Homework 4 - Math 243 **Name:** ________________________

Solve the following initial value problems using the integrating factors method.

1. $\dfrac{dx}{dt} + 2tx = 4e^{-t^2}$, $x(0) = 5$.

2. $\dfrac{dy}{dt} + \dfrac{y}{t+1} = 2$, $y(0) = 3$.

3. A 30-gallon tank initially contains 10 gallons of water and 4 pounds of dissolved salt. A concentrated salt water solution containing 1 pound of salt per gallon is added to the tank at a rate of 2 gallons per minute, while a well-mixed solution is drained from the tank at a rate of 1 gallon per minute. Find a differential equation to model the amount of salt $y(t)$ in the tank. You do not need to solve the differential equation.

Find all equilibrium solutions for the following systems of equations.

4. $$\begin{aligned} \frac{dx}{dt} &= x - 2y + 4 \\ \frac{dy}{dt} &= 2x + y - 7 \end{aligned}.$$

5. $$\begin{aligned} \frac{dR}{dt} &= 5R - 0.5RF \\ \frac{dF}{dt} &= -4F + 0.1RF \end{aligned}.$$

6. Consider the differential equation $\frac{dv}{dt} + 6v = e^{2t}$.

(a) What is the corresponding homogeneous differential equation?

(b) What is the general solution of the homogeneous differential equation?

(c) Find one particular solution for the original equation of the form $v(t) = Ae^{2t}$. Then combine the particular solution and the homogeneous solution to express the general solution.

7. Consider the differential equation $\frac{dy}{dt} + y = 6\cos x$.

(a) Find constants A and B such that $y(t) = A\cos t + B\sin t$ is a solution to this ODE.

(b) Find the general solution of $\frac{dy}{dt} + y = 6\cos x$.