Auxiliary Practice Problems

Please report any typos to Ben Chugg at ben.chugg@alumni.ubc.ca

These problems may or may not be aligned with course material. I just think they're fun.

1. A metric space is a tuple (X, ρ) where X is a set and $\rho : X \times X \to \mathbb{R}_{\geq 0}$ is a function assigning "distances" to elements of X which obeys the following properties: for all $x, y, z \in X$, (i) $\rho(x, y) =\geq 0$ with equality iff $x = y$, (ii) $\rho(x, y) = \rho(y, x)$, (iii) $\rho(x, y) \leq \rho(x, z) + \rho(z, y)$ (triangle inequality).

2. An integer linear program (ILP) is a linear program in which the variables are restricted to integers. That is, in standard form an ILP is written as $\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x}$ s.t. $A\mathbf{x} \leq \mathbf{b}$, $\mathbf{x} \geq 0$, $\mathbf{x} \in \mathbb{Z}$.

   (a) Given a graph $G = (V, E)$, the *independent set problem* is to find a maximum set of vertices such that there are no edges connecting the vertices. The independent set problem is NP-Hard. Prove that ILP is NP-Hard.

   (b) Call a *boolean linear program* a LP in which the variables are restricted to $\{0, 1\}$. Maximizing a boolean linear program is NP-hard. Prove that ILP is NP-Hard.

   (c) *Mized integer linear programming* is a generalization of an ILP in which some variables are restricted to integer values, and some are not. Why is it obvious that mixed integer linear programming is NP-Hard?