Stability of multi-agent learning dynamics

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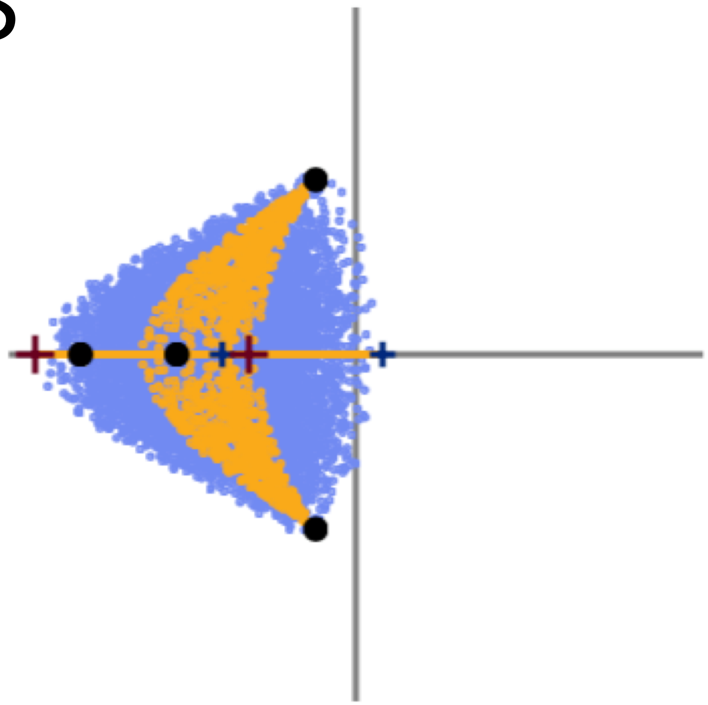


Introduction

- Motivating examples of
  - Brain-machine interface
  - Assistive rehabilitation devices
    - “you don’t know what you need to get better”
  - Robust machine learning?
    - Protect models from adversaries
  - “Biocompatible” learning?
- tldr; convince audience of wide-ranging applications of this theory

Optimization landscape

- Main point: there is a difference between

a)
$$\min_{\theta_1, \theta_2} \; f(\theta_1, \theta_2)$$

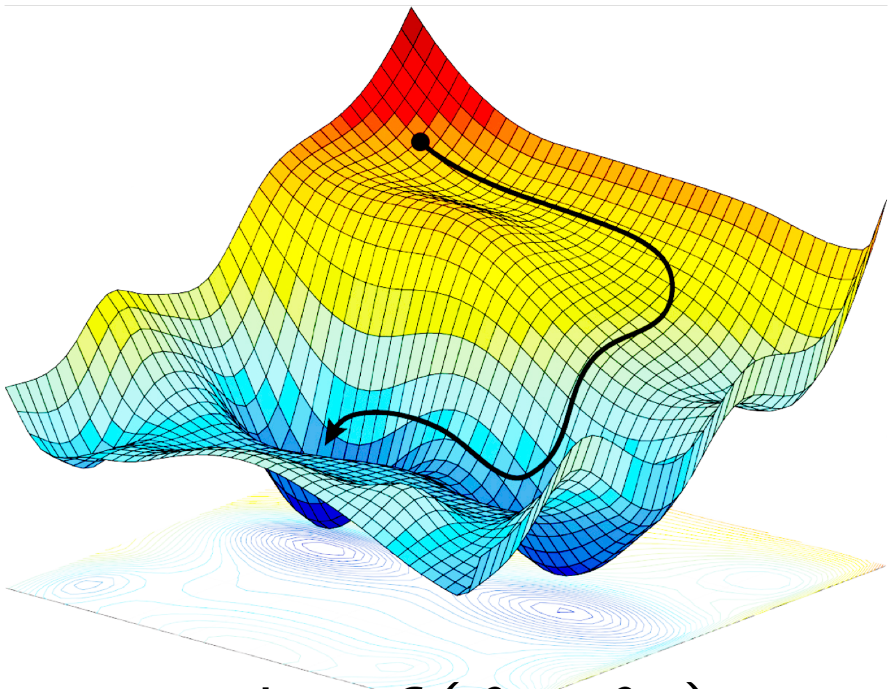
b)
$$\min_{\theta_1} \; f_1(\theta_1, \theta_2), \;\; \min_{\theta_2} \; f_2(\theta_1, \theta_2)$$

Optimization landscape

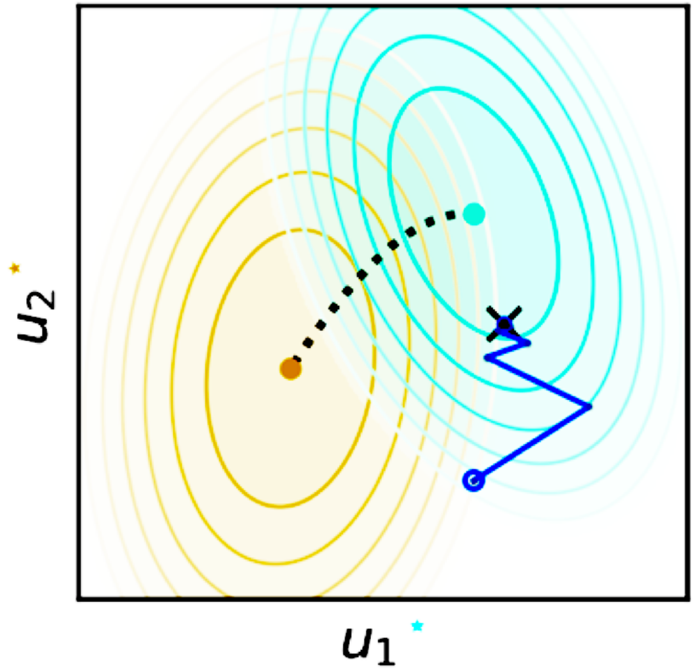
- a) cooperative setting: x1 and x2 can **coordinate** with each other
- b) non-cooperative setting: x1 and x2 are **restricted** to the subspace of their own decisions
  - More realistic due to constraints on information, trust, communication…

Figures:

Optimization landscape

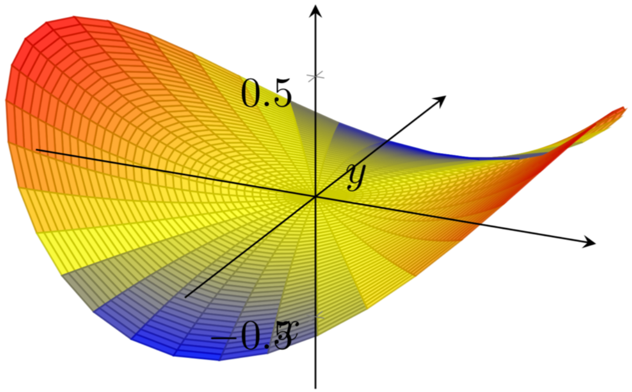


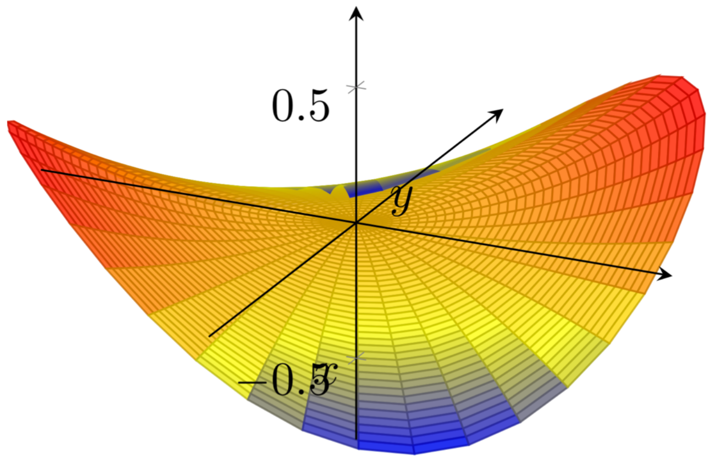
$$\min_{\theta_1,\theta_2} f(\theta_1,\theta_2)$$

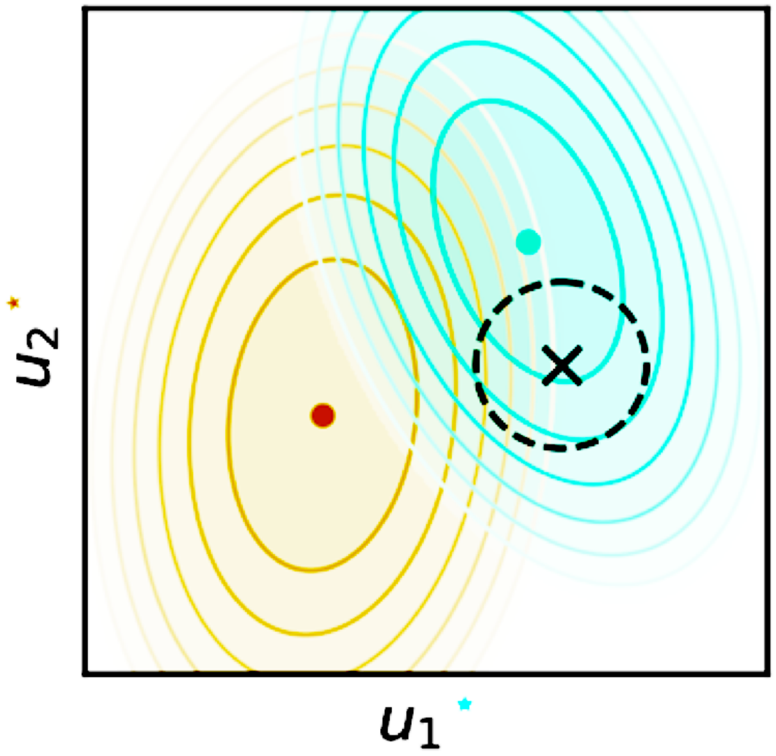


$$\min_{\theta_1} f_1(\theta_1,\theta_2),\ \min_{\theta_2} f_2(\theta_1,\theta_2)$$

Optimization landscape







$$\min_{\theta_1} \; f_1(\theta_1, \theta_2), \;\; \min_{\theta_2} \; f_2(\theta_1, \theta_2)$$

Stability of learning dynamics

Linearized dynamics about fixed point:

$$\dot{\theta} = J(\tilde{\theta})\theta,$$

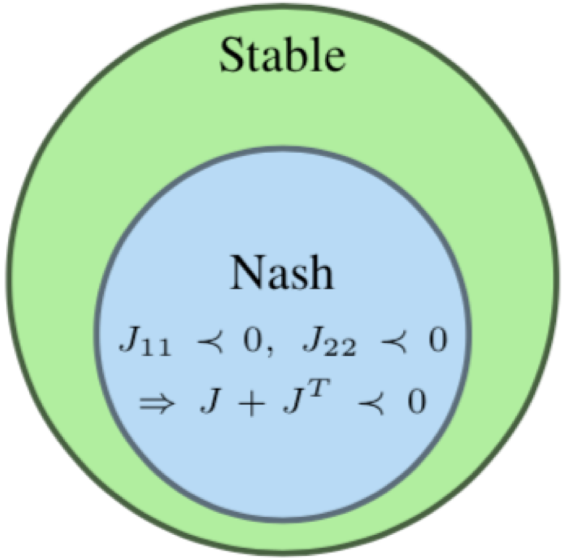
Matrix with block structure

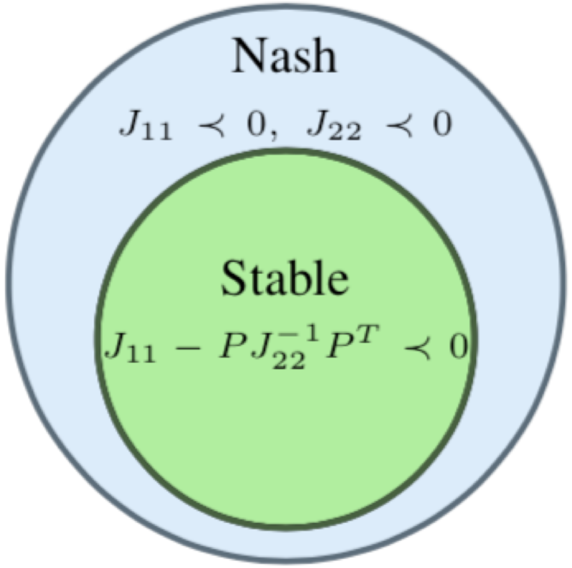
$$g(\tilde{\theta}) = 0, \;\; J(\tilde{\theta}) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

A taxonomy

|---|

Adversarial *Potential*





Order of play

|-----------------------------------|---|

Simultaneous *Leader-follower* *Leader-leader*

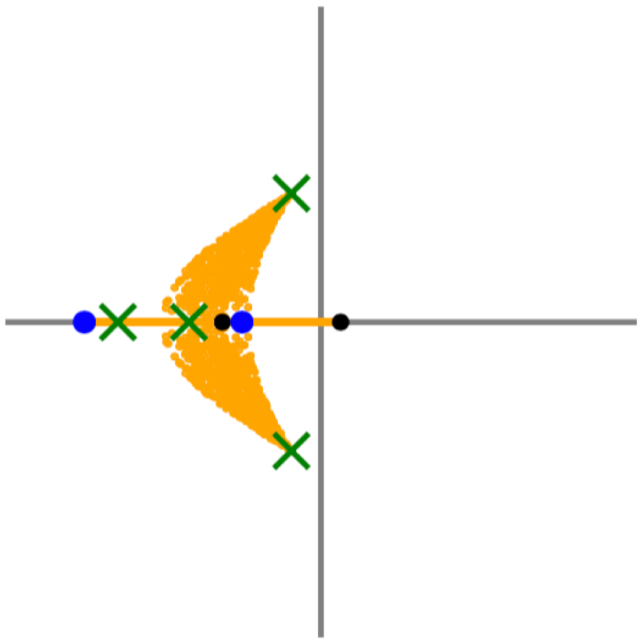
$$\boldsymbol{x}_1^* \in \arg\min \boldsymbol{f}_1(\boldsymbol{x})$$
$$\boldsymbol{x}_2^* \in \arg\min \boldsymbol{f}_2(\boldsymbol{x})$$

$$\boldsymbol{x}_1^* \in \arg\min \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{\xi}_1(\boldsymbol{x}))$$
$$\boldsymbol{\xi}_1(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_2(\boldsymbol{x})$$
$$\boldsymbol{x}_2^* \in \arg\min \boldsymbol{f}_2(\boldsymbol{x})$$

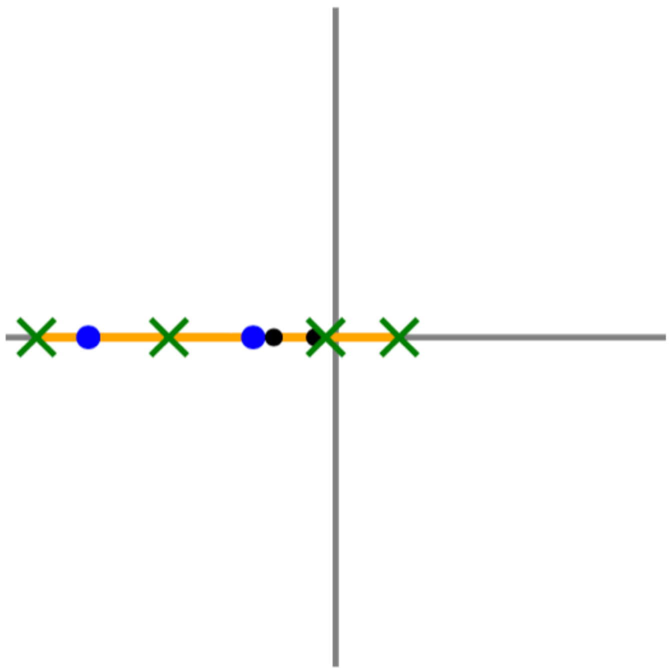
$$\boldsymbol{x}_1^* \in \arg\min \boldsymbol{f}_1(\boldsymbol{x}_1, \boldsymbol{\xi}_1(\boldsymbol{x}))$$
$$\boldsymbol{\xi}_1(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_2(\boldsymbol{x})$$
$$\boldsymbol{x}_2^* \in \arg\min \boldsymbol{f}_2(\boldsymbol{\xi}_2(\boldsymbol{x}), \boldsymbol{x}_2)$$
$$\boldsymbol{\xi}_2(\boldsymbol{x}) \in \arg\min \boldsymbol{f}_1(\boldsymbol{x})$$

Eigenstructure

$$J(\tilde{\theta}) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$



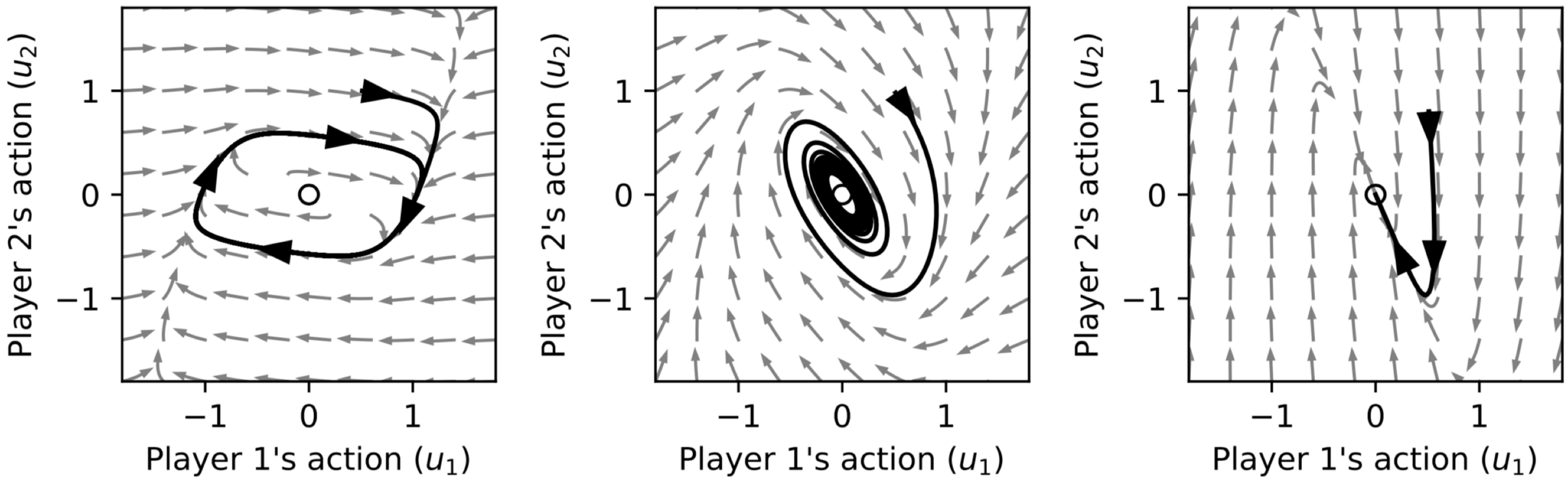
$J_{12} = -J_{21}^\top$



$J_{12} = J_{21}^\top$

Effects of learning rates

- $R_1(\theta_1, \theta_2) = \frac{1}{4}\theta_1^4 - \frac{1}{2}\theta_1^2 - \theta_1\theta_2$
- $R_2(\theta_1, \theta_2) = \frac{1}{2}\theta_2^2 + 2\theta_1\theta_2$



Predictions

- Convergence to…
  - Nash?
  - Stackelberg?
  - Limit cycle?
- Non-convergence..
  - Chaos
  - Time average
    - Regret

Future work