# When, and to what, does human-machine coadaptation converge?


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## Introduction and motivation

Machines are designed to optimize a performance metric.

**A recent trend:** machines run an optimization scheme in the loop **with other adaptive components**.
Example: *human-in-the-loop optimization* of assistive devices. The optimizer tries to improve the human's gait or metabolic usage by actuating their limbs.

**Challenges**

- the machine does not consider the human's adaptation
- the human's response is not stationary (it depends on what the machine is doing)
- un-modeled strategic components lead to sub-optimal performance (slow convergence or cycling)

## Mathematical model

We consider a game played by two strategic agents – a human and a machine. Agents iteratively descend *cost* functions and make small adjustments to their strategy by using gradients.

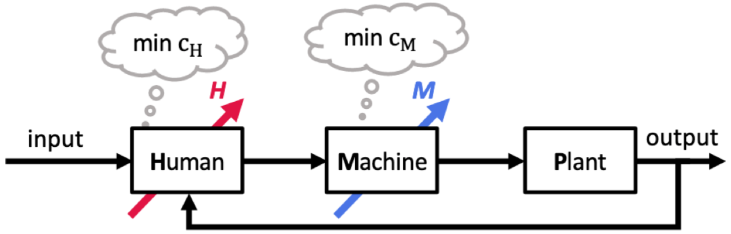
| | |
|---|---|
| Cost functions | $c_H, c_M,$ |
| Decision variables | $(H, M) \in \mathcal{H} \times \mathcal{M}.$ |
| Learning rates | $\alpha, \beta$ |

Human (agent 1): $$\frac{\partial}{\partial t} H = -\alpha \frac{\partial c_H}{\partial H}(H, M),$$

Machine (agent 2): $$\frac{\partial}{\partial t} M = -\beta \frac{\partial c_M}{\partial M}(H, M),$$

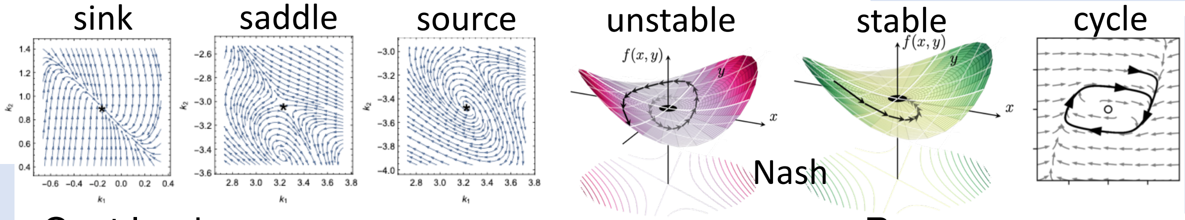
## Coadaptive closed-loop system

The human-machine adaptation is closed-loop dynamical system.




## Dynamics of gradient-based learners

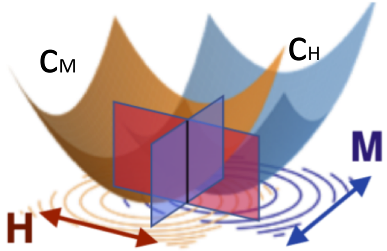
The **learning dynamics** explain the rich set of agent behaviors:




## Cost landscape

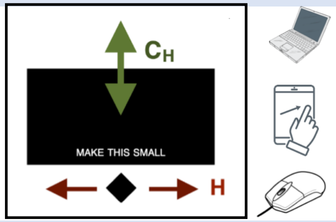
In the neighborhood of a *local equilibrium*, the cost landscape can be approximated by quadratic costs. Agents seek *stationary points* that are *minimizers* of their individual costs.

Agents can choose only their own strategy, but other strategies will affect their cost. This is the central challenge of studying adaptation in games.




## Experimental setup

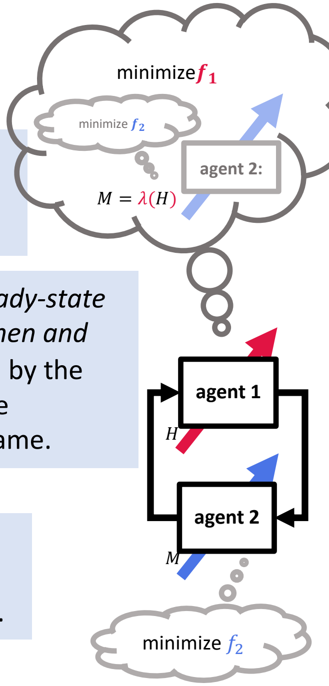
Each player chooses a scalar/vector valued action H,M.
The interaction is determined by the game *vector field*.




**Machine**: gradient descent with fixed step size
**Human**: move cursor to decrease cost
**Cost display**: prescribed through the interface

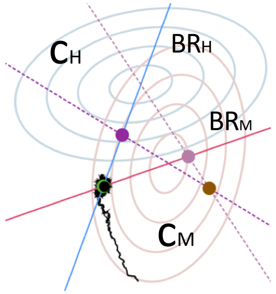
**Conclusion:** The *steady-state performance (i.e. when and what)* is determined by the characteristics of the *equilibrium* of the game.

## Modeling others…




## Response maps

If agents have quadratic costs, then their *best responses* are linear. This abstraction allows us to use geometry to analyze agent convergence.

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