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Comparisons of surface wave amplitude decays as measured by rotation and translation sensors

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SUMMARY
Current broad-band surface wave magnitude equations relate magnitude with station-receiver distance and the vertical component of peak ground velocity, such that only contributions from the vertical component of Rayleigh waves are present. With the advent of rotational ground motion data from instruments such as ring laser gyroscopes and fiber-optic gyroscopes, it is possible to determine peak amplitudes of rotations about the vertical axis, which is theoretically only sensitive to the transverse nature of Love waves, unaffected by the horizontal component of Rayleigh waves. We use this concept to study the amplitude decay of rotations versus translations, and determine the necessity of a separate surface wave magnitude equation for Love waves. Utilizing a large database of rotation ground motion events, collected in Wettzell, Germany, we empirically define decay constants for measured observables: rotation rate, rotation, vertical velocity and transverse velocity. Results indicate that measured rotation amplitudes decay faster over distance compared to velocity amplitudes, both on vertical and transverse components. Observations are corroborated with synthetic seismograms produced on a full scale 3D global model with crustal and Moho topography models. Synthetics were created with the spectral-element method Salvus, and suggest that ...

Key words: magnitude, rotational ground motions, seismic instrumentation, amplitude decay

1 INTRODUCTION

For over a decade, the application of ring laser gyroscope technology to the field of seismology has allowed for near-continuous, direct, measurements of rotational ground motions. An ever growing number of observations from seismic events of varying size, distance and source mechanism, has been collected in an expansive catalog of waveform recordings for both direct rotation, and collocated translation measurements. Previous work on this unique waveform dataset includes phase comparisons of translations and rotations [Igel 2005] and estimations of local horizontal phase velocities. Much of this preceding work however, focuses on single events, or collections of non-earthquake sources.

In this paper, we aim to characterize and understand the differences in amplitude decay behavior of rotational and translational ground motion for a large number of events. To do this, we make use of a sizable catalog of seismic event data, measured by an observatory based ring laser gyroscope and collocated broadband translation sensor. By processing rotation and translation observations in an identical manner, we are able to directly compare processed results over a large set of event magnitudes, epicentral distances and backazimuths. We additionally seek to use this information to better understand decay characteristics of Love and Rayleigh waves.

This paper builds on work done by [Igel 2007], where the question was posed, whether observed peak rotation amplitudes matched with expected values given by the surface wave magnitude equation. At the time, a lack of events lead to a limited study size, however a much larger number of events is now currently available. In this work, we attempt to readdress this question, while also approaching the problem from the unique perspective of deriving magnitude scales for peak amplitudes over distance, in an attempt to similarly quantify the decay characteristics of rotations and translations.

Due to the uniqueness of our instrumental setup, we were restricted to observations at a single station. As a result, Global 3D synthetic simulations were run with the wave propagation code Specfem3D Globe, such that we could compare with synthetics. Numerically generated seismograms for source-receiver pairs similar or identical to the real world observations were created. Outputs of synthetic rotations and translations were fitted with magnitude scales, and their decay characteristics were compared against observations. Real seismic event locations and source parameters were used, as well as real station locations, in order to provide the most comparable synthetic setup to our observations.

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1.1 Rotational Ground Motions

Rotational ground motions induced by seismic events can currently be measured in two ways: by array methods and by direct measurement. In the former, spatial gradients are taken, for measurements in an adequately spaced array of translation sensors, in order to derive components of the strain tensor [Spudich ?]. In the latter, unique instrument designs allow for direct gradient measurements [Schreiber? Others?].

In this study, data was recorded by the Großring (G-ring) [Schreiber 2005, 2006], a 4x4m helium-neon ring laser gyroscope, located in Wettzell, Germany (49.144°N, 12.87°E). The G-ring operates on the principle of Sagnac interferometry [Stedman, 1997], which relates interference of counter propagating light beams to absolute rotation rate, through the following equation,

$$\delta f = \frac{4A}{\lambda P}\mathbf{n} \cdot \mathbf{\Omega}, \tag{1}$$

where the constants are given by instrument area A, perimeter P and operating light wavelength λ. Equation 1 relates an observable beat frequency δf to rotation rate Ω.

It is important to note that given stable instrument geometry and lasing, changes to the beat frequency δf, can only be introduced by changes to the inner product of the plane normal $\mathbf{n}$ with $\mathbf{\Omega}$ (i.e. through instrumental tilt), and through externally induced rotations (i.e. the passing of seismic waves). It has been shown that changes to the inner product as produced by tilt are one to two orders of magnitude smaller than rotations produced by passing seismic waves [Igel? Schreiber?]. This gives G the very unique benefit that it is theoretically insensitive to translations, and only sensitive to externally induced rotations.

1.1.1 Phase velocity relation

As shown in [Igel 2005], for a simple plane wave assumption, the amplitudes of vertical rotation rate and transverse acceleration can be related through the equation $\frac{\ddot{u}_t}{\Omega_z} = -2c$, where c represents an apparent horizontal phase velocity. This relationship shows that for a sufficient distance from an event to assume a plane wave, rotations are similarly sensitive to the transverse component of translation, represented by surface horizontal waves (either SH or Love waves). It also states that waveforms of transverse acceleration and vertical rotation rate will theoretically be in phase for passing horizontal waves.

1.1.2 Peak correlation coefficient

Correlations are a useful measure of similarity between waveforms. It has been shown previously that for collocated measurements of rotation rate and transverse acceleration, high values for zero lag correlations can be retrieved in time windows centered around surface wave arrivals. Peak values of zero lag correlation coefficients are routinely processed for new events measured by the G ring [Salvermoser 2017], and are used in this study as a filtering tool to separate out events with low signal to noise ratio or dissimilar waveforms which may arise due to non-physical effects (i.e. instrumental effects).

1.2 Magnitude scales

Amplitude based magnitude scales provide empirically derived relationships between maximum trace amplitudes and source-receiver distances. Magnitude scales offer useful and quick estimates of relative sizes of earthquakes in a standard and easily-understandable manner. The International Association of Planets Seismology and Earths Interior (IASPEI) Working Group on Magnitudes has proposed a modified version of the original surface wave magnitude equation proposed by Karnik et al. and Vanek et al., which is compatible for use with modern day broadband seismic instruments.

In this work, we adhere strictly to the standard procedures provided by IASPEI as an outline for defining our own empirical magnitude scales. We use these derived scales as a tool for quantifying amplitude decay for different measured observables, in a standard fashion.

1.2.1 Standard Procedures

The Working Group on Magnitudes' standard procedures states that the revised surface wave magnitude equation for broad-band instruments is given as,

$$M_S^{BB} = log_{10}(V/2\pi) + B \cdot log_{10}(\Delta) + C, \tag{2}$$

where the constants $B = 1.66$ and $C = 0.3$ control amplitude decay and order of magnitude, respectively. The parameter V should be the maximum trace amplitude (in nanometers per second) in the surface wave train, for a seismogram proportional to velocity, measured on the vertical component.

Further criteria given by IASPEI posit that the period of the surface wave should lie within 3 s $<$ T $<$ 60 s, while epicentral distances should be between $2^\circ < \Delta < 160^\circ$. It is further recommended that only shallow focus earthquakes should be considered, as medium to deep events are less capable of generating strong surface waves [Borrmann and Bergman 2000]. Maximum trace amplitudes are described as one half the largest peak to adjacent trough deflection, and associated period are given as two times the temporal difference between peak and adjacent trough. All events and processing steps in this paper adhere to these guidelines.

1.2.2 Instrumental proxies for Love and Rayleigh waves

Standard procedures for determining surface wave magnitude pose that amplitudes should be measured on vertical component translation instruments. The reason given is that vertical translation should only be sensitive to the vertical motion caused by Rayleigh waves, whereas vector sums of horizontal components can have influence from both Love and Rayleigh waves. In the same vein, velocity measured on a transverse component should be insensitive to the radial component of Rayleigh waves, showing only sensitivity to Love waves. This is however less commonly done in practice, potentially due to the necessity of prior knowledge of the event backazimuth, and rotation of the horizontal components to the correct backazimuth.

The G-ring, which is 1) insensitive to translations and 2) proportional in phase and amplitude to transverse acceleration, should also only be sensitive to Love waves in the surface wave train, irrespective of backazimuth. In this study we use the respective instrument or component as a physical filter, in order to separate phases in the surface wave train. This allows us to study the influences of Love waves and Rayleigh waves individually. In this paper we take advantage of this idea by comparing the vertical and transverse components of a three component translation sensor, to the vertical component of rotation in order to understand, by proxy, the wave types they are sensitive to.

1.2.3 Application of rotations to magnitude scales

The surface wave magnitude equation is only defined for peak vertical velocity amplitudes in the surface wave train. In order to give a fair comparison using derived magnitude scales, a complementary rotation parameter to vertical velocity is necessary. In Section 1.1.1, an equation is given that relates rotation rates Ω with accelerations $\ddot{u}$. It would make the most sense then, to compare and velocities with rotations ω, if one integrates both sides. Without precedence and for completeness, however, we present here both rotations and rotation rates.

2 EVENT CHOICE

The G-ring has been continuously recording at its current resolution since May, 2009. The time range for events used in this study spans from June, 2009 to September, 2016. A general catalog was fetched from the Harvard Global Centroid Moment Tensor [Ekström et al. 2012 ?], and events were sorted by acceptable distance and depth criteria. Only events with moment magnitude values $6 \leq M_w < 8$ were considered, as surface wave magnitude and moment magnitude are approximately equal in this range [Shearer 2009]; we impose the restriction that our derived magnitudes should be as close to the given surface wave magnitudes as possible, this insures that our derived scales do not stray far from established values.

Events were filtered by peak correlation coefficient $0.7 \leq$ PCC ≤ 1.0, to ensure well behaved waveform. These bounds narrowed the catalog down to less than 500 events in the given time period. Each event was appropriately filtered and processed (Section 3.1), and waveforms were individually inspected. Waveforms that exhibited anomalous behavior (i.e. unexpected high amplitude peaks outside the surface wave train, high signal-to-noise ratio etc.) were rejected. A final event catalog of 243 events was reached.

3 METHODS

3.1 Data Processing

Events were processed in a similar fashion as [Salvermoser 2017]. Instrument response was corrected for on translation instruments to give broadband seismograms proportional to velocity (nm/s). Epicentral distances and theoretical backazimuth values were derived from station-receiver latitude and longitude pairs. Horizontal components originally in a North, East system were rotated into the transverse, radial coordinate system by the appropriate backazimuth. Measurements from ring laser gyroscope instruments do not require frequency dependent instrument correction [Sagnac?], therefore only a simple scale factor was necessary to retrieve seismograms proportional to rotation rate (nrad/s). Rotation rate traces were integrated to provide measurements of rotation (nrad).

A bandpass filter was applied to all traces for periods between 3 and 60 seconds to isolate dominant periods of surface waves. Peak amplitudes were chosen by finding the minimum and maximum values in the trace and the largest associated peak or trough. The larger value of the two was then taken, along with the associated arrival time and dominant period. Theoretical considerations used to restrict search to the surface wave train proved inconsistent over a large number of events, so maximum amplitudes in the entire trace were considered. Through manual inspection, picked amplitudes that fell outside the surface wave train were rejected, in order to stick closely to standard procedures.

3.2 Curve fitting

3.2.1 Linear regression

To quantify amplitude decay, magnitude scale coefficients were fit to the data using a simple linear regression. Equation 2 represents a relationship between magnitude and amplitude for a single event. A collection of n events can be represented in the form,

$$\begin{pmatrix} log_{10}(\Delta_1) & 1 \\ log_{10}(\Delta_2) & 1 \\ \vdots & \vdots \\ log_{10}(\Delta_n) & 1 \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} M_{w_1} - log_{10}(V_1/2\pi)_{max} \\ M_{w_2} - log_{10}(V_2/2\pi)_{max} \\ \vdots \\ M_{w_n} - log_{10}(V_n/2\pi)_{max} \end{pmatrix}, \tag{3}$$

which can be condensed as the matrix vector product, $\mathbf{Gm} = \mathbf{d}$. The unknowns B and C are represented in the vector $\mathbf{m}$, and can be solved for through the normal equation $\mathbf{m} = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{d}$. Solving for B and C, we create an empirical magnitude scale that best describes the behavior of our events. In Equation 3, we impose that our derived magnitude value should be as close to an events given moment magnitude as possible, by setting M_S^{BB} equal to the value of M_w.

3.2.2 Confidence intervals

A 95% confidence interval was constructed for each parameter of the vector $\mathbf{m}$, which gives a measure of error by providing bounds where 95% of repeated measurements would lie. Confidence intervals were constructed by the variance of estimates of the jth parameter of m, by $\hat{m_j} \pm c\sqrt{\hat{var}(\hat{m_j})}$, where the hat denotes an estimate and the value of c is given as 1.96 for a 95% confidence interval.

4 SYNTHETIC SEISMOGRAMS

Because the G-ring is a unique instrumental setup, it is not currently possible to draw on other catalogs of rotational ground motion. One possibility to achieve more observations is through array derived rotations as a substitute for direct rotation measurements. A lack of long-term arrays with the optimal station spacing however, limits this option. In lieu of this, we turn to waveform modelling to generate synthetic seismograms, in order to recreate our experimental setup and provide a comparable set of observations.

The wave propagation code Specfem3D Globe was employed for this purpose. A realistic global model featuring 3D crust and mantle models was used. Real world effects that might influence the the travel and amplitudes of surface waves at periods close to 20 seconds were also included: ocean loading, ellipticity, topography, self gravitation, Earth's rotation and attenuation. Event locations and moment tensors were taken from a handful of real seismic events present in the observation catalogs, so as to provide comparable source receiver pairings. A corner frequency was set to 10 s, which limited the number of events we were capable of propagating. To generate more observations, many stations, both real and synthetic, were included. Real stations were set to the locations of the Global Seismic Network (GSN) with inclusion of the G-ring location of Wettzell, and the observatory station Fürstenfeldbruck. Synthetic stations were generated on an event by event basis, but based on real station location, either temporary or permanent. Most synthetic stations were placed close to the source, for better coverage at smaller epicentral distances, something that was unfortunately lacking in the observational data.

The direct outputs of Specfem3D were adjusted from the standard to give displacement in the transverse, radial and vertical components, as well as a direct output of rotation in the same coordinate system. These traces were differentiated to velocity and rotation rate, respectively. A workflow identical to that used for observations was employed to calculate peak trace amplitudes. A magnitude equation was then fit to the data.

5 RESULTS

5.1 Derived magnitude scales

For rotations and translations, decay characteristics were derived by solving for constants B and C in the magnitude equation. These values are presented in a Table 1. Since the constant C is dependent on the order of magnitude of the units, it is difficult to compare this between the different observation types. The value of B, however, controls the decay of amplitude with distance, and allows us to fairly compare rotations and translations. To check the results, the same processing was performed on observations taken at the geophysical observatory in Fürstenfeldbruck, Germany (48.163°N, 11.275°E), which is located roughly 200 km to the south-west of Wettzell. Though a smaller subset of events was used due to data availability, the results confirmed those given by Wettzell by returning derived constants B and C within 1%.

Consulting Table 1, rotation has the closest decay constant B to the surface wave magnitude equation. Rotation rate exhbiits even higher

5.2 Expected amplitudes

5.3 Synthetic results

6 DISCUSSIONS AND CONCLUSIONS

Long term continuous recordings of collocated rotation and translation has now provided a superb catalog of seismic events useable in comparisons of direct translation measurements and direct measurements of spatial gradients. Phase information was the logical starting point, but now the measurement of amplitudes proves useful in understanding both subsurface velocity structures as well as amplitude decays of surface waves.

The results of this work show the consistency of rotation measurements for a large observatory based ring laser gyroscope, and allow very accurate and stable comparisons of rotations and translations. Since events were manually chosen to exhibit the best characteristics for their waveforms, we can be sure that the derived decay characteristics that are shown here also are stable, albeit for teleseismic waves. The results at face value show that observed rotations for the event catalog used decay faster than observed translations for the same events, and that rotation rate decays faster than the rotation, though this is to be excepted as we are simply looking at time derivative information. What is most interesting is how the rotation and rotation rate decays match much closer to the globally averaged surface wave magnitude as compared to the translation measurements which it is based off. Understandably a global average should be taken to give a more reasonable value for the decay. It should be pointed out, however, that even without global averaging, it is exciting to see the characteristics of this new observable, and by taking comparisons of direct measurements, we are gaining more understanding of the behaviors of this observable.

APPENDIX A: FOR AUTHORS

Table **??** is a list of design macros which are unique to GJI. The list displays each macro's name and description.

APPENDIX B: FOR EDITORS

The additional features shown in Table **??** may be used for production purposes.

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Scale	**Label**	**B**	**C**	**Wave**
IASPEI	M_S^{BB}	1.66	0.3	Rayleigh
Rotation	M_{RT}	1.557 $\pm$ 0.295	4.186 $\pm$ 0.569	Love
Rotation Rate	M_{RR}	1.823 $\pm$ 0.303	4.113 $\pm$ 0.586	Love
Transverse Velocity	M_T	1.45 $\pm$ 0.27	0.527 $\pm$ 0.521	Love
Vertical Velocity	M_Z	1.084 $\pm$ 0.264	1.093 $\pm$ 0.511	Rayleigh

Table 1. Magnitude scales and derived constants with 95% confidence intervals for observations at station WET, for equations of the form $M = log_{10}(V/2\pi) + B \cdot log_{10}(\Delta) + C$. The final column gives consideration to the wave type that each instrument component should provide a proxy for.