Formula Sheet

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Integrating Factors

A first order linear differential equation $\frac{dy}{dt} + f(t)y = g(t)$ has general solution

$$y(t) = \frac{\int e^{F(t)} g(t)\, dt}{e^{F(t)}}$$

where $F(t)$ is any antiderivative of $f(t)$.

Trace-Determinant Formula for the Characteristic Polynomial

For a 2-by-2 matrix A, the characteristic polynomial is

$$\lambda^2 - \lambda \operatorname{tr} A + \det A.$$

Linear Systems

For a linear system $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$:

Straight-line solutions $$\mathbf{x}(t) = Ce^{\lambda t}\mathbf{v}.$$	**Matrix exponential solution** $$\mathbf{x}(t) = e^{At}\mathbf{x}(0).$$
Complex eigenvalues If $\lambda = \alpha \pm i\beta$ is a complex eigenvalue with eigenvector $\mathbf{v}$, then the real and imaginary parts of $$e^{\alpha t}(\cos(\beta t) \pm i \sin(\beta t))\mathbf{v}$$ are both real-valued solutions.	**Repeated eigenvalues** If A is a 2-by-2 matrix with a repeated eigenvalue λ, then the solution is $$\mathbf{x}(t) = e^{\lambda t}(I + t(A - \lambda I))\mathbf{x}(0).$$

Hamiltonian Systems

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}$$
$$\frac{dy}{dt} = -\frac{\partial H}{\partial x}$$