

## Midterm 2 Review - Math 243

1. Determine the type of the equilibrium (sink, source, spiral sink, spiral source, saddle, or center) at the origin for each of the following linear systems.

$$(a) \frac{dX}{dt} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} X$$

$$(b) \frac{dY}{dt} = \begin{bmatrix} -1 & -5 \\ 4 & -2 \end{bmatrix} Y$$

$$(c) \frac{dZ}{dt} = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} Z$$

2. Show that  $\begin{bmatrix} 2 \\ i \end{bmatrix}$  is an eigenvector for the matrix  $A = \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix}$ . What is the corresponding eigenvalue?

3. Use the eigenvalue and eigenvector from the previous problem to find the general solution of the system

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 0 & 4 \\ -1 & 0 \end{bmatrix} \mathbf{x}.$$

4. Consider the one-parameter system  $\frac{d\mathbf{y}}{dt} = \begin{bmatrix} 1 & a \\ 1 & 3 \end{bmatrix} \mathbf{y}$ . Use the trace and determinant to find the values of  $a$  where the type of equilibrium changes. Describe in words how the type of equilibrium depends on  $a$ .

5. Find the eigenvalues for the matrix  $\begin{bmatrix} 2 & -6 \\ -2 & 3 \end{bmatrix}$ . For each eigenvalue, find a corresponding eigenvector.

6. Use the eigenvectors and eigenvalues from the last problem to find the general solution of the linear system

$$\begin{aligned}\frac{dx}{dt} &= 2x - 6y, \\ \frac{dy}{dt} &= -2x + 3y.\end{aligned}$$

7. The system

$$\begin{aligned}\frac{dx}{dt} &= x - 2y, \\ \frac{dy}{dt} &= 3y\end{aligned}$$

is partially decoupled. Suppose that  $x(0) = 1$  and  $y(0) = 2$ . Solve this initial value problem by solving for  $y(t)$  first, and then use  $y(t)$  to find  $x(t)$ .

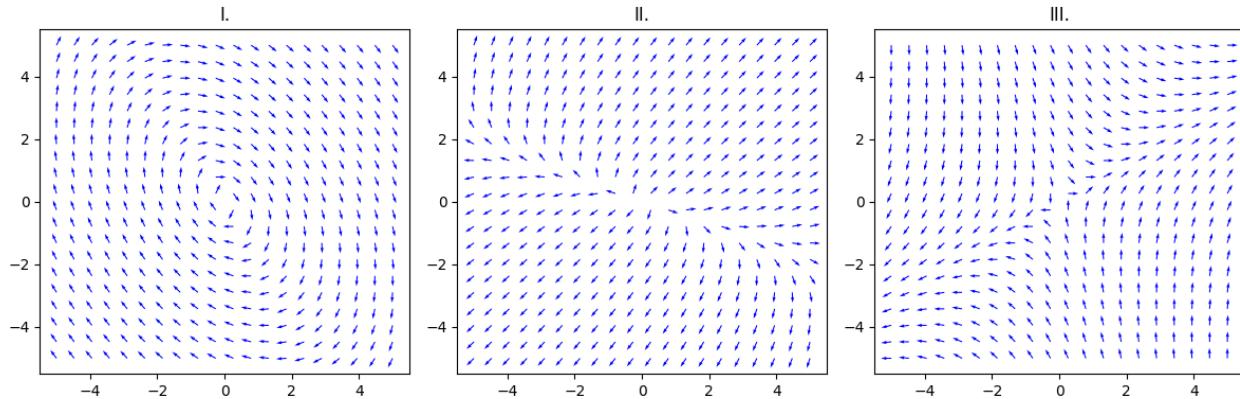
8. The previous example can be expressed using matrix notation as  $\mathbf{x}' = A\mathbf{x}$  where  $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ .

It turns out that

$$e^{At} = \begin{bmatrix} e^t & -e^{3t} + e^t \\ 0 & e^{3t} \end{bmatrix}.$$

Use this to solve the initial value problem above.

9. The following direction fields represent linear systems  $\mathbf{x}' = A\mathbf{x}$ . Match each direction field with the correct matrix from the options below.



A.  $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

B.  $\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$

C.  $\begin{bmatrix} -4 & 0 \\ 1 & -3 \end{bmatrix}$

D.  $\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}$

10. Find the general (real-valued) solution to the system

$$\frac{dx}{dt} = x + 3z,$$

$$\frac{dy}{dt} = -3y,$$

$$\frac{dz}{dt} = 3x + z.$$

11. Find the solution of the last problem that satisfies the initial condition

$$x(0) = 5,$$

$$y(0) = 2,$$

$$z(0) = -1.$$

12. The system  $\frac{dX}{dt} = \begin{bmatrix} -5 & 3 \\ -3 & 1 \end{bmatrix} X$  has  $\lambda = -2$  as a repeated eigenvalue. What is the solution to the system with initial condition  $X(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ ?