Homework 12 - Math 140 **Name:** ______________________

Find both partial derivatives of the following functions.

1. $h(x, y) = x^2 + 2xy + y^2$

   (a) $\dfrac{\partial h}{\partial x} =$ (b) $\dfrac{\partial h}{\partial y} =$

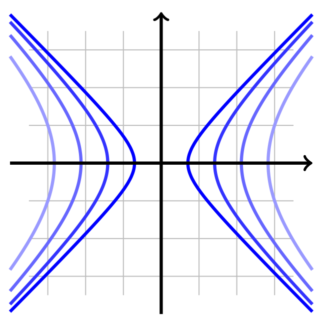
2. $g(x, y) = \dfrac{y}{x + y}$

   (a) $\dfrac{\partial g}{\partial x} =$ (b) $\dfrac{\partial g}{\partial y} =$

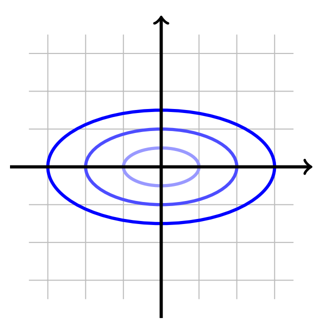
3. $z = (x^2 + y)^{1/2}$

   (a) $\dfrac{\partial z}{\partial x} =$ (b) $\dfrac{\partial z}{\partial y} =$

4. Several level curves for the function $f(x, y) = x^2 - y^2$ are shown below. Find the partial derivatives at the point $(1, 1)$ and draw an arrow starting at the point $(1, 1)$ that shows the direction of steepest ascent.



5. Several level curves for the function $f(x, y) = x^2 + 4y^2$ are shown below. Find the partial derivatives at the point $(-2, 1)$ and draw an arrow starting at the point $(-2, 1)$ that shows the direction of steepest ascent.



6. A cupcake shop can produce $Q(x, y) = 100x^{1/2}y$ dollars worth of cupcakes in a day where x is hours of labor and y is the number of ovens they have running. Find the two partial derivatives Q_x and Q_y when $x = 16$ and $y = 1$. Include the units for each.

7. A factory employs two types of workers. They have x skilled workers and y unskilled workers. The total output of the factory is $Q(x, y) = 10x^{0.6}y^{0.4}$. Find the marginal productivity of skilled and of unskilled workers when $x = 50$ and $y = 50$. The marginal productivity of an input is the partial derivative of output with respect to that input.

8. Find the (x, y)-coordinates of the critical point of $f(x, y) = x^2 + 2y^2 - xy + 14x$.

9. Use the second derivative test to determine if the critical point from the last problem is a local max, local min, or saddle point.

10. Find the (x, y)-coordinates of all critical points of $z = x^2 - 4x + 2y^3 - 9y^2$.

11. Use the second derivative test to classify each critical point from the last problem as a local max, local min, or saddle point.