

**Homework 7 - Math 243****Name:** \_\_\_\_\_

Find the eigenvalues for each of the following systems and then use them to classify the type of equilibrium at the origin.

1.  $\frac{dx}{dt} = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix} x$

2.  $\frac{dy}{dt} = \begin{bmatrix} -1 & -8 \\ 2 & -1 \end{bmatrix} y$

3.  $\frac{dx}{dt} = \begin{bmatrix} 5 & 6 \\ 0 & 3 \end{bmatrix} x$

4.  $\frac{dy}{dt} = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} y$

Find real-valued general solutions for the following linear systems. You can use a computer to find the relevant eigenvectors/eigenvalues.

5.  $\begin{aligned} x' &= 2y \\ y' &= -2x \end{aligned}$

6.  $\begin{aligned} x' &= -x + y \\ y' &= -9x - y \end{aligned}$

Find solutions for the following initial value problems. You should start with the general solutions from the last two problems.

7.  $\begin{aligned} x' &= 2y \\ y' &= -2x \end{aligned} \quad \text{with } x(0) = 1 \text{ and } y(0) = 0.$

8.  $\begin{aligned} x' &= -x + y \\ y' &= -9x - y \end{aligned} \quad \text{with } x(0) = 4 \text{ and } y(0) = -6.$

9. Suppose that a 2-by-2 real matrix  $A$  has eigenvector  $\begin{bmatrix} 1 \\ 2+i \end{bmatrix}$  with corresponding eigenvalue  $\lambda = 1 + 5i$ . Find the general solution of the system  $\mathbf{x}' = A\mathbf{x}$ .