Output-Oblivious Stochastic Chemical Reaction Networks

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Acknowledgements

Independent work by Severson, Haley, and Doty (2019).

Full paper in Proceedings of OPODIS 2018.

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That's it! (Almost ...)

Why YOU should care

- Closely related to distributed computing models such as Population Protocols. Applications in Signal Processing, Graphical Models and modelling cellular processes.

http://www.quickmeme.com/meme/3qmru3

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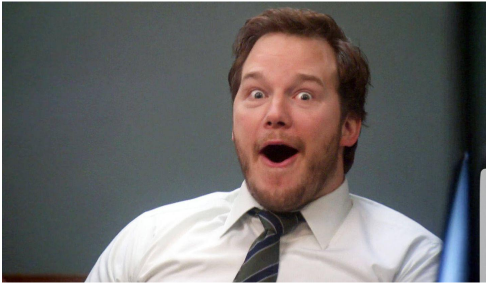
- “Programming Language” for molecular programming and DNA computing. Implementation via Strand Displacement Systems.
- Mathematically fun ...

https://steemit.com/@funmeme

CRN Function Computation

We can view CRNs as computing functions!



https://www.memesmonkey.com/topic/chris+pratt

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Computes the function $f(n) = 2n - 1$.
Function computation is *stable*: will always produce correct answer, no matter the order of computation.

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Essentially, piecewise affine functions. E.g.,

$$f(n) = \begin{cases} 3, & n \leq 5 \\ 3n - 2, & n \text{ even}, \ n \geq 5 \\ 2n + 6, & \textit{otherwise}. \end{cases}$$

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https://me.me/t/houston-we-have-a-problem

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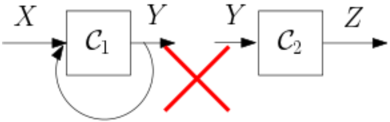
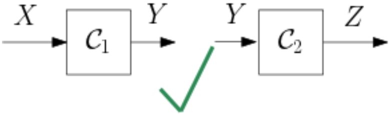
Leads to incorrect results :(.



We can avoid this travesty if the CRNs are *output-oblivious*: Outputs are never reactants.



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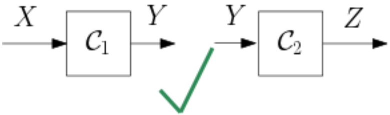

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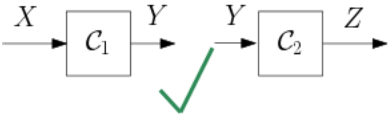

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Call these output-oblivious functions.

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Focus on functions $f : \mathbb{N}^2 \to \mathbb{N}$.

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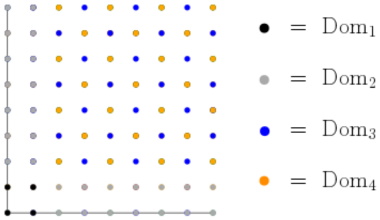
One class of output-oblivious functions: *Grid affine functions*.

These are piecewise affine functions defined on different “grids” (which can be zero-, one-, or two-dimensional).

Grid-Affine functions

Example:

$$f(\boldsymbol{n}) = \begin{cases} 2, & \boldsymbol{n} \in \mathrm{Dom}_1 \cup \mathrm{Dom}_2 \\ 2n_1 + 3n_2 + 1, & \boldsymbol{n} \in \mathrm{Dom}_3 \\ 2n_1 + 3n_2, & \boldsymbol{n} \in \mathrm{Dom}_4. \end{cases}$$



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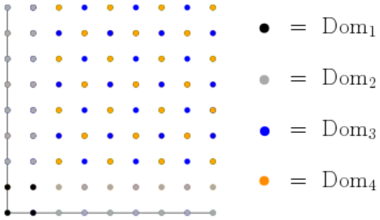

Intuition: You can "track" on which grid the input lies, since grids are (roughly) a constant distance from one another.

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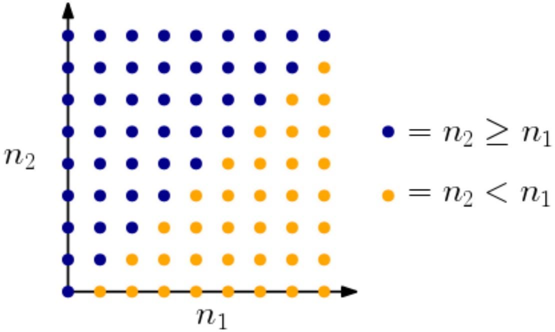
But not grid-affine!



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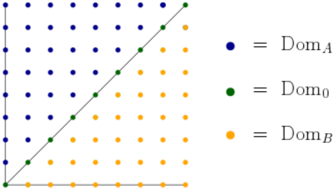
$$f(\boldsymbol{n}) = \begin{cases} 3n_1 + 2n_2 + 1, & \boldsymbol{n} \in \mathrm{Dom}_A, \\ 5n_1, & \boldsymbol{n} \in \mathrm{Dom}_0, \\ 2n_1 + 3n_2 + 2, & \boldsymbol{n} \in \mathrm{Dom}_B. \end{cases}$$



Here f agrees with $\min\{2n_1 + 3n_2 + 2, 3n_1 + 2n_2 + 1\}$, except on the "fissure line" Dom_0.

The main result!

If f_1 and f_2 are output-oblivious, then $\min\{f_1 + f_2\}$ is output oblivious:

$\mathcal{C}_1$ computes f_1 and produces Y_1.
$\mathcal{C}_2$ computes f_2 and produces Y_2.

$$\min(f_1, f_2) : \quad Y_1 + Y_2 \to Z.$$

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$$\min(f_1, f_2): \quad Y_1 + Y_2 \to Z.$$

Therefore, since it's illegal to conclude a theory talk without stating a confusing theorem.

Theorem

An increasing, semilinear function $f : \mathbb{N}^2 \to \mathbb{N}$ is stably computable by an output-oblivious CRN if and only if it is grid-affine or the minimum of finitely many fissure functions.