Migration, Specialization, and Trade: Evidence from the Brazilian March to the West

1 An alternative model with sorting and selection

This is an alternative model to the knowledge diffusion contained in Pellegrina + Sotelo (mimeo). The idea is that it is possible that the positive correlation between migration flows L_{ijkt} and amount of people at the origin region $L_{ik,t-1}$ is generated by more productive farmers moving towards XXX. In this section we consider such case based on Behrens et al (2014). Now the model. There is a fixed amount of people in Brazil $L = 1$. Each farmer from origin region i chooses its destination j, and once is there, it choose which crop k to grow. All farmers are exante identical, except for their talent t and serendipity s. Each farmers initially knows its talent, then choose where to migrate. After moving, serendipity happens. Talent times serendipity determines the farmer's productivity $\phi = t \times s$. Knowing its productivity allows the farmer to select which crop to farm. The number of regions I is discrete and fixed.

2 Selection and Agglomeration

Before dealing with sorting, we deal with selection given that location sorting has already been made. This is where the most important of the analysis is anyway since here is where agglomeration forces appear. Consider some region. People now discover their serendipity, then they know their productivity ϕ. After their discover their productivity, they decide which crop to grow.

2.1 Demand

Each farmer produces a variety ω of the crop k. The demand for that variety comes from a representative consumer maximizing utility subject to its budget constraint:

$$max \quad U = (\sum_k (\int_{\Omega_k} q_k(\omega)^{\frac{\epsilon_k - 1}{\epsilon_k}})^{\frac{\epsilon_k}{\epsilon_k - 1}\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \qquad s.t. \qquad \sum_k \int_{\Omega_k} q_k(\omega) p_k(\omega) \leq X$$

Notice that the elasticity of substitution across varieties for a given crop depends on the crop itself. This generates demand:

$$q_k(\omega) = (\frac{p_k(\omega)}{p_k})^{-\epsilon_k} (\frac{p_k}{p})^{-\sigma} q$$

where:

$$p = (\sum_k p_k^{1-\sigma})^{\frac{1}{1-\sigma}}$$

$$p_k = (\int_{\Omega_k} p_k(\omega)^{1-\epsilon_k} d\omega)^{\frac{1}{1-\epsilon_k}}$$

2.2 Supply

The farmer that grows variety ω of a crop k maximizes profits subject to its demand:

$$max \quad \Pi_k(\omega) = (p_k(\omega) - \frac{c_k}{\phi_k(\omega) A_k}) q_k(\omega) \qquad s.t. \qquad q_k(\omega) = (\frac{p_k(\omega)}{p_k})^{-\epsilon_k} (\frac{p_k}{p})^{-\sigma} q$$

Where c_k are unit costs, $\phi_k(\omega)$ is productivity, and A_k is crop-productivity. Then, we get a price function that depends on a constant markup and unit costs:

$p_k(\omega) = (\frac{\epsilon_k}{\epsilon_k - 1})\frac{c_k}{\phi_k(\omega)A_k}$
Then, aggregated prices are:

$p_k = (\frac{\epsilon_k}{\epsilon_k - 1})\frac{c_k}{\phi_k A_k} \quad where \quad \phi_k = (\int_{\Omega_k} \phi_k(\omega)^{\epsilon_k - 1} d\omega)^{\frac{1}{\epsilon_k - 1}}$

$p = (\frac{\epsilon_k}{\epsilon_k - 1})\frac{1}{\phi} \quad where \quad \phi = (\sum_k (\frac{A_k \phi_k}{c_k}))^{\frac{1}{\sigma - 1}}$

2.3 Demand and Supply

Considering the price function from supply and plugging it in the demand function, we get:
$q_k(\omega) = (\frac{\phi_k(\omega)}{\phi_k})^{\epsilon_k}(\frac{\phi_k}{\phi})^{\sigma} q$
If we plug this and the price function in the profit function, we get:
$\Pi_k(\omega) = (\frac{1}{\epsilon_k})(\frac{c_k}{A_k})(\frac{\phi_k(\omega)}{\phi_k})^{\epsilon - 1}(\frac{\phi_k}{\phi})^{\sigma - 1} X, \qquad where \qquad X = p \times q$

2.4 Produce crop k or k'?

Consider there are $k = \{1, ..., K\}$ crops in Brazil, and they are ordered such that $\epsilon_{k=1}$ is the highest and $\epsilon_{k=K}$ is the lowest. Also assume $\sigma < \epsilon_k, \forall k$. Farmer with productivity ω should be indifferent between producing crops $k = 1$ and $k = 2$ if and only if $\Pi_{k=1}(\omega) = \Pi_{k=2}(\omega)$. This happens at threshold ϕ^*, which is:
$\phi^* = ((\frac{A_{k=1}}{A_{k=2}})(\frac{c_{k=2}}{c_{k=1}})(\frac{\phi_{k'}^{\sigma - \epsilon_{k=2}}}{\phi_k^{\sigma - \epsilon_{k=1}}}))^{\frac{1}{\epsilon_{k=1} - \epsilon_{k=2}}}$
All farmers with productivity above ϕ^* for sure produce crop $k = 1$. Something I'm not sure here is if all farmers with productivity above ϕ^* only produce crop $k = 1$ or other crops too, since we could do the same analysis with respect to $k = 3$ and so on. Let's maintain the analysis between these two crops. Notice that under our parametrization, $\frac{d\phi^*}{d\phi_k} > 0$ so selection to produce crop $k = 1$ is tougher when aggregate productivity for that crop is higher. This is the higher competition effect we expect to happen.