

Homework 8 - Math 243

Name: _____

Each of the following linear systems has a single parameter a . Calculate the trace and determinant for each system, and use them to find the values of a where the type of equilibrium changes.

1. $\frac{d\mathbf{x}}{dt} = \begin{bmatrix} a & -1 \\ 2 & 0 \end{bmatrix} \mathbf{x}.$

2. $\frac{d\mathbf{y}}{dt} = \begin{bmatrix} a & a \\ 1 & 0 \end{bmatrix} \mathbf{y}.$

3. Suppose that we have a linear system $\mathbf{x}' = A\mathbf{x}$ where A is a 2-by-2 matrix. If we replace the matrix A with the matrix $-A$, then how would the equilibrium at the origin change? Use the trace and the determinant to justify your answer.

Use the matrix exponential function to solve the following initial value problems.

4. $\begin{aligned} x' &= 4x + 2y \\ y' &= 1x + 3y \end{aligned}, x(0) = 2 \text{ and } y(0) = 7.$

5. $\begin{aligned} x' &= -3x + 4y \\ y' &= 3x - 2y \end{aligned}, x(0) = 8 \text{ and } y(0) = -6.$

6. $\begin{aligned} x' &= 3x - y \\ y' &= -x + 3y \\ z' &= 5x - 5y - 6z \\ x(0) &= 4 \\ y(0) &= 2 \\ z(0) &= -3. \end{aligned}$

7. $\begin{aligned} x' &= -3x + 4y \\ y' &= 3x - 2y \end{aligned}, x(0) = 8 \text{ and } y(0) = -6.$