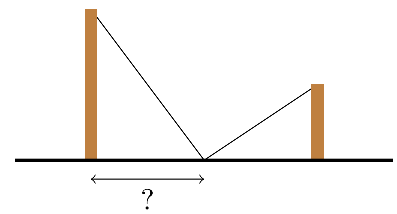
Math 141 - Homework 11 **Name:** ________________________

Solve each of the following optimization problems. Be sure to include confirmation that your solution is really the maximum or the minimum (use the first or second derivative test).

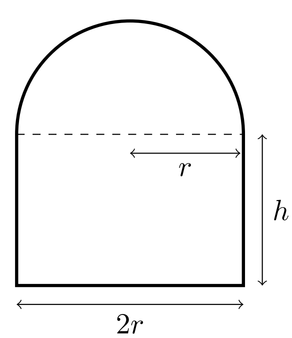
1. Two poles are connected by a wire that is also connected to the ground. The first pole is 20 ft tall and the second pole is 10 ft tall. There is a distance of 30 ft between the two poles. Where should the wire be anchored to the ground to minimize the amount of wire needed?



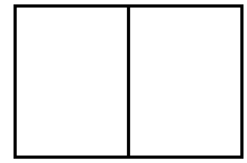
2. The sum of two positive numbers is 10. Find the values of the numbers that maximize their product.

3. What point on the line $3x + 4y = 50$ is closest to the origin?

4. A Norman window is a rectangle with a half-circle on top. If the perimeter of the window is 20 feet, find the dimensions r and h for the Norman window that has the largest possible area.

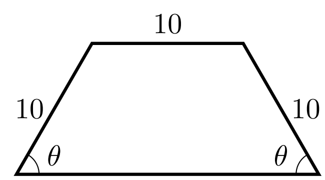


5. A farmer has 600 feet of fencing and wants to create a rectangular enclosure with a fence dividing the middle. Find the dimensions of the enclosure that maximize the area.



6. In economics, if $C(x)$ is the cost to produce x units of a commodity, then $\dfrac{C(x)}{x}$ is the **average cost** per unit produced. The derivative $C'(x)$ is the **marginal cost** of each extra item. Use calculus to show that the average cost is minimized at a level of production x where the average cost is equal to the marginal cost.

7. The trapezoid shown below has 3 short sides that are all 10 cm long and one long base. Find the angle θ that maximizes the area of the trapezoid. Recall that the area of a trapezoid is $A = \frac{1}{2}(a+b)h$ where a is the length of the top side, b is the length of the base, and h is the height (the distance from the top to the bottom).



8. A rectangle is inscribed in a quarter circle of radius 2 (shown below). Use calculus to show that the area of the rectangle is maximized when $\theta = \frac{\pi}{4}$.