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ON THE APPLICABILITY OF THE KOSCHMIEDER VISIBILITY FORMULA

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Abstract—The inverse proportionality between the visibility and the extinction coefficient, first derived by Koschmieder, is only applicable under very limited conditions: the atmosphere must be illuminated homogeneously, the extinction coefficient and the scattering function are not allowed to vary with space, the object must be ideally black and be viewed against the horizon, and the eye of the observer must have a constant contrast threshold. A general formula, taking these facts in account has been derived and used to calculate possible errors which might arise if the simple Koschmieder Formula is used instead. The following results were obtained: inhomogeneous illumination generates errors smaller than 5 per cent. An inhomogeneous distribution only due to different dilutions of the aerosol gives no error if the average of the extinction coefficient is used. Using non black objects as visibility markers can give errors up to 50 per cent if they are illuminated by the sun, but the errors will be below 5 per cent if they are in their own shadow. The errors due to a varying contrast threshold of the eye can be considerable, if the visibility markers are too small. By proper selection of the visibility markers it will be possible to use the Koschmieder Formula to calculate the extinction coefficient from observed visibilities with an error of less than about 10 per cent.

NOMENCLATURE

A, $B_a(x)$	= constant, representing the brightness of a layer of aerosol of unit thickness;
B, B_0	= brightness;
$\bar{B}$	= intrinsic brightness of an object;
B_h	= brightness of the horizon;
B_0	= brightness of object seen by observer;
b, $b(x)$	= extinction coefficient of the aerosol;
b	= average extinction coefficient of the aerosol over the visual range;
dB_0	= infinitesimal brightness of an aerosol layer illuminated by light;
dB	= infinitesimal brightness of an aerosol layer illuminated by light, seen by the observer;
dx, $d\xi$	= infinitesimal thickness of layer of aerosol;
R	= distance from the observer;
x	= distance from observer;
V	= visual range;
V_k	= visibility obtained by using the Koschmieder Formula;
ϵ	= contrast threshold of the eye;
ΔB	= decrease in brightness;
ξ	= distance from observer.

1. INTRODUCTION

SEVERAL papers have been published, relating the visibility to another parameter of air pollution (e.g. FETT, 1967; NOLL *et al.*, 1969; HORVATH and NOLL, 1969; GARLAND, 1969). In most cases the Koschmieder Formula $V = 3{\cdot}9/b$ was used to relate the observed visibilities to the extinction coefficient of the aerosol. Alternately, the extinction coefficient of the aerosol is calculated from the size distribution and the refractive index of the particles, and the Koschmieder Formula is used to calculate the visibility. In deriving this formula, several serious assumptions are made (see e.g. MIDDLETON, 1952). Generally, these are not valid under normal conditions of observation. The purpose of this paper is to investigate the corrections which will have to be made to the Koschmieder Formula under normally occurring conditions, and to determine how methods of observation might be modified to obtain better agreement with the simple Koschmieder theory.

2. THE KOSCHMIEDER FORMULA

The following assumptions are made:

(a) Every volume element of the atmosphere is illuminated by the same amount of light (this occurs for instance if the sky is either completely overcast or cloudless).

(b) Both extinction coefficient and scattering function are assumed to be constant along the path of sight.

(c) The object observed is perfectly black and seen against the horizon.

(d) The contrast threshold of the eye has the value of 0·02.

These are the usual assumptions for visibility calculations. The theory (see e.g. MIDDLETON, 1952) gives the visibility V_k as a function of the extinction coefficient b:

$$V_k = 3 \cdot 9/b. \tag{1}$$

3. A MODIFIED KOSCHMIEDER FORMULA

Let us now assume the object has a brightness $\bar{B}$, the extinction coefficient varies with range ($b = b(x)$), and the brightness of a layer of thickness dx is d$B_0 = B_a(x)$dx. Then (as proved in Appendix 1) the visibility can be calculated by solving the equation

$$\epsilon = \left| \frac{-\int_V^\infty B_a(x) \exp\left(-\int_0^x b(\xi) d\xi\right) dx + \bar{B} \exp\left(-\int_0^v b(\xi) d\xi\right)}{\int_0^\infty B_a(x) \exp\left(-\int_0^x b(\xi) d\xi\right) dx} \right| \tag{2}$$

with respect to V.

This is the most general case and in the following discussion we will take particular cases in order to determine the discrepancies which might arise if the Koschmieder Formula is applied under conditions for which it was not derived.

4. CORRECTIONS OF THE KOSCHMIEDER FORMULA

4.1 *Non-uniform illuminations*

If all other assumptions besides uniform illumination are true equation (2) can be written as

$$\epsilon = \left| \frac{\int_v^\infty B_a(x) \, \mathrm{e}^{-bx} \, dx}{\int_0^\infty B_a(x) \, \mathrm{e}^{-bx} \, dx} \right|. \tag{3}$$

Large variations in the brightness of the aerosol are often encountered, when it is illuminated by the sun, clouds, sky, and ground in one part and only by clouds, sky and ground in other parts of the path of sight. The differences brightness do not usually exceed a factor of 2.

The worst possible case occurs when the path between the observer and the object is illuminated by the sun while the aerosol behind the object is in shadow.

Since the horizon sky actually appears to be darker than usual, a black object will reach a brightness which makes it invisible at a shorter distance, so the visibility is decreased. Putting

$$B_a(x) = \begin{cases} 2.b & \text{for } 0 \leqq x \leqq V \\ b & \text{for } V < x \end{cases} \tag{4}$$

in equation (3) the visual range is decreased by 17·7 per cent. If, on the other hand, the aerosol between the observer and the object is in shadow and the aerosol behind the object is illuminated by the sun, the visibility is increased by 17·7 per cent.

These two cases are extremes which will occur very rarely, so the deviations will usually be smaller. Another example demonstrates this: in the first half of the visual range the aerosol is illuminated, in the second half it is in shadow. Behind the object the aerosol is illuminated for the length of one visual range and for the remaining distance the aerosol is in shadow. The observed visibility is 2 per cent higher than the visibility calculated with the Koschmieder Formula. If the illumination is "turned around" (dark–bright–dark–bright) the visibility is 2 per cent smaller. Deviations of 5 per cent or less are typical of inhomogeneous illumination. Unfortunately the inhomogeneous illumination also varies the brightness of non black objects and this can be the source of considerable deviations from the Koschmieder Formula. This will be treated in detail in Section 4.3.

4.2 *Non-uniform spatial distribution of the aerosol*

Let us first assume that the extinction coefficient of the aerosol varies in space, but the brightness of an aerosol layer of unit length is always proportional to the extinction coefficient, i.e.

$$B_a(x) = Ab(x). \tag{5}$$

This is the case, for instance, if the aerosol size distribution is the same and only the total particle number varies. The visibility of a black object seen under these conditions can be calculated using the formula derived in Appendix 2:

$$\epsilon = \exp\left(- \textstyle\int_0^v b(\xi)d\xi\right), \tag{6}$$

or for $\epsilon = 0{\cdot}02$

$$3{\cdot}9 = \textstyle\int_0^v b(\xi)d\xi. \tag{6a}$$

This is very similar to the Koschmieder Formula (1),

$$3{\cdot}9 = Vb, \tag{7}$$

only $\int_0^v b(\xi)d\xi$ replaces by Vb.
If the average of the extinction coefficient over the visual range,

$$\bar{b} = \frac{1}{V}\int_0^v b(\xi)d\xi, \tag{8}$$

is used in the Koschmieder Formula the correct visibility can be calculated.

On the other hand, if a visibility determination is made and, due to lack of information on the spatial distribution of the aerosol, formula (1) or (7) is used to calculate the extinction coefficient b, this calculated extinction coefficient is the average of the actual extinction coefficient along the path of sight between the observer and the observed visibility marker.

Let us look at a particular case which might occur in practice. Between the observer and the visibility marker is a layer of aerosol which scatters part of the light and absorbs part of the light. This layer could be a tenuous black smoke plume. The visibility obtained by averaging the extinction coefficient and using the Koschmieder Formula will give a visibility that is too small. The reason for this is simple. Averaging the extinction coefficient implies that the brightness is strictly proportional to the extinction coefficient. Therefore the contrast of the object against the horizon is

$$C_1 = \left| \frac{B_0 - B_h}{B_h} \right|. \tag{9}$$

The presence of the absorbing layer means that over a given interval less scattered light is produced. The dark layer is located between the object and the observer, thus the brightness of both the object and the horizon is reduced by the same amount ΔB, therefore the contrast is

$$C_2 = \left| \frac{B_0 - B_h}{B_h - \Delta B} \right|. \tag{10}$$

Obviously the contrast is larger in the second case ($C_2 > C_1$), so the visibility is increased.

Two examples will illustrate the magnitude of this effect. First, take the case of an aerosol with an extinction coefficient of 1 km^{-1} (4 km visibility) and an absorbing layer extending from 0·3 km to 0·5 km. If the extinction coefficient of this absorbing layer is twice the coefficient of its surroundings the visibility is increased by 8 per cent, if it is ten times higher the visibility increases by 18 per cent.

In the second case, we consider the inhomogeneity arising from the curvature of the earth. After a length of 10 km a horizontal path of sight has risen 10 m, and the aerosol concentration may have decreased slightly. In Appendix 2 it is shown that the brightness of the horizon does not depend on the magnitude of the extinction coefficient so, even if the path of sight goes through more diluted aerosols, the brightness of the horizon will be the same. Therefore, the curvature of the earth seems to be of negligible importance for vision below 10 km.

4.3 *Non-black objects*

If the object has an intrinsic brightness $\bar{B}$ and the other conditions ((a), (b) and (d)) of the Koschmieder Formula hold, equation (2) can be transformed to

$$\epsilon = \left| \frac{-A\exp(-bV) + \bar{B}\exp(-bV)}{A} \right| = e^{-bV} \left| \frac{\bar{B}}{A} - 1 \right|, \tag{11}$$

or, provided $|\bar{B}/A - 1| \geqslant \epsilon$,

$$V = \frac{\ln(1/\epsilon)}{b} + \frac{1}{b}\ln|\bar{B}/A - 1|, \tag{12}$$

$$V = 3{\cdot}9/b + 1/b \ln|\bar{B}/A - 1|,$$

If V_k is the visibility resulting from the Koschmieder Formula V can be represented by

$$V = V_k(1 + 1/3{\cdot}9 \ln|\bar{B}/A - 1|). \tag{13}$$

If $[\bar{B}/A - 1] < \epsilon$, the object is invisible at any distance. This is a very hypothetical case because the object must also have exactly the same colour as the horizon. The visibility is smaller than V_k if $\ln|\bar{B}/A - 1| < 0$ or $0 < \bar{B} < 2A$ if it has a brightness between zero and twice the brightness of the horizon. If the object has more than twice the brightness

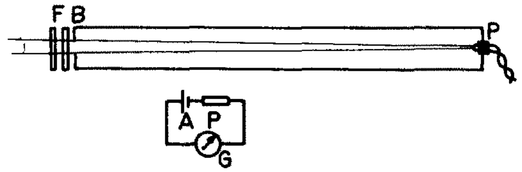
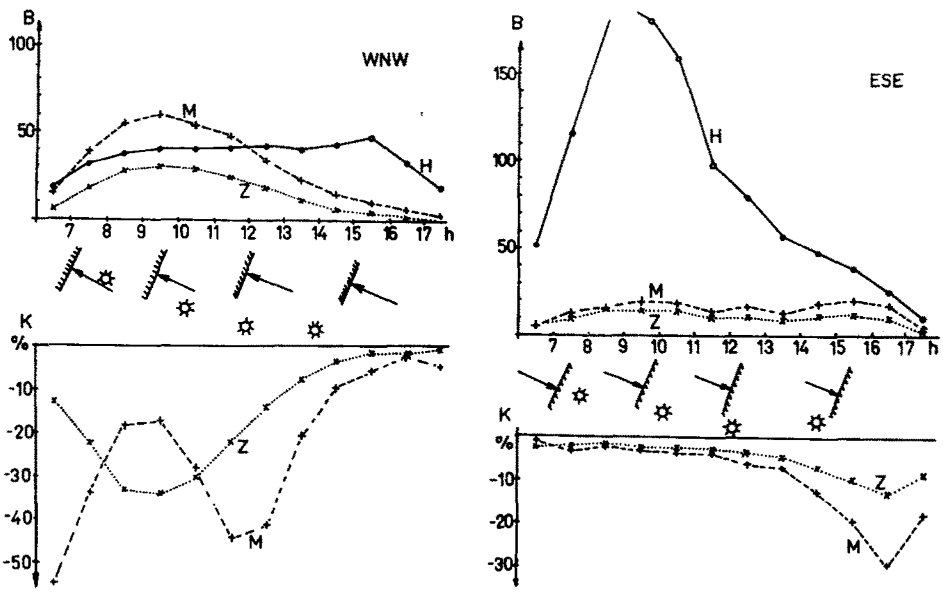


FIG. 1. Simple telephotometer for measuring the brightness of objects. The light goes through the colour filters F (which are selected to produce the same spectral response as the eye), hitting the photoresistor P at the end of the tube. Due to a small rectangular opening the area measured was 2° × 6°. Below: Electric circuit (A, battery; G, Galvanometer).

of the horizon the visibility is increased (obviously searchlights are more visible than black objects). If a visibility marker has a brightness half that of the horizon its visibility is 8 per cent less than the visibility of a black object under the same condition.

To study the influence of the intrinsic brightness on the visibility measurement a simple telephotometer was used to determine the brightness of objects and the horizon. The photometer consisted of a tube (30 cm long and 5 cm in diameter) with a photoresistor on the one end and a rectangular opening on the other end (FIG. 1). The geometric layout was arranged in such a way that an angular area of 2° × 6° could be measured. A colour filter was put in front of the photometer so that the spectral sensitivity of the instrument could be matched with that of the human eye. FIGURES 2a and 2b show the measured intrinsic brightness of a brick wall, a wall covered with grey mortar and the brightness of the horizon during a sunny day. If the



(a) (b)

FIG. 2a and b. Measured brightness B of a brick wall (Z), a mortar covered wall (M), and the horizon (H) during a sunny day (top), the position of the sun, the wall, and the direction of observation (→) (middle), and the necessary corrections (k) of the Koschmieder Formula (bottom).

wall is in its own shadow its brightness is always much less than the brightness of the horizon. If the wall is illuminated by the sun it may reach the brightness of the horizon or even surpass it. The necessary corrections to the Koschmieder Formula (drawn in the bottom of the figures) are therefore large if the non-black object is illuminated by the sun and they are usually below 5 per cent if the marker is in its own shadow. Thus a visibility observation looking in the horizontal direction towards the sun will give the largest distance at which an object can be recognized and will be in best agreement with the Koschmieder Formula.

This is apparently in disagreement with the experience of most people enjoying the view from a mountain on a hazy day. Obviously the amount of light scattered in the forward direction (looking towards the sun) is larger than in the backward direction (looking with the sun). The haze appears much brighter looking towards the sun, the mountains which can be seen in this direction are usually recognized only as dark outlines without any details such as rocks, meadows, trees etc. because they are in their own shadow. Also, since the brightness of the horizon is very large, some time must be taken for the eye to accommodate. Looking with the sun many details of the scenery can be recognized if they are not too far away. The observer considers the visibility looking with the sun better both because he does not want to strain his eyes by looking against the sun and also because he sees more detail looking with the sun. Observations performed by the author and his colleagues on several hikes showed that the visibility, defined as the ability to recognize a mountain outline, is actually greater or at least as great as the direction of the sun as it is away from the sun.

4.4 *Variations of the contrast threshold of the eye*

The contrast threshold of the eye in general is 0·02 but under certain conditions (e.g. looking at objects with a small angular dimension or during dusk or dawn) the contrast threshold may be larger. BLACKWELL (1946) has performed excellent work on

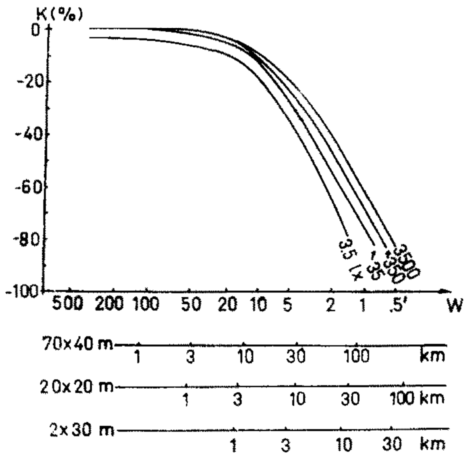


FIG. 3. Necessary correction K to the Koschmieder Formula due to a varying contrast threshold of the eye. On the abscissa the varying angular size of the object is plotted. The brightness of the horizon is given as a parameter (3500 lx: bright day, 3·5 lx: 0·5 h after sunset). Below the distance from the observer is plotted, which an object may have, in order to have the angular size plotted above.

the contrast threshold of observers, considering more than half a million measurements. These data have been used to calculate the deviations from the Koschmieder Formula, if V_k is called the visibility resulting from (1) the actual visibility V is obtained by

$$V = V_k \ln(1/\epsilon)/b. \tag{14}$$

FIGURE 3 shows the possible corrections if the brightness of the horizon and/or the angular size of the object becomes small. These corrections may be of considerable magnitude. By choosing large visibility markers this error can be almost completely reduced.

5. CONCLUSIONS

We have seen that there exist two groups of corrections to the Koschmieder Formula.

1. Corrections in the order of 10 per cent or below. These are due to inhomogeneous illumination of the atmosphere or a non homogeneous spatial distribution of the aerosol. The visibility observer can do nothing to avoid these errors.

2. Corrections, which may exceed 50 per cent, resulting from a non-black object or an object of small angular dimensions. These errors can be avoided by choosing visibility markers which are in their own shadow during the observation and have a large size. If the visibility markers are used in this way the extinction coefficient calculated with the Koschmieder Formula will show the smallest possible deviation from its actual value.

Visibilities estimated in the above mentioned way probably do not agree with the standards required for aviation or meteorological purposes. The main purpose of the proposed visibility measuring method is to obtain a reliable value of the extinction coefficient which then may be used as a parameter characterizing air pollution.

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APPENDIX 1

A modified Koschmieder Formula

The brightness of a layer of thickness dx (Fig. 4) is

$$B_a(x)\,dx. \tag{15}$$

$B_a(x)$ is a function of the light incident on the volume element of the aerosol multiplied by the scattering function. Since the illumination may vary with space as well as the optical properties of the aerosol, $B_a(x)$ is a function of space. The brightness passing through the turbid medium is reduced in a layer of thickness $d\xi$ by an amount

$$dB = b(\xi)\,B\,.\,d\xi$$

doing similar steps as used for the Koschmieder Formula. The contrast of an object of intrinsic luminance $\bar{B}$ seen against the horizon is

$$C = \left|\frac{B_0 - B_H}{B_H}\right| = \left|\frac{-\int_R^\infty B_a(x)\exp\left(-\int_0^x b(\xi)d\xi\right)dx + \bar{B}\exp\left(-\int_0^x b(\xi)d\xi\right)}{\int_0^\infty B_a(x)\exp\left(-\int_0^x b(\xi)d\xi\right)dx}\right|. \tag{16}$$

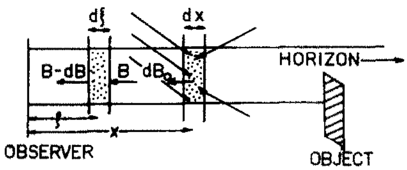

FIG. 4. Light from the sky, clouds, sun and ground is partially scattered into the direction of the observer, giving a layer a brightness dB_0. This brightness is reduced by a thin layer $d\xi$ according to $dB = B\, b\,(\xi)\, d\xi$.

APPENDIX 2

Inhomogeneous atmosphere

Assuming

$$Ba(x) = Ab(x) \tag{17}$$

and a black object, the visibility formula (2) reduces to

$$\epsilon = \frac{\int_V^\infty b(x) \exp(-\int_0^x b(\xi)d\xi)\, dx}{\int_0^\infty b(x) \exp(-\int_0^x b(\xi)d\xi)\, dx}, \tag{18}$$

since $\int_0^x b(\xi)d\xi$ is related to $b(x)$ by

$$\frac{d}{dx}\int_0^x b(\xi)d\xi = b(x) \quad \text{or} \quad d\int_0^x b(\xi)d\xi = b(x)dx \tag{19}$$

we may rewrite (18),

$$\epsilon = \frac{-\int_V^\infty \exp(-\int_0^x b(\xi)d\xi)\, d(-\int_0^x b(\xi)d\xi)}{-\int_0^\infty \exp(-\int_0^x b(\xi)d\xi)\, d(-\int_0^x b(\xi)d\xi)} = \frac{[\exp(-\int_0^x b(\xi)\, d\xi)]_V^\infty}{[\exp(-\int_0^x b(\xi)\, d\xi)]_0^\infty} \tag{20}$$

since $b(x)$ does not become zero, even in pure air (due to the Rayleigh scatter)

$$\int_0^\infty b(\xi)d\xi = \infty,\ \exp(-\int_0^\infty b(\xi)d\xi) = 0.$$

Therefore, (18) becomes

$$\epsilon = \exp(-\int_0^V b(\xi)d\xi). \tag{21}$$

The brightness of the horizon is

$$B_H = A[\mathrm{e}\,(-\int_0^V b(\xi)d\xi)]_0^\infty = A.$$

i.e. even if the aerosol is distributed inhomogeneous the brightness of the horizon does not depend on $b(x)$.