2021

FIS Project1

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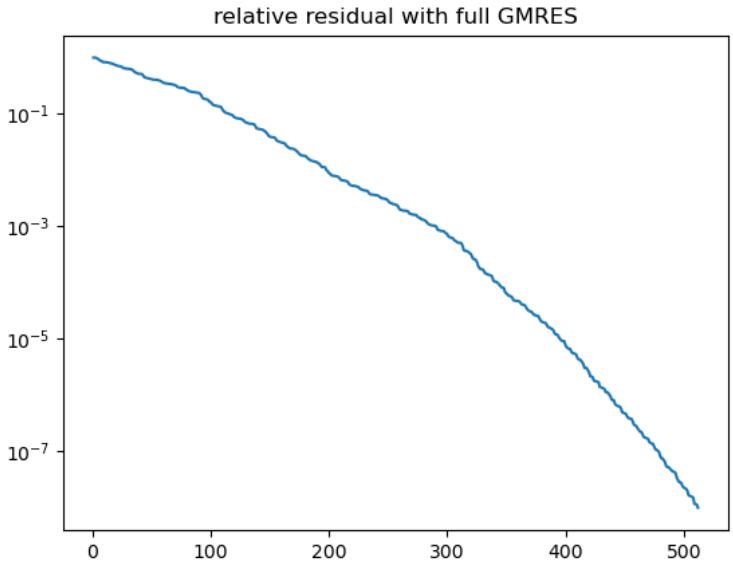
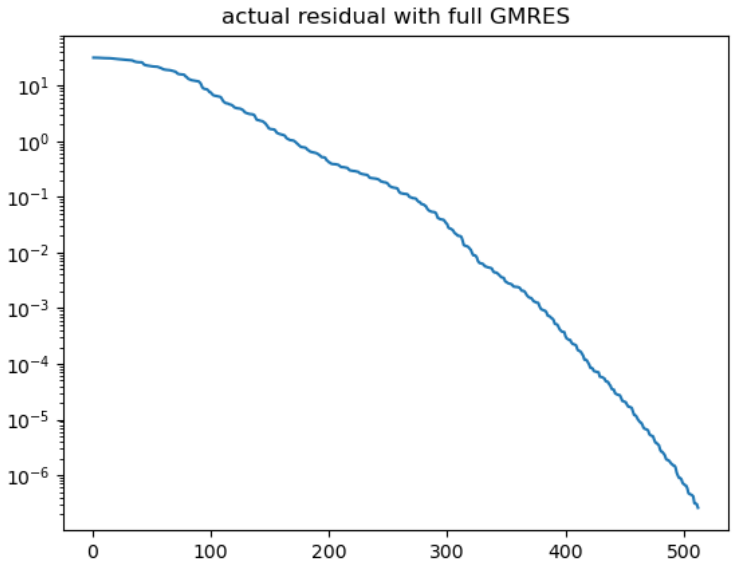
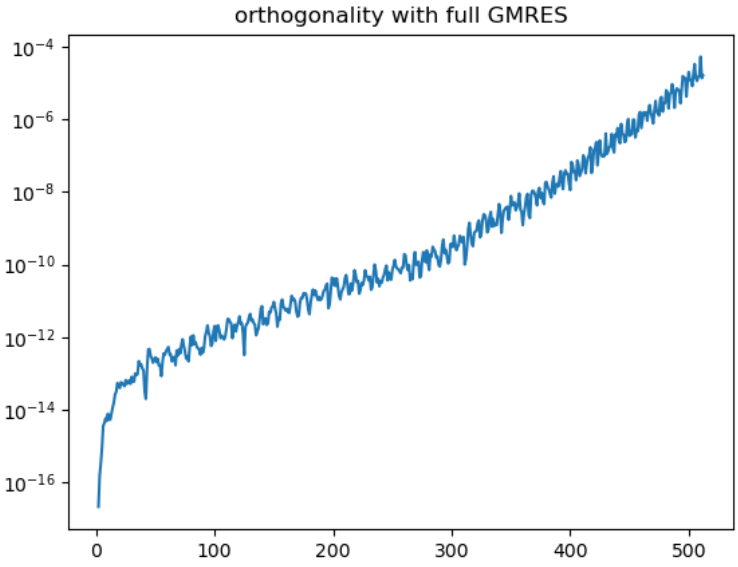
1. Full GMRES

For the full GMRES without preconditioning (set m = 600 to make it converge before restart), the relative error will converge below the threshold 10^{-8} after the 512th iteration. I would display relative residual ($\left\|\frac{r_k}{r_0}\right\|$), actual residual ($\|x_k - x^*\|$) and orthogonality here.

(1) Unpreconditioned Full GMRES





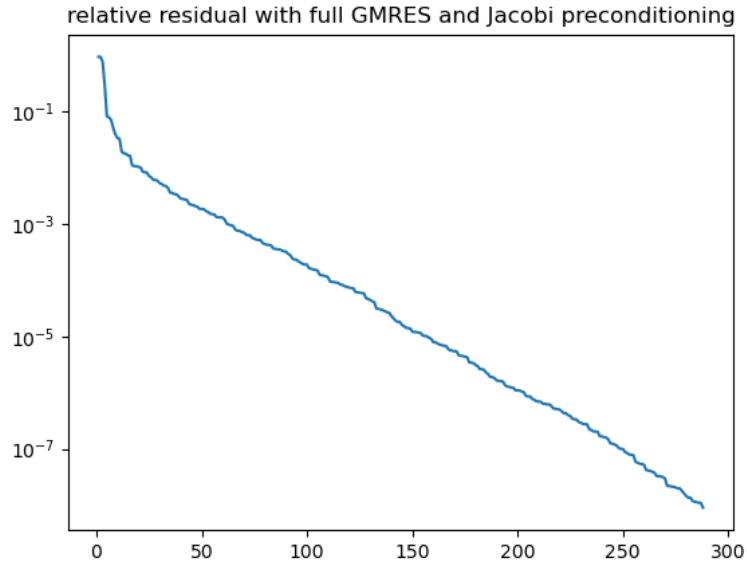



```
511 th iteration
512 th iteration
spent 82.546875 seconds
```

So we know that full GMRES without preconditioning takes 512 vectors to solve the system.

Next, apply Jacobi respectively to see the difference.

(2) Full GMRES with Jacobi as preconditioner



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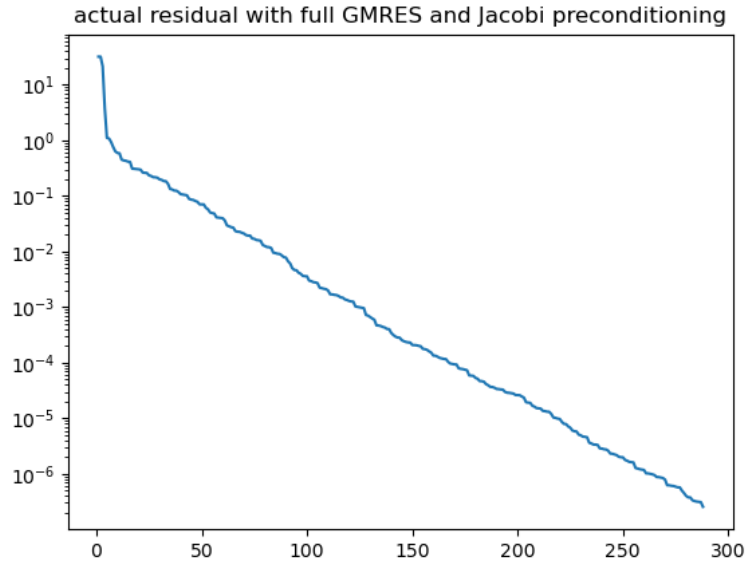
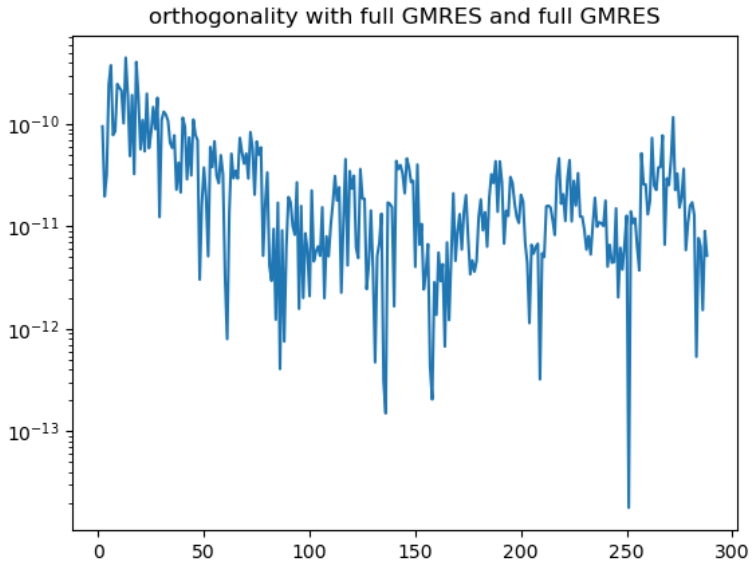


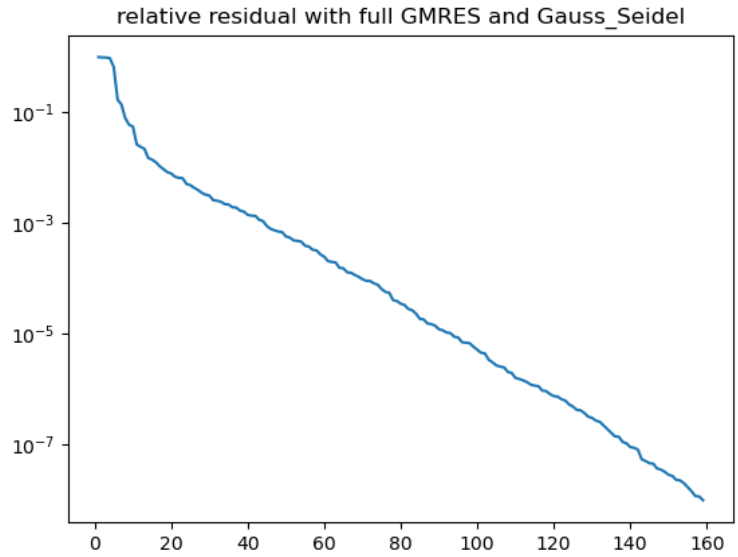


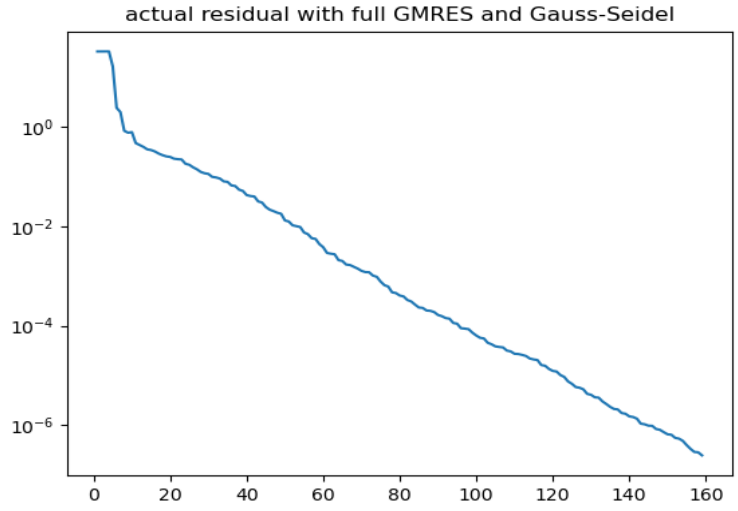

```
287 th iteration
288 th iteration
spent 73.046875 seconds
```

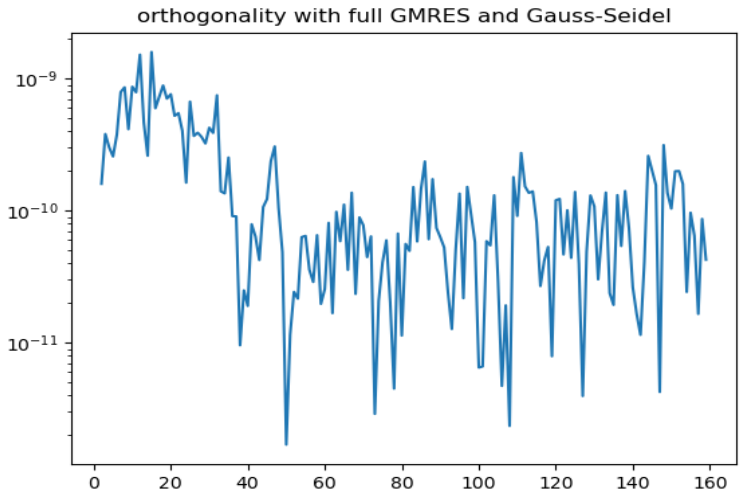
With Jacobi as preconditioner, we need just 288 vectors tot finish the calculation. The number of needed vectors decreased enormously.

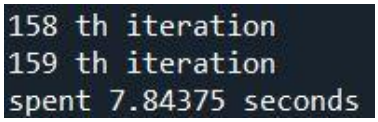
Finally, the GMRES was done with Gauss-Seidel as preconditioner.

(3) Full GMRES with Gauss-Seidel as preconditioner









It took only 159 vectors to solve the same system. Moreover, the spent time is just 1/10 of it without preconditioner.

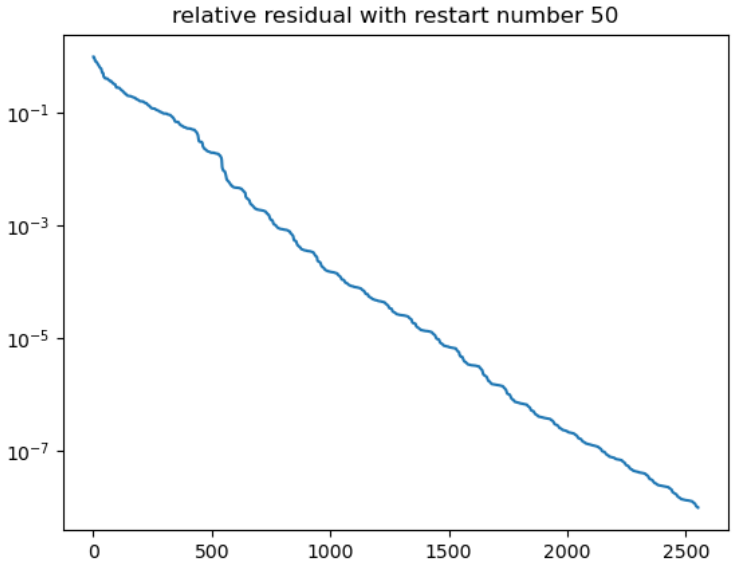
2. Restarted GMRES

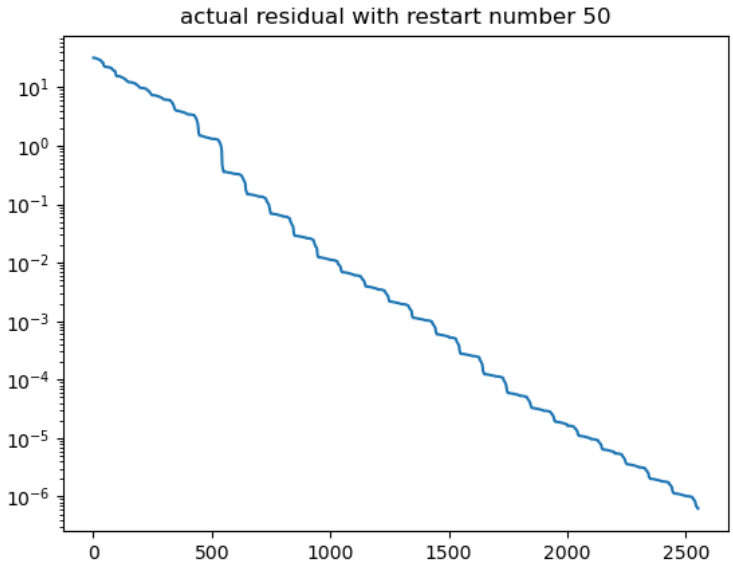
Restarted GMRES with m = 10	Can't converge under the threshold
Restarted GMRES with m = 12	181.5s
Restarted GMRES with m = 30	53.3s
Restarted GMRES with m = 50	42.9s
Restarted GMRES with m = 100	48.9s
Full GMRES (equal to m = 600)	84.5s

From the table above, we notice that Restarted GMRES with any parameter m except for m = 10 are faster than Full GMRES. Restarted GMRES with m = 10 can't converge to a number below the threshold.
I think the reason is that I build the matrix multiply function on my own, which takes many for loop to reach the result. It will consume a huge amount of time to finish the computation, especially when the size of matrix is large. Full GMRES requires multiplying matrix of size bigger than 100. As a result, Full GMRES will take much more time than Restarted GMRES.
In a better case, in which the memory are allocated well, Full GMRES might be faster than restarted GMRES. However, Full GMRES demand a large memory space to store all of the Krylov vectors needed, while restarted GMRES will fresh the vectors after m iteration. Thus, although Full GMRES probably take less time, it requires higher space complexity. We prefer to select Restarted GMRES if the available memory space is not enough.

Since m = 50 is likely to be the best parameter, I also apply those 2 preconditioner with this max iteration number.

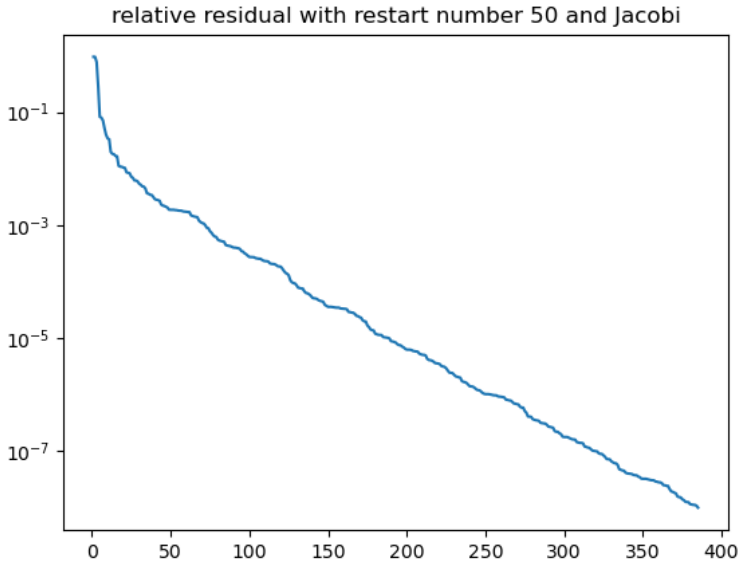
(1) Unpreconditioned with m = 50

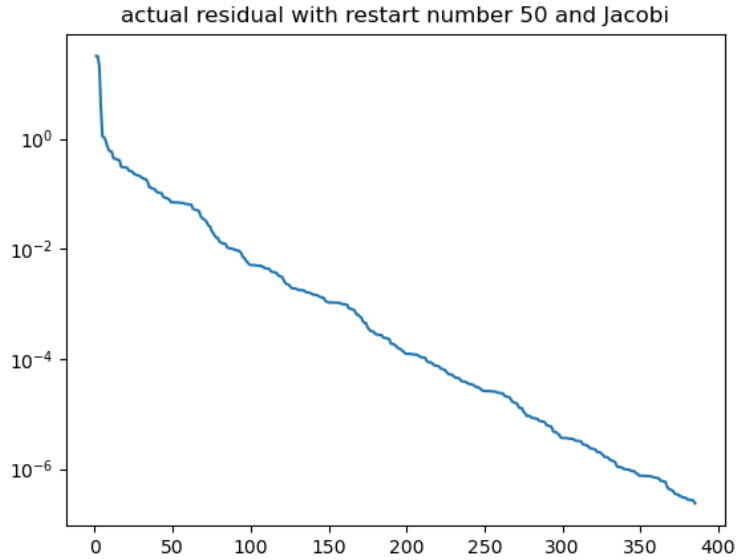





```
2550 th iteration
2551 th iteration
spent 130.953125 seconds
```

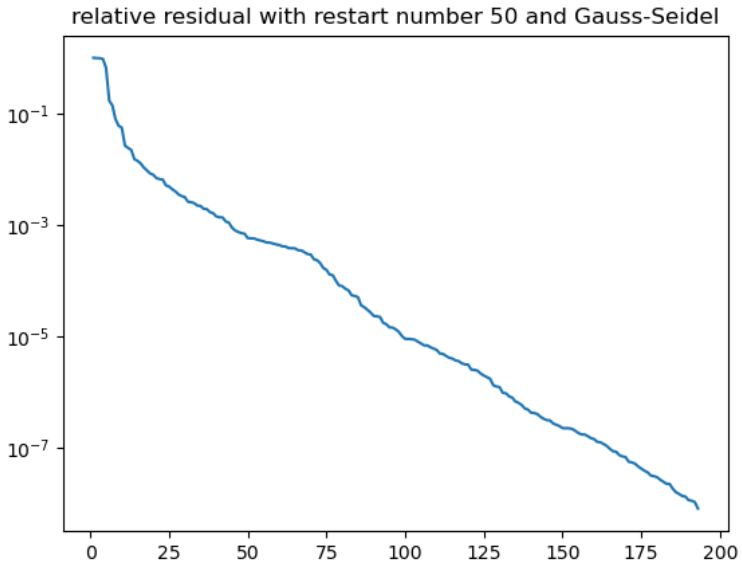
(2) Jacobi as preconditioner with m = 50

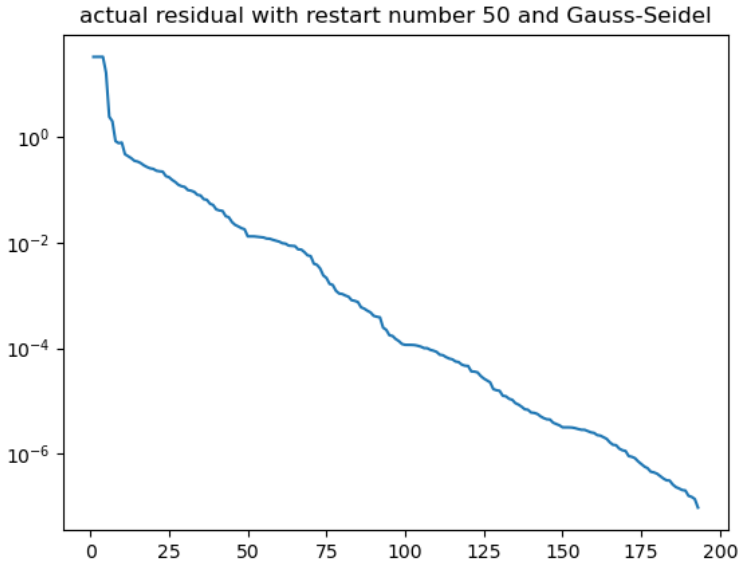





```
384 th iteration
385 th iteration
spent 6.421875 seconds
```

(3) Gauss-Seidel as preconditioner with m = 50






```
192 th iteration
193 th iteration
spent 3.421875 seconds
```

3. CG Method

I finished the CG algorithm within the last hour before the deadline. However, the size of the matrix is too large to be run in time.

I can here just display the iteration number and the residue that shows the downward trend of the residual. And I also submitted the CG method python file.

```
0.004452887435770618
15510 th iteration
0.0051468643961511615
15511 th iteration
0.005942591502632947
15512 th iteration
0.006805335466417703
15513 th iteration
0.006256501970239051
15514 th iteration
0.005342174671685291
15515 th iteration
0.004773982342260929
15516 th iteration
0.004765810186296717
15517 th iteration
0.005465819326144244
15518 th iteration
0.0065016895494914425
15519 th iteration
0.006905301532168831
15520 th iteration
0.007376050067398636
15521 th iteration
0.007611780494449345
15522 th iteration
0.006738457923620225
15523 th iteration
0.005879360588012883
15524 th iteration
0.005724884581484714
15525 th iteration
0.005745898168355631
```