Machine Learning - Assignment 1 (Week ending 31st August)

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1. Total Sample Spaces (possible outcomes) = 36
   Total possible outcomes of dice landing on same numbers = 6
   Hence, Probability of Rolling Doubles,
   P(X) = 6/36 = 1/6

2. Given, P[X,Y] = 0.2, P[X] = 0.5
   Independent Random variable, by formula,
   P[X,Y] = P[X] * P[Y]
   P[Y] = 0.5/0.2 = 2.5

3. Let P(F) be the probability of step taken forward and P(B) be the probability of step taken backward. Now, we are given
   P(F) = 0.6 and P(B) = 0.4
   After 10 steps, the drunkard will be at the starting point since he will take 5 steps forward and 5 steps backward in no particular order. Therefore, probability of him being at the starting point,
   P(X) = $\mathbf{0.6^5 * 0.4^5}$ => 0.000796

4. Given that, X, Y, Z are three random variables. X & Y are independent of each other.
   Also, E[X] = 2, var[X] = 1, E[Y] = 3, $Z = X^2 * Y$
   Now, Variance of a random variable is given by,
   $\mathrm{var[X]} = \mathrm{E}[(\mathrm{X} - \mathrm{E(X)})^2]$
   Solving the RHS and Substituting the values in the formula,
   $1 = E[X^2] - (E[X])^2$
   $1 = E[X^2] - 2^2$
   $E[X^2] = 5$
   Since, $Z = X^2 * Y,$
   $E[Z] = E[X^2] * E[Y]$
   E[Z] = 5 * 3 = 15

5. Given numbers : 1, 6, -1, 4, 10
   Mean, X' = [1 + 6 + (-1) + 4 + 10] / 5 => 4
   Sorting the numbers in ascending order,
   Median of -1, 1, 4, 6, 10 => 4
   Now, according to the formula,
   Variance = $\sum_{1=1}^{n} (X_i - X')/n$
   $= \frac{(1-4)^2 + (6-4)^2 + (-1-4)^2 + (4-4)^2 + (10-4)^2}{5}$
   $= \frac{9 + 4 + 25 + 0 + 36}{5}$ => **14.8**

6. Let P(W) be the probability of winning = 20% = 1/5
   Let P(L) be the probability of losing = 80% = 4/5
   Amount each time the gambler wins = $ 10
   Amount each time the gambler loses = $ 5
   Hence, his expected gain can be calculated using the formula,
   E[Gain] = [P(W) * 10] + [P(L) * (-5)]
   = (1/5 * 10) + (4/5 * -5)
   = 2 – 4 = -2
   If the gambler plays ‘n’ games, the total expected gain will be = **-2n**

7. Total no. of cards in a deck = 52
   Total no. of spades = 13
   Let event X = (1 spade drawn) & P(Y) be the probability of second card drawn be spade
   No. of cards left = 51
   No. of spades left = 12
   Hence, **P(Y) = 12 / 51**

8. Urn 1 contains balls – 2 White and 7 Black
   Urn 2 contains balls – 5 White and 6 Black
   Let P(H) be the probability of Heads as outcome of toss
   Let P(W1) & P(W2) be the probabilities of White ball drawn from Urns 1 & 2 resp.
   Now, P(H) = ½
   P(W1 | H) = 2/(2+7) = 2/9
   P(W2 | H) = 5/(5+6) = 2/9
   P(W) = P(H) * P(W1 | H) + P(H) * P(W2 | H)
   = (1/2 * 2/9) + (1/2 * 5/11) = 67 / 198
   Now, using Bayes Rule, probability of Heads given that a White ball was selected,
   P(H | W) = [P(W | H) * (P(H)] / P(W)
   = (2/9 * 1/2) / (67/198) = **22 / 67**

9. Probability of getting a Head in each toss, *p* =1/2
   Probability of Heads more than 6 off 10 can be calculated using Binomial Distribution
   $P(X) = \binom{n}{k} p^k (1-p)^{n-k}$
   $$\boldsymbol{P(X > 6) = \binom{10}{7}(1/2)^7(1/2)^3 + \binom{10}{8}(1/2)^8(1/2)^2 + \binom{10}{9}(1/2)^9(1/2)^1 + \binom{10}{7}(1/2)^{10}(1/2)^0}$$
   Solving, P(X > 6) = **0.17 = 17%**

10. Probability of winning a bet = *p*
    Probability of winning for the first time after *n* bets can be calculated by **Geometric Distribution**.
    $$\boldsymbol{P(X) = (1-p)^{n-1} * p}$$