

Math 444 - Homework 7**Name:** _____

1. In this problem you will evaluate the integral $\int_{\gamma} |z|^2 dz$ where $\gamma(t)$ is the parabola $\gamma(t) = -t + i(t^2 - 1)$ from $t = -1$ to $t = 1$.

(a) What are the real and imaginary parts of $|\gamma(t)|^2 \cdot \gamma'(t)$?

(b) Use the real and imaginary parts above to evaluate $\int_{\gamma} |z|^2 dz$.

2. Integrate the function $z - \bar{z}$ on the upper half of the unit circle from $z = 1$ to $z = -1$.

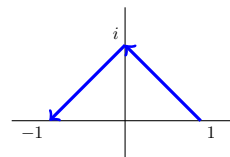
3. Find the length of the path $\gamma(t) = t + \frac{2}{3}t^{3/2}i$, $0 \leq t \leq 3$.

4. Show that $\lim_{n \rightarrow \infty} \left| \int_{\gamma} \frac{1}{z} dz \right| = 0$ when γ is the horizontal line segment from $1 - ni$ to $-1 - ni$. Hint: One way to do this would be to calculate the integral exactly for any n . An easier alternative is to use the inequality

$$\left| \int_{\gamma} f(z) dz \right| \leq \text{length}(\gamma) \cdot \max_{z \in \text{range}(\gamma)} |f(z)|.$$

Use a computer (I recommend SymPy) to calculate the following integrals.

5. $\int_{\gamma} (\bar{z})^3 dz$ on the piecewise path shown below. Hint: in SymPy the complex conjugate function is `conjugate()`. You'll need to parameterize each piece separately.



6. $\int_{\gamma} (\bar{z})^3 dz$ on the piecewise path shown below.

