ECON 358 - Problem Set 3
Suggested Solutions[1]

1 Question 1 - Data

As suggested in the statement, I downloaded GDP data from the EIU webpage. In particular we will use GDP in current US dollars, which is also the units of the trade flow data from Problem Set 1. From now on, all variables are expressed in billions.[2]

In this section I describe what observations we have and how we construct the rest, using the accounting that the model proposes. So far, we have data on GDP, Y_i, and bilateral trade flows in manufactures, X_{ni}^M. We make the assumption that all trade is in manufactures, which means that we also observe overall trade flows $X_{ni} = X_{ni}^M$. From these primitive data, I proceed as follows:

1. Compute total absorption of manufacturing:

$$X_n^M = \sum_{i=1}^{N} X_{ni}^M. \tag{1}$$

   Similarly, we can calculate total production of manufactured goods by

$$Y_i^M = \sum_{n=1}^{N} X_{ni}^M. \tag{2}$$

2. By the definition of deficit, the deficit in manufactures is $D_i^M = X_i^M - Y_i^M$. Imposing that all trade is in manufactured goods, we also have $D_i = D_i^M$. Note that, as discussed in class, these deficits add up to zero by construction. The reason is that we have a unique data source for each bilateral trade flow, since we are measuring manufacture flows only from the importer side (see DEK (2006)). A discrepancy can only arise when we have more than one independent measure for the same observation (say, each country reports its own trade deficit).

[1]Please send me e-mails with corrections, suggestions or questions. The calculations are performed in the matlab file posted on chalk.

[2]A quick comparison of these data to those on Table 1 of DEK (2008), for 2004, shows that they worked with the same series. The only difference is China, which in DEK(2008) is aggregated with Hong Kong. EKNR 2011, however, does not, so in what follows I do not aggregate either, to keep the consistency with the trade data.

3. Calculate overall absorption using the definition of the overall deficit, but solving for X_i:

$$X_i = Y_i + D_i$$

4. Now we can calculate the values of α_i that make the system internally consistent. Use the accounting identity for expenditure on manufactures to solve for α_i:

$$X_i^M = \alpha_i X_i + (1-\beta)\, Y_i^M \Rightarrow \alpha_i = \frac{X_i^M - (1-\beta)\, Y_i^M}{X_i}. \tag{3}$$

Table 1 shows, for each country in the sample, GDP, trade deficit, and the implied share of manufactures. By large, the US runs the largest deficit with 499.7 billion dollars, but not as a percent of GDP, where it is surpassed by countries like Greece (10%) and Romania (11.5%). On the other end, China and Germany run the largest surpluses: 398.8 and 226.1 billions. Their relative position is maintained in terms of GDP. The table also shows that of the 20 countries in the sample, only 7 run a surplus.

The estimates of α_n are between 0.108 (for Germany) and South Korea (0.287), with an average of 0.173. These numbers are rather low, when compared to a richer specification, as the one in DEK (2008), where the range is about 0.2 to 0.5. One big simplification we do here is that we set to zero the share of non-manufactured goods in the production of manufactures (called γ in that paper). Thus, a smaller part of the manufacturing absorption must come from final demand. Also, we consider overall GDP, Y_i, but have an low estimate of the production of manufactures (only 20 countries), which tends to reduce the value of βY_i^M in (3).

2 Question 2,3 - Algorithm and Deficit Reduction

I will not go over the proofs and derivations we saw in class. I suggest you take a look at Professor Kortum's lecture notes, section 4, "Unbalanced Trade: DEK (2007)". The version of the Alvarez-Lucas algorithm that I use to solve this part is also posted in Chalk. What is the nature of the exercise in Question 3? DEK use exogenous deficits to model trade flows between all countries in the world. In this exercise we force those deficits to go down to zero and analyze how the endogenous variables of the model adjust. That is, we are after changes in wages and prices of manufactures, as functions of World wage, as well as production flows that are consistent with a move towards a scenario where trade is balanced. As seen in class, we can boil down the equilibrium changes to changes in $\{\pi_{ni}\}$, $\{\rho_n\}$ and $\{\omega_n\}$. The equations that summarize those values are, from slide 95:

$$y_i^{M'} = \sum_{n=1}^{N} \pi'_{ni} \left(y_n^{M'} + \delta_n^{M'} \right) \tag{4}$$

$$y_i^{M'} = \frac{\alpha_i}{\beta} \left(\hat{\omega}_i y_i + \delta_i^{'} - \frac{1}{\alpha_i} \delta_i^{M'} \right) \tag{5}$$

$$\pi'_{ni} = \frac{\pi_{ni} \left(\hat{\omega}_i^{\beta} \hat{\rho}_i^{1-\beta} \right)^{-\theta}}{\sum_{k=1}^{N} \pi_{nk} \left(\hat{\omega}_k^{\beta} \hat{\rho}_k^{1-\beta} \right)^{-\theta}} \tag{6}$$

$$\hat{\rho}_n = \left[\sum_{k=1}^{N} \pi_{nk} \left(\hat{\omega}_k^{\beta} \hat{\rho}_k^{1-\beta} \right)^{-\theta} \right]^{-1/\theta} \tag{7}$$

Thus, we just need to solve these equations for $\delta_i^{'} = \delta_n^{M\prime} = 0$ and find the solutions for $\{\hat{\omega}_n, \hat{\rho}_n\}$ and $\{\pi'_{ni}\}$. Note that equation (5) is an adaptation of the lecture notes to our model where the share of manufactures in final demand varies by country.

Description of the algorithm

Computationally, this involves a main loop, where we iterate on values of $\hat{\omega}$, and a smaller loop nested within the main loop, where we iterate on values of $\hat{\rho}$. We proceed as follows:

1. Make a guess about the changes in relative wages. As suggested in class, $\hat{\omega}^i = 1$ is a good guess.[3]

2. Using a version of (7), and exploiting the fact that it is a contraction, we calculate the vector $\hat{\rho}\left(\hat{\omega}^i\right)$ consistent with the initial guess:[4]

   (a) Make an initial guess $\hat{\rho}^i$, which implies $\tilde{\rho}^i = \log\left(\hat{\rho}^i\right)$ The actual guess does not matter, since it is a contraction.

   (b) Apply the operator to obtain the $\tilde{\rho}^o$:

   $$\tilde{\rho}_n^o = -\frac{1}{\theta} \ln \left[\sum_{k=1}^{N} \pi_{nk} \exp\left\{ -\theta \left(\beta \tilde{\omega}_k + (1-\beta) \tilde{\rho}_k^i \right) \right\} \right]$$

   (c) Given some tolerance, e.g. $\hat{\rho}^{tol} = 10^{-6}$, decide whether you have achieved conver-

[3]Throughout, I use the **superscript** i for the vector of guesses that we feed into the loop or operator, and the **superscript** o for the vector that comes out as a result. I apologize for the fact that a subindex i is usually used for countries, and may generate confusion.

[4]Slide 99 shows that this operator satisfies Blackwell's sufficient conditions and is therefore a contraction.

gence. In the algorithm, I define convergence as

$$\sum_{n=1}^{N} \left| \exp\left(\tilde{\rho}_n^o\right) - \exp\left(\tilde{\rho}_n^i\right) \right| < \hat{\rho}^{tol}$$

If convergence is achieved, $\hat{\rho}\left(\hat{\omega}^i\right) = \exp\left(\tilde{\rho}^o\right)$. Otherwise, set $\tilde{\rho}^i = \tilde{\rho}^o$ and go back to b. Note that, to do step 2, all the data that we need to know are β, θ, and the original shares of exports. One can think of other more sophisticated convergence criteria. Here, I do not scale by the magnitude of the vectors because we are dealing with percentage changes already.

3. Compute the new shares of exports, that is, those consistent with the guess $\hat{\omega}^i$ and the outcome prices $\hat{\rho}\left(\hat{\omega}^i\right)$. Follow (6), where the right hand side uses $\hat{\omega}^i$ and $\hat{\rho}\left(\hat{\omega}^i\right)$, instead of $\hat{\omega}$ and $\hat{\rho}$. No new data is needed here.

4. See slide 101. We construct a normalized excess demand for the output of each source i. We subtract country i's manufacturing production from World's demand for country i's manufactures: $Z_i\left(\hat{\omega}_i\right) = \frac{1}{\hat{\omega}_i}\left[\sum_{n=1}^{N} \pi'_{ni}\left(y_n^{M'} + \delta_n^{M'}\right) - y_i^{M'}\right]$ To make this a function of $\hat{\omega}$ only, substitute (5) into (4). Note that we will no longer obtain the expressions in the slides, since those assume $\alpha_n = \alpha$. Instead, the excess demand function is

$$\begin{aligned} Z_i &= \frac{1}{\hat{\omega}_i}\left[\sum_{n=1}^{N} \pi'_{ni}\alpha_n\left(\hat{\omega}_n y_n + \delta_n^{'} - \frac{1-\beta}{\alpha_n}\delta_n^{M'}\right)\right. \\ &\quad \left. -\alpha_i\left(\hat{\omega}_i y_i + \delta_i^{'} - \frac{1}{\alpha_i}\delta_i^{M'}\right)\right] \end{aligned}$$

where only β has canceled now.

In this step we use our data on $y, \alpha, \delta', \delta^{M'}$; by combining these data with our guesses $\hat{\omega}^i$ and $\pi'_{ni}\left(\hat{\omega}^i\right)$, we obtain $Z\left(\hat{\omega}^i\right)$. With this vector of excess demands at hand, we apply the mapping described in the slides

$$\hat{\omega}_i^o = T\left(\hat{\omega}_i^i\right) = \hat{\omega}_i\left[1 + \nu Z_i\left(\hat{\omega}^i\right)/y_i\right]$$

Note that a fixed point of this mapping implies zero excess demands

$$\begin{aligned} \hat{\omega}_i &= T\left(\hat{\omega}_i\right) \Leftrightarrow \\ \hat{\omega}_i &= \hat{\omega}_i + \frac{\hat{\omega}_i}{y_i}\nu Z_i\left(\hat{\omega}\right) \Rightarrow Z_i\left(\hat{\omega}\right) = 0 \ \forall i. \end{aligned}$$

Also note that, despite the different values of α_n, it will still satisfy the adding up constraint

$$\begin{aligned}
\sum_{i=1}^{N} \hat{\omega}_i Z_i(\hat{\omega}) &= \\
&= \sum_{i=1}^{N} \left[\sum_{n=1}^{N} \pi'_{ni} \alpha_n \left(\hat{\omega}_n y_n + \delta'_n - \frac{1-\beta}{\alpha_n} \delta_n^{M'} \right) \right. \\
&\quad \left. -\alpha_i \left(\hat{\omega}_i y_i + \delta'_i - \frac{1}{\alpha_i} \delta_i^{M'} \right) \right] \\
&= \sum_{n=1}^{N} \alpha_n \left(\hat{\omega}_n y_n + \delta'_n - \frac{1-\beta}{\alpha_n} \delta_n^{M'} \right) \sum_{i=1}^{N} \pi'_{ni} \\
&\quad - \sum_{i=1}^{N} \alpha_i \left(\hat{\omega}_i y_i + \delta'_i - \frac{1}{\alpha_i} \delta_i^{M'} \right) \\
&= -(1-\beta) \sum_{n=1}^{N} \delta_n^{M'} + \sum_{i=1}^{N} \delta_i^{M'} = 0
\end{aligned}$$

5. Given some tolerance, e.g. $\hat{\omega}^{tol} = 10^{-6}$, decide whether you have achieved convergence. The criterion I use is the same as in 2.c, for the appropriate vectors. If the algorithm converged, $\hat{\omega} = \hat{\omega}^o = \hat{\omega}^i$, $\hat{\rho} = \hat{\rho}\left(\hat{\omega}^i\right)$ and $\pi'_{ni} = \pi'_{ni}\left(\hat{\omega}^i\right)$. If the algorithm has not converged, set $\hat{\omega}^i = \hat{\omega}^o$ and go back to step 2.

We are asked to provide the equilibrium changes for relative wage, manufacturing price levels and real wages. We know how to compute the first two. Real wages, as in problem set 2, are $w_n^R = w_n/P_n = (w_n/p_n)^{\alpha_n} = (\omega_n/\rho_n)^{\alpha}$. We can compute the change in this variable as:

$$\hat{w}_n^R = (\hat{\omega}_n/\hat{\rho}_n)^{\alpha} \tag{8}$$

Table 2 presents the results. The model suggests that countries that run a deficit must export more and import less; their relative wage must fall for this adjustment to take place. The converse is true for countries that run a surplus. The results confirm this intuition. Countries with large deficits relative to GDP bear large relative wage adjustments. Greece's relative wage falls by nearly 15%, while that of the US falls by close to 5%. On the other extreme, wages in Germany and China increase by approximately 10% and 9%. Figure 1 presents changes in relative wages and confirms that this is an general pattern. The price of manufactures changes, too, reflecting the fact that relative wages in different sources have also changed. For example, presumably the US sources most of its manufactures from itself, and thus the price of its manufactures decreases, together with its relative wage. The total effect on welfare is small, with variation in the real wage in the range of -1% to 1%. The main reason is that our estimated α_n are quite low, meaning that the change in the price of manufactures allmost fully offsets the change in the wages. Figure 2, presents the same pattern as Figure 1 for real wages, but note the changes in the magnitudes.

3 Question 4. Technology Improvements in the U.S.

For this question we need to derive again equations (6) and (7). Denote the vector of technology in the counterfactual relative to the baseline by $\hat{T}$, so $\hat{T}_i = T_i'/T_i$. In this exercise, we will set $\hat{T}_i = 1.2$ if $i = US$, and $\hat{T}_i = 1$ otherwise. In the counterfactual equilibrium,

$$\rho_n' = \gamma \left[\sum_{k=1}^{N} T_k' \left(\kappa \omega_k'^{\beta} \rho_k'^{1-\beta} d_{nk} \right)^{-\theta} \right]^{-1/\theta}.$$

We use the definition of the shares in the original equilibrium $\pi_{ni} = T_i c_i^{-\theta} d_{ni}^{-\theta} / \Phi_{ni}^{-\theta}$ to solve for $d_{ni}^{-\theta} = \pi_{ni}\Phi_n / \left(T_i c_i^{-\theta} \right)$ and substitute in:

$$\begin{aligned}
\rho_n' &= \gamma \left[\sum_{k=1}^{N} T_k' \left(\kappa \omega_k'^{\beta} \rho_k'^{1-\beta} d_{nk} \right)^{-\theta} \right]^{-1/\theta} \\
&= \gamma \left[\sum_{k=1}^{N} T_k' \left(\kappa \omega_k'^{\beta} \rho_k'^{1-\beta} \right)^{-\theta} \pi_{nk}\Phi_n / \left(T_k c_k^{-\theta} \right) \right]^{-1/\theta} \\
&= \gamma \kappa \Phi_n^{-1/\theta} \left[\sum_{k=1}^{N} \pi_{nk} \hat{T}_k \left(\kappa \hat{\omega}_k^{\beta} \hat{\rho}_k^{1-\beta} \right)^{-\theta} \right]^{-1/\theta} \\
\Rightarrow \quad \hat{\rho}_n &= \left[\sum_{k=1}^{N} \pi_{nk} \hat{T}_k \left(\kappa \hat{\omega}_k^{\beta} \hat{\rho}_k^{1-\beta} \right)^{-\theta} \right]^{-1/\theta} \qquad (9)
\end{aligned}$$

where the last line uses the equilibrium result that $p_n = \gamma \kappa \Phi_n^{-1/\theta}$.

The new trade shares will also reflect changes in technology. In the counterfactual equilibrium

$$\pi_{ni}' = \frac{T_i' \left(c_i' d_{ni} \right)^{-\theta}}{\sum_k T_k' \left(c_k' d_{nk} \right)^{-\theta}} \qquad (10)$$

We substitute again for $d_{ni}^{-\theta}$ in (10) to obtain:

$$\begin{aligned}
\pi'_{ni} &= \frac{T'_i\left(c'_i\right)^{-\theta} d_{ni}^{-\theta}}{\sum_k T'_k\left(c'_k\right)^{-\theta} d_{nk}^{-\theta}} \\
&= \frac{T'_i\left(c'_i\right)^{-\theta} \pi_{ni}\Phi_n/\left(T_i c_i^{-\theta}\right)}{\sum_k T'_k\left(c'_k\right)^{-\theta} \pi_{nk}\Phi_n/\left(T_k c_k^{-\theta}\right)} \\
&= \frac{\hat{T}_i\left(\hat{c}_i\right)^{-\theta}\pi_{ni}}{\sum_k \pi_{nk}\hat{T}_k\left(\hat{c}_k\right)^{-\theta}} \\
&= \frac{\hat{T}_i\pi_{ni}\left(\hat{\omega}_i^{\beta}\hat{\rho}_i^{1-\beta}\right)^{-\theta}}{\sum_{k=1}^{N}\hat{T}_k\pi_{nk}\left(\hat{\omega}_k^{\beta}\hat{\rho}_k^{1-\beta}\right)^{-\theta}} \qquad (11)
\end{aligned}$$

Note that if we set $\hat{T}_i = 1$, $\forall i$, we are back to the system of equations in the previous question. To continue, we need to find the values of $\hat{\omega}$, $\hat{\rho}$ and $\{\pi'_{ni}\}$ that solve the system of equations conformed by (4), (5), (9) and (11). Now we only need to repeat steps 1 through 5 in the previous question. Note that in this question we isolate the effect of a technological change, and thus we set $\delta' = \delta^{M'} = \delta^M = \delta$ (which means that the US absolute deficit will grow).

Table 3 presents the results. The price index of manufactures decreases for each country, as a result of more efficient production in the US. Also, the wage in terms of the price index of manufactures increases for all countries, most notably for the US, where it rises up to almost 7%, while in most other countries this change is already an order of magnitude below. Finally, changes in welfare are muted, given that the shares of manufactures in total expenditure are small for all countries. The US ends up being the only substantial winner. This reflects increased productivity in the US and the fact that deficits are kept in place, so the US must absorb large part of the technological improvement, otherwise it would flood the market with its goods and the deficit would be reversed.

Compare this to the results in EK (2002), Table XI, column 1. There, the only country with comparable gains was Canada, although all other magnitudes tended to be larger than in our exercise, too. Here, we retain the result that countries that are closer (in the gravity sense) to the US are the ones who benefit the most. The same is true of countries that run deficits (since we do not force them to go to zero and thus they have access to cheaper manufactures). The exercise is different to the one in EK because here trade is not balanced, and the values of α and β are different. Figure 3 gives a sense of the role of distance and initial deficits, excluding the US. Note that Germany is the only loser with the improvement in the US.

Table 1. Variables by country

Country	GDP (USD bill)	Deficit (USD bill)	Deficit (pct of GDP)	Alpha
Austria	372	2.0	0.5	0.137
Canada	1424	33.5	2.4	0.148
China	3494	-398.8	-11.4	0.247
Czech Republic	174	2.4	1.4	0.275
Denmark	311	6.4	2.1	0.109
Finland	246	-3.8	-1.6	0.171
France	2599	9.8	0.4	0.126
Germany	3335	-226.1	-6.8	0.108
Greece	311	31.2	10.0	0.164
India	1187	12.6	1.1	0.163
Italy	2116	-13.6	-0.6	0.170
Japan	4378	-167.4	-3.8	0.158
Mexico	1036	17.8	1.7	0.164
Poland	425	23.2	5.5	0.220
Romania	171	19.7	11.5	0.220
South Korea	1049	-46.4	-4.4	0.287
Spain	1444	81.5	5.6	0.193
Sweden	462	-7.8	-1.7	0.138
United Kingdom	2812	124.1	4.4	0.123
United States	14062	499.7	3.6	0.141

Table 2. Effect of eliminating deficits

Country	Relative Wage (relative to baseline)	Mfg. Price (relative to baseline)	Real Wage (relative to baseline)
Austria	0.988	1.027	0.995
Canada	0.972	0.985	0.998
China	1.088	1.067	1.005
Czech Republic	0.998	1.021	0.994
Denmark	0.968	1.014	0.995
Finland	1.028	1.025	1.000
France	1.005	1.010	0.999
Germany	1.099	1.038	1.006
Greece	0.845	0.936	0.983
India	0.997	1.005	0.999
Italy	1.017	1.017	1.000
Japan	1.070	1.056	1.002
Mexico	0.978	0.990	0.998
Poland	0.967	1.006	0.991
Romania	0.901	0.978	0.982
South Korea	1.054	1.045	1.003
Spain	0.953	0.988	0.993
Sweden	1.031	1.021	1.001
United Kingdom	0.935	0.985	0.994
United States	0.953	0.980	0.996

Table 3. Effect of 20 Pct increase in US technology			
Country	Mfg. Price (relative to baseline)	Wage/Mfg. Price (relative to baseline)	Real Wage (relative to baseline)
Austria	0.978	1.002	1.000
Canada	0.977	1.006	1.001
China	0.977	1.000	1.000
Czech Republic	0.978	1.001	1.000
Denmark	0.978	1.002	1.000
Finland	0.978	1.000	1.000
France	0.978	1.001	1.000
Germany	0.977	1.000	1.000
Greece	0.980	1.004	1.001
India	0.978	1.001	1.000
Italy	0.978	1.000	1.000
Japan	0.977	1.000	1.000
Mexico	0.977	1.005	1.001
Poland	0.978	1.001	1.000
Romania	0.979	1.003	1.001
South Korea	0.977	1.000	1.000
Spain	0.979	1.001	1.000
Sweden	0.978	1.000	1.000
United Kingdom	0.978	1.002	1.000
United States	0.974	1.069	1.010

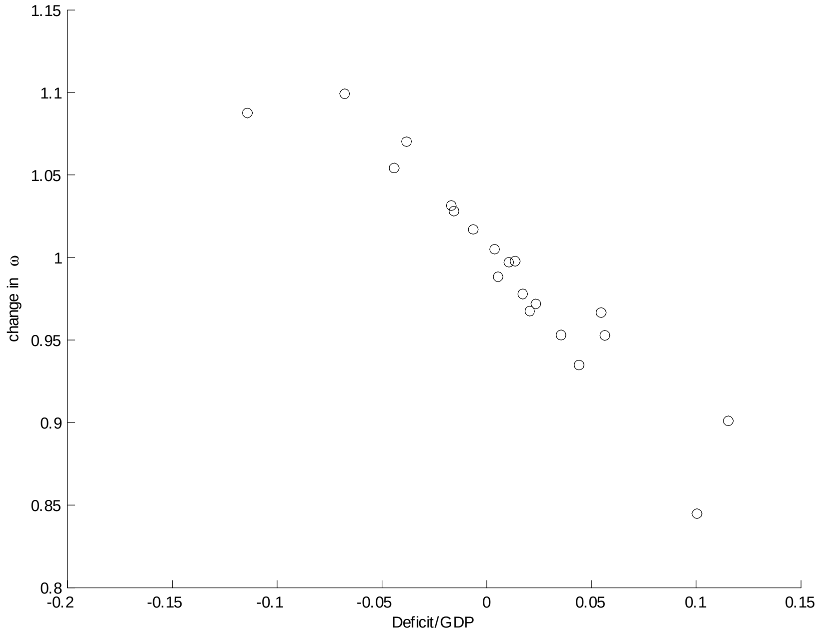

Figure 1

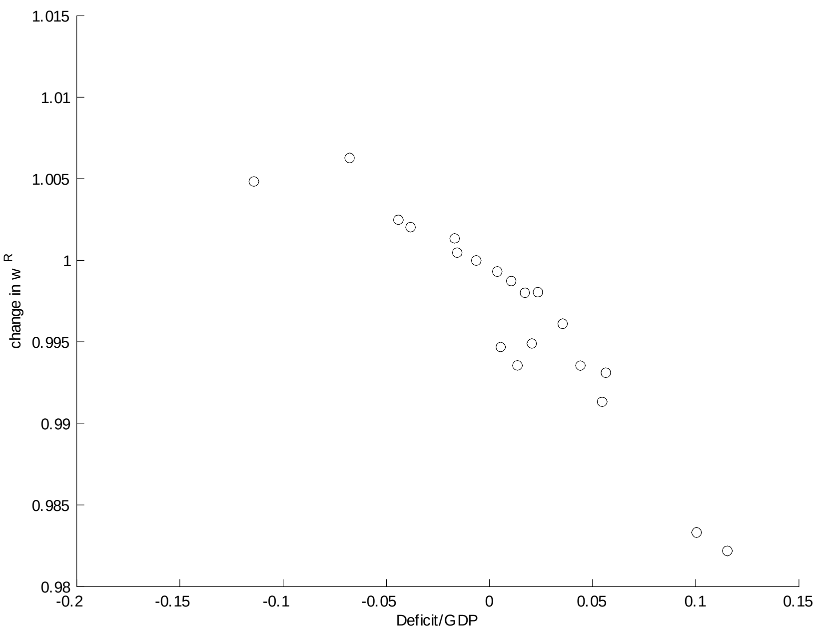

Figure 2

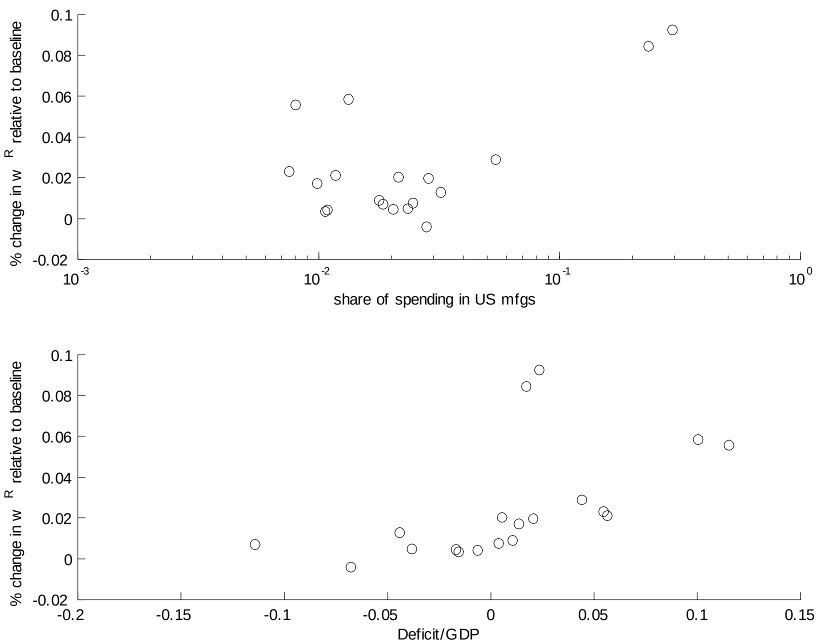
% change in w R relative to baseline
share of spending in US mfgs
Deficit/GDP

Figure 3