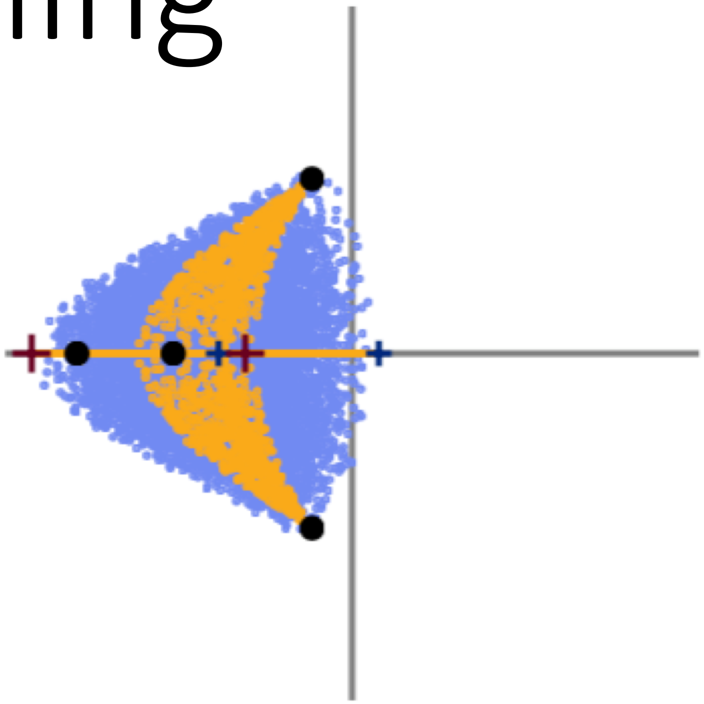
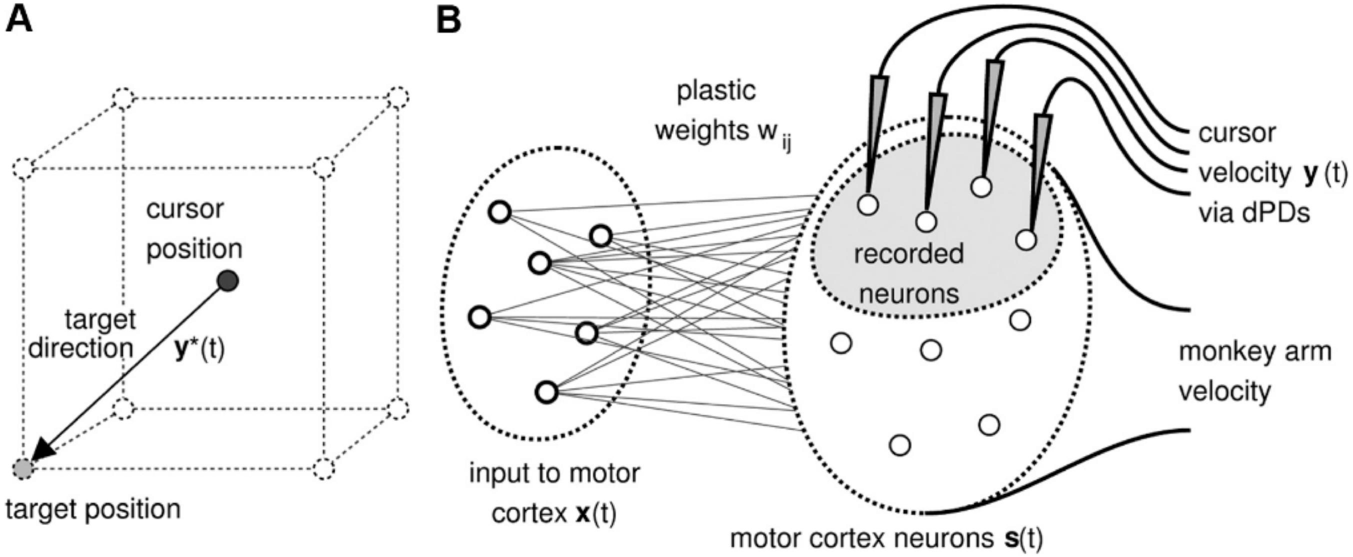
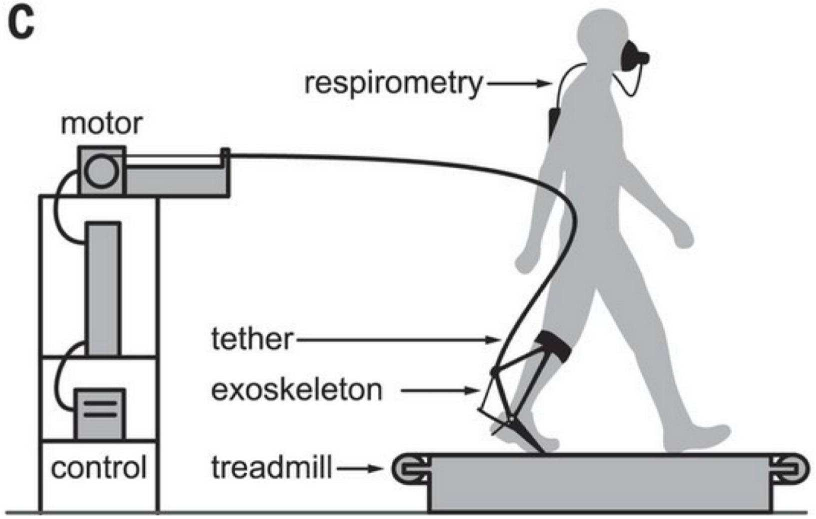
Dynamics of co-adaptation and multi-agent learning

Benjamin Chasnov
Tanner Fiez
Dan Calderone
Samuel A. Burden
Lillian J. Ratliff



Co-adaptation and learning

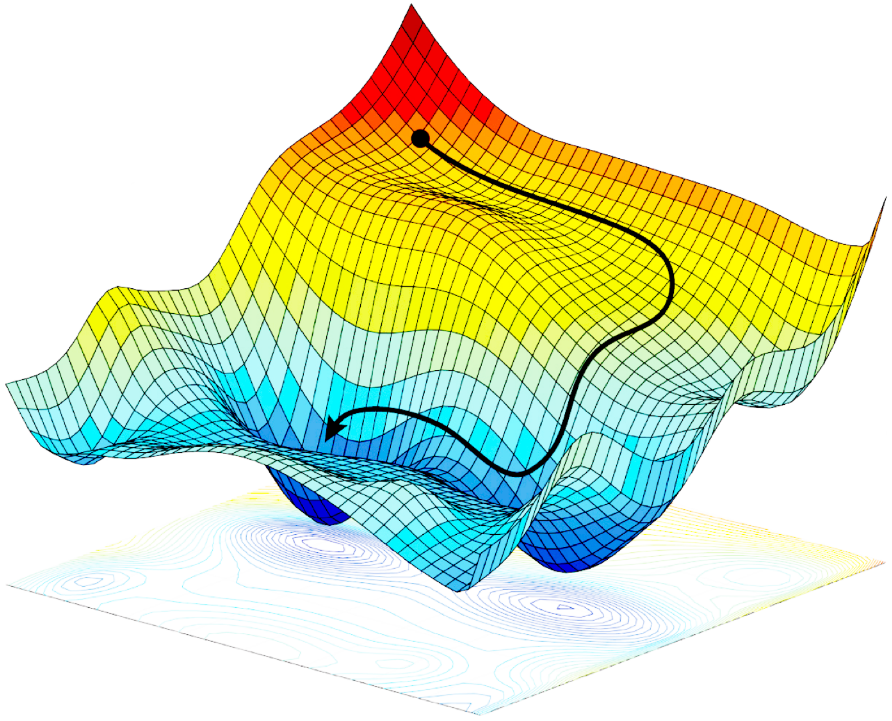




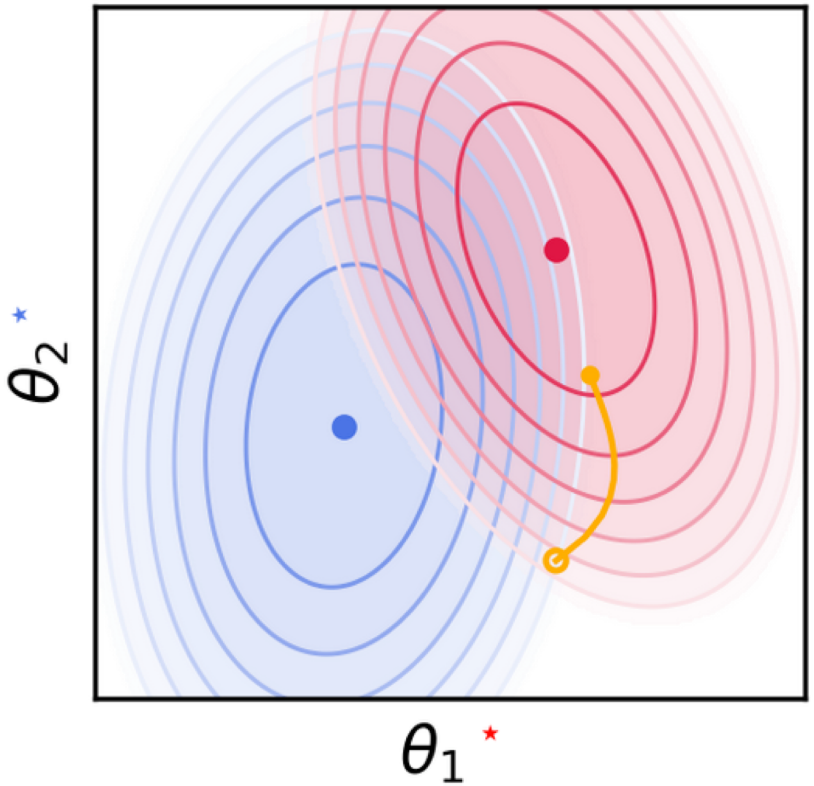
Legenstein, Chase, Schwartz, Maass. A reward-modulated hebbian learning rule can explain experimentally observed network reorganization in a brain control task. Journal of Neuroscience. 2010

Zhang, Fiers, Witte, Jackson, Poggensee, Atkeson, Collins. Human-in-the-loop optimization of exoskeleton assistance during walking. Science. 2017

Optimization landscape

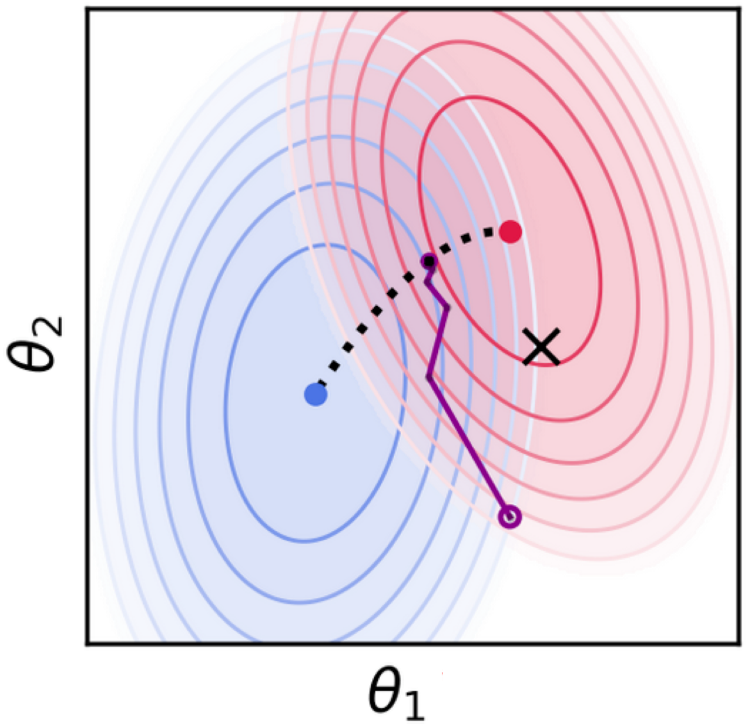


$$\min_{\theta} \; f(\theta)$$

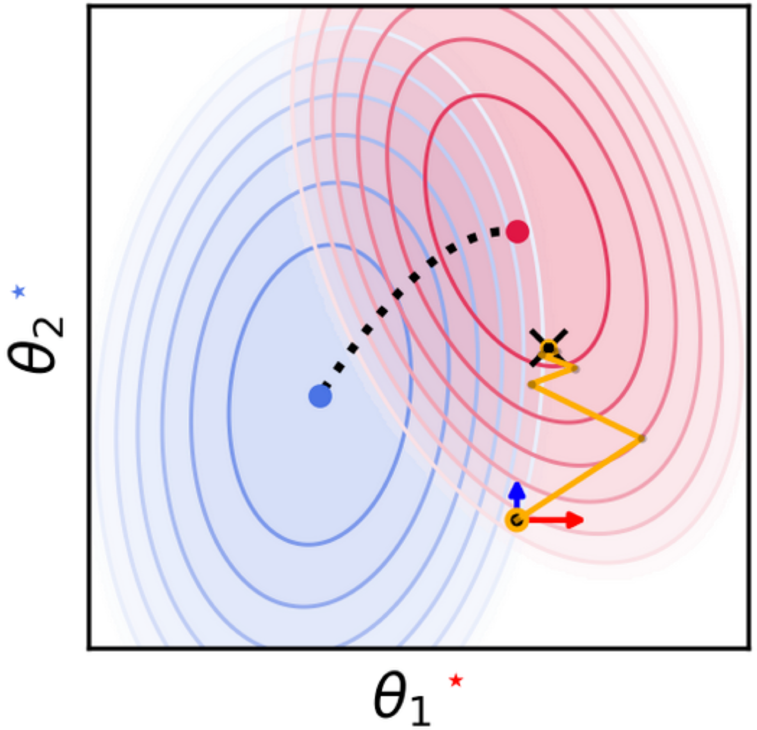


$$\min_{\theta_1} \; f_1(\theta_1, \theta_2), \quad \min_{\theta_2} \; f_2(\theta_1, \theta_2)$$

Decision subspace



$$\min_{\theta_1,\theta_2} f(\theta_1,\theta_2)$$



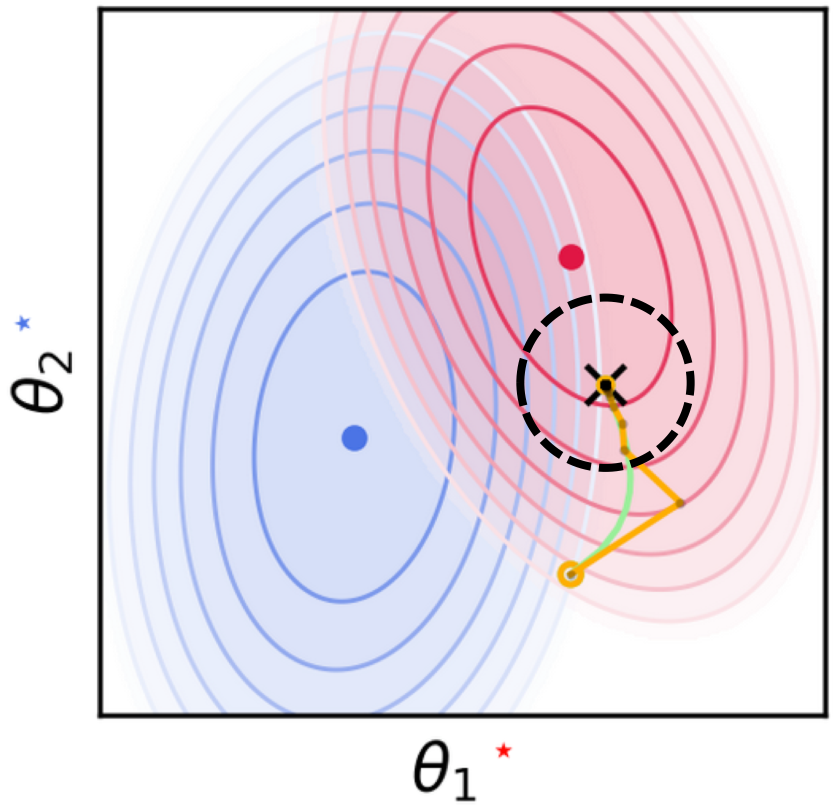
$$\min_{\theta_1} f_1(\theta_1,\theta_2), \quad \min_{\theta_2} f_2(\theta_1,\theta_2)$$

Simultaneous gradient descent

$$\theta_1^+ = \theta_1 - \gamma_1 \frac{\partial}{\partial \theta_1} f_1(\theta_1, \theta_2)$$

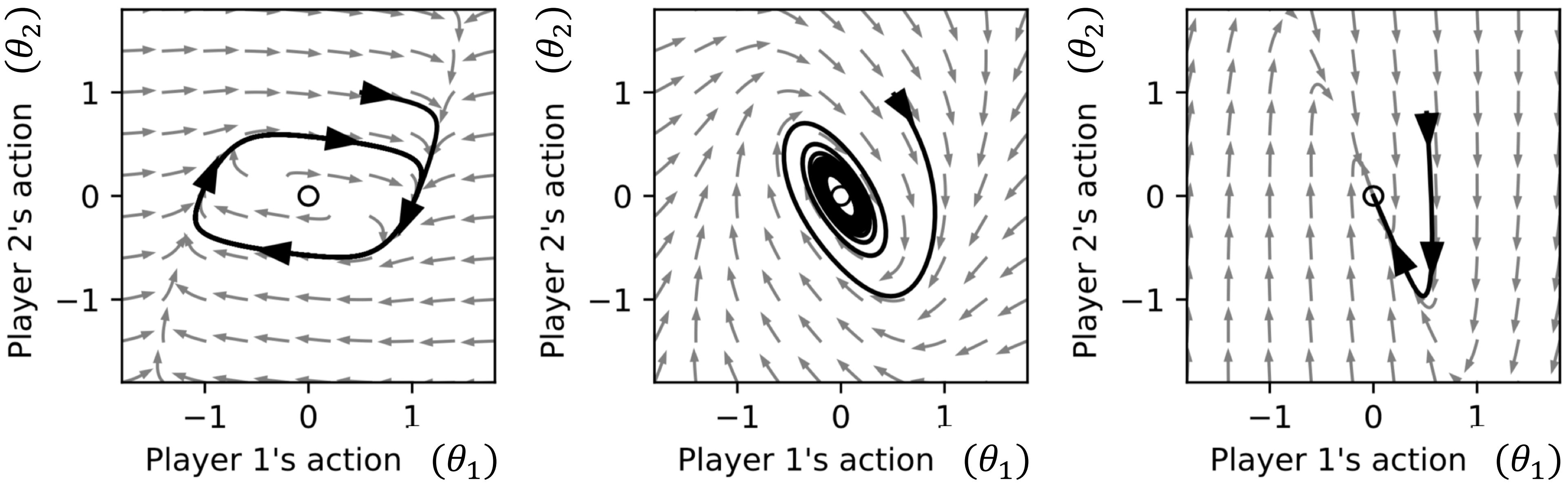
$$\theta_2^+ = \theta_2 - \gamma_2 \frac{\partial}{\partial \theta_2} f_2(\theta_1, \theta_2)$$

$$\dot{\theta} = \gamma g(\theta)$$



Effects of learning rates

- $R_1(\theta_1, \theta_2) = \frac{1}{4}\theta_1^4 - \frac{1}{2}\theta_1^2 - \theta_1\theta_2$
- $R_2(\theta_1, \theta_2) = \frac{1}{2}\theta_2^2 + 2\theta_1\theta_2$



Stability of learning dynamics

Linearized dynamics about fixed point:

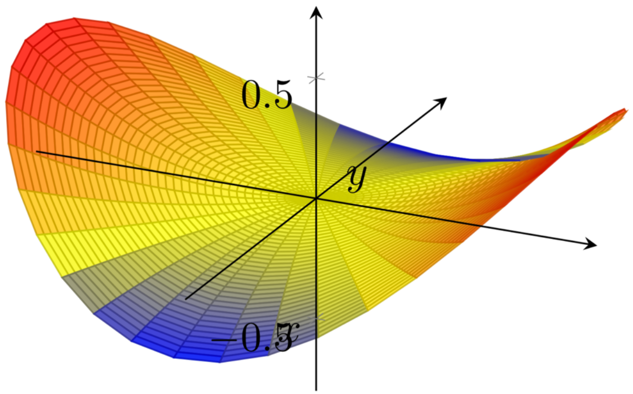
$$\dot{\theta} = J(\tilde{\theta})\theta,$$

Matrix with block structure

$$g(\tilde{\theta}) = 0, \; J(\tilde{\theta}) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

Classes of games

Adversarial / zero-sum

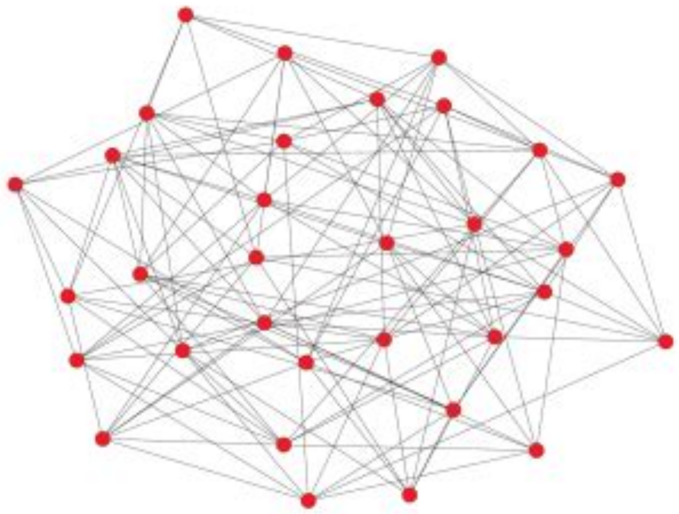


$$\min_{\theta_1} \; f(\theta_1, \theta_2)$$

$$\max_{\theta_2} \; f(\theta_1, \theta_2)$$

Von Neumann, Morgenstern. Theory of games and economic behavior. Princeton university press; 1953.

Potential game



Monderer, Shapley.
Potential Games. 1996

Relationship between stability and optimality

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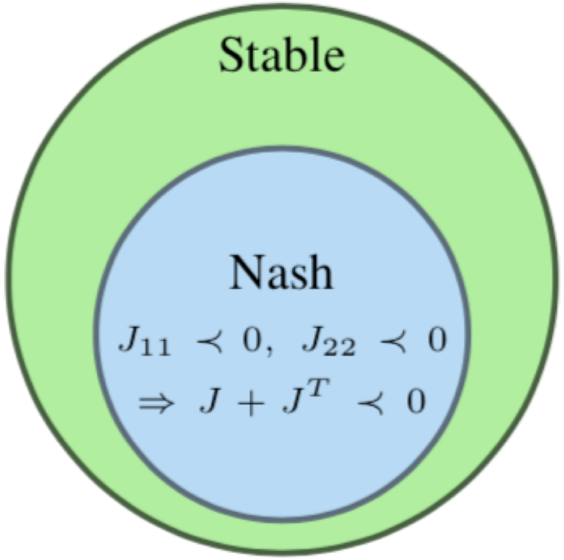
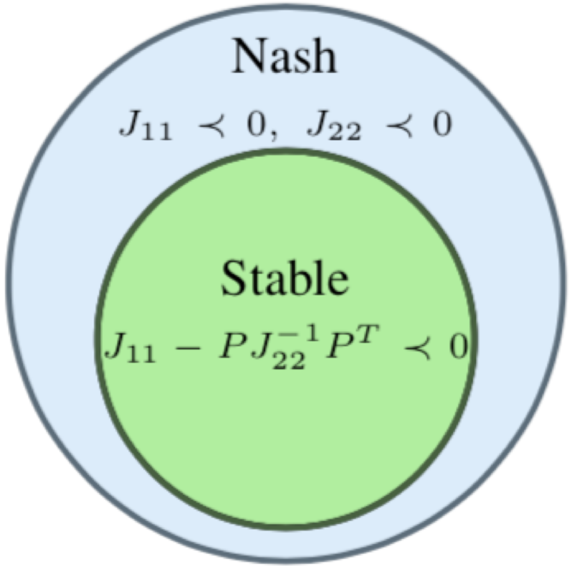
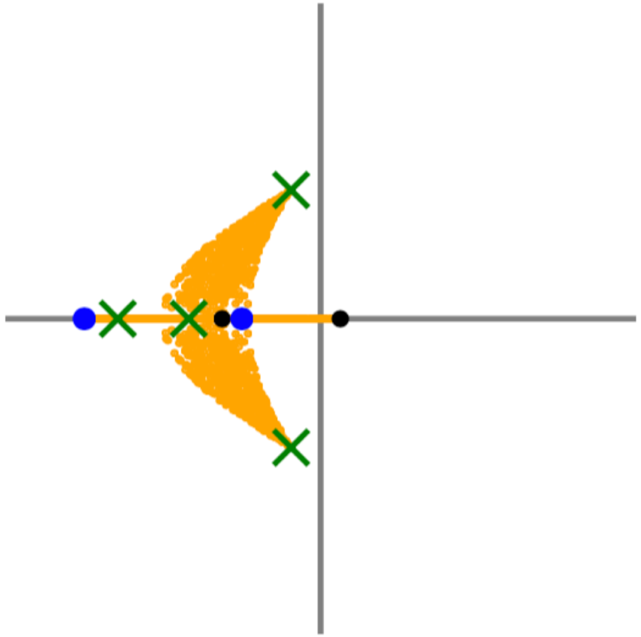
Adversarial *Potential*

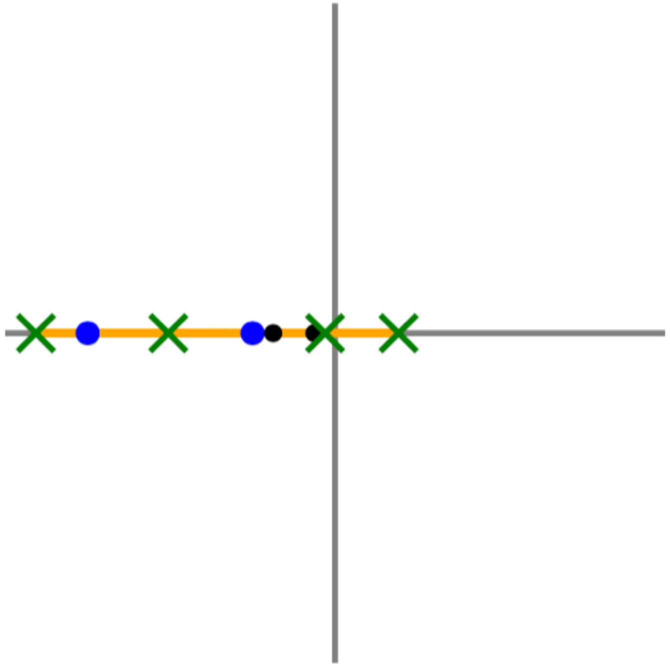




Eigenstructure



$$\begin{bmatrix} A & B \\ -B^\top & C \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ B^\top & D \end{bmatrix}$$

Order of play

|------------------------------------|--|

Simultaneous — *Leader-follower* — *Leader-leader*

$$\boxed{\begin{aligned} \boldsymbol{x}_1^* &\in \arg\min \boldsymbol{f}_1(\boldsymbol{x}) \\ \boldsymbol{x}_2^* &\in \arg\min \boldsymbol{f}_2(\boldsymbol{x}) \end{aligned}}$$

$$\boxed{\begin{aligned} \boldsymbol{x}_1^* &\in \arg\min \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{\xi}_1(\boldsymbol{x})) \\ &\qquad \boldsymbol{\xi}_1(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_2(\boldsymbol{x}) \\ \boldsymbol{x}_2^* &\in \arg\min \boldsymbol{f}_2(\boldsymbol{x}) \end{aligned}}$$

$$\boxed{\begin{aligned} \boldsymbol{x}_1^* &\in \arg\min \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{\xi}_1(\boldsymbol{x})) \\ &\qquad \boldsymbol{\xi}_1(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_2(\boldsymbol{x}) \\ \boldsymbol{x}_2^* &\in \arg\min \boldsymbol{f}_2(\boldsymbol{\xi}_2(\boldsymbol{x}), \boldsymbol{x}_2) \\ &\qquad \boldsymbol{\xi}_2(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_1(\boldsymbol{x}) \end{aligned}}$$

Thank you!