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# IMPROVING OUTCOME PREDICTIONS FOR PATIENTS RECEIVING MECHANICAL CIRCULATORY SUPPORT BY OPTIMIZING IMPUTATION OF MISSING VALUES

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## ABSTRACT

**Background** Risk predictions play an important role in clinical decision making. When developing risk prediction models, practitioners often impute missing values to the mean. The purpose of this article is to evaluate the impact of applying different strategies to impute missing values on the prognostic accuracy of prediction models fitted to the imputed data. A secondary objective was to compare the accuracy of different imputation methods. To complete these objectives, we used data from the Interagency Registry for Mechanically Assisted Circulatory Support (INTERMACS).

**Methods and Results** We applied thirteen different strategies to impute missing values in combination with three different strategies to fit a risk prediction model for mortality and transplant after receiving mechanical circulatory support. Model performance was measured by 12-month discrimination, calibration and net reclassification index. Results indicated that multiple imputation consistently provided prediction models with greater prognostic accuracy than other missing data strategies, particularly imputation to the mean.

**Conclusion** Selecting an optimal strategy to handle missing values impacts the prognostic accuracy of downstream models. In the current analysis, multiple imputation emerged as an optimal strategy to handle missing values in the INTERMACS data. The current study shows that evaluation and selection of an optimal strategy to impute missing data has the potential to improve prognostic accuracy of risk predictions for other longitudinal registries.

**Keywords** Missing Data, · INTERMACS, · imputation, · heart failure, · mortality, · risk prediction

## 1 Introduction

1 Heart disease is a leading cause of death in the United States. Heart failure, a primary component of heart disease, affects  
2 over 6 million Americans, and for ~10% of these patients medical management is no longer effective [1, 2]. Mechanical  
3 circulatory support (MCS) is a surgical intervention in which a mechanical device is implanted in parallel to the heart to  
4 improve circulation [3]. Typically, MCS is used while a patient waits for a heart transplant (bridge-to-transplant) or in  
5 some cases as an alternative to transplant (destination therapy) [4]. Over 250,000 patients could benefit from MCS  
6 [5]. However, less than 4,000 new patients receive a long-term MCS device each year, with widely heterogeneous  
7 outcomes [6]. The 2-year survival on MCS ranges from 61% for destination therapy to 78% for bridge-to-transplant [3].  
8 Therefore, there is great need for reliable predictions of patient-specific risk to experience adverse events after receiving  
9 MCS. This information can be used to improve patient selection for MCS, inform the design of next generation pumps,  
10 and refine patient management strategies.

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\*Source code available at <https://github.com/bcjaeger/INTERMACS-missing-data>

The Interagency Registry for Mechanically Assisted Circulatory Support (INTERMACS) was launched to improve MCS patient outcomes through the collection of patient characteristics, medical events, and long term outcomes at a nation wide level. Currently, INTERMACS comprises over 20,000 patients who have received a MCS device. As the largest registry for data on patients receiving MCS devices, INTERMACS has been leveraged to develop numerous risk prediction models for mortality and other types of adverse events that may occur after receiving a device [7, 8, 9]. A critical step for developing risk prediction models is handling missing data, which is common in a “real world” database that is not designed for the rigor of data completeness found in a clinical trial. While some missing values are related to data entry error, others may relate to instability of a patient’s circulatory status at the time of implant. For instance, hemodynamic laboratory values are missing for patients in whom invasive catheterizations were not performed. Patients without invasive catheterizations may be too ill (unstable) to tolerate the procedure. Instead, such patients may, after determination of basic hemodynamics, proceed directly to device placement.

The primary aim of this article is to quantify how much the prognostic value of a risk prediction model developed from the INTERMACS registry depends on the strategy that was applied to impute missing data prior to developing the model. A secondary aim is to measure imputation accuracy of each strategy by introducing varying levels of artificial missing data and then imputing it. Because imputation to the mean has been a standard method for multiple annual summaries of the INTERMACS data, we measure the potential improvement in prognostic value of a risk prediction model when other strategies are applied to impute missing data *instead of imputation to the mean*. The over-arching aim is to clarify which imputation strategies are most likely to improve risk prediction (and by extension, quality of care) for patients who receive MCS devices. This investigation can directly inform future analyses of INTERMACS data and provide evidence quantifying the benefit of imputing missing data with sound methodology.

## 2 Methods

Here we describe the INTERMACS Registry (Section 2.1) and give a broad overview of general objectives for imputation of missing data in the context of statistical inference and supervised statistical learning (Section 2.2). We then describe thirteen strategies to handle missing values (Section 2.3) and three strategies to develop risk prediction models after missing data have been imputed (Section 2.4). Next, we describe how risk prediction models and by extension the imputation strategy applied before training the model were evaluated in Section 2.5. Last, we outline an internal validation procedure that was applied to compare combinations of imputation and modeling strategies using hierarchical Bayesian models in Section 2.6.

### 2.1 INTERMACS Registry

INTERMACS is a North American observational registry for patients receiving MCS devices that began as a partnership between the National Heart, Lung, and Blood Institute, US Food and Drug Administration, the Centers for Medicaid and Medicare Services, industry, and individual hospitals with the mission of improving MCS outcomes. In 2018, INTERMACS became an official Society of Thoracic Surgeons database.

The current analysis was conducted using publicly available data provided by the National Heart, Lung, and Blood Institute. We included a contemporary cohort of 14,738 patients who received continuous flow LVAD from 2012-2017. Patient follow-up begins after implantation of a durable, long term MCS device and continues while the device is in place. Registry endpoints include death on a device, heart transplantation, or cessation of support (for recovery and non-recovery reasons). INTERMACS collects pre-implant patient characteristics, medical status, laboratory values, and many other variables. Data is collected at regularly scheduled follow-up as well as during adverse events such as re-hospitalization. This is a secondary analysis of de-identified data obtained from the National Heart Lung and Blood Institute. Primary data collection is approved through University of Alabama Institutional Review Board and at individual sites.

### 2.2 Statistical Inference and Learning with Missing data

**Statistical Inference** Previous research has thoroughly investigated statistical inference in the context of missing data. In this setting, analysts often create multiple imputed datasets, replicate an analysis in each of them, and then pool their results to obtain valid test statistics for hypothesis testing [10]. Imputation strategies that create a single imputed dataset (*e.g.*, imputation to the mean) have been shown to increase type I errors (*i.e.*, rejecting a true null hypothesis) for inferential statistics by artificially reducing the variance of observed data and ignoring the uncertainty attributed to missing values [11]. When very few data are missing, analysts may apply listwise deletion, which removes any observation with at least one missing value and produces a dataset with fewer observations and no missing values.

**Statistical Learning** In the presence of missing data, the goal of supervised statistical learning is to develop a prediction function for an outcome variable that accurately generalizes to testing data, which may or may not contain missing values [12]. Because testing data may contain missing values, listwise deletion is not a feasible strategy for statistical learning tasks. In contrast to statistical inference, strategies that create a single imputed dataset are often used for statistical learning [13]. Previous work has emphasized the importance of imputation strategies with greater accuracy, suggesting that more accurate imputation strategies lead to better performance of downstream models (*i.e.*, models fitted to the imputed data) [14]. Others have shown that imputation to the mean, model-based imputation, and multiple imputation can provide Bayes-optimal prediction models provided certain assumptions are true, but these assumptions are difficult to validate in applied settings [15].

## 2.3 Missing Data Strategies

**Single and multiple imputation** Several of the imputation methods we considered allowed for creation of one or many imputed datasets. For downstream models fitted to multiple sets of imputed data, we applied the modeling technique to each imputed training set, separately, and then used a pooling technique to generate a single set of predictions based on multiple imputed testing data. Specifically, we created 10 imputed training and testing sets, then applied a modeling procedure to each imputed training set and computed model predictions on the corresponding imputed testing set, which led to 10 sets of predictions. To aggregate these predictions, we computed the median for each observation. Informal experiments where the mean was used instead of the median to aggregate predictions showed little or no difference in the model’s prediction accuracy.

**Imputation to the mean** Imputing data to the mean involves three steps. First, numeric and nominal variables are identified. Second, for each numeric variable, the mean is computed and used to impute missing values in the corresponding variable. Third, for each nominal variable, the mode is computed (because one cannot compute a mean for categorical variables) and used to impute missing values in the corresponding variable. The computed means and modes are then stored for future imputation of testing data. Because imputation to the mean is frequently used in practice, we use it as a reference for comparison of all other strategies to impute missing values,

**Bayesian Regression** Imputation with Bayesian regression draws imputed values from the posterior distribution of model parameters, accounting for uncertainty in both model error and estimation [10]. Multiple imputation with chained equations (MICE), a well known technique for generating multiply imputed data [16, 11, 17]. In each iteration of MICE, each specified variable in the dataset is imputed using the other variables in the dataset. These iterations are run until convergence criteria have been met.

**Predictive Mean Matching (PMM)** PMM computes predicted values from a pre-specified model that treats one incomplete column in the data,  $X$ , as a dependent variable. For each missing value in  $X$ , PMM identifies a set of candidate donors based on distance in predicted values of  $X$  [18]. One donor is randomly drawn from the candidates, and the observed value of the donor is taken to replace the missing value. When missing values were imputed using PMM, we applied MICE to form multiple imputed datasets. For consistency with other approaches, we also generated a single imputed dataset using PMM by randomly selecting one of the multiple datasets imputed.

**K-Nearest-Neighbors (KNN)** KNN imputation identifies  $k$  ‘similar’ observations (*i.e.*, neighbors) for each observation with a missing value in a given variable [19]. A donor value for the current missing value is generated by sampling or aggregating the values from the  $k$  nearest neighbors. In the current analysis, we identified 10 nearest neighbors using Gower’s distance [20]. When imputing a single dataset using KNN, we aggregated values from nearest neighbors using the median for numeric variables and the mode for categorical variables. When imputing multiple datasets using KNN, we sampled one value at random from nearest neighbors for each imputed set.

**Hot Deck** Similar to KNN, hot deck imputation finds  $k$  similar observations for each observation with a missing value in a given variable [21]. However, hot deck imputation uses a less computationally intensive approach, either identifying neighbors at random or using a subset of variables to find similar observations. When imputing a single dataset using hot deck imputation, we used a selection of 5 variables simultaneously to identify nearest neighbors. When imputing multiple datasets using hot deck imputation, we used a separate numeric variable for each dataset.

**Random forests** The random forest framework can be used for imputation as well as prediction. In the current analysis, we implement `miceRanger`, an R package similar to `missForest` that implements MICE with random forests by implementing PMM [22]. `miceRanger` follows an iterative procedure where one random forest is fit to each variable in a dataset, separately, and then used to impute missing values in that variable. The procedure may repeat

until estimated generalization error of the random forest collection stabilizes. When imputing a single dataset with `mi ceRanger`, we did not use PMM and instead imputed missing values with a prediction. When imputing multiple datasets, we implemented PMM with 10 donor candidates.

**Missingness incorporated as an attribute (MIA)** MIA adds another observed category to nominal variables: “Missing”. For numeric variables, MIA creates two columns: one where missing values are imputed with positive infinity and the other with negative infinity. When a decision tree uses a finite cut-point to split a given numeric variable, it will assess the cut-point using both the positive and negative infinite imputed columns, and utilize whichever column provides the best split of the current data. This procedure translates to sending all missing values to the left or to the right when forming two new nodes of the decision tree, using whichever direction results in a better split. In the current analysis, we used the MIA strategy only for ensembles of boosted decision trees, as it is a well known and frequently used strategy for these models [23, 24].

**Surrogate splitting** When decision trees search for a variable and cut-point to use for growing two new nodes in a decision tree, surrogate splitting will temporarily ignore missing values while searching for an initial split. Once an initial split is chosen, surrogate splitting searches for a split on another variable with observed values for the observations with missing values on the initial splitting variable and creates a surrogate split based on the new variable that is similar to the original split. In the current analysis, we used a maximum of three surrogate splits and sent any remaining missing values to the daughter node with higher sample size. Additionally, we only used surrogate splitting for conditional inference forests, as this strategy is well known and frequently used for these models.

**Assumptions** Each missing data strategy poses different assumptions regarding the data and the mechanisms that lead to missing values in the data. Imputation using Bayesian regression makes the same distributional assumptions as the Bayesian models that are applied. The `mi ceRanger` algorithm does not make any formal distributional assumptions, as random forests are non-parametric and can thus handle skewed and multi-modal data as well as categorical data that are ordinal or non-ordinal. Hot deck and KNN imputation also do not make distributional assumptions, but implicitly assume that a missing value for a given observation can be approximated by aggregating observed values from the  $k$  most similar observations. PMM makes a similar implicit assumption, but is slightly more robust to skewed or multi-modal data because imputed values are sampled directly from observed ones. MIA operates based on an implicit assumption that missingness itself is informative and surrogate splitting presumes that variables with missing values are related to other observed variables used by the decision tree. It is difficult to validate these assumptions in applied settings and also likely that downstream models will perform poorly if an imputation technique’s assumptions are invalid.

## 2.4 Risk Prediction Models

We applied three modeling strategies after imputing missing values:

- Cox proportional hazards (PH) model with forward stepwise variable selection
- gradient boosted decision trees (hereafter referred to as ‘boosting’)
- Conditional inference forest (CIF, an extension of random forests)

A thorough description of stepwise variable selection, boosting, and random forests can be found in Sections 6.1.2, 8.2.2, and 8.2.3, respectively, of *Introduction to Statistical Learning* [25]. Sir David Cox’s PH model is one of the most frequently applied methods for the analysis of right-censored time-to-event outcomes [26]. According to the PH assumption, the effect of a unit increase in a predictor is multiplicative with respect to a baseline hazard function. Boosting grows a sequence of decision trees, each using information from the previous trees in an attempt to correct their errors [27, 28]. In contrast, random forests grow an ensemble of de-correlated decision trees, where each tree is grown using a bootstrapped replicate of the original training data [29]. The CIF obtains unbiased recursive partitioning by applying early-stopping and permutation tests and is one of several methods that extends random forests to right-censored time-to-event data [30, 31, 32, 33, 34].

## 2.5 Evaluation of Predictions

**The Brier score** The prognostic value of each risk prediction model was primarily assessed using the Brier score, which depends on both the discrimination and calibration of predicted risk values [35, 36]. Let  $\tilde{Y}_i(t)$  represent the observed status of individual  $i$  at time  $t > 0$  in a testing set of  $M$  observations. Suppose  $\tilde{Y}_i(t) = 1$  if there is an

observed event at or before  $t$  and  $\tilde{Y}_i(t) = 0$  otherwise. The Brier score is computed with

$$\widehat{\text{BS}}(t) = \frac{1}{M} \sum_{i=1}^M \widehat{W}_i(t) \left\{ \tilde{Y}_i(t) - \widehat{S}(t | \mathbf{x}_i) \right\}^2, \quad (1)$$

where, for the  $i^{\text{th}}$  observation,  $\widehat{S}(t | \mathbf{x}_i)$  is the estimated probability of survival at time  $t$  according to a given risk prediction model,  $\mathbf{x}_i$  is the set of input values for predictor variables in the model, and  $\widehat{W}_i(t)$  is the inverse proportional censoring weight at time  $t$  [37]. Thus, the Brier score is the mean squared difference between observed event status and expected event status according to a RPE at time  $t$ . Throughout the current analysis, we set  $t = 6$  months after receiving MCS to focus on short term risk prediction. Models have been developed to simultaneously predict short term and long term mortality risk after receiving MCS, but these are beyond the scope of the current study [38].

**The scaled Brier score** The Brier score is dependent on the rate of observed events, which can make it a difficult metric to interpret. It is often more informative to scale the Brier score based on the Brier score of a naive model. More specifically, for a given risk prediction model, the scaled Brier score is computed as

$$\text{Scaled } \widehat{\text{BS}}(t) \text{ of model} = 1 - \frac{\widehat{\text{BS}}(t) \text{ of model}}{\widehat{\text{BS}}(t) \text{ of naive model}}.$$

As the Brier score for risk prediction is analogous to mean-squared error for prediction of a continuous outcome, the scaled Brier score is analogous to the  $R^2$  statistic. Similar to the  $R^2$  statistic, a scaled  $\widehat{\text{BS}}(t)$  of 1.00 and 0.00 indicate a perfect and worthless model, respectively. In our analyses, a Kaplan-Meier estimate based on the training data (*i.e.*, a risk prediction model that did not use any predictor variables) provided the naive prediction.

## 2.6 Internal Validation via Monte-Carlo Cross-Validation (MCCV)

To assess the prognostic value of each missing data strategy, we internally validated a total of FILL IN modeling algorithm based on combinations of imputation strategies and modeling strategies described in Sections 2.3 and 2.4. For convenience, we use the term ‘modeling algorithm’ to denote the combination of a missing data strategy and a modeling strategy (*e.g.*, imputation using mean/mode values followed by fitting the PH model with stepwise variable selection) [39]. We conducted internal validation using 150 replicates of Monte-Carlo cross validation (MCCV), a resampling technique for internal validation (Figure ??).

**Steps taken in each MCCV replicate** In each replicate, 50% of the available data were used for model training and testing. Imputation was conducted in the training and testing sets, separately, for each imputation strategy. Some imputation strategies (*e.g.*, KNN and random forests) used only the training data to generate imputed values, whereas others (*e.g.*, Bayesian regression, PMM, and hot deck)

After imputation, each modeling algorithm (*i.e.*, stepwise PH models, GBDTs, and the cIF) was applied, separately, to develop a RPE for death or transplant that incorporated up to 50 predictor variables from the entire collection of covariates available.

## References

- [1] Emelia J Benjamin, Michael J Blaha, Stephanie E Chiuve, Mary Cushman, Sandeep R Das, Rajat Deo, J Floyd, M Fornage, C Gillespie, CR Isasi, et al. Heart disease and stroke statistics-2017 update: a report from the american heart association. *Circulation*, 135(10):e146–e603, 2017.
- [2] National Center for Health Statistics. *Health, United States, 2016, With chartbook on long-term trends in health*. Number 2017. Government Printing Office, 2017.
- [3] Chetan B Patel, Jennifer A Cowger, and Andreas Zuckermann. A contemporary review of mechanical circulatory support. *The Journal of Heart and Lung Transplantation*, 33(7):667–674, 2014.
- [4] Mark S Slaughter, Joseph G Rogers, Carmelo A Milano, Stuart D Russell, John V Conte, David Feldman, Benjamin Sun, Antone J Tatroles, Reynolds M Delgado III, James W Long, et al. Advanced heart failure treated with continuous-flow left ventricular assist device. *New England Journal of Medicine*, 361(23):2241–2251, 2009.
- [5] Leslie W Miller. Left ventricular assist devices are underutilized. *Circulation*, 123(14):1552–1558, 2011.
- [6] Garrick C Stewart and Lynne W Stevenson. Keeping left ventricular assist device acceleration on track. *Circulation*, 123(14):1559–1568, 2011.

- [7] James K Kirklin, Francis D Pagani, Robert L Kormos, Lynne W Stevenson, Elizabeth D Blume, Susan L Myers, Marissa A Miller, J Timothy Baldwin, James B Young, and David C Naftel. Eighth annual intermacs report: Special focus on framing the impact of adverse events. *The Journal of Heart and Lung Transplantation*, 36(10):1080–1086, 2017.
- [8] Robert L Kormos, Jennifer Cowger, Francis D Pagani, Jeffrey J Teuteberg, Daniel J Goldstein, Jeffrey P Jacobs, Robert S Higgins, Lynne W Stevenson, Josef Stehlik, Pavan Atluri, et al. The society of thoracic surgeons intermacs database annual report: Evolving indications, outcomes, and scientific partnerships. *The Journal of Heart and Lung Transplantation*, 38(2):114–126, 2019.
- [9] Luigi Adamo, Yuanyuan Tang, Michael E. Nassif, Eric Novak, Philip G. Jones, Shane LaRue, John A. Spertus, and Douglas L. Mann. The heartmate risk score identifies patients with similar mortality risk across all intermacs profiles in a large multicenter analysis. *JACC: Heart Failure*, 4(12):950–958, 2016.
- [10] Donald B Rubin. *Multiple imputation for nonresponse in surveys*, volume 81. John Wiley & Sons, 2004.
- [11] Stef Van Buuren. *Flexible Imputation of Missing Data*. Chapman and Hall/CRC, 2018.
- [12] Trevor Hastie, Robert Tibshirani, and Jerome Friedman. *The elements of statistical learning: data mining, inference, and prediction*. Springer Science & Business Media, 2009.
- [13] Max Kuhn and Kjell Johnson. *Feature engineering and selection: A practical approach for predictive models*. CRC Press, 2019.
- [14] José M Jerez, Ignacio Molina, Pedro J García-Laencina, Emilio Alba, Nuria Ribelles, Miguel Martín, and Leonardo Franco. Missing data imputation using statistical and machine learning methods in a real breast cancer problem. *Artificial intelligence in medicine*, 50(2):105–115, 2010.
- [15] Julie Josse, Nicolas Prost, Erwan Scornet, and Gaël Varoquaux. On the consistency of supervised learning with missing values. *arXiv preprint arXiv:1902.06931*, 2019.
- [16] Melissa J Azur, Elizabeth A Stuart, Constantine Frangakis, and Philip J Leaf. Multiple imputation by chained equations: What is it and how does it work? *International journal of methods in psychiatric research*, 20(1):40–49, 2011.
- [17] Stef Van Buuren, Jaap PL Brand, Catharina GM Groothuis-Oudshoorn, and Donald B Rubin. Fully conditional specification in multivariate imputation. *Journal of statistical computation and simulation*, 76(12):1049–1064, 2006.
- [18] Lawrence R Landerman, Kenneth C Land, and Carl F Pieper. An empirical evaluation of the predictive mean matching method for imputing missing values. *Sociological Methods & Research*, 26(1):3–33, 1997.
- [19] Jiahua Chen and Jun Shao. Nearest neighbor imputation for survey data. *Journal of Official Statistics*, 16(2):113, 2000.
- [20] J. C. Gower. A general coefficient of similarity and some of its properties. *Biometrics*, 27(4):857–871, 1971.
- [21] Rebecca R Andridge and Roderick JA Little. A review of hot deck imputation for survey non-response. *International statistical review*, 78(1):40–64, 2010.
- [22] Daniel J. Stekhoven and Peter Bühlmann. MissForest—non-parametric missing value imputation for mixed-type data. *Bioinformatics*, 28(1):112–118, 10 2011.
- [23] Bhakisipho Twala. An empirical comparison of techniques for handling incomplete data using decision trees. *Applied Artificial Intelligence*, 23(5):373–405, 2009.
- [24] Yufeng Ding and Jeffrey S Simonoff. An investigation of missing data methods for classification trees applied to binary response data. *Journal of Machine Learning Research*, 11(Jan):131–170, 2010.
- [25] Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani. *An introduction to statistical learning*, volume 112. Springer, 2013.
- [26] David G Kleinbaum and Mitchel Klein. *Survival analysis*, volume 3. Springer, 2010.
- [27] Jerome H Friedman. Greedy function approximation: a gradient boosting machine. *Annals of statistics*, pages 1189–1232, 2001.
- [28] Tianqi Chen and Carlos Guestrin. Xgboost: A scalable tree boosting system. In *Proceedings of the 22Nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, KDD ’16, pages 785–794, New York, NY, USA, 2016. ACM.
- [29] Leo Breiman. Random forests. *Machine Learning*, 45(1):5–32, Oct 2001.

- 1 [30] Torsten Hothorn, Peter Buehlmann, Sandrine Dudoit, Annette Molinaro, and Mark Van Der Laan. Survival  
2 ensembles. *Biostatistics*, 7(3):355–373, 2006.
- 3 [31] Carolin Strobl, Anne-Laure Boulesteix, Achim Zeileis, and Torsten Hothorn. Bias in random forest variable  
4 importance measures: Illustrations, sources and a solution. *BMC Bioinformatics*, 8(25), 2007.
- 5 [32] Carolin Strobl, Anne-Laure Boulesteix, Thomas Kneib, Thomas Augustin, and Achim Zeileis. Conditional  
6 variable importance for random forests. *BMC Bioinformatics*, 9(307), 2008.
- 7 [33] Hemant Ishwaran, Udaya B Kogalur, Eugene H Blackstone, Michael S Lauer, et al. Random survival forests. *The*  
8 *annals of applied statistics*, 2(3):841–860, 2008.
- 9 [34] Byron C Jaeger, D Leann Long, Dustin M Long, Mario Sims, Jeff M Szychowski, Yuan-I Min, Leslie A  
10 Mcclure, George Howard, Noah Simon, et al. Oblique random survival forests. *The Annals of Applied Statistics*,  
11 13(3):1847–1883, 2019.
- 12 [35] Erika Graf, Claudia Schmoor, Willi Sauerbrei, and Martin Schumacher. Assessment and comparison of prognostic  
13 classification schemes for survival data. *Statistics in medicine*, 18(17-18):2529–2545, 1999.
- 14 [36] Kaspar Rufibach. Use of brier score to assess binary predictions. *Journal of clinical epidemiology*, 63(8):938–939,  
15 2010.
- 16 [37] Thomas A Gerds and Martin Schumacher. Consistent estimation of the expected brier score in general survival  
17 models with right-censored event times. *Biometrical Journal*, 48(6):1029–1040, 2006.
- 18 [38] Eugene H Blackstone, David C Naftel, and Malcolm E Turner Jr. The decomposition of time-varying hazard into  
19 phases, each incorporating a separate stream of concomitant information. *Journal of the American Statistical*  
20 *Association*, 81(395):615–624, 1986.
- 21 [39] Max Kuhn and Kjell Johnson. *Applied predictive modeling*, volume 26. Springer, 2013.