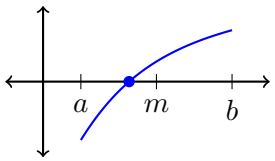
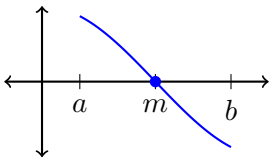
The Bisection Method **Math 342**

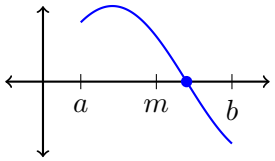
This is a simple algorithm to find a root of any continuous function $f(x)$. If $f(x)$ is positive at one end of an interval $[a, b]$ and negative on the other end, then it must cross somewhere in the middle. Let $m = \frac{1}{2}(a + b)$ be the midpoint of the interval. There are three possible cases:



$f(x)$ changes sign between a and m



$f(m) = 0$ exactly



$f(x)$ changes sign between m and b

You can tell which possibility occurs by looking at the sign (positive, negative, or zero) of $f(a)$ and $f(m)$. This leads to an algorithm for finding the root:

Step 0. Start with a continuous function f and the endpoints of an interval $[a, b]$ such that $f(a)$ and $f(b)$ have opposite signs. Also choose a maximum number of repetitions n.

Step 1. Compute the midpoint $m = (a + b)/2$ and find $f(m)$. If $f(m) = 0$, then stop since you have found a root; otherwise go to the next step.

Step 2. Check whether $f(a)$ and $f(m)$ have the same sign or not.

- If they are different, then there is a root between a and m.
- If they are the same, then there is a root between m and b.

Step 3. Replace $[a, b]$ with either $[a, m]$ or $[m, b]$ depending on step 2.

Step 4. Repeat steps 1-3 up to n times. Then return the midpoint of the final interval.

1. Complete the Python function below to implement the bisection algorithm by replacing the comments (lines starting with #) with code.

```
from math import *
from numpy import sign

def bisection(f, a, b, n):
    for i in range(n):
        # define m
        if sign(m) == 0:
            return m
        elif sign(a) == sign(m):
            # update a
        else:
            # update b
    # After the for-loop ends, be sure to return an answer
```

2. Use your bisection function with $n = 20$ to find the roots of the following functions.

   (a) $f(x) = \tan(x) - 1$ on the interval $[0, 1]$. Hint: define $f(x)$ using a lambda function like this: `f = lambda x: tan(x) - 1`

   (b) $x^3 - 2$ on the interval $[0, 2]$.

3. Why wouldn't your bisection function work to find the root of $\tan(x)$ on the interval $[1, 2]$? What happens if you run it anyway?