Last time, we proved that if $f \in C^2[a,b]$ has a root $r \in [a,b]$ and there are constants L, M > 0 such that $|f'(x)| \ge L$ and $|f''(x)| \le M$ for all $x \in [a,b]$, then

$$|x_{n+1} - r| \le \frac{M}{2L}|x_n - r|^2$$

whenever $x_n \in [a, b]$. From this theorem, we also showed that

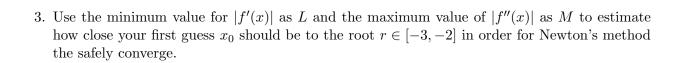
$$|x_n - r| \le \left(\frac{M}{2L}\right)^{2^n - 1} |x_0 - r|^{2^n}$$

for all n as long as the Newton's method iterates x_n stay inside the interval [a, b].

- 1. Suppose we have a function $f \in C^2(\mathbb{R})$ such that $|f'(x)| \geq 2$ and $|f''(x)| \leq 5$ for all x.
 - (a) How bad could the error $|x_n r|$ get after n = 10 iterations of Newton's method if we start with an initial guess x_0 such that $|x_0 r| = 1$?

(b) How bad could the error $|x_n-r|$ get after n=10 iterations of Newton's method if $|x_0-r|=0.5$?

2. One of the roots of the polynomial $x^3 - 5x + 3$ is in the interval [-3, -2]. On this interval, what is the minimum value of |f'(x)|? What is the maximum value of |f''(x)|?



4. Use Newton's method to find the root of $x^3 - 5x + 3$ in the interval [-3, -2]. Your answer should be accurate to at least 10 decimal places.

5. Find the other two roots of $x^3 - 5x + 3$ using Newton's method.