COMS 461 - Midterm 2 Review

1. (12 points) Create a context free grammar that generates the following language.

$$L = \{w \in \{a, b\}^* : w \text{ starts and ends with different symbols}\}.$$

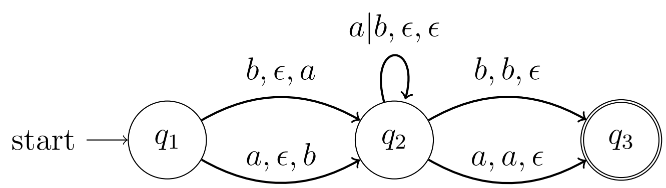
Solution:

$$S \to aXb \mid bXa$$
$$X \to aX \mid bX \mid \epsilon$$

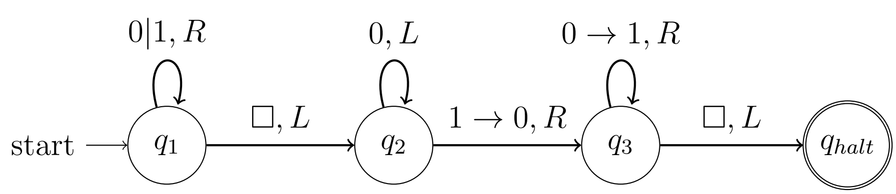
2. (12 points) Draw a state diagram for a nondeterministic pushdown automata (NPDA) that recognizes

$$L = \{w \in \{a, b\}^* : w \text{ starts and ends with different symbols}\}.$$

Solution:



3. (12 points) Consider the Turing machine with state diagram shown below.



(a) What will this TM output if the input tape initially contains the string `101000` and the head is initially pointing at the left-most digit? For your answer, write down the final tape contents and indicate the position of the head when the TM halts.

Solution: The tape contains `100111` with head pointed at the rightmost 1.

(b) This TM corresponds to a simple function on binary numbers. What is that function?

Solution: Binary decrement by 1 function.

4. (12 points) Consider the context free grammar below.

$$S \to aB|bA$$
$$A \to a|aS|AAB$$
$$B \to b|bS|BBA$$

This grammar generates the language of all strings in $\{a, b\}^*$ with an equal number of a's and b's. Prove that this grammar is abiguous by finding two different left derivations of the string $abba$. Draw parse trees for the two different derivations that makes it clear that they are different.

5. (20 points) The following statements are all false. For each one, explain why it is false.

(a) For any function $f : \{0, 1\}^* \to \{0, 1\}$, you can always find a Turing machine that accepts a string $w \in \{0, 1\}^*$ if and only if $f(w) = 1$, but the Turing machine might loop forever on w if $f(w) = 0$.

> **Solution:** Let $L = \{w \in \{0, 1\}^* : f(w) = 1\}$. Then L is a language which might not be Turing computable. If it is not Turing computable, then there won't be any Turing machine that accepts it, let alone decides it.

(b) All decidable languages are regular.

> **Solution:** In the Chomsky hierarchy of languages, all regular languages are Turing decidable, but not vice versa.

(c) Let $P = \{\langle M \rangle : M \text{ is a Turing machine with only one state}\}$. Then Rice's theorem implies that P is an undecidable language because both P and its complement $\overline{P}$ (in the language of all TM encodings) are nonempty sets.

> **Solution:** Rice's theorem does not apply since there are TM's M_1 and M_2 that accept the same language, but M_1 has only one state while M_2 has two.

(d) No Turing machine M can ever accept its own encoding $\langle M \rangle$.

> **Solution:** If M is a TM that accepts all strings, then M will accept $\langle M \rangle$.

6. (12 points) Let

$$\text{FINITE} = \{\langle M \rangle : M \text{ is a TM and the set of strings accepted by } M \text{ is finite}\}.$$

Explain clearly why FINITE is an undecidable language.

7. (20 points) Let

$$L_1 = \{a^n\, b\, a^m\, b\, a^n \;:\; m, n \in \mathbb{N}\}$$

and let

$$L_2 = \{a^n\, b\, a^n\, b\, a^n \;:\; n \in \mathbb{N}\}.$$

(a) Prove that L_1 is context free.

(b) Use the pumping lemma to show that L_2 is not context free.