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Compensation Techniques for Servos with Friction

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Abstract

Friction compensation is a broad area that has recently been surveyed in [3]. The survey considers over 250 references from the journals of controls, tribology, lubrication engineering, acoustics, and general engineering and physics and is perhaps 20 times too large to fit in this conference paper and its companion. Rather than treating one or a few topics to the neglect of many others; this paper is composed as an outline of topics and literature references important for the analysis and compensation of servos with friction.

I. Analysis Tools That Have Been Applied To Systems With Friction

1. Describing functions:
   Describing functions seem like a natural tool for treating systems with friction, and many formulations and several theorems have been presented; but when their predictions are compared with simulated or experimental results, the degree of agreement can leave much to be desired.
   a. Memoryless element describing function [21, 24, 25, 110, 151, 156, 28, 29, 31, 58].
   b. Integrated Plant/Model describing function [150, 138, 139, 140, 161].
   c. Extensive testing of the describing function [21, 151].
2. Algebraic analysis:
   Some constructions of the friction and control problem permit exact analysis of the resulting differential equations. Other constructions permit the use of approximate algebraic techniques.
   a. Solution of differential equations [46, 138, 139].
   b. Perturbation Analysis [4, 5].
   c. Lyapunov Stability [143].
   d. Other Techniques [37, 38, 53].
3. Phase plane analysis: [138, 139, 98, 151, 128].
   The phase plane can be used both as a graphical tool for presentation of ideas, and as a means develop or prove theorems.
4. Analysis by simulation:
   Among the most challenging simulation problems are systems with fast and slow dynamics, hard nonlinearities and coupling of the highest order derivative into the differential equation (algebraic loops, perhaps nonlinear). Simulation of systems with friction is just such a case.
   a. General simulation issues, standard friction models [61, 112, 147, 14, 133, 94,].
   b. Simulation with discontinuous friction models [61, 112, 53].
   c. Alternate friction models [147, 14, 133, 71, 94, 89, 165].
   d. Load dependent friction [5, 28, 53, 69, 70, 147].
   e. Issues of existence and uniqueness of solutions to the equations of motion [117, 102, 133, 129, 109, 54, 55].

The majority of the discussions of analysis tools cited above have focused on the question of the existence of stick-slip limit cycle. The answer to the question hinges on the choice of task, whether steady sliding is to be achieved or an end position accurately attained, and controller and friction model. Table 1 presents twelve different possible combinations of these variables and principal references relating to each one. The most challenging case is a controller that adds state (PID or Lead-Lag) applied to the tracking task and a full nonlinear friction model incorporated. Many of the cases have been only partially addressed, and the most challenging case remains to be treated.

	TASK / CONTROLLER			
	Positioning		Tracking	
FRICTION MODEL	PD	PID or Lag(2)	PD	PID or Lag†
Coulomb	NO Kubo, Radcliffe, Wallenborg	NO Radcliffe	NO Kubo	?
Coulomb + Static	NO Tou(1)	YES Tou(1) Radcliffe(1)	YES Derjaguin(3)	YES Shen(1)
Coulomb + Stribeck	NO Radcliffe(1)	YES Radcliffe(1)	YES Dupont(4) Armstrong(3)(4)	YES

LEGEND:

(1) Specific system parameters considered.

(2) Reasonable parameters assumed.

(3) Incorporates Rising Static Friction.

(4) Incorporates Frictional Memory.

Tou: [Tou 53; Tou *et al.* 53]
Derjaguin: Derjaguin *et al.* [56, 57]
Shen: [Shen *et al.* 64]
Radcliffe: [Radcliffe *et al.* 90]
Kubo: [Kubo *et al.* 86]
Wallenborg: [Wallenborg *et al.* 88]
Dupont: [Dupont 91]
Armstrong: [Armstrong-Hélouvry 91, 93]

Table 1 Papers providing analytic prediction of stick slip in machines; the indications NO or YES refer to the possibility of stick slip determined by the cited analyses.

II. Compensation Techniques For Machines With Friction

Compensation tasks: A classification of compensation tasks appears in the table below. Of the four tasks, one is the regulator and the remaining three are versions of the tracking problem. They are listed along with the associated controller error and the dominant friction effect. A specific machine application could involve several of these tasks.

	Compensation Task	Control Error	Dominant Frictional Contributor
I.	Regulator (position)	Steady-state error, Hunting (limit cycle around fixed point)	Stiction.
II.	Tracking with Velocity Reversal	Stand still, lost motion	Stiction.
III.	Tracking at Low Velocities	Stick Slip	Negatively-sloped Stribeck curve; Stiction.
IV.	Tracking at High Velocities	Large Tracking Errors	Viscous Behavior of Lubricant.

Table 2 Compensation Tasks.

Compensation techniques:

1. Problem avoidance: design for control [124, 136, 141].
   A common method in practice of dealing with friction induced problems is to modify the machine or its lubricant.
   a. Lubricant selection [12, 95].
   b. Bearings [15, 36, 121, 83, 115].
   c. Stiffness and Actuation [7, 151, 21, 48].
   d. Inertia [59, 144, 66].
2. Non-model-based compensation for friction:
   Non-model-based control methods are those that do not require any explicit knowledge of friction for design or implementation. They include standard control designs where parameters may be adjusted to compensate for friction. There has been relatively little attention given to the study of nonlinearities in integral control, such as deadbands, resetting functions or sign reversals, even though these are often used in practice to compensate for the nonlinearities of friction.
   a. Stiff PD control:
      - Single valued friction models [131, 73, 46, 37, 26, 10, 6, 65].
      - Models with frictional memory [4, 57, 56].
   b. Integral Control [139, 145, 21].
   c. Dither [18, 2, 105, 9, 113, 166, 167, 32, 13, 100].
      There is more to dither than first meets the eye.
      - Tangential and normal dither [63, 68, 80, 81, 108].
      - Depth of discontinuity [32, 84, 100].
   d. Impulsive control [164, 82, 145, 5, 47].
      Impulsive control refers to the application of brief, high—amplitude torque commands to overcome static friction. Nanometer motions of sliding contacts have been achieved.
   e. Joint torque control [162, 103, 120, 155, 93, 75].
      Joint torque control refers to direct sensing of the torque or force delivered at the end of the drive train and closing a stiff control loop around the friction elements. Reductions of 30:1 in apparent friction are reported.
      - Sensing [120, 162, 103, 76].
      - Controller design [120, 155, 76].
      - Performance [103, 120, 76, 93].
3. Model-based compensation for friction
   When a model of friction is available, it is possible to compensate for friction by applying a force/torque command equal and opposite to the instantaneous friction force. This presumes that force or torque actuation of adequate bandwidth is available and is stiffly coupled to the friction element. In many cases the dominant friction sources are the motor and transmission, and so adequate stiffness is assumed.
   a. Fixed compensation [67, 157, 78, 1, 39, 28, 30, 31, 149, 130, 23, 22, 21, 5, 89, 145, 52].
      Fixed friction compensations designs vary according to the friction model used and the choice of velocity signal. The most common friction model is simply Coulomb friction, which is fedforward (e.g., [67, 23]). This is found in both research and industrial applications. Because the Coulomb friction is discontinuous at zero velocity, the choice of velocity signal has important implications for stability, possibilities considered in the literature are: sensed velocity, estimated velocity and desired velocity.
   b. Friction identification and adaptive control:
      A great variety of friction identification schemes have been demonstrated. They vary according to the complexity of the friction model being identified, whether the parameters of the model are identified individually or in unison and the nature of the identification algorithm. [28] has addressed some issues of mis-identification when the friction model is incomplete.
      i. Off-line identification [122, 123, 20, 89, 151].
      ii. Full model identification [34, 5, 79, 89].
      iii. Adaptive control:
         - Recursive least squares and least mean squares algorithms [67, 157, , 39, 28, 30, 31].
         - Model reference adaptive control [67, 8, 22, 137, 77, 106, 107].
         - Lyapunov methods [62].
         - Learning control [40, 99].
4. Input from engineers in industry who have controlled machines with friction:
   Engineers with companies and government laboratories in Europe, Japan and the United States have been surveyed to learn what techniques are being applied to control machines with friction. A great breadth of techniques were found to be in service:
   - System hardware Modification
   - High servo gains (stiff position and velocity control)
   - Modifications to integral control
   - Linear adaptive control

- Model-based friction compensation
  - fixed
  - adaptive
- Dither
- Table lookup compensation
- Learning control
- Joint torque control
- Variable structure control

III. Conclusions

A summary of control tasks and the associated compensation techniques is seen in figure 1.

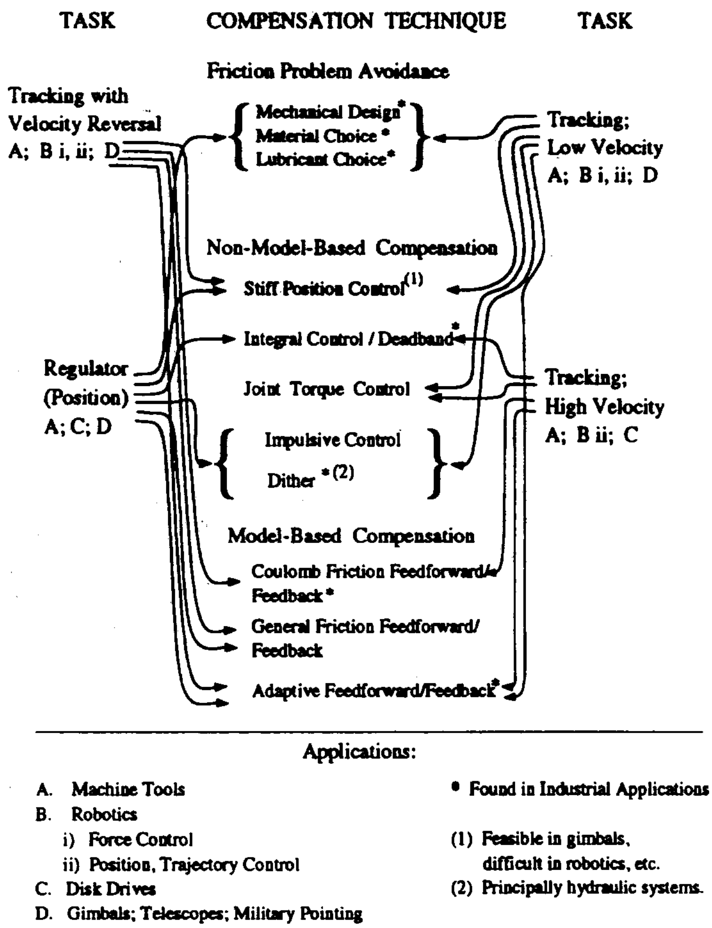

Figure 1 Tasks and their associated compensation techniques as reported in the literature. Typical task applications appear in the legend.

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