

IS622 - Homework 7

Brian Chu / Oct 11, 2015

```
support <- 5
abovesupport <- character()
total_itemcount <- 0

for (i in 1:100) {
  itemcount <- 0
  basket <- character()

  for (j in 1:100) {
    if (i %% j == 0) {
      itemcount <- itemcount + 1
      basket <- c(basket, j)
    }
  }
  basket <- paste(basket, collapse=" ")

  total_itemcount <- total_itemcount + itemcount

  if (itemcount >= 5) {
    abovesupport <- c(abovesupport, i)
  }
  print(paste("basket:", i, "| frequency:", itemcount, "| items:", basket))
}
```

```
## [1] "basket: 1 | frequency: 1 | items: 1"
## [1] "basket: 2 | frequency: 2 | items: 1 2"
## [1] "basket: 3 | frequency: 2 | items: 1 3"
## [1] "basket: 4 | frequency: 3 | items: 1 2 4"
## [1] "basket: 5 | frequency: 2 | items: 1 5"
## [1] "basket: 6 | frequency: 4 | items: 1 2 3 6"
## [1] "basket: 7 | frequency: 2 | items: 1 7"
## [1] "basket: 8 | frequency: 4 | items: 1 2 4 8"
## [1] "basket: 9 | frequency: 3 | items: 1 3 9"
## [1] "basket: 10 | frequency: 4 | items: 1 2 5 10"
## [1] "basket: 11 | frequency: 2 | items: 1 11"
## [1] "basket: 12 | frequency: 6 | items: 1 2 3 4 6 12"
## [1] "basket: 13 | frequency: 2 | items: 1 13"
## [1] "basket: 14 | frequency: 4 | items: 1 2 7 14"
## [1] "basket: 15 | frequency: 4 | items: 1 3 5 15"
## [1] "basket: 16 | frequency: 5 | items: 1 2 4 8 16"
## [1] "basket: 17 | frequency: 2 | items: 1 17"
## [1] "basket: 18 | frequency: 6 | items: 1 2 3 6 9 18"
## [1] "basket: 19 | frequency: 2 | items: 1 19"
## [1] "basket: 20 | frequency: 6 | items: 1 2 4 5 10 20"
## [1] "basket: 21 | frequency: 4 | items: 1 3 7 21"
## [1] "basket: 22 | frequency: 4 | items: 1 2 11 22"
## [1] "basket: 23 | frequency: 2 | items: 1 23"
## [1] "basket: 24 | frequency: 8 | items: 1 2 3 4 6 8 12 24"
## [1] "basket: 25 | frequency: 3 | items: 1 5 25"
```

```

## [1] "basket: 26 | frequency: 4 | items: 1 2 13 26"
## [1] "basket: 27 | frequency: 4 | items: 1 3 9 27"
## [1] "basket: 28 | frequency: 6 | items: 1 2 4 7 14 28"
## [1] "basket: 29 | frequency: 2 | items: 1 29"
## [1] "basket: 30 | frequency: 8 | items: 1 2 3 5 6 10 15 30"
## [1] "basket: 31 | frequency: 2 | items: 1 31"
## [1] "basket: 32 | frequency: 6 | items: 1 2 4 8 16 32"
## [1] "basket: 33 | frequency: 4 | items: 1 3 11 33"
## [1] "basket: 34 | frequency: 4 | items: 1 2 17 34"
## [1] "basket: 35 | frequency: 4 | items: 1 5 7 35"
## [1] "basket: 36 | frequency: 9 | items: 1 2 3 4 6 9 12 18 36"
## [1] "basket: 37 | frequency: 2 | items: 1 37"
## [1] "basket: 38 | frequency: 4 | items: 1 2 19 38"
## [1] "basket: 39 | frequency: 4 | items: 1 3 13 39"
## [1] "basket: 40 | frequency: 8 | items: 1 2 4 5 8 10 20 40"
## [1] "basket: 41 | frequency: 2 | items: 1 41"
## [1] "basket: 42 | frequency: 8 | items: 1 2 3 6 7 14 21 42"
## [1] "basket: 43 | frequency: 2 | items: 1 43"
## [1] "basket: 44 | frequency: 6 | items: 1 2 4 11 22 44"
## [1] "basket: 45 | frequency: 6 | items: 1 3 5 9 15 45"
## [1] "basket: 46 | frequency: 4 | items: 1 2 23 46"
## [1] "basket: 47 | frequency: 2 | items: 1 47"
## [1] "basket: 48 | frequency: 10 | items: 1 2 3 4 6 8 12 16 24 48"
## [1] "basket: 49 | frequency: 3 | items: 1 7 49"
## [1] "basket: 50 | frequency: 6 | items: 1 2 5 10 25 50"
## [1] "basket: 51 | frequency: 4 | items: 1 3 17 51"
## [1] "basket: 52 | frequency: 6 | items: 1 2 4 13 26 52"
## [1] "basket: 53 | frequency: 2 | items: 1 53"
## [1] "basket: 54 | frequency: 8 | items: 1 2 3 6 9 18 27 54"
## [1] "basket: 55 | frequency: 4 | items: 1 5 11 55"
## [1] "basket: 56 | frequency: 8 | items: 1 2 4 7 8 14 28 56"
## [1] "basket: 57 | frequency: 4 | items: 1 3 19 57"
## [1] "basket: 58 | frequency: 4 | items: 1 2 29 58"
## [1] "basket: 59 | frequency: 2 | items: 1 59"
## [1] "basket: 60 | frequency: 12 | items: 1 2 3 4 5 6 10 12 15 20 30 60"
## [1] "basket: 61 | frequency: 2 | items: 1 61"
## [1] "basket: 62 | frequency: 4 | items: 1 2 31 62"
## [1] "basket: 63 | frequency: 6 | items: 1 3 7 9 21 63"
## [1] "basket: 64 | frequency: 7 | items: 1 2 4 8 16 32 64"
## [1] "basket: 65 | frequency: 4 | items: 1 5 13 65"
## [1] "basket: 66 | frequency: 8 | items: 1 2 3 6 11 22 33 66"
## [1] "basket: 67 | frequency: 2 | items: 1 67"
## [1] "basket: 68 | frequency: 6 | items: 1 2 4 17 34 68"
## [1] "basket: 69 | frequency: 4 | items: 1 3 23 69"
## [1] "basket: 70 | frequency: 8 | items: 1 2 5 7 10 14 35 70"
## [1] "basket: 71 | frequency: 2 | items: 1 71"
## [1] "basket: 72 | frequency: 12 | items: 1 2 3 4 6 8 9 12 18 24 36 72"
## [1] "basket: 73 | frequency: 2 | items: 1 73"
## [1] "basket: 74 | frequency: 4 | items: 1 2 37 74"
## [1] "basket: 75 | frequency: 6 | items: 1 3 5 15 25 75"
## [1] "basket: 76 | frequency: 6 | items: 1 2 4 19 38 76"
## [1] "basket: 77 | frequency: 4 | items: 1 7 11 77"
## [1] "basket: 78 | frequency: 8 | items: 1 2 3 6 13 26 39 78"
## [1] "basket: 79 | frequency: 2 | items: 1 79"

```

```
## [1] "basket: 80 | frequency: 10 | items: 1 2 4 5 8 10 16 20 40 80"
## [1] "basket: 81 | frequency: 5 | items: 1 3 9 27 81"
## [1] "basket: 82 | frequency: 4 | items: 1 2 41 82"
## [1] "basket: 83 | frequency: 2 | items: 1 83"
## [1] "basket: 84 | frequency: 12 | items: 1 2 3 4 6 7 12 14 21 28 42 84"
## [1] "basket: 85 | frequency: 4 | items: 1 5 17 85"
## [1] "basket: 86 | frequency: 4 | items: 1 2 43 86"
## [1] "basket: 87 | frequency: 4 | items: 1 3 29 87"
## [1] "basket: 88 | frequency: 8 | items: 1 2 4 8 11 22 44 88"
## [1] "basket: 89 | frequency: 2 | items: 1 89"
## [1] "basket: 90 | frequency: 12 | items: 1 2 3 5 6 9 10 15 18 30 45 90"
## [1] "basket: 91 | frequency: 4 | items: 1 7 13 91"
## [1] "basket: 92 | frequency: 6 | items: 1 2 4 23 46 92"
## [1] "basket: 93 | frequency: 4 | items: 1 3 31 93"
## [1] "basket: 94 | frequency: 4 | items: 1 2 47 94"
## [1] "basket: 95 | frequency: 4 | items: 1 5 19 95"
## [1] "basket: 96 | frequency: 12 | items: 1 2 3 4 6 8 12 16 24 32 48 96"
## [1] "basket: 97 | frequency: 2 | items: 1 97"
## [1] "basket: 98 | frequency: 6 | items: 1 2 7 14 49 98"
## [1] "basket: 99 | frequency: 6 | items: 1 3 9 11 33 99"
## [1] "basket: 100 | frequency: 9 | items: 1 2 4 5 10 20 25 50 100"
```

```
length(abovesupport)
```

```
## [1] 38
```

```
abovesupport
```

```
## [1] "12" "16" "18" "20" "24" "28" "30" "32" "36" "40" "42"
## [12] "44" "45" "48" "50" "52" "54" "56" "60" "63" "64" "66"
## [23] "68" "70" "72" "75" "76" "78" "80" "81" "84" "88" "90"
## [34] "92" "96" "98" "99" "100"
```

There are 38 baskets with at least 5 items.

```
support <- 5
frequent <- character()
total_basketcount <- 0

for (i in 1:100) {
  basketcount <- 0
  item <- character()

  for (j in 1:100) {
    if (j %% i == 0) {
      basketcount <- basketcount + 1
      item <- c(item, j)
    }
  }
  item <- paste(item, collapse=" ")

  total_basketcount <- total_basketcount + basketcount
}
```

```

if (basketcount >=5) {
  frequent <- c(frequent, i)
}
print(paste("item:", i, "| frequency:", basketcount, "| baskets:", item))
}

```

```

## [1] "item: 1 | frequency: 100 | baskets: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
## [1] "item: 2 | frequency: 50 | baskets: 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 4
## [1] "item: 3 | frequency: 33 | baskets: 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54 57 60 63
## [1] "item: 4 | frequency: 25 | baskets: 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84
## [1] "item: 5 | frequency: 20 | baskets: 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100"
## [1] "item: 6 | frequency: 16 | baskets: 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96"
## [1] "item: 7 | frequency: 14 | baskets: 7 14 21 28 35 42 49 56 63 70 77 84 91 98"
## [1] "item: 8 | frequency: 12 | baskets: 8 16 24 32 40 48 56 64 72 80 88 96"
## [1] "item: 9 | frequency: 11 | baskets: 9 18 27 36 45 54 63 72 81 90 99"
## [1] "item: 10 | frequency: 10 | baskets: 10 20 30 40 50 60 70 80 90 100"
## [1] "item: 11 | frequency: 9 | baskets: 11 22 33 44 55 66 77 88 99"
## [1] "item: 12 | frequency: 8 | baskets: 12 24 36 48 60 72 84 96"
## [1] "item: 13 | frequency: 7 | baskets: 13 26 39 52 65 78 91"
## [1] "item: 14 | frequency: 7 | baskets: 14 28 42 56 70 84 98"
## [1] "item: 15 | frequency: 6 | baskets: 15 30 45 60 75 90"
## [1] "item: 16 | frequency: 6 | baskets: 16 32 48 64 80 96"
## [1] "item: 17 | frequency: 5 | baskets: 17 34 51 68 85"
## [1] "item: 18 | frequency: 5 | baskets: 18 36 54 72 90"
## [1] "item: 19 | frequency: 5 | baskets: 19 38 57 76 95"
## [1] "item: 20 | frequency: 5 | baskets: 20 40 60 80 100"
## [1] "item: 21 | frequency: 4 | baskets: 21 42 63 84"
## [1] "item: 22 | frequency: 4 | baskets: 22 44 66 88"
## [1] "item: 23 | frequency: 4 | baskets: 23 46 69 92"
## [1] "item: 24 | frequency: 4 | baskets: 24 48 72 96"
## [1] "item: 25 | frequency: 4 | baskets: 25 50 75 100"
## [1] "item: 26 | frequency: 3 | baskets: 26 52 78"
## [1] "item: 27 | frequency: 3 | baskets: 27 54 81"
## [1] "item: 28 | frequency: 3 | baskets: 28 56 84"
## [1] "item: 29 | frequency: 3 | baskets: 29 58 87"
## [1] "item: 30 | frequency: 3 | baskets: 30 60 90"
## [1] "item: 31 | frequency: 3 | baskets: 31 62 93"
## [1] "item: 32 | frequency: 3 | baskets: 32 64 96"
## [1] "item: 33 | frequency: 3 | baskets: 33 66 99"
## [1] "item: 34 | frequency: 2 | baskets: 34 68"
## [1] "item: 35 | frequency: 2 | baskets: 35 70"
## [1] "item: 36 | frequency: 2 | baskets: 36 72"
## [1] "item: 37 | frequency: 2 | baskets: 37 74"
## [1] "item: 38 | frequency: 2 | baskets: 38 76"
## [1] "item: 39 | frequency: 2 | baskets: 39 78"
## [1] "item: 40 | frequency: 2 | baskets: 40 80"
## [1] "item: 41 | frequency: 2 | baskets: 41 82"
## [1] "item: 42 | frequency: 2 | baskets: 42 84"
## [1] "item: 43 | frequency: 2 | baskets: 43 86"
## [1] "item: 44 | frequency: 2 | baskets: 44 88"
## [1] "item: 45 | frequency: 2 | baskets: 45 90"
## [1] "item: 46 | frequency: 2 | baskets: 46 92"
## [1] "item: 47 | frequency: 2 | baskets: 47 94"

```

```
## [1] "item: 48 | frequency: 2 | baskets: 48 96"
## [1] "item: 49 | frequency: 2 | baskets: 49 98"
## [1] "item: 50 | frequency: 2 | baskets: 50 100"
## [1] "item: 51 | frequency: 1 | baskets: 51"
## [1] "item: 52 | frequency: 1 | baskets: 52"
## [1] "item: 53 | frequency: 1 | baskets: 53"
## [1] "item: 54 | frequency: 1 | baskets: 54"
## [1] "item: 55 | frequency: 1 | baskets: 55"
## [1] "item: 56 | frequency: 1 | baskets: 56"
## [1] "item: 57 | frequency: 1 | baskets: 57"
## [1] "item: 58 | frequency: 1 | baskets: 58"
## [1] "item: 59 | frequency: 1 | baskets: 59"
## [1] "item: 60 | frequency: 1 | baskets: 60"
## [1] "item: 61 | frequency: 1 | baskets: 61"
## [1] "item: 62 | frequency: 1 | baskets: 62"
## [1] "item: 63 | frequency: 1 | baskets: 63"
## [1] "item: 64 | frequency: 1 | baskets: 64"
## [1] "item: 65 | frequency: 1 | baskets: 65"
## [1] "item: 66 | frequency: 1 | baskets: 66"
## [1] "item: 67 | frequency: 1 | baskets: 67"
## [1] "item: 68 | frequency: 1 | baskets: 68"
## [1] "item: 69 | frequency: 1 | baskets: 69"
## [1] "item: 70 | frequency: 1 | baskets: 70"
## [1] "item: 71 | frequency: 1 | baskets: 71"
## [1] "item: 72 | frequency: 1 | baskets: 72"
## [1] "item: 73 | frequency: 1 | baskets: 73"
## [1] "item: 74 | frequency: 1 | baskets: 74"
## [1] "item: 75 | frequency: 1 | baskets: 75"
## [1] "item: 76 | frequency: 1 | baskets: 76"
## [1] "item: 77 | frequency: 1 | baskets: 77"
## [1] "item: 78 | frequency: 1 | baskets: 78"
## [1] "item: 79 | frequency: 1 | baskets: 79"
## [1] "item: 80 | frequency: 1 | baskets: 80"
## [1] "item: 81 | frequency: 1 | baskets: 81"
## [1] "item: 82 | frequency: 1 | baskets: 82"
## [1] "item: 83 | frequency: 1 | baskets: 83"
## [1] "item: 84 | frequency: 1 | baskets: 84"
## [1] "item: 85 | frequency: 1 | baskets: 85"
## [1] "item: 86 | frequency: 1 | baskets: 86"
## [1] "item: 87 | frequency: 1 | baskets: 87"
## [1] "item: 88 | frequency: 1 | baskets: 88"
## [1] "item: 89 | frequency: 1 | baskets: 89"
## [1] "item: 90 | frequency: 1 | baskets: 90"
## [1] "item: 91 | frequency: 1 | baskets: 91"
## [1] "item: 92 | frequency: 1 | baskets: 92"
## [1] "item: 93 | frequency: 1 | baskets: 93"
## [1] "item: 94 | frequency: 1 | baskets: 94"
## [1] "item: 95 | frequency: 1 | baskets: 95"
## [1] "item: 96 | frequency: 1 | baskets: 96"
## [1] "item: 97 | frequency: 1 | baskets: 97"
## [1] "item: 98 | frequency: 1 | baskets: 98"
## [1] "item: 99 | frequency: 1 | baskets: 99"
## [1] "item: 100 | frequency: 1 | baskets: 100"
```

```
length(frequent)
```

```
## [1] 20
```

```
frequent
```

```
## [1] "1" "2" "3" "4" "5" "6" "7" "8" "9" "10" "11" "12" "13" "14"  
## [15] "15" "16" "17" "18" "19" "20"
```

Each of items 1-20 appear in at least 5 baskets. This is also intuitive since $100/20 = 5$.

```
f <- expand.grid(1:20, 1:20)  
f2 <- f[f[,2] > f[,1],]  
f2 <- f2[order(f2[,1]),]  
  
freqcount <- 0  
for (i in 1:nrow(f2)) {  
  doublecount <- 0  
  for (j in 1:100) {  
    if ((j %% f2[i,1] == 0) & (j %% f2[i,2] == 0)) {  
      doublecount <- doublecount + 1  
    }  
  }  
  
  if (doublecount >= 5) {  
    freqpair <- paste("(", f2[i,1], ",", f2[i,2], ")", sep="")  
    print(paste("pair:", freqpair, "| frequency:", doublecount))  
    freqcount <- freqcount + 1  
  }  
}
```

```
## [1] "pair: (1,2) | frequency: 50"  
## [1] "pair: (1,3) | frequency: 33"  
## [1] "pair: (1,4) | frequency: 25"  
## [1] "pair: (1,5) | frequency: 20"  
## [1] "pair: (1,6) | frequency: 16"  
## [1] "pair: (1,7) | frequency: 14"  
## [1] "pair: (1,8) | frequency: 12"  
## [1] "pair: (1,9) | frequency: 11"  
## [1] "pair: (1,10) | frequency: 10"  
## [1] "pair: (1,11) | frequency: 9"  
## [1] "pair: (1,12) | frequency: 8"  
## [1] "pair: (1,13) | frequency: 7"  
## [1] "pair: (1,14) | frequency: 7"  
## [1] "pair: (1,15) | frequency: 6"  
## [1] "pair: (1,16) | frequency: 6"  
## [1] "pair: (1,17) | frequency: 5"  
## [1] "pair: (1,18) | frequency: 5"  
## [1] "pair: (1,19) | frequency: 5"  
## [1] "pair: (1,20) | frequency: 5"  
## [1] "pair: (2,3) | frequency: 16"
```

```
## [1] "pair: (2,4) | frequency: 25"
## [1] "pair: (2,5) | frequency: 10"
## [1] "pair: (2,6) | frequency: 16"
## [1] "pair: (2,7) | frequency: 7"
## [1] "pair: (2,8) | frequency: 12"
## [1] "pair: (2,9) | frequency: 5"
## [1] "pair: (2,10) | frequency: 10"
## [1] "pair: (2,12) | frequency: 8"
## [1] "pair: (2,14) | frequency: 7"
## [1] "pair: (2,16) | frequency: 6"
## [1] "pair: (2,18) | frequency: 5"
## [1] "pair: (2,20) | frequency: 5"
## [1] "pair: (3,4) | frequency: 8"
## [1] "pair: (3,5) | frequency: 6"
## [1] "pair: (3,6) | frequency: 16"
## [1] "pair: (3,9) | frequency: 11"
## [1] "pair: (3,12) | frequency: 8"
## [1] "pair: (3,15) | frequency: 6"
## [1] "pair: (3,18) | frequency: 5"
## [1] "pair: (4,5) | frequency: 5"
## [1] "pair: (4,6) | frequency: 8"
## [1] "pair: (4,8) | frequency: 12"
## [1] "pair: (4,10) | frequency: 5"
## [1] "pair: (4,12) | frequency: 8"
## [1] "pair: (4,16) | frequency: 6"
## [1] "pair: (4,20) | frequency: 5"
## [1] "pair: (5,10) | frequency: 10"
## [1] "pair: (5,15) | frequency: 6"
## [1] "pair: (5,20) | frequency: 5"
## [1] "pair: (6,9) | frequency: 5"
## [1] "pair: (6,12) | frequency: 8"
## [1] "pair: (6,18) | frequency: 5"
## [1] "pair: (7,14) | frequency: 7"
## [1] "pair: (8,16) | frequency: 6"
## [1] "pair: (9,18) | frequency: 5"
## [1] "pair: (10,20) | frequency: 5"
```

```
print(freqcount)
```

```
## [1] 56
```

There are 56 pairs that are in at least 5 baskets together.

```
print(total_itemcount)
```

```
## [1] 482
```

```
print(total_basketcount)
```

```
## [1] 482
```

Exercise 6.1.5 : For the data of Exercise 6.1.1, what is the confidence of the following association rules?

(a) $\{5,7\} \rightarrow 2$.

5 and 7 appear in two baskets together (35, 70). Of these, 2 appears in one of these baskets (70). Therefore, the confidence is $1/2 = 50\%$.

(b) $\{2,3,4\} \rightarrow 5$.

```
for (i in 1:100) {
  if ((i %% 2 == 0) & (i %% 3 == 0) & (i %% 4 == 0)) {
    print(i)
  }
}
```

```
## [1] 12
## [1] 24
## [1] 36
## [1] 48
## [1] 60
## [1] 72
## [1] 84
## [1] 96
```

2, 3, and 4 appear in 8 baskets together. Of these, 5 appears in just one (60). Therefore, the confidence is $1/8 = 12.5\%$.

Suppose the support threshold is 4. On the first pass of the PCY Algorithm we use a hash table with 11 buckets, and the set $\{i,j\}$ is hashed to bucket $i \times j \bmod 11$.

(a) By any method, compute the support for each item and each pair of items.

```
baskets <- list(c(1,2,3), c(2,3,4), c(3,4,5), c(4,5,6),
               c(1,3,5), c(2,4,6), c(1,3,4), c(2,4,5),
               c(3,5,6), c(1,2,4), c(2,3,5), c(3,4,6))

for (i in 1:6) {
  count <- 0
  for (b in baskets) {
    if (i %in% b) {
      count <- count + 1
    }
  }
  print(paste("item:", i, "| baskets/support:", count))
}

p <- expand.grid(1:6, 1:6)
p2 <- p[p[,2] > p[,1],]
p2 <- p2[order(p2[,1]),]
```



```

fcount <- 0
for (i in 1:nrow(p2)) {
  dcount <- 0
  for (j in 1:length(baskets)) {
    if ((p2[i,1] %in% baskets[[j]]) & (p2[i,2] %in% baskets[[j]])) {
      dcount <- dcount + 1
    }
  }

  pair <- paste("(", p2[i,1], ",", p2[i,2], ")", sep="")
  print(paste("pair:",pair, "| baskets/support:",dcount))
}

```

```

## [1] "item: 1 | baskets/support: 4"
## [1] "item: 2 | baskets/support: 6"
## [1] "item: 3 | baskets/support: 8"
## [1] "item: 4 | baskets/support: 8"
## [1] "item: 5 | baskets/support: 6"
## [1] "item: 6 | baskets/support: 4"
## [1] "pair: (1,2) | baskets/support: 2"
## [1] "pair: (1,3) | baskets/support: 3"
## [1] "pair: (1,4) | baskets/support: 2"
## [1] "pair: (1,5) | baskets/support: 1"
## [1] "pair: (1,6) | baskets/support: 0"
## [1] "pair: (2,3) | baskets/support: 3"
## [1] "pair: (2,4) | baskets/support: 4"
## [1] "pair: (2,5) | baskets/support: 2"
## [1] "pair: (2,6) | baskets/support: 1"
## [1] "pair: (3,4) | baskets/support: 4"
## [1] "pair: (3,5) | baskets/support: 4"
## [1] "pair: (3,6) | baskets/support: 2"
## [1] "pair: (4,5) | baskets/support: 3"
## [1] "pair: (4,6) | baskets/support: 3"
## [1] "pair: (5,6) | baskets/support: 2"

```

(b) Which pairs hash to which buckets?

```

for (i in 1:nrow(p2)) {
  hb <- (p2[i,1] * p2[i,2]) %% 11
  pair <- paste("(", p2[i,1], ",", p2[i,2], ")", sep="")
  print(paste("pair:",pair, "| bucket:",hb))
}

```

```

## [1] "pair: (1,2) | bucket: 2"
## [1] "pair: (1,3) | bucket: 3"
## [1] "pair: (1,4) | bucket: 4"
## [1] "pair: (1,5) | bucket: 5"
## [1] "pair: (1,6) | bucket: 6"
## [1] "pair: (2,3) | bucket: 6"
## [1] "pair: (2,4) | bucket: 8"
## [1] "pair: (2,5) | bucket: 10"
## [1] "pair: (2,6) | bucket: 1"
## [1] "pair: (3,4) | bucket: 1"

```

```
## [1] "pair: (3,5) | bucket: 4"
## [1] "pair: (3,6) | bucket: 7"
## [1] "pair: (4,5) | bucket: 9"
## [1] "pair: (4,6) | bucket: 2"
## [1] "pair: (5,6) | bucket: 8"
```

```
for (i in 0:10) {
  hbcount <- 0
  pairs <- character()
  for (j in 1:nrow(p2)) {
    hb <- (p2[j,1] * p2[j,2]) %% 11
    if (hb == i) {
      hbcount <- hbcount + 1
      pair <- paste("(", p2[j,1], ",", p2[j,2], ")", sep="")
      pairs <- c(pairs, pair)
    }
  }

  pairs <- paste(pairs, collapse=", ")
  print(paste("bucket:", i, "| count:", hbcount, "| pair(s):", pairs))
}
```

```
## [1] "bucket: 0 | count: 0 | pair(s): "
## [1] "bucket: 1 | count: 2 | pair(s): (2,6), (3,4)"
## [1] "bucket: 2 | count: 2 | pair(s): (1,2), (4,6)"
## [1] "bucket: 3 | count: 1 | pair(s): (1,3)"
## [1] "bucket: 4 | count: 2 | pair(s): (1,4), (3,5)"
## [1] "bucket: 5 | count: 1 | pair(s): (1,5)"
## [1] "bucket: 6 | count: 2 | pair(s): (1,6), (2,3)"
## [1] "bucket: 7 | count: 1 | pair(s): (3,6)"
## [1] "bucket: 8 | count: 2 | pair(s): (2,4), (5,6)"
## [1] "bucket: 9 | count: 1 | pair(s): (4,5)"
## [1] "bucket: 10 | count: 1 | pair(s): (2,5)"
```

(c) Which buckets are frequent?

None are frequent for support = 4. However, buckets 1, 2, 4, 6, and 8 are frequent with $s > 1$.

(d) Which pairs are counted on the second pass of the PCY Algorithm?

Bucket 1: pair(2,6): support = 1: NO
 Bucket 1: pair(3,4): support = 4: YES
 Bucket 2: pair(1,2): support = 2: NO
 Bucket 2: pair(4,6): support = 3: NO
 Bucket 4: pair(1,4): support = 2: NO
 Bucket 4: pair(3,5): support = 4: YES
 Bucket 6: pair(1,6): support = 0: NO
 Bucket 6: pair(2,3): support = 3: NO
 Bucket 8: pair(2,4): support = 4: YES
 Bucket 8: pair(5,6): support = 2: NO

Pairs (3,4), (3,5), and (2,4) are counted on the second pass.

Exercise 6.3.2: Suppose we run the Multistage Algorithm on the data of Exercise 6.3.1, with the same support threshold of 4. The first pass is the same as in that exercise, and for the second pass, we hash pairs to nine buckets, using the hash function that hashes $\{i, j\}$ to bucket $i + j \bmod 9$. Determine the counts of the buckets on the second pass. Does the second pass reduce the set of candidate pairs? Note that all items are frequent, so the only reason a pair would not be hashed on the second pass is if it hashed to an infrequent bucket on the first pass.

```
pass1 <- list(c(2,6), c(3,4), c(1,2), c(4,6),
             c(1,4), c(3,5), c(1,6), c(2,3),
             c(2,4), c(5,6))

pass1 <- matrix(c(2,3,1,4,1,3,1,2,2,5,
                 6,4,2,6,4,5,6,3,4,6),
               nrow=10, ncol=2)

for (i in 0:9) {
  hbcount2 <- 0
  pairs2 <- character()
  for (j in 1:nrow(pass1)) {
    hb <- (pass1[j,1] + pass1[j,2]) %% 9
    if (hb == i) {
      hbcount2 <- hbcount2 + 1
      pair2 <- paste("(", pass1[j,1], ",", pass1[j,2], ")", sep="")
      pairs2 <- c(pairs2, pair2)
    }
  }

  pairs2 <- paste(pairs2, collapse=" ")
  print(paste("bucket:", i, "| count:", hbcount2, "| pair(s):", pairs2))
}
```

```
## [1] "bucket: 0 | count: 0 | pair(s): "
## [1] "bucket: 1 | count: 1 | pair(s): (4,6)"
## [1] "bucket: 2 | count: 1 | pair(s): (5,6)"
## [1] "bucket: 3 | count: 1 | pair(s): (1,2)"
## [1] "bucket: 4 | count: 0 | pair(s): "
## [1] "bucket: 5 | count: 2 | pair(s): (1,4), (2,3)"
## [1] "bucket: 6 | count: 1 | pair(s): (2,4)"
## [1] "bucket: 7 | count: 2 | pair(s): (3,4), (1,6)"
## [1] "bucket: 8 | count: 2 | pair(s): (2,6), (3,5)"
## [1] "bucket: 9 | count: 0 | pair(s): "
```

Yes, the second pass reduces the candidate pairs to 6, compared to 8 with PCY. Pairs (1,2) and (4,6) were removed on the second pass.