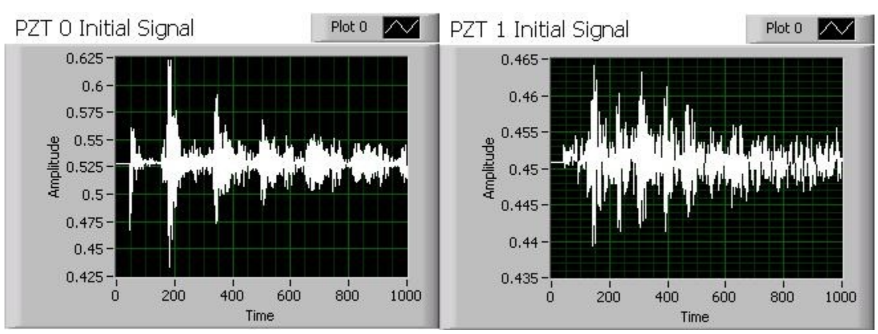

Figure 14. Graphs of the signals read by each of the end PZTs

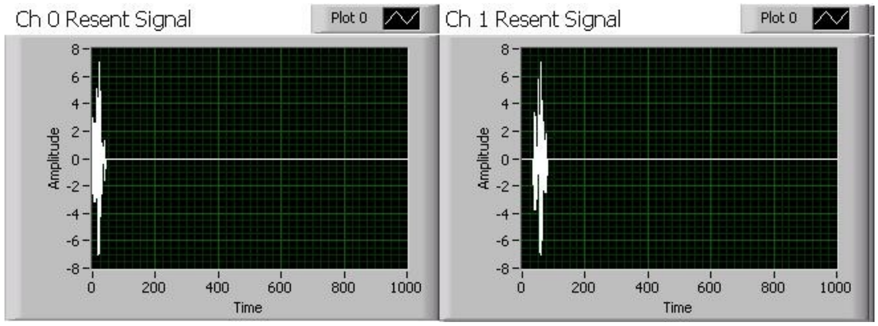

Figure 15. Graphs of the signals that remain after applying a normalized correlation filter. These signals are then resent by their respective channels during the time reversal phase

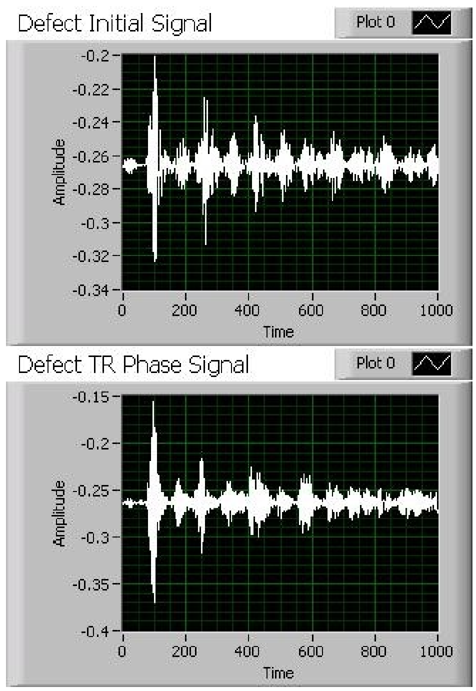

Figure 16. Graphs of the signal that is read at the defect PZT during the initial phase and the time reversal phase of the focusing algorithm

Defect PZT Initial Phase Amplitude	122.497 mV
Defect PZT Time Reversal Phase Amplitude	210.388 mV
Defect PZT Amplitude with only PZT1 replaying a signal	80.444 mV
Defect PZT Amplitude with only PZT0 replaying a signal	151.428 mV

Figure 17. Chart showing the amplitude at the defect PZT averaged over 5 tests

II.C.4. Iterative Acoustic Focusing Results

The results we received from iterative time reversal show that the response to the desired wave at the defect grows very quickly for the first four iterations. After that, it continues to grow until about the ninth iteration at which time it levels off. The amplitude of the response is almost six times greater at the end of the program than it was at the beginning. We can see that the unwanted noise also grows at a similar, if not slightly faster, rate than the desired signal grows. However, the overall amplitude of the unwanted noise remains fairly small in comparison to the amplitude of the desired signal. This can be seen in figure 18. In figure 19 you can see the signals that are sent and recorded by PZT0. Figure 20 shows the amplitude of the response at the defect through each successive iterations. You can see both the desired wave and the unwanted noise in figure 20.

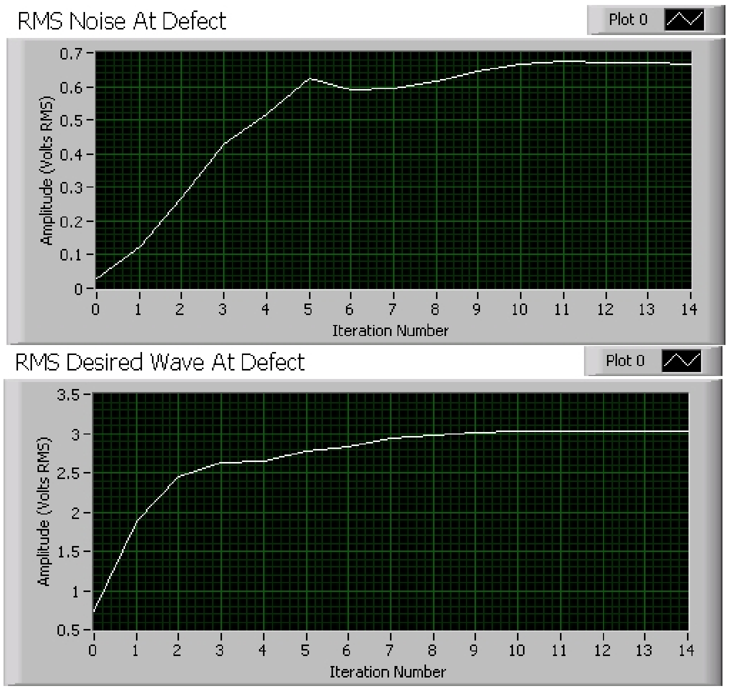

Figure 18. Graphs showing the voltage RMS for both the desired signal and the unwanted noise which is seen by the defect PZT.

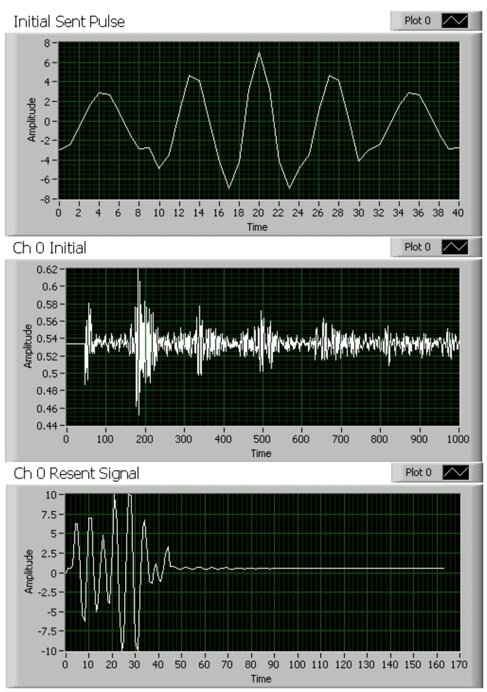

Figure 19. These graphs show the original multi-tone pulse sent by PZT0 during the iterative tests, the signal that is recorded by PZT0, and the signal that is then continuously played back by PZT0

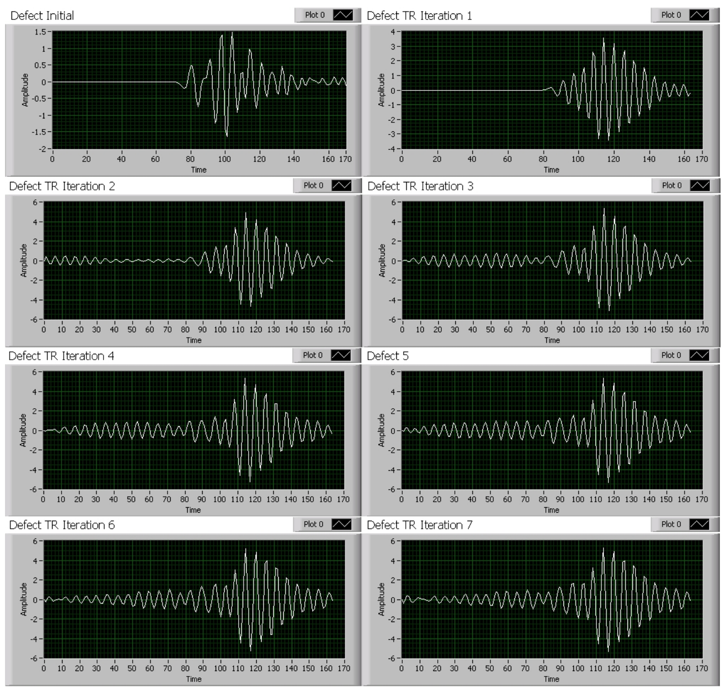

Figure 20. These graphs show the response at the defect PZT through each successive iteration of the time reversal program

III. Conclusion

We have seen in the epoxy curing tests that the rate of curing is accelerated when acoustic energy is introduced. The results from the ringing tests have so far shown that the ringing in the system will diminish rather quickly. We have also shown that when using a time reversal algorithm, we can easily focus acoustic energy at an arbitrary crack location without any actual knowledge of the physical position of the crack. In doing so, we are able to efficiently focus energy only at a desired location while minimizing unwanted vibrations throughout the rest of the system. Our iterative time reversal tests have also indicated that we can effectively combine the energy of waves through successive iterations and achieve a greater wave amplitude within the system. Future work will involve the testing of time-reversal in multiple dimensions, as well as the detection of cracks within the system. Ongoing work also includes further study of the curing response of epoxy and the way in which acoustic energy affects the response characteristics. Our goal is to put these pieces together in order to demonstrate accelerated self-healing through the use of focused acoustic energy at a damaged point.

Acknowledgments

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