Math 444 - Homework 1 **Name:** ______________________

Simplify each of the following expressions as much as you can. Show your work. No calculators.

1. i^{14}
2. $(5i)(-2i)(3i)$
3. $(3+i)^2$
4. $\operatorname{Im}\left(\dfrac{12}{5-i}\right)$
5. $(3-2i)(4+i)$
6. $\dfrac{1-i}{1+i}$
7. $\left|\dfrac{1}{5+12i}\right|$
8. $\overline{(3+4i)(1-i)}$
9. $\overline{e^{i\frac{\pi}{3}}}$

Convert the following from rectangular to polar form.

10. $\frac{1}{2}+\frac{\sqrt{3}}{2}i$
11. $i-1$
12. $\dfrac{i}{1+i}$

Convert the following from polar to rectangular form.

13. $e^{5\pi i/3}$
14. $e^{-\pi i/4}$
15. $(\sqrt{3}\,e^{7\pi i/12})(\sqrt{12}\,e^{29\pi i/12})$

Convert to polar or rectangular form to evaluate the following.

16. $\sqrt{2i}$
17. i^i
18. $\operatorname{Re}\left(2e^{\pi i/6}\right)$
19. $(i-1)^6$

20. We are going to find the roots of the polynomial equation $z^2 + 2z + (1 - i) = 0$ two ways.

   (a) Re-write the equation as $z^2 + 2z + 1 = i$ and factor the left hand side (which is a perfect square). Then take the square root of both sides. Remember that all non-zero complex numbers have two square-roots!

   (b) Now use the quadratic formula $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Do you get the same answer as before?

21. An **n-th root of unity** is a number z such that $z^n = 1$. Prove that the n-th roots of unity are the set $\{e^{2\pi i \frac{k}{n}} : k \in \mathbb{Z}\}$.

22. Find all of the 4th roots of unity. How many are there? Express them in rectangular form.

23. If $z \in \mathbb{C}$ is a root of a polynomial p with real number coefficients, then $\overline{z}$ is also a root of that polynomial because $p(\overline{z}) = \overline{p(z)}$. Find an example to show that this is not true for all polynomials with complex number coefficients.

24. Prove that for every $z \in \mathbb{C}$, $|z|^2 = z\overline{z}$.

25. Prove that $|z| = 1$ if and only if $\overline{z} = \dfrac{1}{z}$.