2021

FIS Project1

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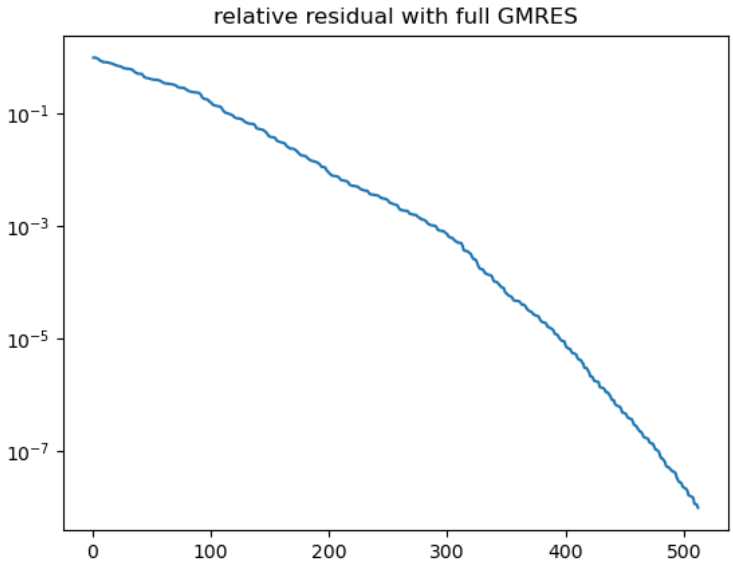
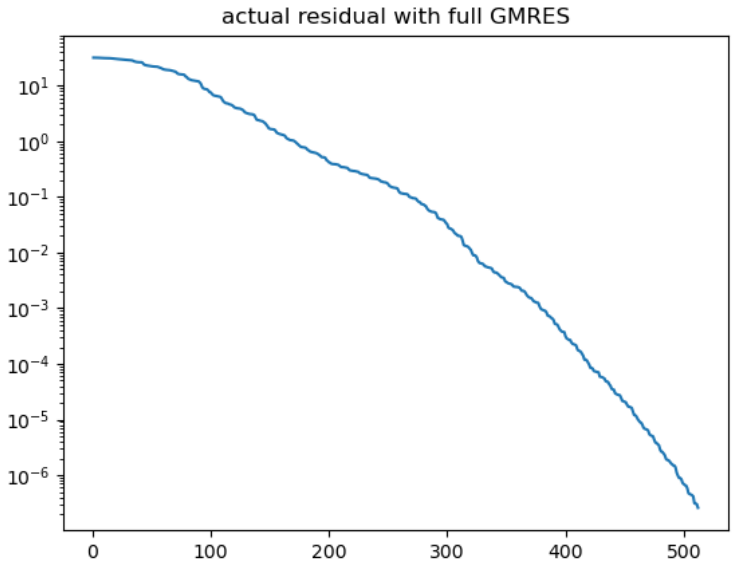
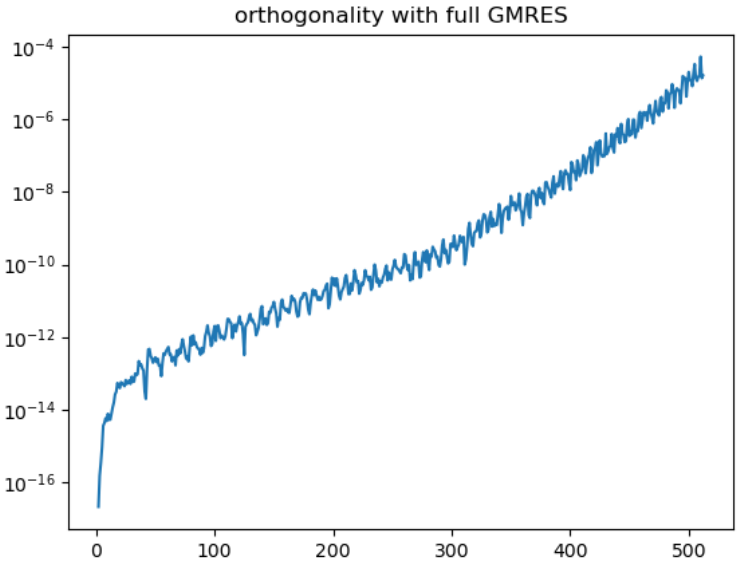
1. Full GMRES

For the full GMRES without preconditioning (set m = 600 to make it converge before restart), the relative error will converge below the threshold 10^{-8} after the 512th iteration. I would display relative residual ($\left\|\frac{r_k}{r_0}\right\|$), actual residual ($\|x_k - x^*\|$) and orthogonality here.

(1) Unpreconditioned Full GMRES








```
511 th iteration
512 th iteration
spent 82.546875 seconds
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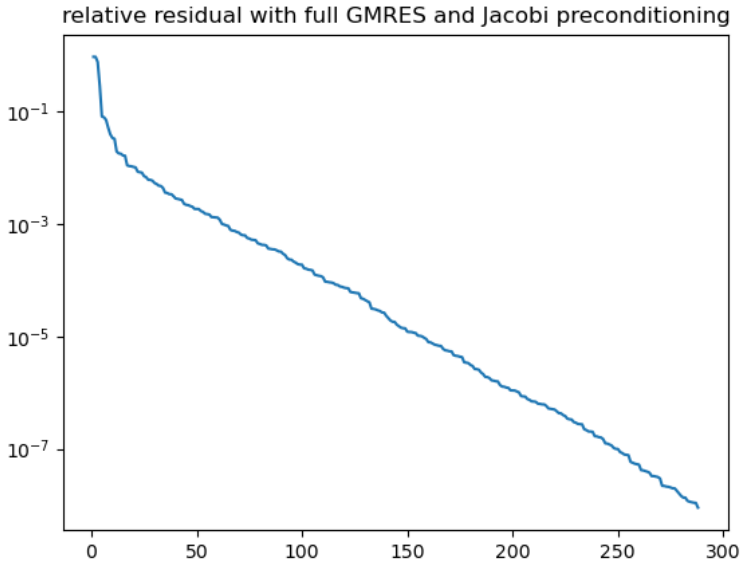
So we know that full GMRES without preconditioning takes 512 vectors to solve the system.

Next, apply Jacobi respectively to see the difference.

(2) Full GMRES with Jacobi as preconditioner



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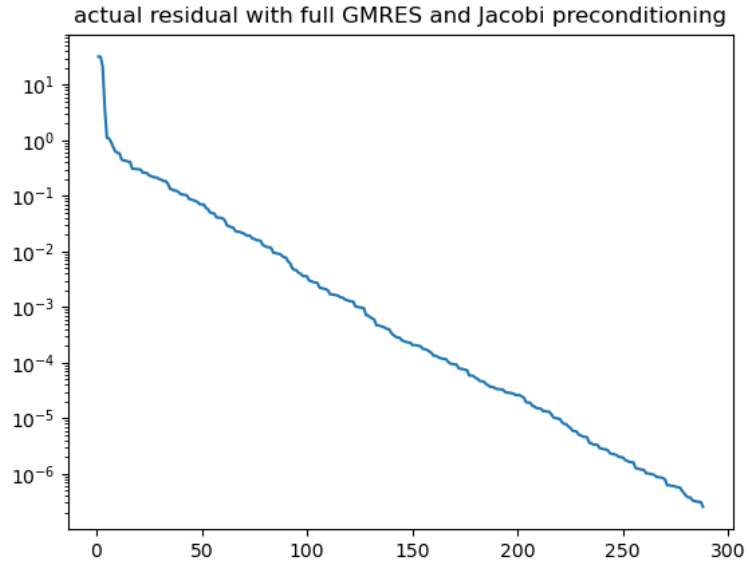

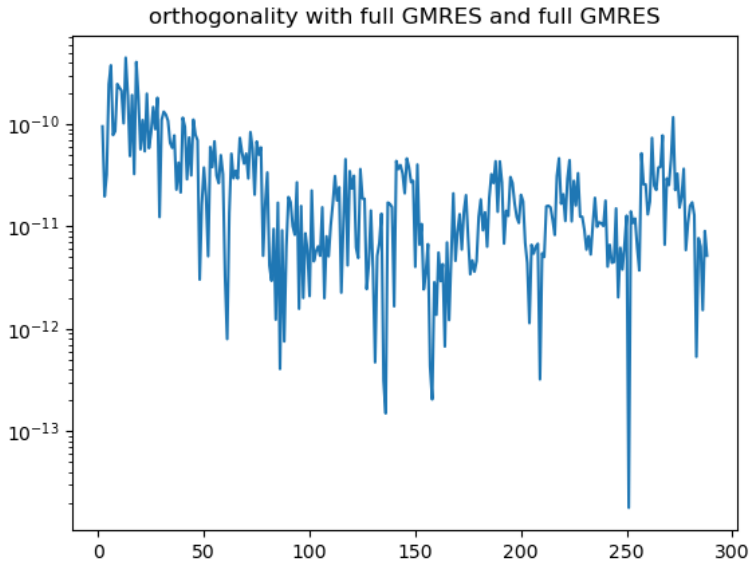


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287 th iteration
288 th iteration
spent 73.046875 seconds
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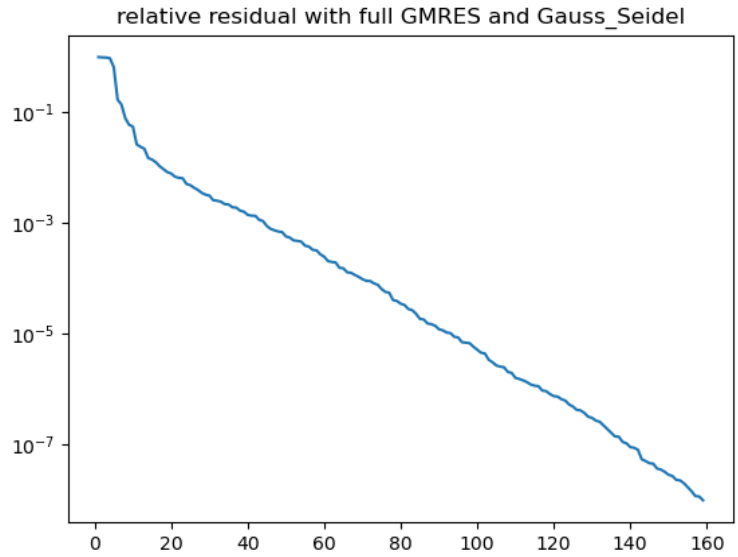
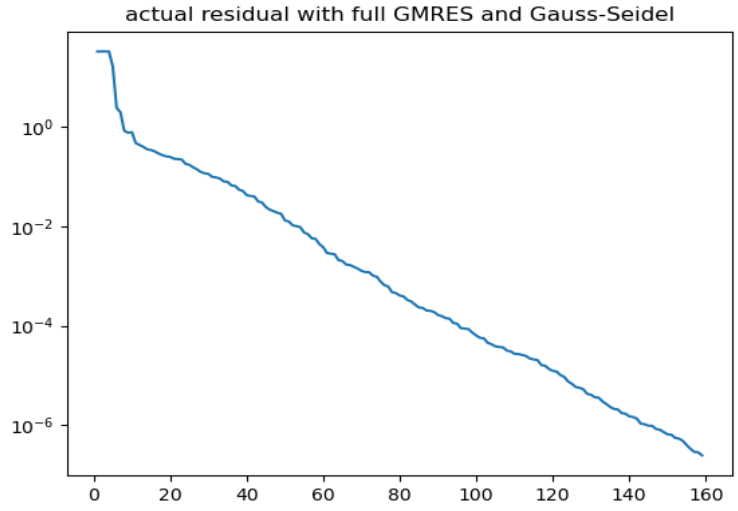
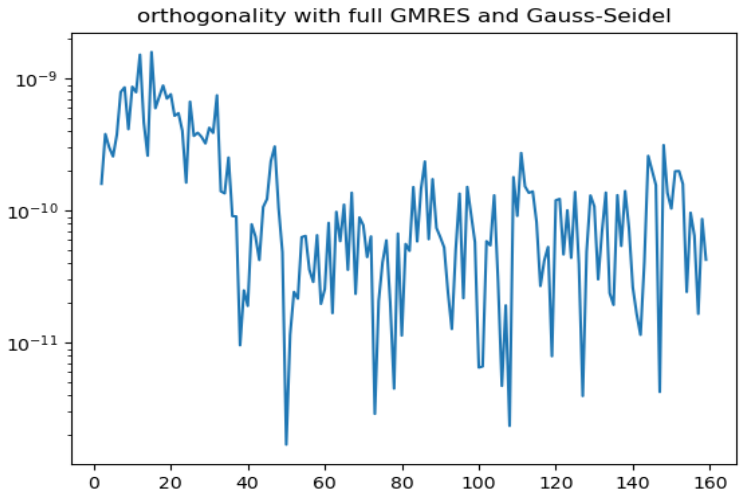
With Jacobi as preconditioner, we need just 288 vectors tot finish the calculation. The number of needed vectors decreased enormously.

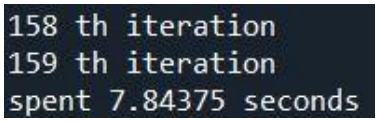
Finally, the GMRES was done with Gauss-Seidel as preconditioner.

(3) Full GMRES with Gauss-Seidel as preconditioner









It took only 159 vectors to solve the same system. Moreover, the spent time is just 1/10 of it without preconditioner.

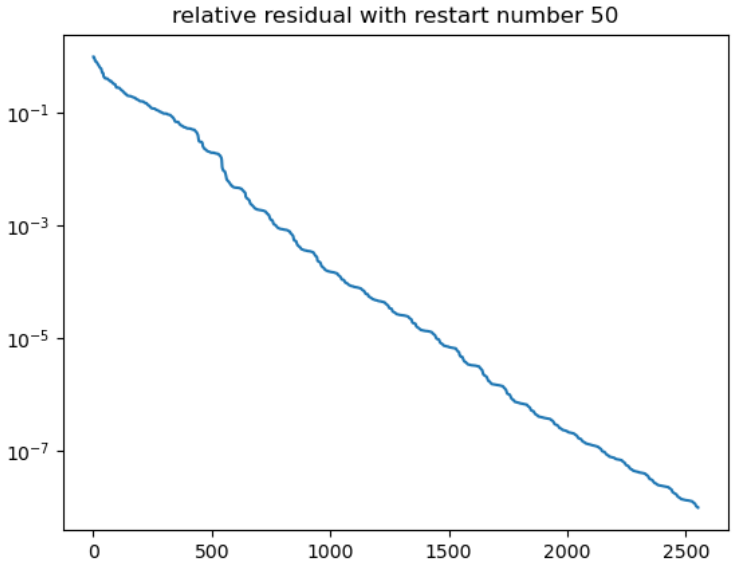
2. Restarted GMRES

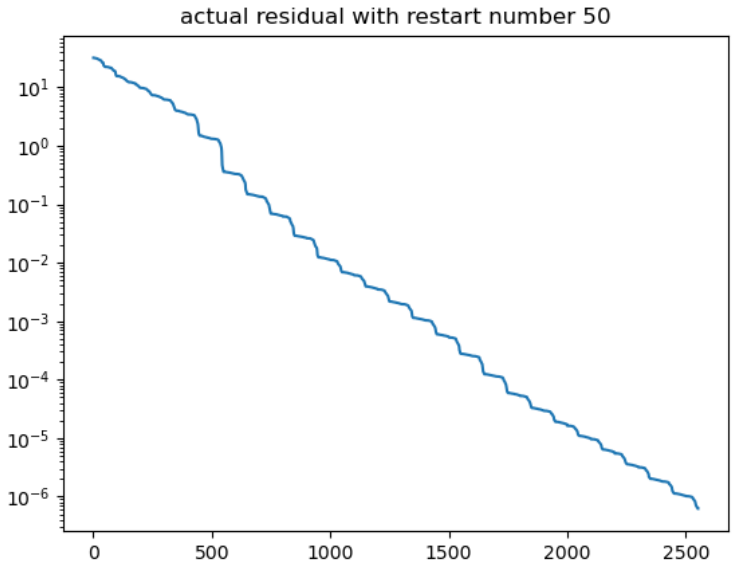
Restarted GMRES with m = 10	Can't converge under the threshold
Restarted GMRES with m = 12	181.5s
Restarted GMRES with m = 30	53.3s
Restarted GMRES with m = 50	42.9s
Restarted GMRES with m = 100	48.9s
Full GMRES (equal to m = 600)	84.5s

From the table above, we notice that Restarted GMRES with any parameter m except for m = 10 are faster than Full GMRES. Restarted GMRES with m = 10 can't converge to a number below the threshold.
I think the reason is that I build the matrix multiply function on my own, which takes many for loop to reach the result. It will consume a huge amount of time to finish the computation, especially when the size of matrix is large. Full GMRES requires multiplying matrix of size bigger than 100. As a result, Full GMRES will take much more time than Restarted GMRES.
In a better case, in which the memory are allocated well, Full GMRES might be faster than restarted GMRES. However, Full GMRES demand a large memory space to store all of the Krylov vectors needed, while restarted GMRES will fresh the vectors after m iteration. Thus, although Full GMRES probably take less time, it requires higher space complexity. We prefer to select Restarted GMRES if the available memory space is not enough.

Since m = 50 is likely to be the best parameter, I also apply those 2 preconditioner with this max iteration number.

(1) Unpreconditioned with m = 50



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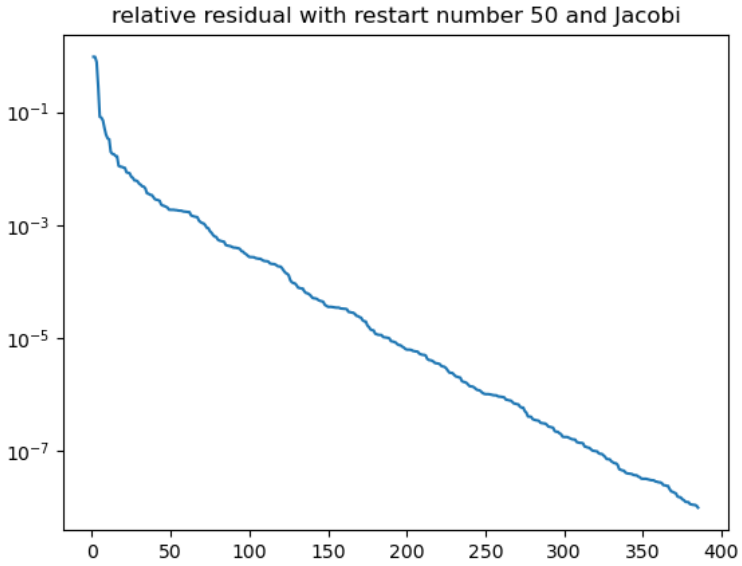
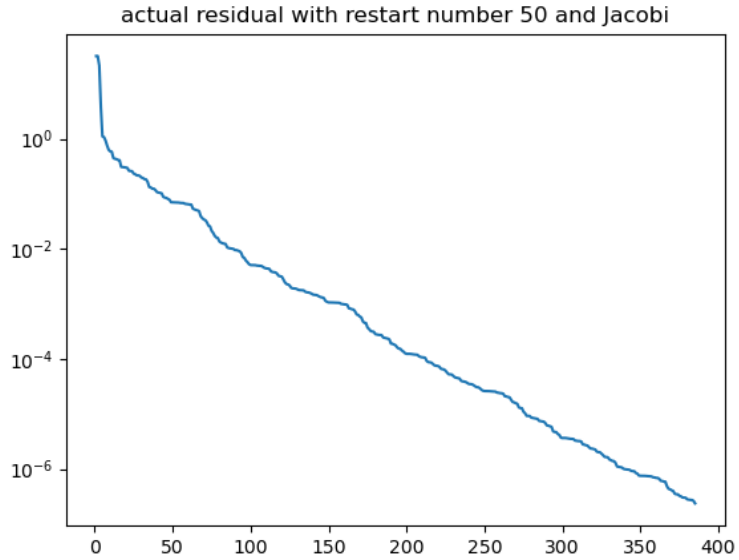
2550 th iteration
2551 th iteration
spent 130.953125 seconds
```

(2) Jacobi as preconditioner with m = 50



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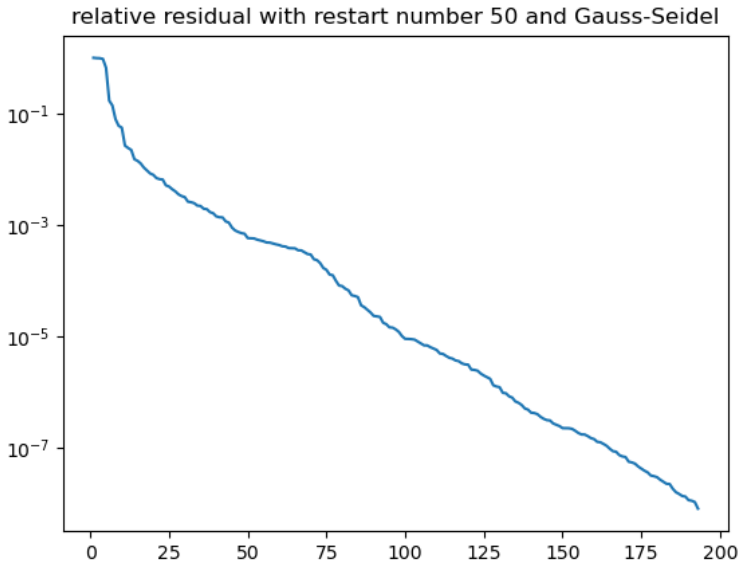
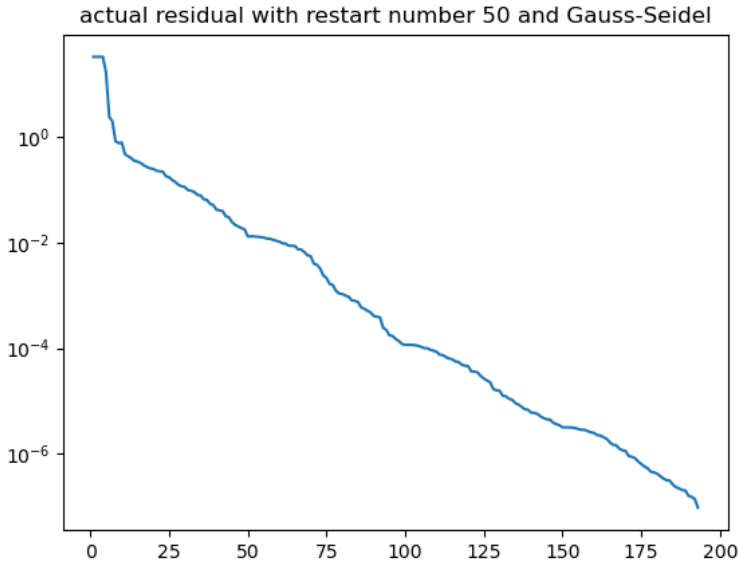
384 th iteration
385 th iteration
spent 6.421875 seconds
```

(3) Gauss-Seidel as preconditioner with m = 50






```
192 th iteration
193 th iteration
spent 3.421875 seconds
```

3. CG Algorithm

Not implemented