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RAPID METHOD FOR ESTIMATING THE UNSATURATED HYDRAULIC CONDUCTIVITY FROM INFILTRATION MEASUREMENTS

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A method is proposed for estimating the unsaturated hydraulic conductivity from observed infiltration data. Infiltration measurements are generally easier to obtain than experimental data required for in situ determination of the hydraulic conductivity. The problem was formulated in terms of a nonlinear least-squares parameter optimization method which combines Philip's two-term infiltration equation with an analytical description of the unsaturated soil hydraulic properties according to Mualem and van Genuchten. Reliable estimates for the hydraulic parameters could be obtained with an inverse procedure when independently measured water retention data were included. The results indicate that soil water content measurements at very low values of the soil water pressure head are especially important to ensure parameter uniqueness. The method provides rapid and cost-effective estimates of the hydraulic properties of field soils.

Numerical models for simulating water flow in the unsaturated zone are now readily available for a wide variety of problems (Campbell 1985; Richter 1988). The accuracy of flow and transport predictions obtained with these models depends to a large extent on the availability of reliable values for the soil hydraulic properties. The required hydraulic properties are the soil water retention curve, $\theta(h)$, the hydraulic conductivity function, $K(h)$ or $K(\theta)$, or the soil water diffusivity curve, $D(\theta)$, where θ is the volumetric water content and h the soil water pressure head. Because of experimental difficulties, current technology for measuring these hydraulic properties has not kept pace with the development of sophisticated modeling techniques. The characterization of field soil hydraulic properties is further hampered by the problems of soil spatial variability across the landscape. Hence, rapid and relatively inexpensive methods for measuring the unsaturated soil hydraulic properties are critically needed to facilitate the application of theoretical models to site-specific subsurface water flow and contaminant transport processes.

The hydraulic properties of unsaturated soils are often described by means of relatively simple analytical functions for $\theta(h)$ and $K(\theta)$. Such analytical functions facilitate a rapid comparison of the hydraulic properties of different soils, and also provide a mechanism to obtain hydraulic properties of individual soils through interpolation between, or extrapolation from, a limited number of hydraulic data points (Russo 1988). One may distinguish two different approaches for estimating the unknown hydraulic parameters in these analytical functions: 1) direct measurement of a limited number of soil water retention and hydraulic conductivity data points followed by fitting a parametric model of the hydraulic functions to the experimental data (e.g., van Genuchten 1980), or 2) estimating the hydraulic parameters from a transient flow experiment by applying some type of inverse procedure (e.g., Kool et al. 1987; Kool and Parker 1988).

In this paper we follow the second approach. A rapid and cost-effective method is described for evaluating the soil hydraulic parameters from field-measured infiltration rates and independently measured soil water retention data. Methods for determining soil hydraulic properties from infiltration measurements have been previously reported by Clothier and White (1981), Reynolds et al. (1985), White and Perroux (1987), and Scotter et al. (1988), among others. Most of these field methods require large amounts of irrigation water and often need water content measurements over periods of several weeks (Jones and Wagenet 1984); hence, these methods appear not well suited for routine field measurement of the unsaturated hydraulic conductivity.

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THEORY

For small to intermediate times, the series expansion for the cumulative infiltration rate, I, during one-dimensional vertical flow may be truncated after two terms (Philip 1987):

$$I(t) = St^{1/2} + At \quad (1)$$

where S is the sorptivity, t is time, and A is a constant. Differentiation of (1) with respect to t yields the instantaneous infiltration rate, i:

$$i(t) = ½\, St^{-1/2} + A \quad (2)$$

These equations are only valid for homogeneous soils with a uniform initial water content, θ_o. Application of Eqs. (1) and (2) is restricted to relatively brief infiltration events and shallow depths of the wetting front.

The coefficient A in Eq. (1) is related to the hydraulic conductivity, K_1, corresponding to the soil water pressure head, h_1, at which the water is being supplied (Philip 1987). Application of Eq. (2) to ponded infiltration conditions leads to the restrictions $h_1 = 0$ and $\theta_1 = \theta_s$ and hence $K_1 = K_s$. The subscript s denotes saturation while 1 refers to conditions at the soil surface. According to Philip (1987), the relationship between A and the field-saturated hydraulic conductivity, K_s, is

$$A = FK_s \quad (3)$$

where the value for F depends on the initial water content, θ_o. For infiltration into saturated soils Philip obtained

$$\theta_o \rightarrow \theta_s,\ A \rightarrow K_s,\ F = 1$$

In relatively dry soils, F is expected to vary between 1/3 and 2/3. Talsma (1969) obtained $F = 0.357$ as an average for several soils.

Generally, S depends on the soil hydraulic properties and the pressure heads h_o and h_1. If the infiltration model by Green and Ampt (1911) is used, the analytical expression for the soil water diffusivity contains the Dirac delta function (Philip 1987). This expression for S leads to an acceptable description of the infiltration process for many soils. The relation between S and the matric flux potential, ϕ, for "delta function soils" is given by (Reynolds et al. 1985; Scotter et al. 1988)

$$S = (g\Delta\theta\phi)^{1/2} \quad (4)$$

where a value of 1.9 was selected for the empirical factor g; $\Delta\theta$ is the difference between θ_s, the saturated water content which was estimated from the "satiated" water content determined in the laboratory by immersing soil cores in water, and the initial water content, θ_o; and ϕ is given by

$$\phi = \int_{h_o}^{0} K(h)dh \quad (5)$$

Equation (5) may be evaluated using any particular model for the unsaturated hydraulic conductivity function, $K(h)$. In this study we used the parametric functions of van Genuchten (1980) for the soil water retention and hydraulic conductivity, i.e.:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{(1 + |\alpha h|^n)^m} \quad (6)$$

and

$$K(h) = K_s \frac{[1 - |\alpha h|^{n-1}(1 + |\alpha h|^n)^{-m}]^2}{(1 + |\alpha h|^n)^{m\ell}} \quad (7)$$

respectively, where θ_r is the residual water content, α and n are shape parameters, $m = 1 - 1/n$, and ℓ is a pore connectivity parameter estimated by Mualem (1976) to be 0.5 as an average of many soils.

The matric flux potential was initially evaluated by substituting (7) into (5) and applying Simpson's rule. Because of the steepness of $K(h)$ in the wet range, a large number of integration intervals was needed to obtain an accurate value for ϕ. Rather than using (7), the expression by Gardner (1958)

$$K(h) = K_s \exp(\beta_j h) \quad h \in [h_{j-1/2}, h_{j+1/2}] \qquad (j = 1,...,N) \quad (8)$$

was used. For the purpose of integration the h-axis is subdivided into N intervals, each having a different β_j. Note that $h_{1/2} = h_o$ and $h_{N+1/2} = 0$. The exponentially based $K(h)$ function tends to linearize the conductivity function which allows a more optimal spacing. Substituting (8) into (5), and summing over all j's, leads to the following numerical approximation for the matric flux potential

$$\phi = \sum_{j=1}^{N} \frac{K_s}{\beta_j} [\exp(\beta_j h_{j+1/2}) - \exp(\beta_j h_{j-1/2})] \quad (9)$$

Preliminary results indicated ϕ to be almost identical for N equal to 50 and 100; subsequently a value of 100 was used. The conductivity function according to Eq. (8) can be expressed in terms of the parameters of Eq. (7) by using suitable expressions for β_j. Equating the two conductivity expressions at the midpoint, h_m, of each interval yields

$$\beta_j = \ln[K(h_m)/K_s]/h_m \tag{10}$$

where $K(h_m)$ is the hydraulic conductivity according to Eq. (7) evaluated at

$$h_m = \tfrac{1}{2}\,(h_{j-1/2} + h_{j+1/2}) \tag{11}$$

The unknown coefficients α, n, ℓ and K_s may be estimated from the observed infiltration rates by comparing measured data with $i(t)$ values calculated with Eq. (2) in which S is calculated with Eqs. (4), (9), and (10). Initial results using infiltration data only failed to provide reliable and reproducible parameter estimates. A well posed inverse problem could be obtained only when the infiltration data were augmented with independently measured water retention data. Consequently, the fitting process was formulated in terms of a nonlinear least-squares parameter optimization problem involving both retention and infiltration data. Parameter estimation was accomplished by minimizing the following objective function, $O(\mathbf{b})$:

$$O(\mathbf{b}) = \sum_{i=1}^{NWC} (w_i[\theta_i - \hat{\theta}_i(\mathbf{b})])^2 + \sum_{i=NWC+1}^{NOB} (W_1 W_2 w_i[i_i - \hat{i}_i(\mathbf{b})])^2 \tag{12}$$

where $\mathbf{b}$ is the vector of unknown parameters, i.e., $\mathbf{b} = \{\theta_r, \theta_s, \alpha, n, \ell, K_s\}$, θ_i and $\hat{\theta}_i$ are the observed and calculated (Eq. 6) water contents, NWC is the number of observed retention data, i_i and $\hat{i}_i$ are the observed and calculated (Eq. 2) infiltration rates, NOB is the total number of retention and infiltration measurements, w_i is a weighting factor reflecting the reliability of each individual measurement, W_1 weights the infiltration data with respect to the retention data in their entirety, and W_2 is calculated internally to account for differences in the number of observations and measurement units between the two different types of data.

In this study Eq. (12) was minimized by applying a modified version of the hydraulic parameter estimation code RETC (Leij et al. 1992) which uses a nonlinear least-squares optimization procedure based on Marquardt's maximum neighborhood method (Marquardt 1963). Hence, in summary, the hydraulic measurement process consists of three steps: 1) measurement of retention data in the laboratory and/or the field, 2) measurement of infiltration rates in the field, and 3) hydraulic parameter estimation according to Eq. (12) in which the water contents, $\hat{\theta}_i$, are evaluated with Eq. (6) and the infiltration rates, $\hat{i}_i$, with Eq. (2) assuming that S is given by Eq. (4) with ϕ evaluated according to Eq. (9).

MATERIALS AND METHODS

The hydraulic data used in this study were obtained as part of a soil tillage study in North Paraná, Brazil, involving both conventional and minimum tillage (Roth et al. 1988). The soil at the experimental site is a ferric oxisol or Rhodic Latossolo Roxo according to the Brazilian classification, with 80% clay, primarily kaolinite and hematite. Some basic properties of this soil are given in Table 1. The soil is composed of very stable aggregates with an effective size between 0.2 and 0.6 mm.

Infiltration measurements were made on three plots with conventional tillage (experiments 1–3) and two plots with minimum tillage (experiments 4 and 5). Each experiment was conducted on a 5 × 5-m subplot located within a larger 10 × 20-m plot. Conventional tillage consisted of disc plowing to a depth of 20–22 cm, followed by disc harrowing, whereas minimum tillage consisted of chisel plowing with packed rings and a cage roller to a depth of 18–20 cm, followed by two discings. Tensiometers were installed horizontally at 10-cm intervals up to a depth of 80 cm below the soil surface. Water was applied as 3-mm drops with a small rainfall simulator to a 74 × 74 cm-area covered with a

TABLE 1

Selected properties of a ferric oxisol from North Paraná (Brazil)

Depth cm	Texture Clay	Silt %	Sand	Carbon content %	Bulk density g/cm^3	Porosity cm^3/cm^3
0–18	79	17	4	1.54	1.01	0.639
18–30	79	17	4	1.18	1.20	0.571
30–60	81	16	3	0.47	1.04	0.629

mulch layer (Roth et al. 1985). To promote one-dimensional flow, measurements were taken in a 50 × 50-cm inner area that was separated from the outer area by a 25-cm deep barrier. Infiltration rates were estimated by taking the difference between the applied rainfall and the runoff. All applied rainfall intensities were approximately 144 cm/d. Runoff usually started about 4–6 min after the initiation of rainfall. The runoff data were first smoothed because of relatively large scatter in these data. The tensiometers yielded information about the initial pressure head, h_o, prior to the infiltration events. The depth of the wetting front was also determined from tensiometer readings while initial and final soil water contents were obtained gravimetrically. The total duration of the infiltration experiments was about 60 min; for our analysis we used only infiltration data from the first 40 min. We collected undisturbed soil samples from each horizon listed in Table 1 for determining soil water retention data with pressure plate extractors in the laboratory at pressures of −60, −100, −330, −1000, −5000, and −15000 cm. Field data of the water retention were obtained from tensiometer readings after infiltration and immediate sampling at corresponding depths to determine the water content. Both field and laboratory observations for $\theta(h)$ were used in Eq. (12).

Furthermore, the hydraulic properties were measured independently with the instantaneous profile method (Hillel et al. 1972). A 2.5 × 2.5-m plot was ponded with water until the soil was saturated up to a depth of 100 cm below the surface. The water content at field capacity was determined gravimetrically from triplicate samples. The plot was then covered with a plastic sheet, and the water redistribution in the soil profile was monitored for 6 weeks with tensiometers also placed to a depth of 80 cm at 10-cm increments.

RESULTS AND DISCUSSION

We first investigated the applicability of Philip's infiltration equation to the infiltration process at our experimental site. For this purpose, Eq. (2) was linearized as follows

$$i = SX + A \text{ with } X = \tfrac{1}{2}\, t^{-1/2} \qquad (13)$$

The values for A and S were calculated by linear regression. As can be seen from the results in Table 2, all infiltration experiments were quite accurately described by Philip's equation, except for experiment 2. Experiments 1 through 3 involve three replications of the infiltration experiment carried out on the conventional tillage plot, whereas experiments 4 and 5 deal with the second and third replication of the experiment on the minimum tillage plot. To keep the residual standard deviation, S_r, reasonably small for this experiment, some observations between 12 and 16 minutes had to be eliminated from the analysis because of a very unstable infiltration process. This instability may have been caused by the presence of compressed air ahead of the wetting front (Philip 1975), and/or sudden breakthrough of water into macropores. The data from experiment 2 were not further analyzed because of the observed irregular infiltration. Except for experiment 2, this part of our study indicated that Philip's infiltration equation accurately described the field infiltration measurements.

TABLE 2

Coefficients in Philip's infiltration equation as derived from infiltration measurements

Experiment no.	t_{max}[a]	S	A	Goodness of fit[b]	
	min	cm/d$^{1/2}$	cm/d	R	S_r cm/d
1	40	7.62	82.7	0.997	1.10
2	18	5.54	94.7	0.964	1.94
3	26	5.11	90.3	0.998	0.80
4	20	1.47	127	0.994	0.28
5	22	3.58	116	0.992	0.57

[a] Total infiltration time for estimating S and A.
[b] R Coefficient of correlation of Eq. (13). S_r Residual standard deviation between observations and the Philip equation.

Next, the modified RETC computer code with the objective function given by Eq. (12) was used to analyze the combined water retention and infiltration measurements. The estimated hydraulic parameters are listed in Table 3 for $F = 0.357$ as suggested by Talsma (1969) and for $F = 0.5$. The parameter ℓ in the hydraulic conductivity function (Eq. 7) was found to be very insensitive to the optimization procedure; values greater than 10 changed the goodness of fit only slightly. For this reason we kept ℓ constant at its average value of 0.5 as suggested by Mualem (1976). Notice that the residual standard deviation, $S_r(i)$, was always between 1 and 2% of the

TABLE 3

Soil hydraulic parameter values estimated from infiltration and soil water retention data

Exp. no.	Experimental conditions				Model parameters					Goodness of fit[a]	
	Depth[b] cm	F	$\Delta\theta$ cm³/cm³	h_o cm	θ_r cm³/cm³	θ_s cm³/cm³	α 1/cm	n	K_s cm/d	$S_r(\theta) \times 10^3$ cm³/cm³	$S_r(i)$ cm/d
1.1	10–20	0.357	0.17	−108	0.239	0.578	0.164	1.378	219	3.55	1.44
1.2	10–20	0.5	0.17	−108	0.244	0.574	0.133	1.412	157	2.90	1.29
3.1	0–10	0.357	0.18	−125	0.218	0.572	0.356	1.360	294	4.22	1.65
3.2	0–10	0.5	0.18	−125	0.221	0.559	0.261	1.391	170	3.65	1.85
4.1	0–10	0.357	0.11	−76	0.207	0.658	2.988	1.267	334	4.21	3.00
4.2	0–10	0.5	0.11	−76	0.208	0.604	1.666	1.275	240	4.04	2.10
5.1	0–10	0.357	0.125	−111	0.218	0.516	0.360	1.342	293	2.79	2.61
5.2	0–10	0.5	0.125	−111	0.221	0.508	0.282	1.365	211	2.37	2.41

[a] S_r Residual standard deviation (observed, fitted).

[b] Depth range for which laboratory measured $\theta(h)$ were used in the estimation procedure.

final measured infiltration rates shown in Fig. 2. We regarded this deviation of only 1 to 2% as acceptable. Because the optimization method estimated the hydraulic parameters from both retention and infiltration data, the standard deviations were found to be somewhat higher than those obtained with a direct fit to the infiltration data only (Table 2). Figures 1 and 2 show the calculated soil water flow retention curves and infiltration rates, respectively, obtained with the parameter values in Table 3. Note that only laboratory data are shown for $\theta(h)$, although both field and laboratory retention data were used in the objective function. The results indicate an acceptable fit to both the soil water retention and infiltration data.

We emphasize that the residual and saturated water contents in this study are treated as unknown parameters and, as such, may have lost some of their physical significance in the optimization process. The view that θ_r and θ_s are essentially empirical parameters is consistent with recent discussions by Nimmo (1991) and Luckner et al. (1991). Nevertheless, as shown by the results in Table 3, application of the least-squares optimization analysis to the four infiltration experiments yielded realistic hydraulic parameter values.

The problem of uniqueness of the inverse solution for unsaturated flow problems was previously discussed by Kool et al. (1987), among others. These authors showed that the optimi-

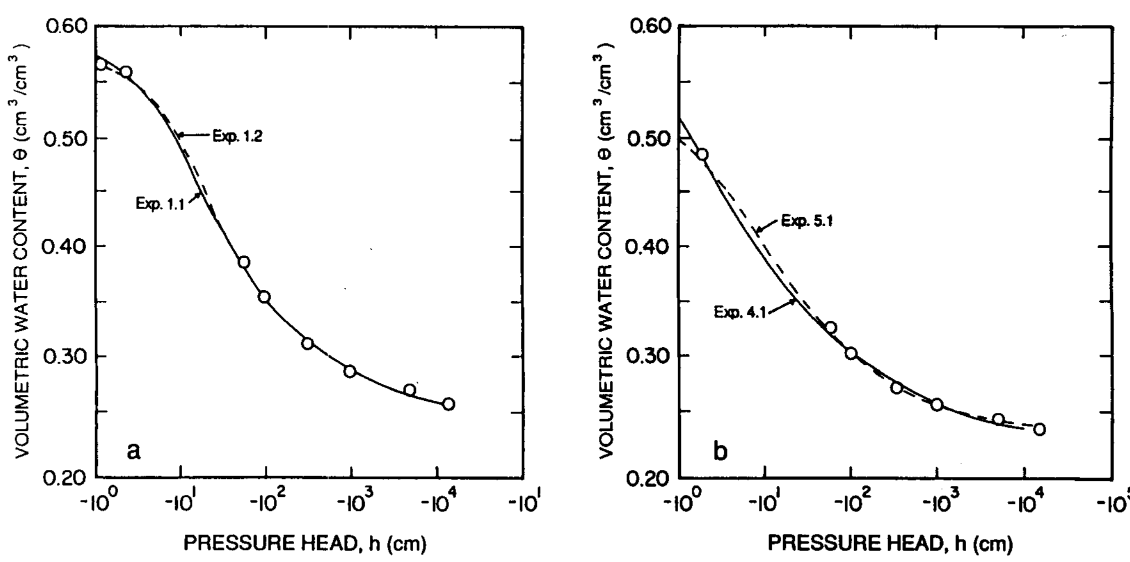

FIG. 1. Observed (open circles) and calculated soil water retention curves for a ferric oxisol: (a) conventional tillage and (b) minimum tillage.

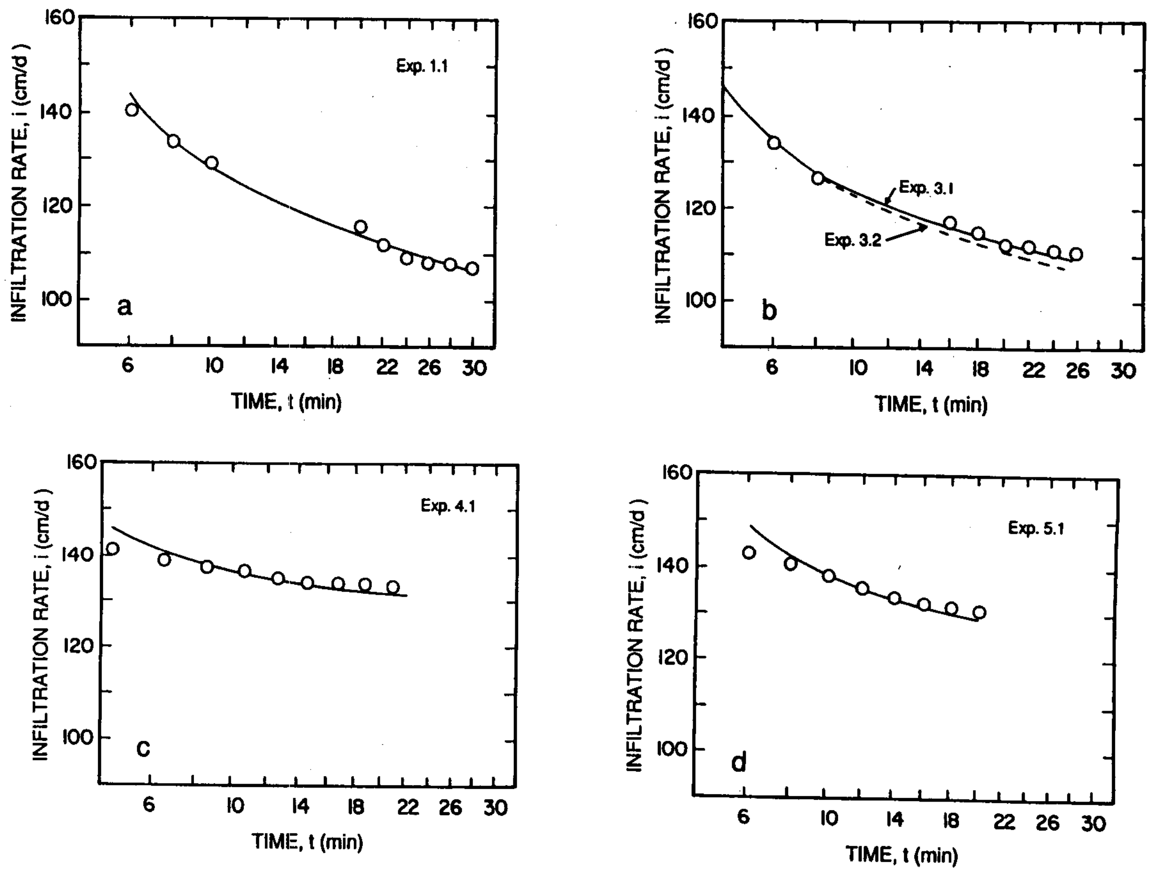

FIG. 2. Measured (open circles) and calculated infiltration rates: (a) conventional tillage (first replication), (b) conventional tillage (second replication), (c) minimum tillage (second replication), and (d) minimum tillage (third replication).

zation results may depend on the assumed initial parameter values. To avoid unrealistic results for θ_s, we used the laboratory-measured "saturated" soil water content at a pressure head of −2 cm as an experimental point. This estimate for θ_s agreed well with observations made in the field during infiltration. Reproducible (unique) estimates for the residual water content, θ_r, could be obtained only if a few measured retention data in the very dry water content range were available. Having one data point at the permanent wilting point (−15,000 cm) will, in most cases (especially for coarse-textured soils), ensure uniqueness in the hydraulic inversion process.

The parameter estimation was carried out with W_1 in Eq. (12) between 0.3 and 0.5, which resulted in approximately equal weights for the retention and infiltration data in the objective function. A logarithmic transformation of the infiltration data in the objective function failed to materially improve the results. The parameter F in Eq. (3) was fixed at a value of 0.357 as suggested by Talsma (1969). Small changes in this parameter led to a nearly proportional shift in the estimated value for the saturated conductivity, K_s (Table 3). The uncertainty involved in F caused deviations in K_s of, at most, a factor of two. We consider such a change in K_s relatively small in view of the often considerable temporal and spatial variability of K_s in naturally heterogeneous field soils. The choice of $F = 0.357$ yielded K_s values that are in good agreement with the independently measured conductivity values. Moreover, the values for A calculated from F and K_s in Table 3 according to Eq. (3), were in fairly close agreement with those obtained by directly fitting Philip's equation to the infiltration data (Table 2).

The parameter values obtained from the retention and infiltration data were used to predict the hydraulic conductivity functions according to Eq. (7). Figure 3 compares the calculated curves with measured data using the instantaneous profile method. We hypothesize that the rather poor fit of the measured vs. calculated curves was caused by a combination of at least three difficulties. First, the instantaneous profile method uses internal drainage data while infiltration is a wetting process. Actually, a better comparison can be made by using conductivity values at the same water content, θ; this is possible if wetting and drying curves for $\theta(h)$ are available. Assuming that there is no hysteresis in $K(\theta)$, it appears that the agreement between calculated and measured conductivity values improves for such a comparison. Second, in spite of the presence of a mulch cover at the soil surface, raindrop induced compaction of the soil surface may have reduced the saturated hydraulic conductivity somewhat during the course of the infiltration experiments. Third, ponding did not occur until 4–6 min after rainfall was started; this situation is not entirely consistent with the application of Philip's equation which assumes ponded infiltration at all times. This last problem could be prevented by using more elaborate infiltration equations (e.g., Kutilek 1980).

Additional improvements in the method may be possible by replacing Eq. (2) with more physically based one- and two-dimensional infiltration equations (e.g., Reynolds et al. 1985; Haverkamp et al. 1990), or perhaps by using numerical solutions for the infiltration process as shown in a recent paper by Russo et al. (1991). One important advantage of numerical solutions is that a variety of nonlinear processes (nonlinear initial and boundary conditions, soil water hysteresis) can be incorporated immediately in the parameter estimation process (Kool and Parker 1988).

SUMMARY AND CONCLUSIONS

The unsaturated soil hydraulic properties are key parameters in any quantitative description of water and solute movement through the vadose zone. Accurate estimates of the hydraulic conductivity are very difficult to obtain, in part because of the problems of spatial variability. A relatively rapid method is proposed for measuring the unsaturated hydraulic conductivity from observed infiltration data. The problem was formulated in terms of a nonlinear least-squares parameter optimization method which combines Philip's two-term infiltration equation with an analytical description of the unsaturated soil hydraulic properties. Reliable hydraulic parameter estimates results could be obtained only when independently measured water retention data were included in the inverse procedure. The method provides rapid and cost-effective esti-

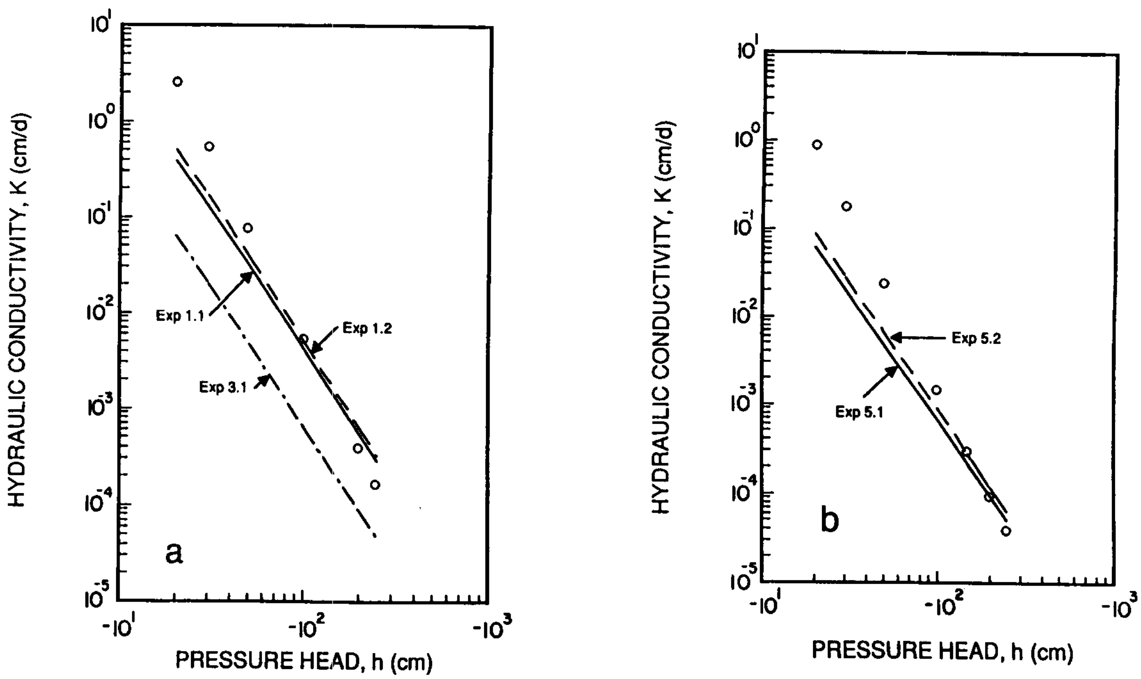

FIG. 3. Measured (open circles) and calculated unsaturated hydraulic conductivity functions: (a) conventional tillage (first replication), and (b) minimum tillage (third replication).

mates of the hydraulic properties of field soils. Further refinements may be possible by coupling the parameter optimization method with more elaborate analytical or numerical solutions for one- and multi-dimensional infiltration.

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