

**Homework 10 - Math 243**

Name: \_\_\_\_\_

*Complexify the forcing term to find a particular solution for each of the following differential equations.*

1.  $y'' + 9y = \sin 4t$

2.  $x'' + 2x' + 10x = e^t \cos t$

Hint:  $e^t \cos t$  is the real part of  $e^{(1+i)t}$ .*Find a particular solution for each of the following non-homogeneous linear equations.*

3.  $x'' + 7x' + 10x = e^{-5t}$

4.  $y'' + 9y = \sin 3t$

5. If a linear differential equation

$$ay'' + by' + cy = f(t) + g(t)$$

has two different forcing functions  $f$  and  $g$  added together, then (by linearity) you can find a particular solution by adding together particular solutions when you only use  $f$  as the forcing function and when you only use  $g$  as the forcing function. Use this to find a particular solution to

$$y'' + 100y = e^t + \cos t.$$

6. The nonlinear system

$$\begin{aligned}x' &= 5y - xy \\y' &= 2x - xy\end{aligned}$$

has two equilibrium points  $(0, 0)$  and  $(2, 5)$ . Calculate the Jacobian matrix at each equilibrium point and use it to determine whether the equilibrium is a source/sink, spiral source/sink, or a saddle.

7. Consider the nonlinear system

$$\begin{aligned}x' &= y - x^2, \\y' &= y - 4.\end{aligned}$$

Find all equilibrium points and classify each equilibrium as a saddle, source/sink, or spiral source/sink.