

# Formula Sheet

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## Integrating Factors

A first order linear differential equation  $\frac{dy}{dt} + f(t)y = g(t)$  has general solution

$$y(t) = \frac{\int e^{F(t)} g(t) dt}{e^{F(t)}}$$

where  $F(t)$  is any antiderivative of  $f(t)$ .

## Trace-Determinant Formula for Eigenvalues

For a 2-by-2 matrix  $A$ ,

$$\lambda = \frac{\text{tr } A \pm \sqrt{(\text{tr } A)^2 - 4 \det A}}{2}.$$

## Linear Systems

For a linear system  $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$ :

<b>Straight-line solutions.</b> $\mathbf{x}(t) = Ce^{\lambda t}\mathbf{v}.$	<b>Matrix exponential solution.</b> $\mathbf{x}(t) = e^{At}\mathbf{x}(0).$
<b>Complex eigenvalues</b> If $\lambda = \alpha + i\beta$ is a complex eigenvalue with eigenvector $\mathbf{v}$ , then the real and imaginary parts of $e^{\alpha t}(\cos(\beta t) + i \sin(\beta t))\mathbf{v}$ are both real-valued solutions.	<b>Repeated eigenvalues</b> If $A$ is a 2-by-2 matrix with repeated eigenvalue $\lambda$ . $\mathbf{x}(t) = e^{\lambda t}(I + t(A - \lambda I))\mathbf{x}(0).$