Math 141 - Homework 4 **Name:** _____

Calculate the following limits exactly.

1. $\lim_{x\to 0} \dfrac{1}{2+\sin x}$

2. $\lim_{x\to -1} \dfrac{2x-1}{x+2}$

3. $\lim_{x\to 2} \dfrac{x-2}{x^2-2x}$

4. $\lim_{x\to 5} \dfrac{x^2-3x-10}{x-5}$

5. $\lim_{x\to 0} \dfrac{\sin x}{1+\cos x}$

6. $\lim_{h\to 0} \dfrac{(1+h)^2-1}{h}$

7. Find $\lim_{h\to 0} \dfrac{\frac{1}{a(a+h)} - \frac{1}{a^2}}{h}$ where a is a non-zero constant.

8. Determine the point(s), if any, at which each of the following functions is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

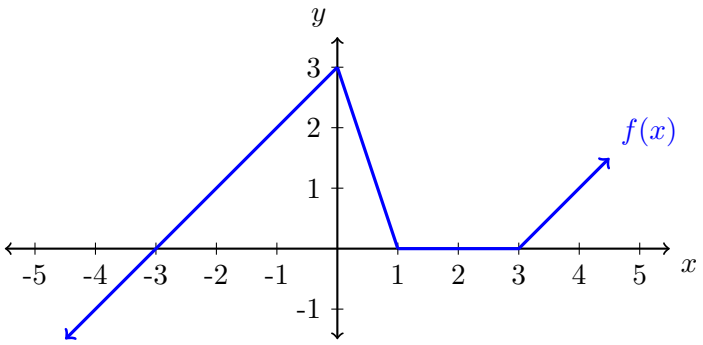
   (a) $f(x) = \dfrac{x}{x^2-x}$ (b) $g(x) = \cot 2x$ (c) $h(t) = t^{-1} + 1$

9. Use the formula $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ to find the derivative of $f(x) = x^2$ at $a = 3$.

10. Expand the polynomial $(x+h)^3$, i.e., multiply the factors $(x+h)(x+h)(x+h)$, then use your answer to find the derivative

$$\frac{d}{dx} x^3 = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}.$$

11. Use the graph below to find the following derivatives, or explain why they do not exist.



(a) $f'(-1)$ (b) $f'(0.5)$ (c) $f'(1)$ (d) $f'(2)$

12. Sketch a rough graph of the derivative of the function shown in the graph below. Be sure to include numbers on the x and y-axes.