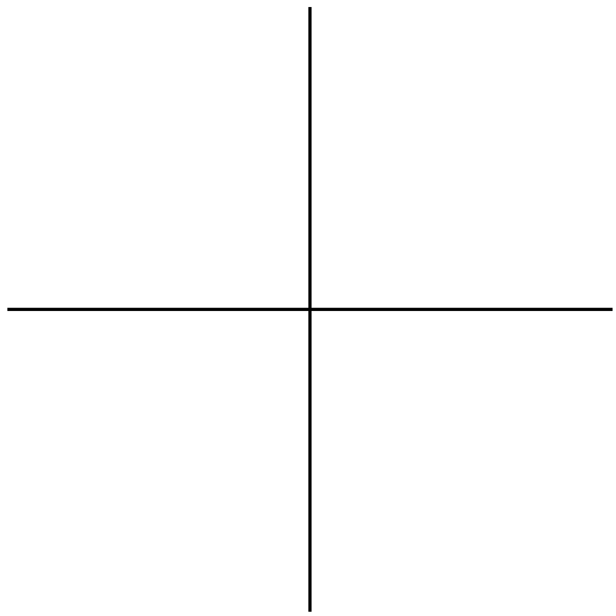
Homework 2 - Math 243 **Name:** ________________________

1. Suppose that a simple electric circuit has a resistor with resistance R in ohms (Ω), a capacitor with capacitance C in farads (F) and a (time-dependent) voltage source that provides $E(t)$ volts (V). The voltage drop across the capacitor E_C satisfies the differential equation
$$RC\frac{dE_C}{dt} + E_C = E(t).$$
Solve this differential equation if $E(t)$ is a constant 5 volts, and $R = 2\ \Omega$ and $C = 1$ F with initial condition $E_C(0) = 10$ V.

2. Draw a rough sketch of a slope field for the following differential equations.



3. Substitute e^{at} for $y(t)$ in each of the following differential equations. Then simplify and find all values of the constant a such that $y = e^{at}$ is a solution.

   (a) $y'' + 2y' - 15y = 0.$

   (b) $y''' - 4y'' + 4y' = 0.$

4. A stockpile of nuclear waste initially contains 0.8 kilograms of radium. The radium decays exponentially at rate r, but new radium waste is added to the stockpile at a rate of 0.02 kilograms per year. Write an differential equation modeling the mass of radium in the stockpile. You don't need to look up the value of the constant r.

Solve the following separable equations with the given initial values.

5. $x\,dx - y^2\,dy = 0$, with $y(0) = 1$.

6. $\dfrac{dy}{dt} = \dfrac{\cos t}{y}$, with $y(0) = 5$.

7. $xy' = \sqrt{1 - y^2}$, with $y(1) = 0$.

   Hint: recall that $\dfrac{d}{dy}\arcsin(y) = \dfrac{1}{\sqrt{1-y^2}}$.

8. $\dfrac{dy}{dt} = y^2$ with $y(0) = 2$.