COMS 461 - Midterm 2 Review

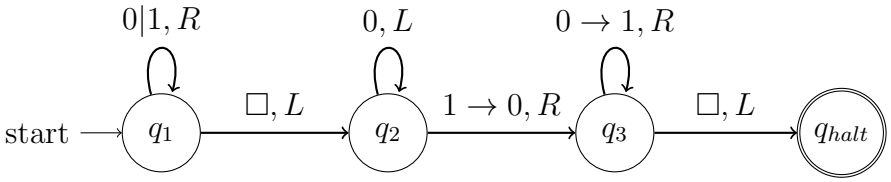
1. Create a context free grammar that generates the following language.

$$L = \{w \in \{a, b\}^* : w \text{ starts and ends with different symbols}\}.$$

2. Draw a state diagram for a nondeterministic pushdown automata (NPDA) that recognizes

$$L = \{w \in \{a, b\}^* : w \text{ starts and ends with different symbols}\}.$$

3. Consider the Turing machine with state diagram shown below.



   (a) What will this TM output if the input tape initially contains the string `101000` and the head is initially pointing at the left-most digit? For your answer, write down the final tape contents and indicate the position of the head when the TM halts.

   (b) This TM corresponds to a simple function on binary numbers. What is that function?

4. Consider the context free grammar below.

$$S \to aB|bA$$
$$A \to a|aS|AAB$$
$$B \to b|bS|BBA$$

This grammar generates the language of all strings in $\{a, b\}^*$ with an equal number of a's and b's. Prove that this grammar is abiguous by finding two different left derivations of the string $abba$. Draw parse trees for the two different derivations that makes it clear that they are different.

5. The following statements are all false. For each one, explain why it is false.

   (a) The is an uncountable number of Turing machines that can be defined with a given input alphabet Σ and tape alphabet Γ.

   (b) For any function $f : \{0, 1\}^* \to \{0, 1\}$, you can always find a Turing machine that accepts a string $w \in \{0, 1\}^*$ if and only if $f(w) = 1$, but the Turing machine might loop forever on w if $f(w) = 0$.

   (c) All Turing decidable languages are regular.

6. Let

$$L_1 = \{a^n\, b\, a^m\, b\, a^n \ : \ m, n \in \mathbb{N}\}$$

and let

$$L_2 = \{a^n\, b\, a^n\, b\, a^n \ : \ n \in \mathbb{N}\}.$$

(a) Prove that L_1 is context free.

(b) Use the pumping lemma to show that L_2 is not context free.

7. Describe a Turing machine that accepts the language

$$L = \{w \in \{a, b\}^* : w \text{ has a different number of a's and b's}\}.$$

Hint: Use a 2-tape Turing machine.