Practice Final Exam - Math 140

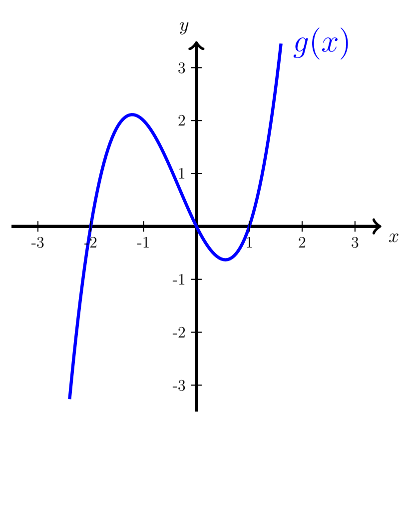
1. Find the intervals of increase/decrease for $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 6x^2$.

2. Use the logarithm rules to simplify, then differentiate $y = \ln\left(\dfrac{x}{(x+4)^2}\right)$.

3. Find a formula for the linear function $f(x)$ with $f(0) = -1$ and $f(3) = 5$.

4. The graph of a function $y = g(x)$ is shown. Use the graph to find the following.

   (a) What are the roots of $g(x)$?



   (b) What is $g(-1)$?

   (c) Is $g'(0)$ positive, negative, or zero?

5. Solve $\frac{1}{x} - \frac{2}{x+4} = 0$.

6. Find the following derivatives.

(a) $\frac{d}{dx} x^{1/5}$

(b) $\frac{d}{dx} e^{(x^2+3x)}$

(c) $\frac{d}{dx} \frac{x+5}{x^2-4}$

(d) $\frac{d}{dx} x^2 e^x$

7. Find the (x, y) coordinates of the local max of $f(x, y) = 9 - x^2 - y^2 + 6y$.

8. What is the slope of the tangent line to $h(x) = \frac{1}{3}x^3 - 3x^2 + 5x$ at the point $(3, -3)$?

9. Find the differential of $y = e^x$ at $x = 0$ when $dx = 0.1$ and use it to estimate the value of $e^{0.1}$.

10. Simplify $(\sqrt{2} + \sqrt{50})^2$. *Remember: Powers don't distribute to terms!*

11. Find the absolute maximum and minimum y-values of the function $y = x + \dfrac{100}{x}$ on the interval $[5, 25]$.

12. Suppose that the population of a town is growing by 5% per year and the current population is 10,000, so the formula for the population after t years is

$$P(t) = 10{,}000(1.05)^t.$$

How long will it be until the population of the town reaches 30,000 people? It is okay to write your answer as a formula using logarithms.

13. Solve $2e^{5x} = 64$.

14. A fruit importer will sell $q(p) = 800 - 20p$ boxes of fruit when the price of a box is p dollars. What price would maximize the fruit importer's revenue?

15. Calculate the following logarithms.

(a) $\log_3(9)$

(b) $\log_2\left(\frac{8}{\sqrt{2}}\right)$

16. Find the partial derivatives (both f_x and f_y) for $f(x, y) = x^2 + y^4 + 2xy^3$.

17. The production level at a factory is $Q(x, y) = 40x^{0.25}y^{0.75}$ where x is hours of labor and y is capital expenditure. Find the partial derivatives of Q when $x = 100$ and $y = 100$.

18. A box with a square bottom and an open top needs to have exactly 4 cubic feet of volume. Find the dimensions of the box with the smallest possible surface area. Recall that the volume of a box is $V = x^2y$ where x is the length and width and y is the height. The surface area is $S = 4xy + x^2$.