Markov Chains 2 **Math 111**

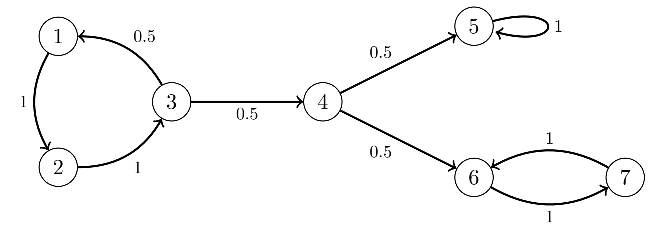
1. **Gambler's Ruin**. A gambler has \$2. Each round, he bets one dollar on red on a roulette wheel. The probability that he wins is 18/38, the rest of the time he loses (20/38 chance). If he wins, then he gets a dollar, otherwise he loses a dollar. He keeps playing until he either runs out of money, or doubles his money (i.e., has \$4).

   (a) Draw and label a graph for this Markov chain situation. Use the amount of money (from \$0 to \$4) that the gambler might have as the states.

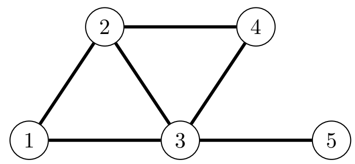
   (b) Find the transition matrix Q for this Markov chain.

   (c) Use a matrix calculator to compute Q^{100}. Based on Q^{100}, what is the probability that the gambler will lose all of his money? What is the probability that he will walk away with \$4?

2. How many strongly connected components does the Markov chain below have? How many are final?



3. **Random Walk**. A bug is wandering around the un-directed graph below. Each round, the bug moves from one vertex to a neighboring vertex along an edge. The bug is equally likely to choose any of the neighboring vertices.



(a) What is the transition matrix for this Markov chain?

(b) Estimate the stationary distribution for this Markov chain. In the long run, what fraction of the time will the bug spend on vertex 3?

4. Suppose that the transition matrix for a Markov chain is

$$Q = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) Draw and label a graph for this Markov chain.

(b) Is there a power k such that Q^k has all positive entries? What is the smallest power that works? Use a matrix calculator to find out.

(c) Find the stationary distribution for this Markov chain. Hint: Raise Q to a large power. If all the rows are the same, then they are all equal to the stationary distribution.