

### Midterm 1 Review - Math 243

1. Consider the initial value problem

$$\frac{dy}{dt} - 3(y-1)^{2/3} = 0, \quad y(0) = 1$$

- (a) Verify that  $y(t) = 1$  is a solution of the initial value problem above.

- (b) Verify that  $y(t) = t^3 + 1$  is also a solution to the initial value problem.

- (c) Why doesn't this contradict the Uniqueness Theorem? Explain.

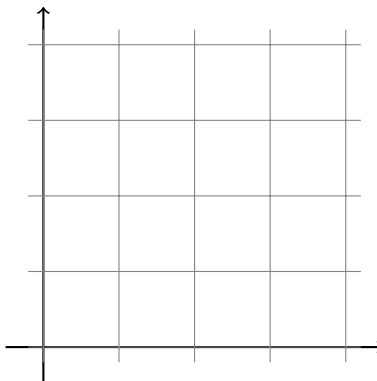
2. Solve the initial value problem

$$y' = \frac{2t+1}{y^2}; \quad y(1) = 2.$$

3. Consider the equation

$$\frac{dy}{dt} = (y - 1)^2 + t.$$

- (a) Sketch the slope field for this differential equation. Just use the points  $(t, y)$  with integer coordinates and  $0 \leq t \leq 4$  and  $0 \leq y \leq 4$ .



- (b) Use Euler's method with step size  $\Delta t = 1$  to estimate the values of  $y(t)$  for  $t = 1, 2, 3$  given the initial condition  $y(0) = 1$ . Add a sketch the Euler's method solution to the slope field above.

4. The following coupled system is a predator-prey population model.

$$\frac{dA}{dt} = 5A - \frac{A^2}{1000} - 3AB$$

$$\frac{dB}{dt} = B\sqrt{A}$$

- (a) Which of the variables  $A$  or  $B$  represents the predator and which represents the prey? Explain your answer.
- (b) In the model, what would happen to the predator population if the prey is extinct?
- (c) What would happen to the prey population if there were no predators?

5. Let  $\frac{dy}{dt} = f_\alpha(y)$  be a family of autonomous differential equations parametrized by  $\alpha$  where  $f_\alpha(y) = y^2 - 2y + \alpha$ .

(a) Draw phase lines for  $\alpha = 0$ ,  $\alpha = 1$ ,  $\alpha = 2$ . In each case identify the equilibria and say whether they are stable, unstable, or nodes.

(b) Use the phase lines in part (a) to sketch solutions to the initial value problem  $y(0) = 0$  for the three cases,  $\alpha = 0$ ,  $\alpha = 1$ , and  $\alpha = 2$ . (Use a different graph for each  $\alpha$ ).

(c) What is (are) the bifurcation value(s) for this family of equations?

(d) Draw the bifurcation diagram.

6. Find the general solutions for the following differential equations.

(a)  $\frac{dy}{dt} + \frac{2}{t}y = 4t^2$ .

(b)  $\frac{dy}{dt} + 2y = 2t + 1$

7. Solve the following initial value problems.

(a)  $y' - 5y = e^{5t}$ ,  $y(0) = 4$ .

(b)  $\frac{dy}{dx} = x(y - 1)$ ,  $y(0) = 3$

(c)  $\frac{dy}{dt} + \frac{y}{t+1} = 6t$ ,  $y(1) = 4$ .

8. Find the general solution to the partially coupled system

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = -4y.$$

9. Suppose the population of fish in a pond obeys a logistic growth model  $\frac{dP}{dt} = 0.3 \left( 1 - \frac{P}{2000} \right)$  where  $t$  is measured in years.

(a) How would you change the model if 100 fish were harvested from the pond each year?

(b) How would you change the model if a quarter of the fish were harvested from the pond each year?

10. Consider the 2nd order homogeneous linear differential equation

$$y'' + 2y' + 5y = 0.$$

(a) Find the general solution.

(b) Find the particular solution that satisfies the initial conditions  $y(0) = 3$  and  $y'(0) = 2$ .