

## Homework 4 - Math 243

Name: \_\_\_\_\_

*Solve the following initial value problems using the integrating factors method.*

1.  $\frac{dx}{dt} + 2tx = 4e^{-t^2}, x(0) = 5.$

2.  $\frac{dy}{dt} + \frac{y}{t+1} = 2, y(0) = 3.$

3. A 30-gallon tank initially contains 10 gallons of water and 4 pounds of dissolved salt. A concentrated salt water solution containing 1 pound of salt per gallon is added to the tank at a rate of 2 gallons per minute, while a well-mixed solution is drained from the tank at a rate of 1 gallon per minute. Find a differential equation to model the amount of salt  $y(t)$  in the tank. You do not need to solve the differential equation.

*Find all equilibrium solutions for the following systems of equations.*

4. 
$$\begin{aligned}\frac{dx}{dt} &= x - 2y + 4 \\ \frac{dy}{dt} &= 2x + y - 7\end{aligned}$$

5. 
$$\begin{aligned}\frac{dR}{dt} &= 5R - 0.5RF \\ \frac{dF}{dt} &= -4F + 0.1RF\end{aligned}$$

6. Consider the differential equation  $\frac{dv}{dt} + 6v = e^{2t}$ .

(a) What is the corresponding homogeneous differential equation?

(b) What is the general solution of the homogeneous differential equation?

(c) Find one particular solution for the original equation of the form  $y(t) = Ae^{2t}$ . Then combine the particular solution and the homogeneous solution to express the general solution.

7. Consider the differential equation  $\frac{dy}{dt} + y = 6 \cos x$ .

(a) Find constants  $A$  and  $B$  such that  $y(t) = A \cos t + B \sin t$  is a solution to this ODE.

(b) Find the general solution of  $\frac{dy}{dt} + y = 6 \cos x$ .