

## Banzhaf Power Index

## Math 111 - Workshop

1. List all winning coalitions and find the Banzhaf power for each player in this weighted voting system:  $[4:3,2,1]$ .
  
2. Four college students start a company. Each invests \$10,000 except the last who only invests \$9,000. Why would it be unfair to use the weighted voting system  $[30:10,10,10,9]$ ? Find the Banzhaf power for each student and use it to explain your answer.
  
3. Find the Banzhaf power distribution of the weighted voting system  $[10:6,5,4,2]$ .
  
4. **U.N. Security Council.** There are 5 permanent members of the United Nations Security Council (the U.S, U.K, France, Russia, and China). There are also 10 non-permanent members that change from year to year. A resolution from the security council requires all the permanent members to support it plus at least 4 of the non-permanent members.
  - (a) Explain why every country in a 9 country winning coalition is critical.
  
  - (b) Explain why a permanent member of the security council is always a critical member of any winning coalition.
  
  - (c) Out of the 848 possible winning coalitions, there are 210 winning coalitions with 9 members. If you wrote down every coalition and circled the critical members, how many circles would there be? How many of the circles would belong to the five permanent security council members?
  
  - (d) How much of the power is in the hands of the permanent members of the U.N. security council? How much power does each temporary member have?

5. **Family Feud.** I have two kids. When my kids were little we would sometimes vote to pick a family activity. To win the vote required a majority, unless both adults (my wife and I) voted against both kids (then the adults win). What was the Banzhaf power distribution in my family?
6. **Choose Carefully.** A small company is started by a group of friends who each invest money. Alice invests \$6,000, Bob invests \$5,000, Carol invests \$4,000, and Dave invests \$2,000. They agree that each person should get 1 vote for every thousand dollars they invested. Now they are trying to decide what the minimum vote threshold should be in order to pass a motion. They want to choose a threshold so that (1) everyone gets some say, but (2) people who invested more money should have more power than people who invested less.
- (a) Why is 9 a bad choice for the threshold? Use Banzhaf power to explain your answer.
- (b) Why is 17 a bad choice for the threshold? Explain.
7. Suppose that the player with 5 votes (call them A) in the weighted voting system  $[8:5,4,3,2]$  hates the player with 2 votes (call them D), so A always votes the exact opposite as D. What are the possible winning coalitions given this restriction, and how much power does each player have?
8. **(Extra credit)** Can you find a weighted voting system where having an enemy like in the last problem could make a player more powerful? Give details.