A Water Droplet Evaporation and Temperature Model

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ABSTRACT

A model for predicting evaporation and temperature changes in water drops traveling through air was developed and evaluated with laboratory data. The model uses combined sensible heat transfer and diffusion theory in an energy balance to simultaneously calculate evaporation as droplet temperature approaches wet bulb temperature of the air. Predicted droplet temperatures agreed closely with measured temperatures. The evaporation portion of the model was evaluated by measuring loss of water over longer time periods after the droplet reached wet bulb temperature. A volumetric method using microliter syringes was used to measure evaporation loss from 0.5 to 2 mm diameter drops. Model predictions generally agreed with measured rates.

INTRODUCTION

Spray evaporation and wind drift is a major problem in sprinkler irrigation. It may be acute with low pressure spray nozzles which are being used extensively with center pivot irrigation. The small droplet sizes cause the spray type heads (those with the greatest energy savings) to be more susceptible to evaporation and wind drift. The increase in drop surface area per unit volume of water delivered with the smaller droplet surface area per unit volume of water delivered with the smaller droplet sizes increases evaporation, and the smaller average mass of the smaller drops also increases the distance they can be displaced by wind. Higher pressure impact type sprinklers also have a small drop size fraction. Drop size distribution data are becoming available for use in predictive models (Solomon et al. 1985, Kohl and DeBoer 1985). Edling (1985), modelled evaporation and drift losses from low pressure spray nozzles using the model of Williamson and Threadgill (1974). Kinzer and Gunn (1951) developed an equation for droplet evaporation for droplets falling at terminal velocity. Shirai et al. (1971, a and b) analyzed droplet temperature changes theoretically and experimentally, and found that the final droplet temperature was slightly lower than the wet bulb temperature.

Many investigators have lumped losses due to evaporation and spray drift together into "Spray losses". This approach has been used largely because of

difficulties encountered with the measurement techniques necessary to separate these losses. A model for accurate prediction and separation of the losses due to evaporation and wind drift under varying climatic conditions is needed and would be of considerable value to designers of sprinkler systems.

The overall objective of this study was to develop and verify a mathematical computer model to predict the losses associated with sprinkler nozzles, and specifically; (a) to develop a model to predict droplet evaporation and temperature changes, (b) to verify the model with laboratory data, and (c) to investigate the role of droplet or water supply temperature in the droplet evaporation process. A subsequent paper will combine a droplet trajectory model with the evaporation model to predict field losses, and the model will be evaluated with field data.

EVAPORATION THEORY

The heat and mass transfer analogy approach offers a sound theoretical basis for the explanation of evaporation from falling sprinkler droplets. This model is combined with particle dynamics theory to give a trajectory model of the falling drop to account for the possible effects of wind drift.

Numerous investigators have used the heat transfer analogy theory to describe the evaporation from droplets, and most of these have referenced the work of Ranz and Marshall (1952). Their model was based on Fröessling's (1938) boundary layer equations and the equations for heat and mass transfer.

In the case of a sprinkler droplet falling through a moving airstream, forced convection is the process under which evaporation takes place. For this situation, Fröessling (1938) developed the following empirical relation for the mass transfer number, Nu':

$$Nu' = 2.0 + 0.60 \, Sc^{1/3} \, Re^{1/2} \quad \dots \dots \dots \dots [1]$$

where

Re $= DV/v =$ Reynold's no.
Sc $= v/K =$ Schmidt no.
D $=$ diameter of droplet, m
V $=$ velocity of droplet relative to air, m/s
v $=$ kinematic viscosity of air, m²/s
K $=$ diffusivity of water vapor in air, m²/s

According to the heat transfer analogy proposed by Ranz and Marshall (1952), the heat transfer number (Nusselt number) should be correlated with heat transfer data according to:

$$Nu = HD/k = 2.0 + 0.60 \, Pr^{1/3} \, Re^{1/2} \quad \dots \dots \dots \dots [2]$$

where

Pr = Prandtl no. = $C_p \mu / k$
H = heat transfer coefficient J m^{-2} K^{-1} s^{-1}
C_p = heat capacity of air at constant pressure, J kg^{-1} K^{-1}
μ = dynamic viscosity of air, kg s^{-1} m^{-1}
k = thermal conductivity of air, J s^{-1} m^{-1} K^{-1}

These equations meet the theoretical requirement that Nu' = Nu = 2.0 at Re = 0.0. Knudsen and Katz (1958) also described equtions 1 and 2 and give the following ranges for the parameters.

(1) 1 < Re <70,000
(2) 0.6 < Pr <400
(3) 0.6 < Sc <400

Marshall (1954) gave the following empirical equation the mass transfer of water vapor from spheres in forced convection:

$$k_g = K \rho_a Nu'/(M_m DP_f) \qquad [3]$$

where

k_g = the mass transfer coefficient
M_m = the mean molecular weight of the gas mixture in the transfer path = 29.0 for air
P_f = partial pressure of air, kPa
ρ_a = density of air, kg m^{-3}

This analysis was based on the following assumptions: (a) air temperature and pressure were constant, (b) evaporation did not affect ambient humidity (there was a large volume of air per droplet), (c) there was no turbulence in the air as the droplets fell through it, and (d) the droplets were spherical and were pure water. Of these assumptions, all but c were reasonable. With fine drops (D≤1 mm), turbulent effects are not negligible, and detracted considerably from the accuracy of later researchers' experimental results (Goering et al., 1972).

Goering et al. (1972) made a slight modification to the Marshall equation, and using geometric and mass definitions derived the following equation for the rate of diameter change for an evaporating spray droplet:

$$dD/dt = -2 (M_v/M_m)(K/D)(\rho_a/\rho_d)(\Delta P/P_f) N_u' \ldots [4]$$

where M_v = the molecular weight of the diffusing water vapor = 18.0
ΔP = vapor pressure difference, kPa
ρ_d = density of liquid in drop, = 1000 kg/m^3 for water.

All of the quantities in parentheses are dimensionless, with the exception of K/D, which has dimensions of L/T. The diffusivity K, is a function of both air temperature and pressure and is taken from List (1963) as:

$$K = (101.3/P_a) 8.8 \times 10^{-10} T_k^{1.81} \ldots [5]$$

where

T_K = water temperature in °K
P_a = atmospheric pressure in kPa

Previous authors (Goering et al. 1972, Williamson and Threadgill 1974, and Edling, 1985) have assumed diffusivity is a function of temperature alone. Equation [5] gave better agreement between the model and the data collected in this study than did the previously used

functions, which gave lower values of diffusivity (and evaporation rates). The air pressure was determined by

$$P_a = 101.3 (1 - 2.257 \times 10^{-5} E)^{5.255} \ldots [6]$$

where E = the elevation of the test site, m (1200 m in this study).

In the Goering (1972) model, the droplet temperature was assumed to be the same throughout and equal to the wet bulb temperature. Consequently, the vapor pressure difference is:

$$\Delta P = P_s - P_v \ldots [7]$$

where P_s is the saturation pressure at the wet bulb temperature of the air, and P_v is vapor pressure at the dry bulb temperature (or saturation pressure at the dewpoint).

Because the airstream contains only air and water vapor, the total pressure P_a (atmospheric) is the sum of the partial pressures of the air and water vapor, and

$$\Delta P/P_f = (P_s - P_v)/(P_a - P_v) \ldots [8]$$

All quantities on the right side of equation 4 are now known, and the evaporation rate dD/dt can be calculated for any time step dt, knowing the initial droplet size D.

Goering et al. (1972) used the Ranz and Marshall (1952, 1954) theory of evaporation and the Smith (1970) trajectory theory to develop an evaporation-drift model. The experimental data of Roth and Porterfield (1965) were used to verify the model. Williamson and Threadgill (1974) also used the mass diffusion equation in a form similar to equation [4]. They obtained good agreement between measured and predicted evaporation rates for drop diameters between 0.1 and 0.2 mm. Their form of the equation was found to give results nearly identical to equation [4] in this study.

TEMPERATURE PREDICTION

The existing theory relating to evaporation from small drops assumes that the temperature of the droplet reaches the wet bulb temperature of the air instantaneously as the droplets leave the nozzles. For spray applications (spraying of agricultural chemicals) or evaporative cooling applications where the droplets involved are relatively small (d<0.55 mm), this is probably a very good assumption. This assumption precludes the possibility of latent heat transfer resulting in condensation, however. Pair et al. (1969) showed that the droplets involved in sprinkler irrigation did not necessarily reach the wet bulb temperature instantaneously upon leaving the nozzle. In any case, an experiment was undertaken to determine how quickly various size droplets reach the wet bulb temperature, which is explained in detail below. This section explains the theoretical equations necessary to describe the evaporation process both prior to and after the droplet comes to the wet bulb temperature from an energy balance standpoint.

Consider the energy balance of a drop leaving a nozzle at temperature T_1 and approaching the wet bulb temperature. The energy or heat balance of the drop over

a small time increment Δt can be written:

$$H_s + \Delta S - \lambda \Delta M = 0 \quad \dots \dots \dots \dots \dots \dots \dots [9]$$

where H_s is the sensible heat transfer from the air to the drop, $\Delta S = M\Delta T$, the change in heat stored in the drop, M is the drop mass, and λ is the latent heat of vaporization.

Starting with the latent heat transfer, we have an expression from the mass diffusion equation (equation 4, but with the saturation pressure at the drop surface temperature, rather than wet bulb temperature) for the change in drop diameter, and mass ΔM.

$$D_2 - D_1 = \Delta t(K/D_a) \, C_1 \, (P_s \, (T_1) - P_v)/(P_a - P_v)N_u'$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots [10]$$

where D_1 and D_2 are the drop diameters at the beginning and end of the time increment Δt, respectively, and D_a is the average drop diameter during the time increment. The constant $C_1 = (M_v/M_m) \, (\rho_a/\rho_d)$. $P_s \, (T_1)$ is the saturation pressure (kPa) at the drop temperature, T_1. N_u' is the mass transfer number defined by equation [1]. Solving equation [10] for D_2, the drop mass can be calculated (drops are assumed spherical).

Next, H_s, the sensible heat transfer, can be calculated by the Nusselt equation [2].

$$H_s = \Delta t \, (k/D_a) \, (T_1 - T_a) \, \pi \, D_a^2 \, N_u \quad \dots \dots \dots \dots [11]$$

where $k = 1.93 \times 10^{-6} \, (T_K)^{0.86}$ is the thermal conductivity of the air (J s^{-1} m^{-1} K^{-1}).

Now equation [9] can be solved for the change in drop temperature over the small time increment Δt:

$$\Delta T = T_2 - T_1 = 2 \, (\lambda \Delta M - H_s)/(M_2 + M_1) \quad \dots \dots \dots [12]$$

By alternately calculating the change in drop size (evaporation) from the mass transfer equation [10], and then calculating the change in drop temperature using the heat transfer equation for each small time step Δt, both drop size and temperature changes with time can be accurately modeled. These equations, [10], [11], and [12]), comprise the evaporation-temperature model.

EXPERIMENTAL PROCEDURES

Evaporation Measurement With Single Drops

To verify the evaporation portion of the model, the evaporation rates of individual droplets were measured when subjected to different temperature, humidity and air velocity conditions. A small wind tunnel was constructed which delivered air at velocities up to 10 m/s, comparable to the initial (and maximum) velocities of droplets leaving spray type sprinkler heads. Tests were conducted in a small room in which the air temperature and humidity could be controlled.

Steady-state temperature in the wind tunnel was measured with thermo-couples and dry bulb thermometers, and wet bulb temperature was measured with an aspirated electric psychrometer ("Psychron"). Air velocity was measured with a thermal anemometer

sensor (Kurz Instrument Company Model 441)*. Velocity was measured at a point immediately upstream of the droplet.

Because drop temperature changes occur within a few seconds, it was not possible to measure temperature change and evaporation simultaneously. The temperature change tests were run separately from the evaporation tests which were conducted at constant drop temperature, and for longer time periods.

The internal temperature of the droplets was measured with a copper-constant thermocouple (wire diameter 0.05 mm) according to the procedures outlined in Ranz and Marshall (1952a). A Nickolet Model 2093 recorder with printer was used by Longley (1984) to monitor the time rate of change of the internal temperature of the droplet.

To produce a changing drop temperature, water was drawn with a syringe from a beaker at a temperature either higher or lower than the wet bulb temperature. The droplet was then deposited on the thermocouple in the moving airstream. The temperature response of the droplet in the moving airstream was then recorded. it was necessary to use very hot (boiling) or cold (ice bath) water to obtain a droplet of suitable temperature after extrusion through the needle. Additional details of the procedure are given in Longley (1984).

The process of measuring the evaporation rate for a single drop involves suspending a drop in an airstream and noting the change in the droplet diameter over time as evaporation takes place. Ranz and Marshall (1952a) describe a method whereby the droplet is suspended on a fine wire thermocouple or a drawn glass bead, and the change in diameter is measured with photographs taken at regular intervals. This method was used by Longley (1984). The photographic technique gave reasonbly good results, but a more accurate method was needed.

The following volumetric technique proved to be more accurate than photographic diameter measurement. A 5-μL syringe was fitted with a micrometer (reading to 0.025 mm (0.001 inch)) to directly measure displacement of the plunger and volumetric loss. The syringe was mounted with the needle pointed downward into the airstream at the outlet of the wind tunnel. A drop of known initial volume was then extruded from the needle. After allowing the drop to evaporate for a time period of 30 to 120 s, the drop was then drawn back into the syringe. The net loss was measured by the net plunger displacement required to bring the water surface back to its initial position at the tip of the needle.

The loss rate was computed for the average drop size during the test. The actual change in drop size was relatively small. For drops between 0.3 mm and 2 mm diameter, a 5-μL syringe (Hamilton no. 95) with a 0.18 mm diameter silica needle was used. Tests were also run with 1 and 2-μL syringes with stainless steel needles (0.5-mm diameter). For drop diameters of 1.5 to 2.5 mm, a 10-μL syringe was used.

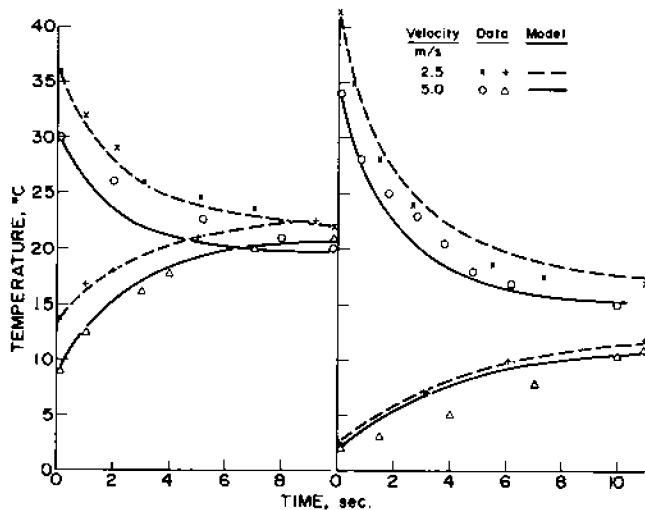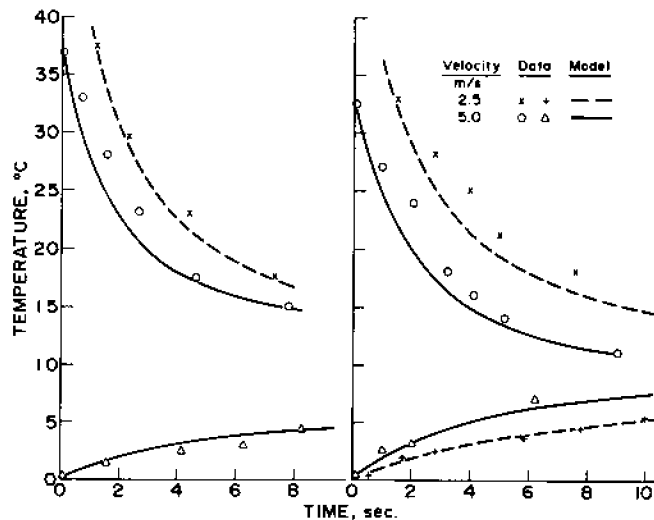Fig. 1—Drop temperature - time for high air temperature (33°C).

Fig. 3—Drop temperature - time for low air temperatures (10 to 15°C).

RESULTS

Drop Temperature Prediction

The temperature predictions with the combined heat transfer-diffusion model were evaluated with laboratory droplet temperature data obtained by the procedure explained above. The results indicated excellent agreement between model drop temperature prediction and the laboratory drop temperature data for a wide variety of conditions in the wind tunnel.

Fig. 1 shows this comparison for high air temperature (33°C). Figs. 2 and 3 show the same comparisons for medium (23°C) and low (13°C) air temperature conditions, respectively. The initial droplet temperature is shown at zero time for each test. Tests were run with initial temperature above and below the wet bulb temperature. The wet bulb temperatures were slightly different for each test. The drop size was about 2 mm for these tests. In most tests the model prediction follows the actual data to within 2°C. In some of the tests shown, the model approached wet bulb temperature faster than did the data. The overall accuracy of the model temperature predictions appears to be acceptable.
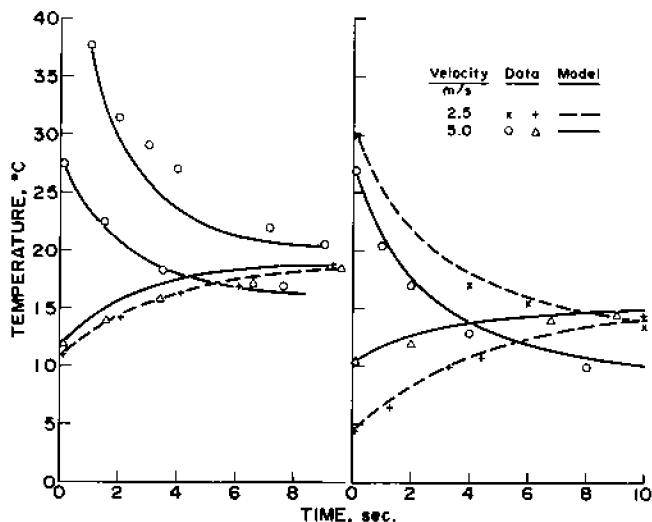
The model actually predicts that the droplet temperature drops slightly below the wet bulb temperature. The magnitude of this temperature depression depends upon what diffusivity function is used. With diffusivity defined by equation [5], and E = 1200 m, the model predicted final drop temperatures about 0.2 to 0.4°C below the wet bulb. This theoretical prediction could not be verified by the experimental data taken in this study. However, Shirai et at. (1971a,b) also found this effect and verified it experimentally. Kinzer and Gunn (1951) also found a tendency for measured drop temperatures to be slightly lower than wet bulb temperature. Therefore, this effect appears to be real. The explanation for this may be that a water droplet may evaporate and cool more efficiently than a wick-covered thermometer, and the droplet temperature may be closer to the true wet bulb temperature.

Figs. 4 and 5 show the effects of droplet size and velocity relative to the air on the time required to

Fig. 2—Drop temperature - time for medium air temperature (22 to 25°C).
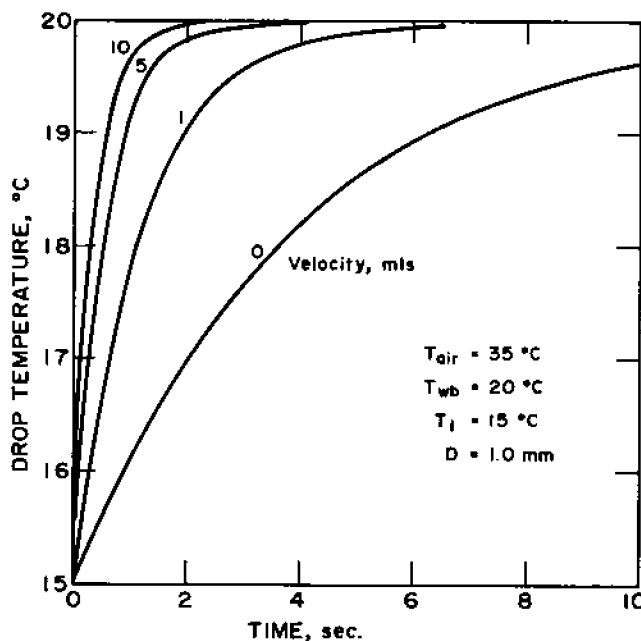

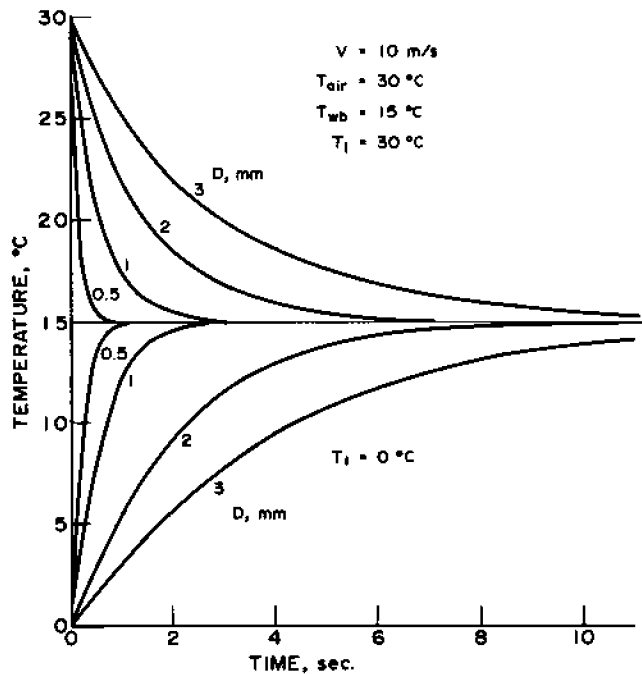Fig. 4—The effect of droplet relative velocity on the time required for the drop to come to wet bulb temperature.

Fig. 5—Effect of drop size on time required for drop to approach wet bulb temperature.

Fig. 6—Evaporation rate vs. drop size for high air temperature and low humidity.

approach wet bulb temperature. Drop size has a large effect on the rate of temperature change, whereas velocity has a relatively small effect.

Evaporation Rates at Constant Droplet Temperature

Initially, results obtained using the diffusion model (equation [4]) and heat transfer model (equation [2]) were compared separately with experimental data to determine which model would most accurately predict droplet evaporation. These comparisons were based on the assumption that the droplet had already reached constant temperature, a fact that will later be explored in greater detail. The heat transfer model (equation [2]) calculates sensible heat transfer, which can be converted to equivalent latent heat or mass loss as described in equation [11].

Comparison of the two models showed that they yield nearly identical results when drop temperature is assumed equal to wet bulb temperature. The unsteady state case showed that the sensible and latent heat components balance as the drop approaches a constant temperature slightly below the wet bulb temperature. The droplet temperature depression has a relatively small effect on the overall heat transfer rate. Therefore, for the steady state case, either model will give reasonable predictions of evaporation rates.

To evaluate the model for prediction of evaporation rates, the microliter syringe method was used. Drop sizes in the range of 0.3 to 1.5-mm diameter were tested. This is the range which occurs with low pressure spray systems and the low end of the impact sprinkler drop size distributions. Drops larger than about 1.5 mm had insignificant evaporation, and drops smaller than 0.3 mm could not be measured accurately. The evaporation rate is expressed in terms of percent mass loss per second as shown in Figs. 6 to 10. This gives a meaningful representation of the increasing relative evaporation loss as drop size decreases.
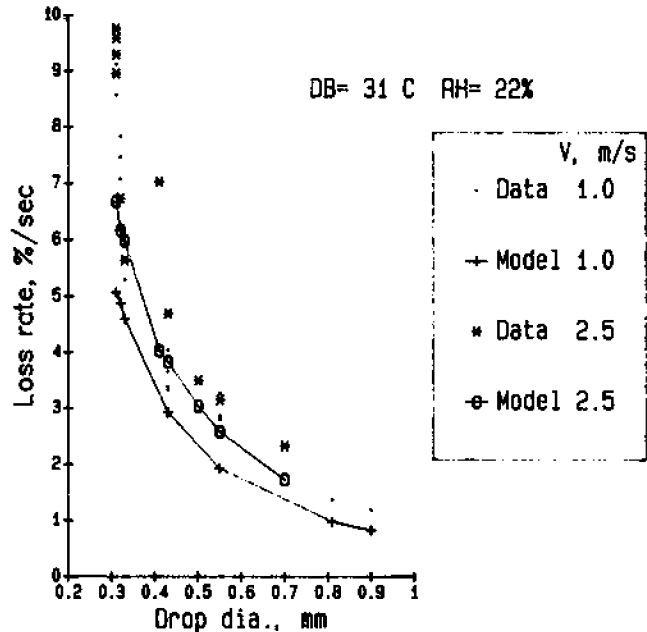
Fig. 6 shows measured and model predicted evaporation rates for high air temperature (31°C) and low (22% relative) humidity conditions. Fig. 7 shows the effect of drop size on evaporation for medium temperature (22°C) and high humidity (81%). In these cases the measured loss rates were slightly higher than those predicted by the model. Figs. 8, 9 and 10 show measured and computed rates plotted for various temperature and velocity conditions. Model predictions were reasonably accurate but there is a tendency for the model to underpredict loss rates for the smallest drops measured (0.3 to 0.5 mm). Some of the difference may be due to experimental error in measuring loss from the smallest drops.
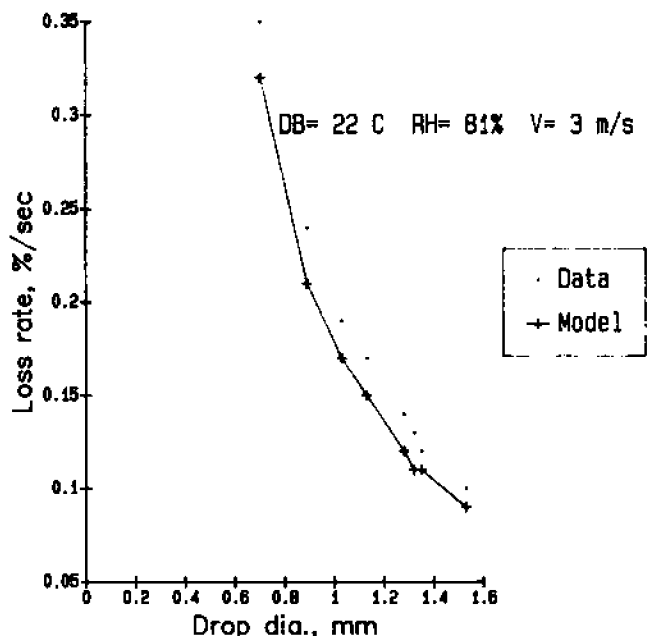

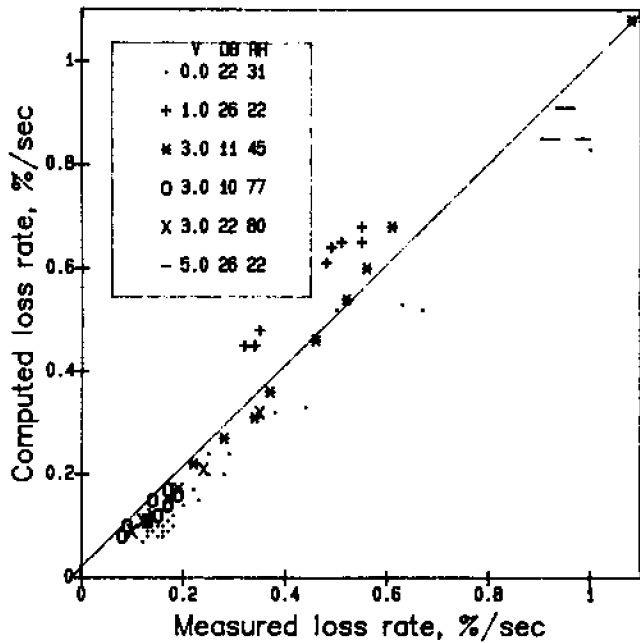Fig. 7—Evaporation rate vs. drop size for medium temperature and high relative humidity.

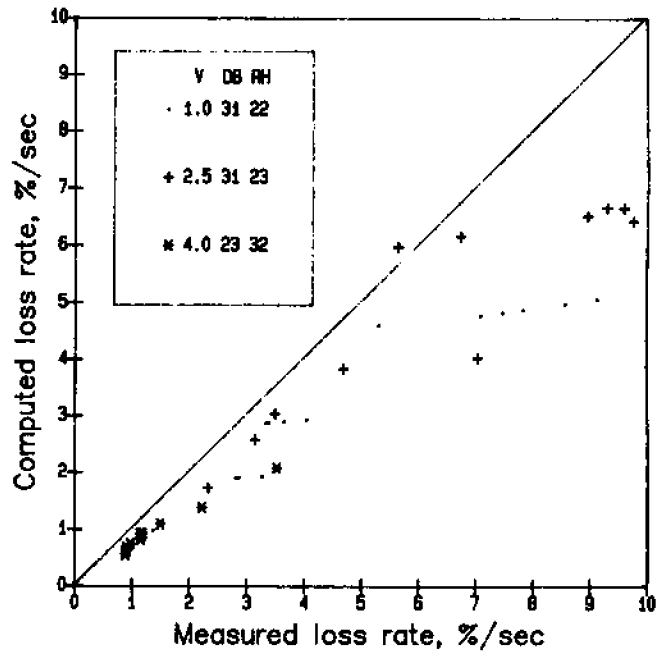**Fig. 8—Comparison of measured and computed evaporation rates (low range).**

Fig. 10—Comparison of measured and computed evaporation rates (high range).

Factors Affecting Droplet Evaporation

In order to better understand the evaporation process as it affects sprinkler droplets and the variables that are important in predicting this evaporation (and wind drift also), it is instructive to consider changes in the model inputs and their effect on evaporation prediction. To do this, let us first consider the effects of changing ambient atmospheric conditions under which sprinklers might be operating, and then some of the physical characteristics of the droplets themselves, such as airspeed, drop size, and drop temperature.

The effect of the dry bulb temperature of the air evaporation is the most distinct. From Figs. 6 and 7, it is

Fig. 9—Comparison of measured and computed evaporation rates (medium range).

obvious that droplet evaporation is more rapid at higher air temperatures. Since droplet temperature has already come to wet bulb temperature, all sensible heat transferred to the droplet must be dissipated as latent heat lost in the evaporation process. Consequently, the evaporation (latent heat lost) is exactly balanced by the sensible heat input to the droplet from the air.

If the droplet is not at wet bulb when it exits the nozzle, however, the situation is much more complicated, which is normally the case. As we can see from the drop temperature charts (Figs. 1 to 5), it may take as much as 8 s for the droplet to reach the wet bulb temperature. When we consider that typical droplet flight times are on the order of 1 to 2 s, unless the water supply is close to the wet bulb temperature of the air, it is unlikely that any but the smallest droplets will reach wet bulb temperature before hitting the ground.

The relative velocity of the droplet to the air determines the Reynolds number in the mass diffusion and heat transfer equations. The heat transfer rate is proportional to velocity to the 1/2 power. Thus the model is less sensitive to velocity. The velocities used in the tests varied from zero to about twice the terminal velocities of the drop sizes tested. Higher velocities were desirable, however, droplets would not stay on the syringe needles at higher velocities. Drops leave sprinkler nozzles at higher velocities with the smaller drops (which tend to evaporate significantly) rapidly approaching terminal velocity relative to the air.

CONCLUSIONS

Two approaches were tested in the evaporation portion of the model, using existing theory of heat and mass transfer. After testing these separately, the heat transfer, and mass diffusion equations were combined in an energy balance to predict droplet temperature changes with time. The combined model predicted that droplets approach and actually drop slightly below the wet bulb
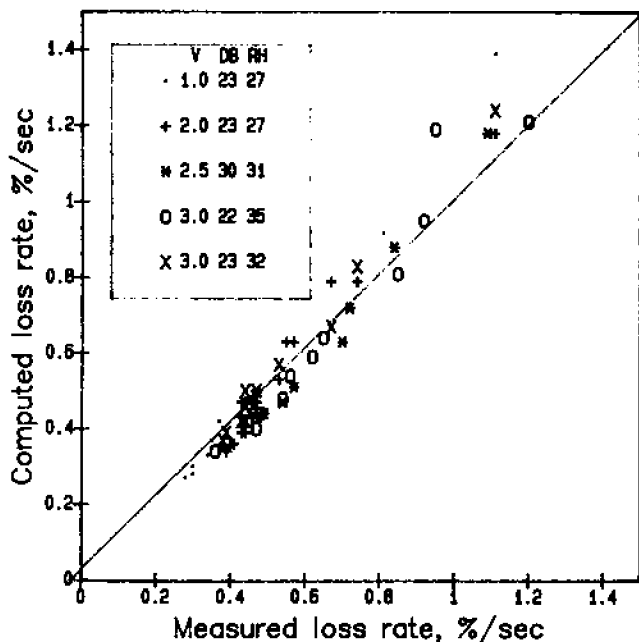
temperature. Temperature measurements agreed with the model quite closely.

Laboratory data were collected to verify the accuracy of the evaporation model under the steady state temperature case (where the droplet remained at constant temperature). A volumetric microliter syringe method was found to give accurate measurements of evaporation loss from drops between 0.3 and 1.5 mm diameter, the size range in which evaporation losses become significant. Measured loss rates were slightly higher than model computed rates.

The latest objectives of this investigation was to determine the importance of the water supply temperature for the sprinkler on evaporation. The results emphasize that the relationship of the water supply temperature to the wet bulb temperature of the air is important in determining evaporation from sprinkler droplets—a fact that was either unknown or ignored in previous sprinkler evaporation studies. A feature of this simulation model is that droplet temperature changes are accurately accounted for throughout the flight period, which can significantly increase the accuracy of evaporation predictions.

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