# Sensorimotor game dynamics in coupled human-machine tasks

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### Summary

While interacting with a machine, humans will naturally formulate beliefs about the machine's behavior, and these beliefs will inform the humans' actions. Conversely, machines will be designed to infer the humans' intentions to optimize performance. A natural model for this coupled interaction is a game, which has been investigated extensively from the perspective of economic game theory and discrete decision-making. However, there is little work on continuous sensorimotor games that arise when humans interact in a dynamic closed loop with machines. We study the dynamics of these games both theoretically and experimentally, deriving predictive models for steady-state (i.e. equilibrium) and transient (i.e. learning) behaviors of humans interacting with other agents (humans and machines). We consider experiments wherein agents are instructed to control a linear system to minimize a given quadratic cost functional. Using our recent results on gradient-based learning in continuous games, we derive predictions regarding steady-state and transient play.

## Introduction

Since Nash's 1951 seminal paper on non-cooperative games [4], research in this field has largely focused on studying steady-state equilibria that arise from rational agents playing a game. At a Nash equilibrium, agents have no incentive to unilaterally deviate their strategy to improve their own objective. The agents' objectives may not be aligned with each other, hence the term non-cooperative.

Despite the appeal of Nash equilibria, little is known about how agents may arrive at such equilibria. It has been shown that simple uncoupled learning dynamics—updates that do not depend on the objectives of the other agent—are not guaranteed to converge to a Nash [3]. The study of learning in games [2] and game dynamics [5] therefore aims to understand the coupled transient behaviors of agents trying to improve an outcome of their task, either by minimizing cost or maximizing utility.

### Results and Discussion

We restrict our attention to the class of sensorimotor games with linear dynamics and quadratic costs. This special class of dynamic games has been extensively studied [1], producing a complete characterization of their Nash equilibria that we leverage in our simulations and experiments.

We show that when multiple agents myopically descend the gradient of their own cost with respect to their own choice variable, counter-intuitive transient behaviors arise. For instance, agents' costs may in fact increase after each iteration due to the coupling of cost functions. Agents' learning dynamic may also converge to limit cycles, where neither agent end up converging to a stationary strategy. Finally, we also show that entire classes of stable equilibria (of the learning dynamics) exist where several agents may be at local maxima of their own cost. These results shed light on the complex behaviors that may occur in coupled sensorimotor learning of multiple agents, and point towards many open questions in this domain for further investigation.

## Acknowledgements

This work was supported in part by Award CNS-1836819 from the Cyber-Physical Systems (CPS) Program in the Directorate for Computer & Information Science & Engineering (CISE) at the National Science Foundation (NSF), by the Center for Amplifying Motion and Performance (AMP Center) Strategic Research Initiative (SRI) in the University of Washington's College of Engineering (UW-CoE), and by the Computational Neuroscience Graduate Training Program at the University of Washington.

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