Math 444 - Homework 4 **Name:** ______________________

Let D be an open subset of $\mathbb{C}$. The derivative of a function $f : D \to \mathbb{C}$ at a point $z \in D$ is defined to be

$$f'(z) = \lim_{h \to 0} \frac{f(z+h) - f(z)}{h}.$$

1. Use the definition of derivative to show that the function $f(z) = \mathrm{Im}(z)$ is not differentiable anywhere.

2. Use the definition of derivative to find the derivative of $f(z) = \dfrac{1}{z}$.

3. Find the sum of the following geometric series and determine the values of $z \in \mathbb{C}$ for which it converges.

$$\sum_{k=0}^{\infty} \frac{1}{(z-i)^k}.$$

4. Suppose that $\lim_{z \to z_0} f(z) = a$ and $\lim_{z \to z_0} g(z) = b$. Use the ϵ-δ definition to prove that

$$\lim_{z \to z_0} f(z) + g(z) = \left(\lim_{z \to z_0} f(z) \right) \left(\lim_{z \to z_0} g(z) \right) = ab.$$