

Due Wed, Jan 22. You can send the code for the programming problem to `blins@hsc.edu`.

1. Write a logical expression using the operators  $\wedge$  (AND),  $\vee$  (OR), and  $\neg$  (NOT) and the Boolean variables  $x_0, x_1, x_2$  so that the expression is TRUE when an even number of the variables are TRUE and FALSE otherwise.

2. Translate the following mathematical short-hand into English:

$$\forall a, b \in \mathbb{Z} \text{ with } b \neq 0, \exists N \in \mathbb{N} : N > a/b \wedge N > b/a.$$

3. Describe the following sets in words. Just give a brief description of what objects are in the sets, you do not need to list all of the elements.

(a)  $[9] \times [9] \times [9]$ .

(b)  $\{x \in \{0, 1\}^{2n} : x_i = x_{i+n} \ \forall i \in [n]\}$ .

4. Write a computer program to verify that  $n^3 + (n+1)^3 + (n+2)^3$  is divisible by 9 for every integer  $0 \leq n < 100$ .

5. Use mathematical induction to prove that  $n^3 + (n+1)^3 + (n+2)^3$  is divisible by 9 for every integer  $n \geq 0$ .