Homework 5 - Math 243 **Name:** ____________________

Solve the following partially coupled systems analytically.

1.
$$\frac{dx}{dt} = -4x$$
$$\frac{dy}{dt} = 3x + 2y$$

2.
$$\frac{dx}{dt} = xy$$
$$\frac{dy}{dt} = y + 1$$

3. In the ocean, cod eat krill and seals eat both cod and krill. Write a system of three differential equations to model the populations of the krill K, the cod C, and the seals S. Use lower case letters for any constants you need and you can assume that the krill population would obey a constrained growth model (i.e., a logistic model) in the absence of predators.

4. Suppose that $\mathbf{F}(1,2) = (2,3)$. If you apply Euler's method to the system of differential equations
$$\frac{d\mathbf{Y}}{dt} = \mathbf{F}(\mathbf{Y})$$
with initial condition $\mathbf{Y}_0 = (1,2)$, then what is the value of $\mathbf{Y}$ after one step with $h = 0.1$?

5. Write the following 2nd order differential equation as a system of first order differential equations. You do not need to solve it.

$$2y'' - 5ty' + \sin y = 0.$$

6. The **Van der Pol equation** is

$$\frac{d^2x}{dt} - (1 - x^2)\frac{dx}{dt} + x = 0.$$

We can study this equation numerically by converting to the system of equations

$$\begin{aligned}\frac{dx}{dt} &= v \\ \frac{dv}{dt} &= (1 - x^2)v - x.\end{aligned}$$

Use Euler's method to approximate the solution of this equation with initial condition $(x_0, v_0) = (1, 1)$, and a step size of $h = 0.01$ after $N = 1{,}000$ steps. What do you get for $x(10)$ and $v(10)$?

7. Use Euler's method with $h = 0.1$ and $N = 50$ steps to approximate the values of $x(5)$ and $y(5)$ if $(x_0, y_0) = (0, 2)$ and Get an example!

$$\begin{aligned}\frac{dx}{dt} &=? \\ \frac{dy}{dt} &=?\end{aligned}$$