

## Math 444 - Homework 3

Name: \_\_\_\_\_

1. Let  $a_n$  and  $b_n$  be sequences in  $\mathbb{C}$ . If  $a_n$  converges to zero and  $b_n$  is bounded, prove that the sequence  $a_n \cdot b_n$  converges to zero.
2. If  $a_n$  converges to  $a \in \mathbb{C}$  and  $b_n$  converges to  $b \in \mathbb{C}$ , then prove that  $a_n \cdot b_n$  converges to  $a \cdot b$ . Hint:  
 $a_n \cdot b_n - a \cdot b = (a_n - a) \cdot b_n + a \cdot (b_n - b)$ .
3. Let  $A \subseteq \mathbb{C}$ . We can define a relation  $\sim$  on  $A$  where  $z \sim w$  if there is a (continuous) path  $\gamma : [a, b] \rightarrow A$  such that  $\gamma(a) = z$  and  $\gamma(b) = w$ . Prove that  $\sim$  is an equivalence relation on  $A$ . Recall that to prove that a relation is an equivalence relation, you need to show that it is
  - (a) Reflexive:  $z \sim z$  for all  $z \in A$ .
  - (b) Symmetric: If  $z \sim w$ , then  $w \sim z$ .
  - (c) Transitive: If  $u \sim v$  and  $v \sim w$ , then  $u \sim w$ .

4. Let  $f(z) = z^2$ . Sketch a graph of the curve  $f(2 + ti)$  for  $t \in \mathbb{R}$ . Be sure to include any points where the curve intersects the real or imaginary axes on your graph. Hint: Simplify  $f(2 + ti)$  before you try to graph it.
5. Suppose that  $A_1, \dots, A_n$  are open sets in  $\mathbb{C}$ . Prove that the intersection  $\bigcap_{1 \leq k \leq n} A_k$  is an open set.
6. Give an example of an infinite collection of open sets  $A_1, A_2, \dots$  in  $\mathbb{C}$  such that  $\bigcap_{k \geq 1} A_k$  is not an open set.
7. Evaluate  $\sum_{n \geq 1} \left( \frac{1+i}{2} \right)^n$  and simplify your answer.