The Visibility of Distant Objects

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For thousands of years, thousands of mariners have sighted thousands of ships, and have made appropriate entries in their logs. Even so, this mass of miscellaneous information is of little use in predicting the range at which a specified object will be just visible under a new set of circumstances. The purpose of this paper is to identify the principal factors involved in the visibility of an object, to indicate how each factor affects the range of visibility, and to supply charts which, by combining these factors, enable the limiting range to be found under any set of prevailing conditions. (This paragraph has been lifted, almost verbatim, from some material prepared during the war by Professor Arthur C. Hardy, then Chief of the Camouflage Section (16.3) of the NDRC.)

1. INTRODUCTION

During the recent war the Camouflage Section (16.3) of the National Defense Research Committee undertook an extensive study of the visibility of distant objects.[1] One result of this research was the construction of nomographic (alignment) charts by means of which laboratory data concerning contrast thresholds for the human eye can be combined with data describing the physical stimulus reaching the observer to predict the limiting range at which a given object will be visually detectable. It is the purpose of this paper to present these charts and to illustrate the manner of their use in predicting the limiting range of visibility for uniform circular objects.** The development of the visibility charts was by no means the sole work of the author; rather, it stems from the combined efforts of many of the personnel of NDRC Section 16.3, its contractors, its liason officers, and its friends. In accordance with the wishes of these contributors the author has prepared this paper.

The visibility of a uniformly luminous object depends upon the apparent contrast between the object and its background, the angular size of the object, its shape, the contrast threshold of the observer at the level of luminance to which his eyes are adapted, and the conditions and technique of observing. Throughout the researches in visibility conducted under the supervision of the NDRC Camouflage Section it was the basic plan to investigate the various factors controlling the apparent contrast of objects, to ascertain from laboratory experiments the contrast thresholds of the human observer, and to devise means for combining this knowledge in such a manner that the limiting range at which an object is visually detectable can be readily predicted. The apparent contrast of distant objects has been discussed by the author,[2–4] contrast thresholds have been reported by Blackwell,[5] and the nomographic visibility charts presented in this paper combine these results in convenient form.[6]

2. THE NATURE OF THE PROBLEM

Both the apparent contrast and the angular size of an object depend upon the distance of the observer, but in accordance with different laws. For example, at a distance of R yards, a circular object of area A square feet subtends an angle α given by:

$$\alpha = 1293A^{\frac{1}{2}}/R \text{ minutes of arc.} \tag{1}$$

The apparent contrast C_R of any object at a

* Formerly Technical Aide, NDRC Section 16.3.
[1] Summary Technical Report of NDRC Division 16, Volume 2.
** The nomographic visibility charts were prepared by the staff of the Louis Comfort Tiffany Foundation under contract OEMsr 597 with the Office of Scientific Research and Development. The directions for their use, the illustrative examples, and many of the paragraphs of discussion have been taken from the writings of the author in Volume 2 of the Summary Technical Report of NDRC Division 16. Oral presentations have included the formal opening of the Boston University Optical Research Laboratory, December 13, 1946.

[2] S. Q. Duntley, J. Opt. Soc. Am. **36**, 359(A) (1946).
[3] S. Q. Duntley, J. Opt. Soc. Am. **36**, 713(A) (1946).
[4] S. Q. Duntley, J. Opt. Soc. Am. **38**, 179 (1948).
[5] H. R. Blackwell, J. Opt. Soc. Am. **36**, 624 (1946).
[6] S. Q. Duntley, J. Opt. Soc. Am. **36**, 359(A) (1946).

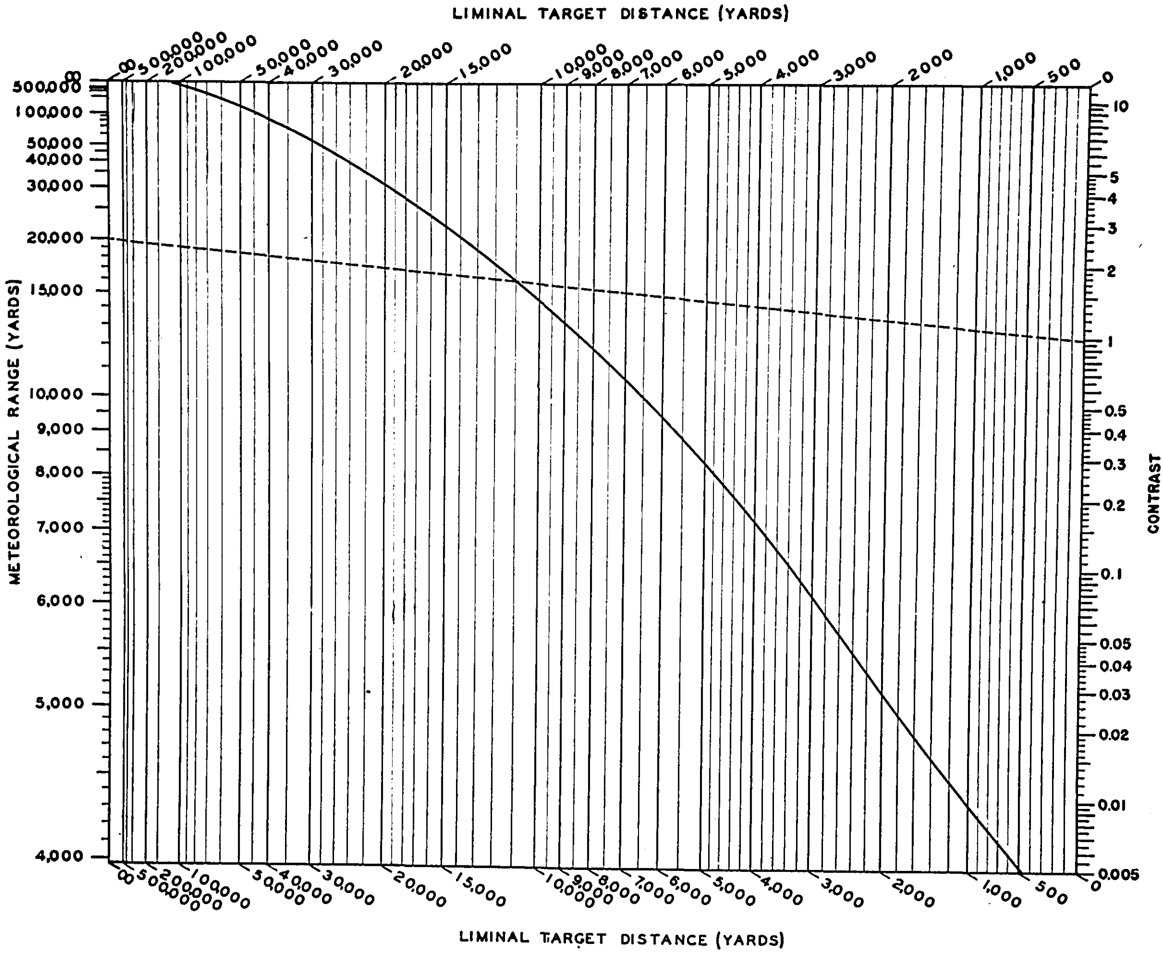

FIG. 1. Nomographic visibility chart. The curve represents visibility data for a uniform, circular object 100 square feet in area when $B_H = 1000$ footlamberts.

distance R has been shown[4] to be related to its inherent contrast C_O by the relation:

$$C_R = (B_O/B_R)C_O e^{-3.912\bar{R}/v}, \tag{2}$$

where B_O is the inherent luminance of the background, B_R is the apparent luminance of the background, $\bar{R}$ is the optical slant range,[4] and v is the meteorological range. The latter is defined as that distance for which the contrast transmittance[4] of the atmosphere is two percent.

In the special case of an object seen against a background of horizon sky, $B_O = B_R$, and Eq. (2) reduces to:

$$C_R = C_O e^{-3.912R/v}. \tag{3}$$

Since the contrast threshold of the human observer depends simultaneously upon α and C_R in the manner shown by Fig. 16 of the recent paper by Blackwell,[5] any calculation intended to determine the range at which an object is just detectable must consist of a series of successive approximations. The following example illustrates such a calculation.

Illustrative Example. Let it be required to find the distance at which a uniform circular object, having a projected area of 100 square feet and a luminance of 10 footlamberts, will be liminally visible*** on a day when the meteorological range is 20,000 yards, assuming the object to be viewed along a homogeneous, horizontal path of sight**** against a uniform background of

*** See Section 8 for a definition of liminal object distance.

**** A homogeneous, horizontal path of sight is one for which the luminous density q, the attenuation coefficient β, and the scattering-rate coefficient σ are the same at all points. (Reference 4.)

horizon sky, the luminance of which is 1000 footlamberts. The inherent contrast of the object is:

$$C_O = 10 - 1000/1000 = -0.990.$$

This value indicates that, to a nearby observer, the object appears as a nearly black silhouette. Negative contrast signifies an object darker than its background; positive contrast denotes an object more luminous than its background. It will be noted that negative contrast can never exceed unity ($C_O = -1.00$), whereas there is no upper limit to positive contrast. An important conclusion from the results reported by Blackwell[5] of the Tiffany Foundation is that, for practical purposes, objects darker than the background are as visible as congruent objects lighter than the background when their respective contrasts are numerically equal.†

Since the meterological range is 20,000 yards, it may be assumed that a very large black object would be liminally visible at approximately that range. However, at 20,000 yards the angle subtended by the target is shown by Eq. (1) to be only 0.646 minute. The Tiffany data[5] show the liminal contrast for an object of this angular size to be -0.355. From the definition of meteorological range, however, the apparent contrast of the object is -0.020. Hence, the object is invisible at 20,000 yards.

Although the liminal object distance is now known to be less than 20,000 yards, its actual value must be found by trial: Assume the object

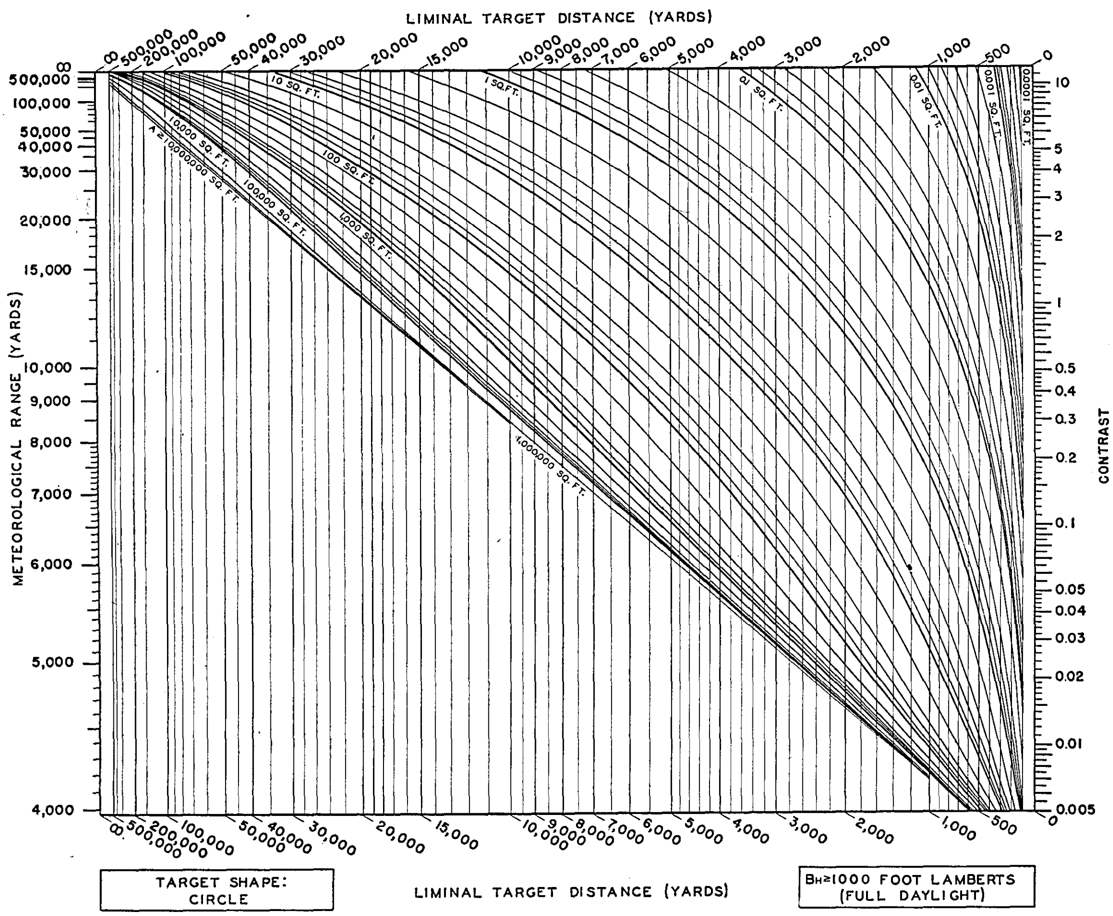

FIG. 2.

† The Tiffany results show a minor exception to this rule for values of adaptation luminance below 10^{-3} footlambert. At these night-time levels large objects darker than their backgrounds are slightly more visible than corresponding objects of numerically equal positive contrast. (Reference 5.)

to be at 10,000 yards. At this distance it subtends an angle of 1.292 minutes, and its apparent contrast is shown by Eq. (3) to be −0.145. From the Tiffany data, the liminal contrast for an object of this angular size is −0.0966. Therefore the object can be seen at 10,000 yards and beyond.

It is evident that the distance at which the object is liminally visible has been "bracketed." If the bracketing process is continued, the answer finally attained is 11,000 yards. It should be evident, however, that such calculations are cumbersome, time consuming, and invite mistakes. Fortunately, they can be avoided completely by the use of nomographic (alignment) charts.

3. THE NOMOGRAPHIC METHOD

Figure 1 shows a nomographic chart capable of making simultaneous allowances for all of the variables. The curved line which crosses the center of the figure represents Tiffany data[5] for the contrast thresholds of a typical observer when his eyes are adapted to full daytime sky brightness, and when the object is uniformly luminous, circular, and 100 square feet in area. An object of any other area could be represented by a different curve, and subsequent nomographic charts in this paper contain a family of curves corresponding to a billion-fold range of areas.

To solve the illustrative example of the preceding section by means of Fig. 1, simply lay a straightedge across the chart in such a manner that it connects 20,000 yards on the scale of meteorological range with 0.99 on the scale of contrast. The position of the straightedge is indicated on Fig. 1 by the dashed line. From the point where the curve is intersected by the straightedge move straight up or straight down

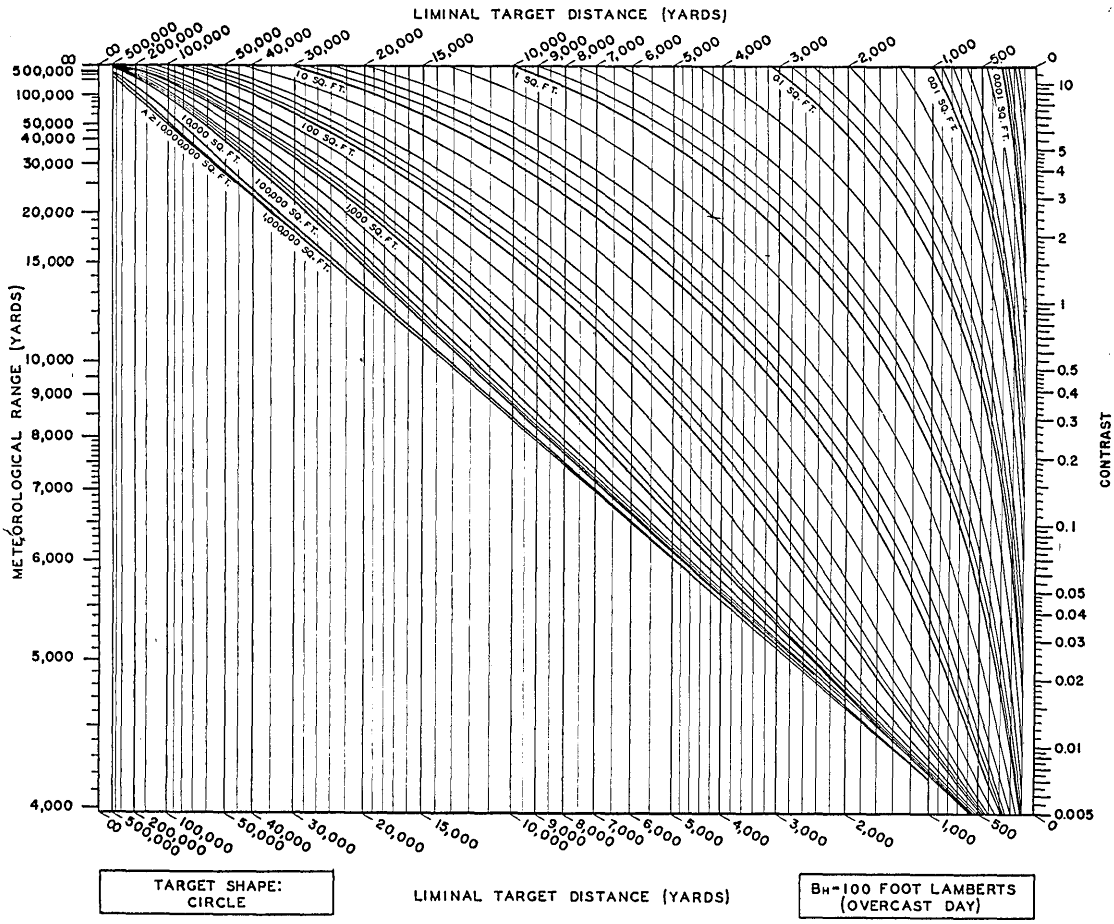

FIG. 3.

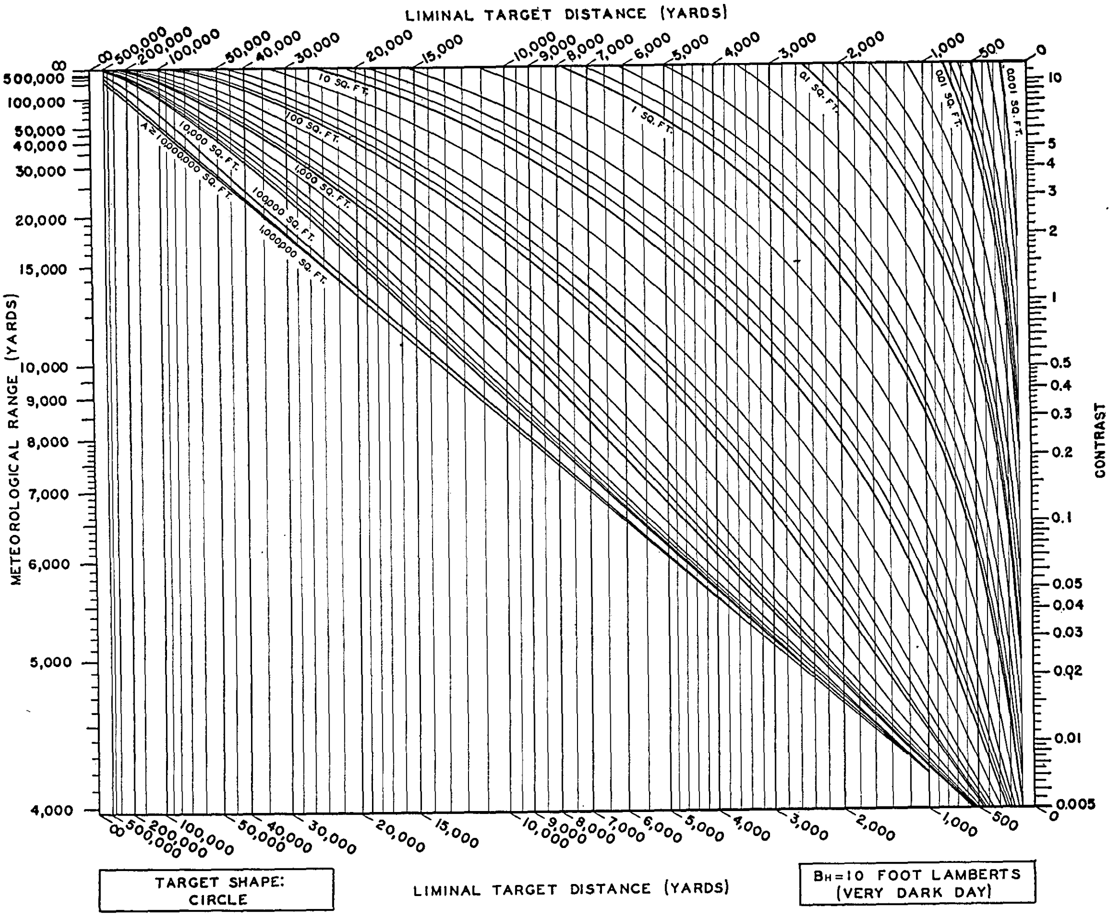

FIG. 4.

to the scale of "liminal target distance."†† The answer, 11,000 yards, read from this scale agrees with the liminal object distance previously obtained by bracketing.

Special Cases

The scales of meteorological range and liminal target distance in Fig. 1 may be multiplied by any factor, provided the value of area assigned to the curved line is multiplied by the square of the factor. This convenient property of the nomographic visibility charts enables them to be used over very wide ranges of their variables. For example, let the scales of meteorological range and liminal target distance be multiplied by $\frac{1}{10}$. The former then applies to fogs in which the meteorological range is as short as 400 yards, and the numbered divisions of the latter begin with 50 yards. The curved line, however, now applies to objects 1 square foot in area. Obviously, if Fig. 1 bore a curve corresponding to an object 10,000 square feet in area on the basis of the scales as originally numbered, the curve would apply to an object 100 square feet in area when the chart is used in the manner just described.

In dealing with objects of very large area, visible at very long distances, the scales of the visibility charts may advantageously be multiplied by 10. The curved line in Fig. 1 then applies to circular objects 10,000 square feet in area, and the dashed line indicates that such an object will be liminally visible at 110,000 yards on a day when the meteorological range is 200,000 yards, provided the inherent contrast of the object is ±0.99.

†† Synonymous with "liminal object distance."

Fig. 5.

Exact Values of Object Area

Since the factor by which the scales are multiplied may have *any* value, the curved line in Fig. 1 can be made to apply to *any* area. For example, let it be required to find the liminal object distance for an object whose area is 64 square feet, assuming other conditions as before. Since the area represented by the curve must be multiplied by 0.64, the scales must be multiplied by 0.80. This means that a meteorological range of 20,000 yards will be represented by the division numbered "25,000" on the chart. If this point is connected by a straightedge (not shown) to 0.99 on the contrast scale, the intersection of the curved line and the straightedge indicates 12,400 on the scale of liminal target distance. After multiplying by the scale factor, 0.80, the liminal object distance is found to be 9920 yards.

The family of curves, each representing a target of different area, which appear on all of the subsequent nomographic visibility charts, is intended to make unnecessary the type of calculation just described. However, for the construction of tables, or for special computations requiring great precision, the method described above should be used.

Other Uses of the Nomograph

The nomographic visibility chart may be considered as a special plot of the Tiffany data[5] on contrast thresholds for the human observer. For example, let it be required to find the liminal apparent contrast of a target 100 square feet in area and 10,000 yards from the observer. Place a straightedge across Fig. 1 in such a manner that it connects the infinity point at the top of the meteorological range scale with the intersection of the curve and the vertical line representing 10,000 yards. The straightedge then intersects the contrast scale at ±0.097, the value of liminal contrast for this angular subtense.†††

The nomographic visibility chart can be used to solve Eq. (3). For example, let it be required to find the apparent contrast of an object of inherent contrast ±0.99 when it is 10,000 yards from the observer on a day when the meteorological range is 20,000 yards. Place a straightedge across Fig. 1 in the position shown by the dashed line. Place the point of a pencil at the intersection of the straightedge with the vertical target-distance line for 10,000 yards. Rotate the straightedge until it passes through the infinity point at the top of the meteorological range scale. The straightedge now intersects the contrast scale at ±0.145, the apparent contrast of the object. Obviously, this technique can be employed to solve for any of the four quantities in Eq. (3).

4. NOMOGRAPHIC VISIBILITY CHARTS

More than two million observations of uniform circular visual targets were made by a homogeneous group of observers at the Tiffany Foundation under its OSRD contract, and a statistical summary of 450,000 of these observations has been reported by Blackwell.[5] No visibility experiment of comparable magnitude or thoroughness is known to have been reported. The Tiffany data are believed to be by far the most reliable existing information on the contrast thresholds of unrestricted binocular human vision. It is appropriate, therefore, to present these

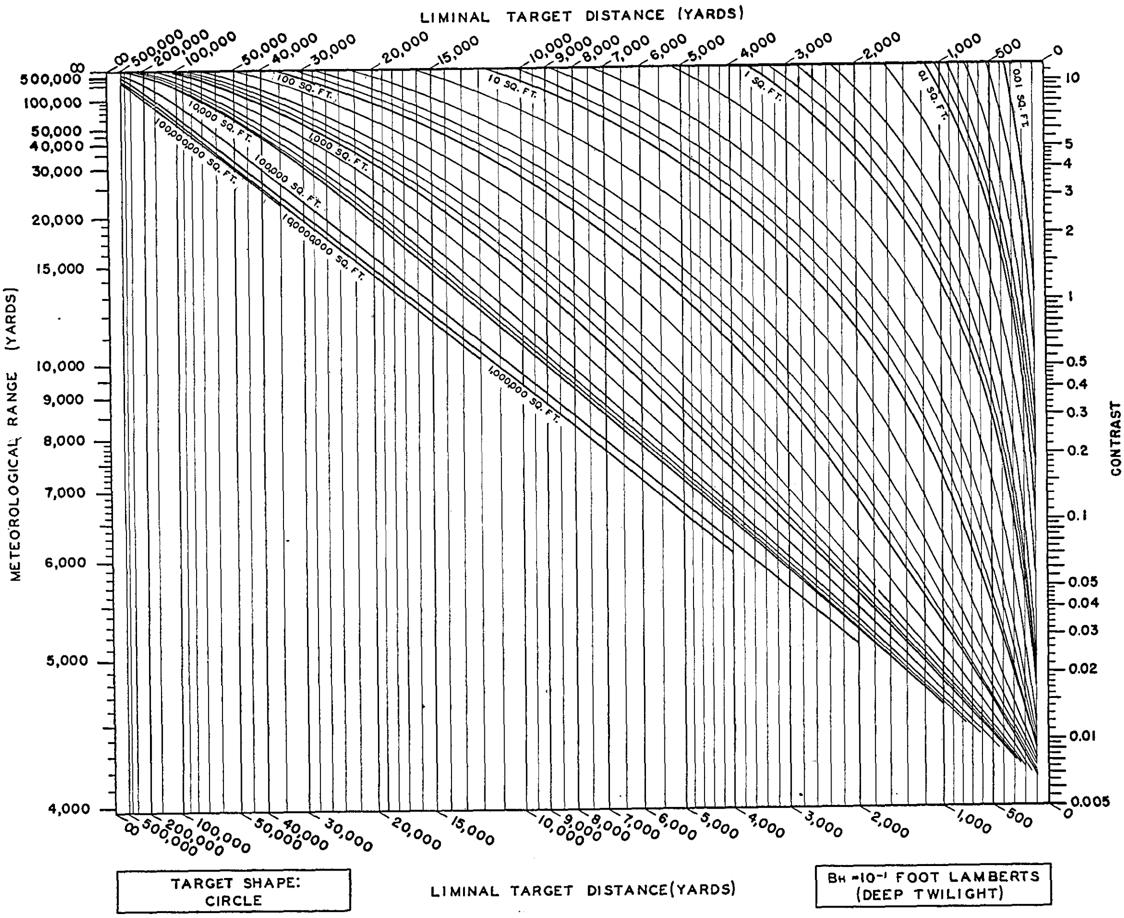

FIG. 6.

††† It will be noted that the locus of liminally visible circular targets subtending the same angle at the eye of the observer is a straight line connecting the infinity point on the meteorological range scale with the point on the contrast scale representing the liminal apparent contrast.

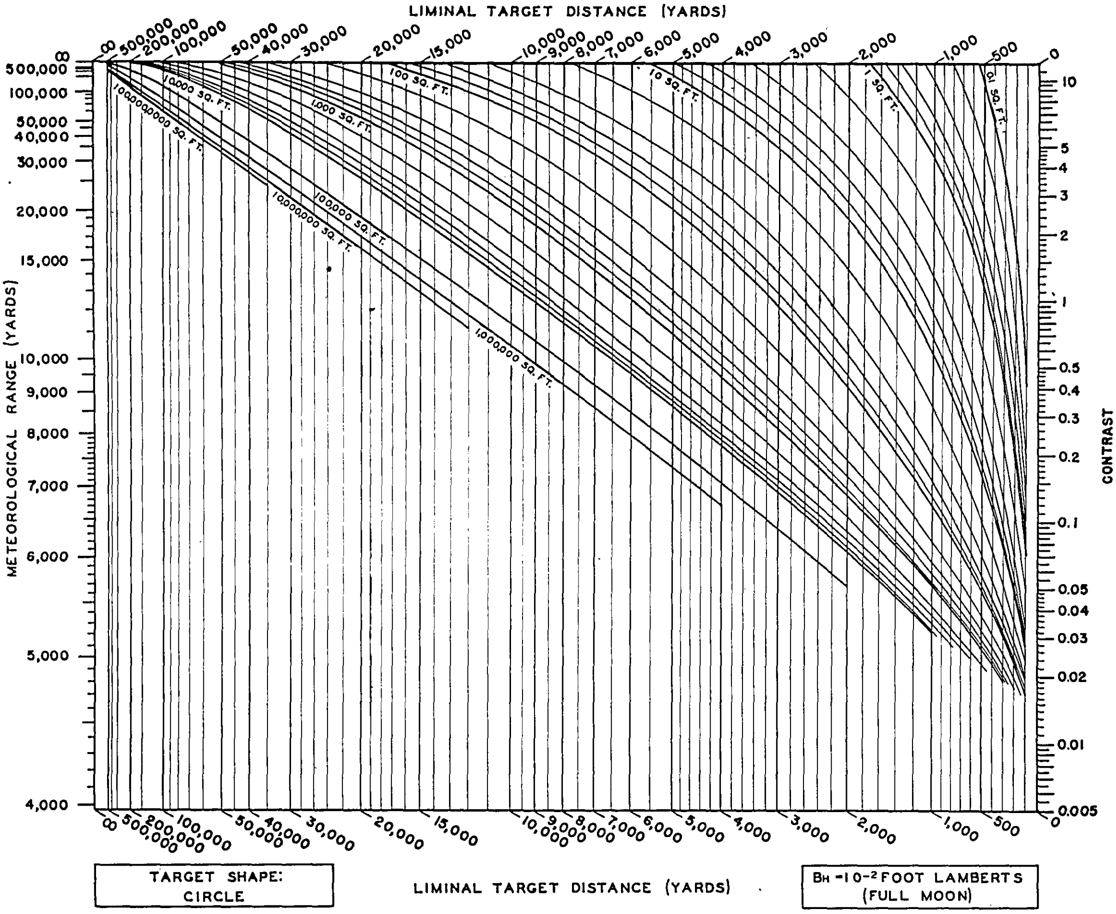

FIG. 7.

data in the form of nomographic charts similar to Fig. 1.

Each of the nine figures (Figs. 2 through 10) is a nomographic visibility chart for uniform circular objects seen against a uniform background of horizon sky having the luminance B_H indicated in the lower right corner of the diagram. Descriptive phrases, such as *overcast day* or *quarter-moon*, have been added to serve as a rough guide in selecting the proper chart for use in a given problem. The charts cover adaptation luminances (B_H) from 10^{-5} footlambert to 1000 footlamberts in decimal steps.

5. VISIBILITY OF NON-CIRCULAR UNIFORM OBJECTS

One of the first experiments performed by the Tiffany Foundation compared the visibility of the silhouette of a ship with that of a uniform circular object having the same area. This experiment suggested that uniform objects of equal area and equal apparent contrast are equally visible regardless of their shape. Later experiments showed, however, that in certain extreme cases correction for object shape is required. Although such corrections will not be treated in this paper, it is important to note that when the time for search is unrestricted no uniform object, regardless of its shape, is more visible than a uniform circular object of equal area and equal inherent contrast.

6. VISIBILITY THROUGH BINOCULARS

The distance at which a specified object is liminally visible through perfect binoculars can be found from the nomographic visibility charts by multiplying the area of the object by the square of the magnifying power of the binoculars before entering the data on the chart. For example, suppose a perfect 7-power binocular is used by the observer in the illustrative example of Section 3. Since the area of the object is 100 square feet, the area used in entering the chart (Fig. 2) is 4900 square feet and the resulting liminal object distance is 22,500 yards. Recalling that the liminal object distance for the unaided eye is 11,000 yards in this example, it will be noted that the 7-power glasses increased the liminal object distance by a factor of approximately two. Only when the meteorological range is infinite do perfect binoculars having a magnifying power M permit objects to be seen M times as far as they can be seen by the naked eye.

The limiting performance of perfect telescopic systems under optimum conditions has been discussed by Hardy,[7] who showed that binoculars are most effective in increasing the limiting range of detection when the object is small. It should be noted that Hardy's treatment applies only to optimum conditions, while the nomographic visibility charts enable the performance of perfect binoculars to be predicted under virtually any conditions.

The foregoing discussion applies to *perfect* binoculars, by which is meant an instrument whose only effect is to increase the apparent angular size of the object. Actually, even the best binoculars fall somewhat short of the ideal, so that, unless data concerning the contrast rendition[8] of the instrument are used, the liminal object distance predicted by means of the nomographic visibility charts represents a limiting value, never exceeded but sometimes approached by observers using real binoculars.

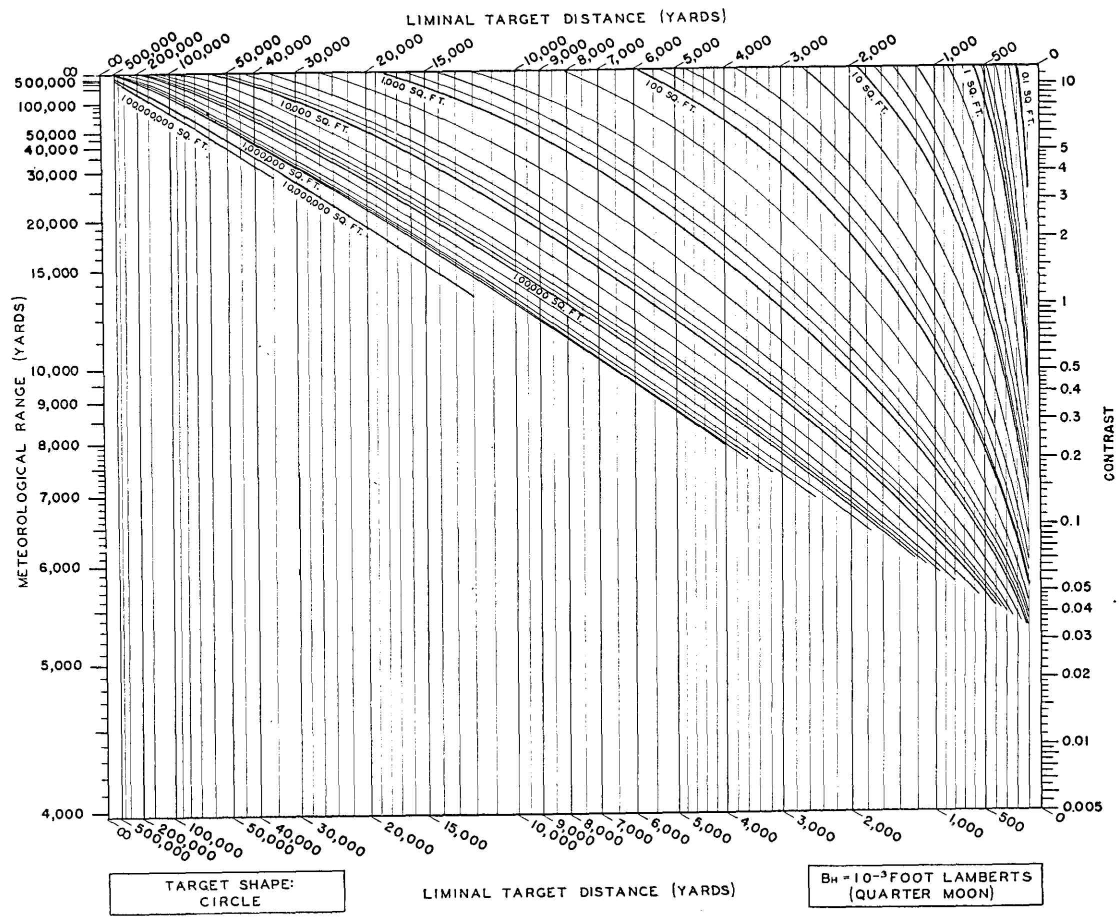

FIG. 8.

[7] A. C. Hardy, J. Opt. Soc. Am. **36**, 283 (1946).
[8] H. S. Coleman, J. Opt. Soc. Am. **37**, 434 (1947).

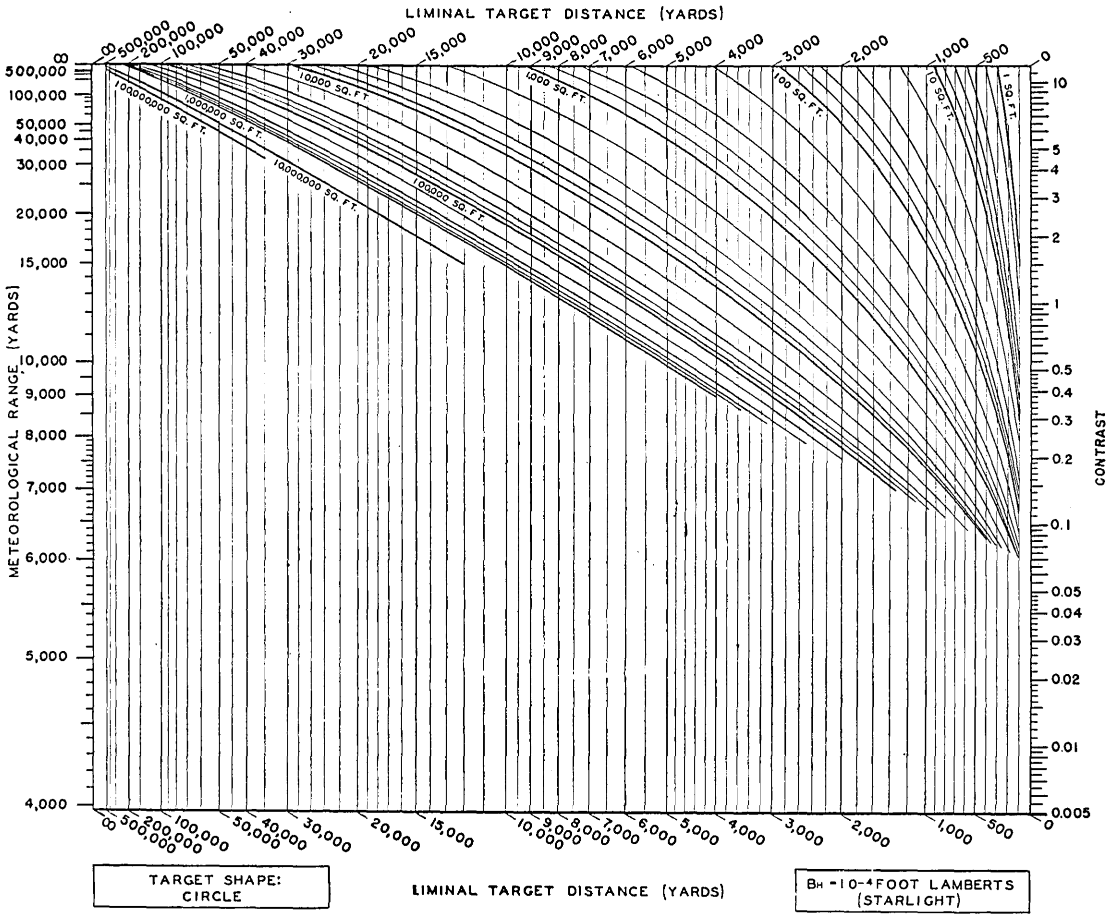

FIG. 9.

7. VISIBILITY ALONG INCLINED PATHS OF SIGHT

The preceding sections have discussed only the special visual task of sighting an object along a horizontal path with the horizon sky for a background. The reduction of the apparent contrast of distant objects along inclined paths of sight and against backgrounds of any luminance has been discussed by the author in another paper.[4] The nomographic visibility charts are useful in solving visibility problems of this type, provided that the parameters of the charts are properly interpreted. Of these, the scale of meteorological range needs no special discussion. The scales of liminal target distance, area, and contrast, however, require the following special interpretation:

Liminal target distance. If the path of sight is other than horizontal the reduction in apparent contrast is dependent upon the optical slant range[4] $\bar{R}$. The scale of liminal target distance should, therefore, be regarded as applying to values of $\bar{R}$.

Area. Because the atmosphere becomes progressively more rarified as altitude is increased, the true slant range ordinarily exceeds the optical slant range. Therefore, the object actually subtends a smaller angle at the eye of the observer than if it were at distance $\bar{R}$. This may be allowed for by entering the nomographic visibility charts with a *reduced* value of object area. The reduced or *effective object area* $\bar{A}$ is related to the object area A by the expression:

$$\bar{A} = (\bar{R}/R)^2 A. \tag{4}$$

Contrast. Since the nomographic visibility charts are constructed around Eq. (3), their use in problems involving Eq. (2) requires that the scale of contrast be identified with the quantity

$(B_O C_O/B_R)$. This is not, however, a convenient parameter for use in visibility problems. More usable variables occur in a modified form[4] of Eq. (2):

$$C_R = C_O[1-(B_H/B_O)(1-e^{3.912\bar{R}/v})]^{-1}, \quad (5)$$

where B_O is the inherent luminance of the background, and B_H is the equilibrium luminance along the path of sight. The quantity B_H/B_O is referred to as the *sky-ground ratio*.[4]

A modified form of the nomographic visibility charts is shown in Fig. 11. This chart, based on Eq. (5), differs from the preceding nomographs only by the addition of a scale of sky-ground ratio along the inside left margin and by a new scale of inherent contrast which is located near the center of the figure. The manner of using this chart will be illustrated by re-solving the example of Section 3 for the case of the target viewed against a background having a luminance of 200 footlamberts. Since the luminance of the horizon sky was assumed to be 1000 footlamberts, the sky-ground ratio is 5.0. The inherent contrast of the object against its background is:

$$C_O = 10 - 200/200 = -0.95.$$

Place a straightedge across Fig. 11 in such a manner that it connects 5.0 on the sky-ground ratio scale with 0.95 on the contrast scale. Place the point of a pencil at the intersection of the straightedge with the right-hand vertical boundary of the chart, and rotate the straightedge until it connects this point with 20,000 yards (11.4 miles) on the meteorological range scale. From the intersection of the straightedge and the curve for a target area of 100 square feet, proceed vertically to a reading of 6900 yards on

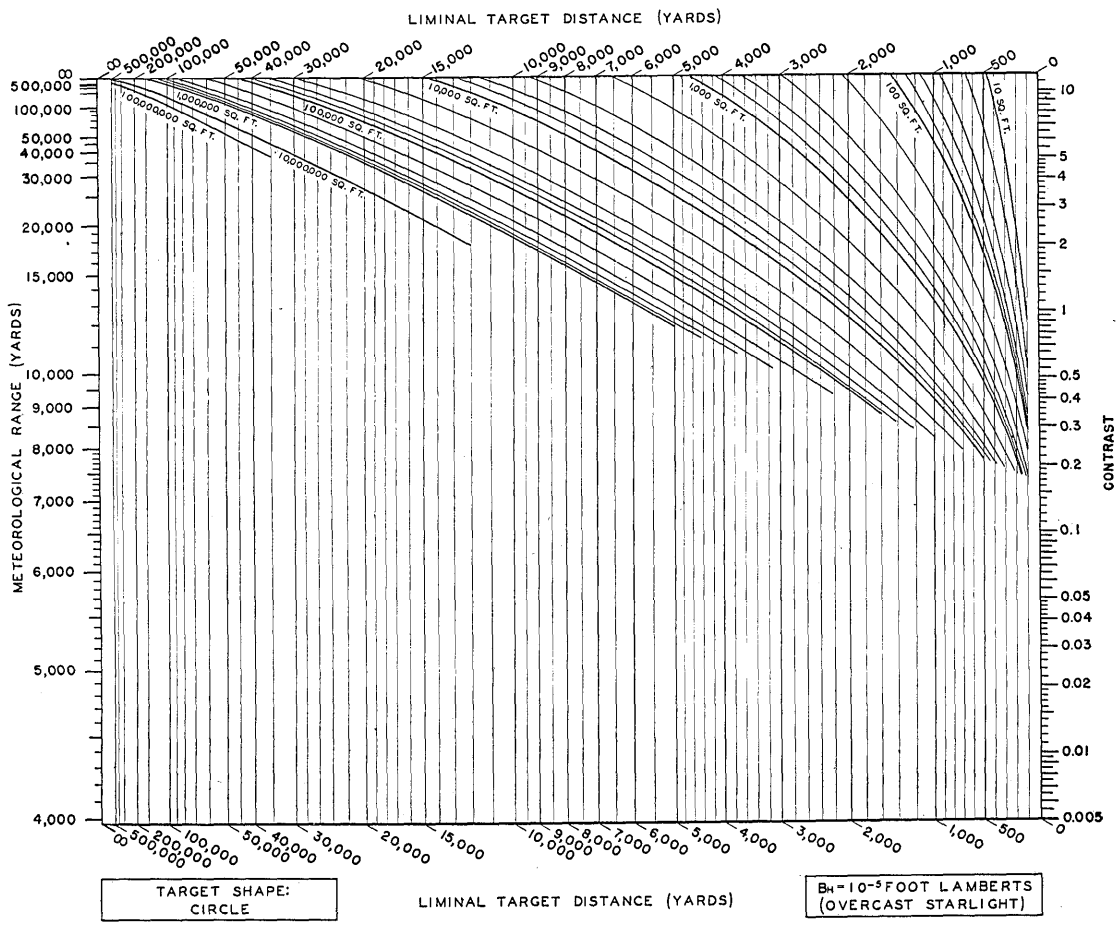

FIG. 10.

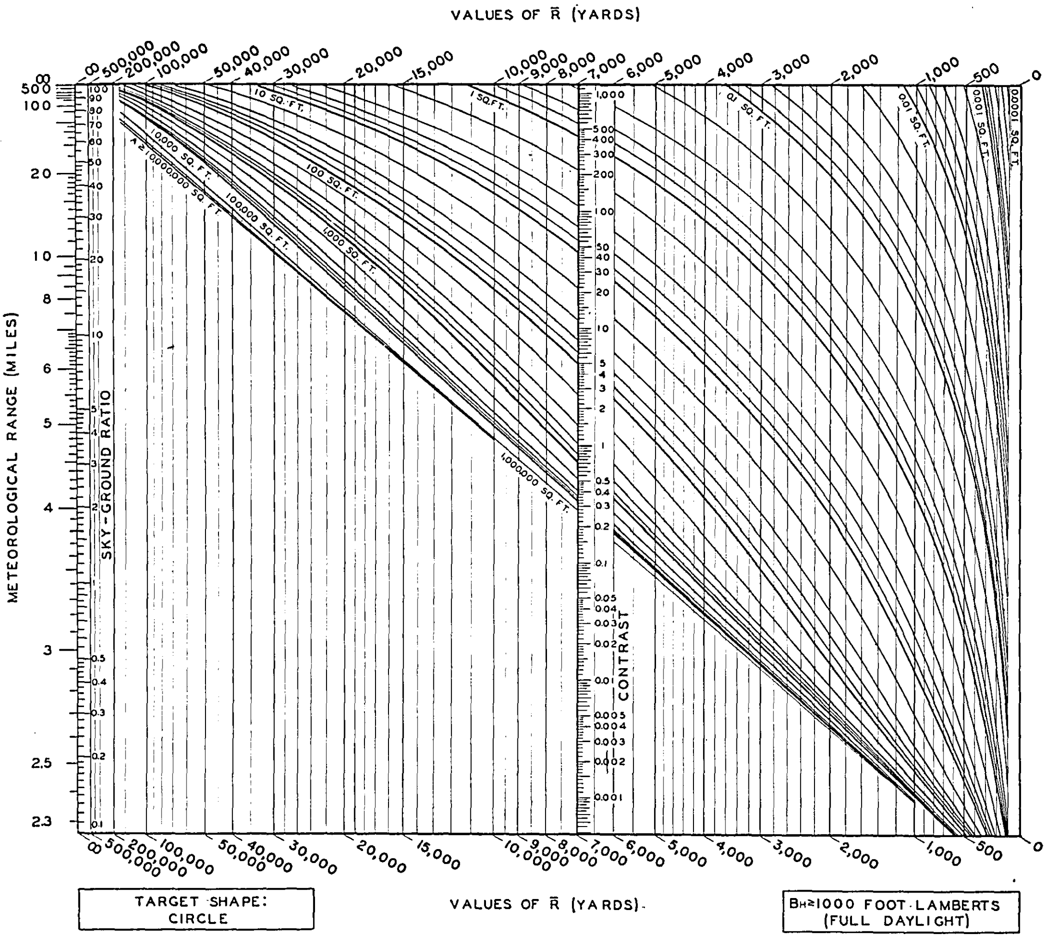

FIG. 11.

the scale marked "*Values of* $\bar{R}$." Along horizontal paths of sight $\bar{R}=R$. Hence, the liminal object distance is 6900 yards.

When the path of sight is not horizontal, it is ordinarily impossible to solve directly for the liminal object distance, since $\bar{R}\neq R$. In such cases, values of liminal inherent contrast for assumed values of slant range R can be determined from Fig. 11, and the liminal object distance can then be found by interpolation.

8. INTERPRETATION OF LIMINAL OBJECT DISTANCE

The nomographic visibility charts presented in this paper are based upon the contrast thresholds for the human observer published by Blackwell of the Tiffany Foundation.[5] The criterion adopted by Blackwell in reporting these thresholds is such that an object is liminally visible when an observer who is *forced* to judge whether the object is present or absent is as likely to be right in his judgment as he is likely to be wrong, due allowance having been made for chance. This criterion, commonly used in psychometric work, is well suited to the needs of the nomographic visibility charts because it has precise physical significance, and because liminal object distance, when so defined, is unlikely to be exceeded in practice when conditions correspond with those upon which the charts are based. It should be borne in mind, however, that the nomographs are based upon data representing the performance of excellent observers under nearly ideal observing conditions. Because of fatigue, discomfort, distraction, and the necessity for search it is to be expected that, in most instances, actual sightings will occur at ranges somewhat less than those indicated by the charts. On the other hand, the atmosphere is sometimes so inhomogeneous that the actual sighting range may exceed the range indicated by the nomographs. Experience in their use is the best guide to the allowances that should be made for departures frcm the conditions upon which the charts are based.

Threshold of Confidence

Blackwell[5] has pointed out that the observer of a liminally visible object is quite unaware that half of his forced responses are correct; he has no confidence that he has seen the object. When the contrast of the object is doubled, however, the average observer experiences a threshold of confidence, and will voluntarily report having seen the object.‡ Therefore, at some range, less than the liminal object distance, the observer becomes aware of seeing the object. This has been called the *sighting range.*

The sighting range can be predicted from the nomographic visibility charts by dividing the inherent contrast of the object by 2 before entering the data on the chart. Thus, in the illustrative example discussed in Sections 2 and 3, if the straightedge connects 20,000 yards on the meteorological range scale of Fig. 1 with 0.50 on the contrast scale, an object distance of 9260 yards is indicated. At approximately this distance the object should be seen with threshold confidence. The nomographic visibility charts have been constructed to indicate the liminal target distance rather than the sighting range, because the former quantity has a precise physical significance not shared by the latter.

9. SUMMARY

The limiting range at which a uniformly luminous object can be detected by unaided vision or with perfect binoculars can be predicted from laboratory data on the contrast thresholds of the human observer, and photometric data concerning the luminance of the object, its background, and the optical state of the atmosphere. Nomographic (alignment) charts are presented which eliminate laborious successive approximations and permit the visibility of objects along either horizontal or inclined paths of sight to be predicted.

‡ According to Blackwell (reference 5), the threshold of confidence corresponds approximately with a 90 percent probability of making a correct forced response.

10. ACKNOWLEDGMENT

As stated in the Introduction, the author was but one of a number of contributors to the development of the nomographic visibility charts. Early in the war, at the suggestion of Professor Arthur C. Hardy, then Chief of NDRC Section 16.3, the author began the construction of alignment diagrams for use in visibility problems. These charts were demonstrated to numerous persons, and many contributed suggestions. The first nomograph which successfully eliminated successive approximations was constructed by Dr. George Kimball. This was such an important advance that the Tiffany Foundation was asked to engage Professor Raymond D. Douglass of the Massachusetts Institute of Technology to construct a series of such nomographs. Because these charts suffered from a complex cycle of operations and from precision difficulties in certain ranges of their variables, much effort was expended by all concerned to develop a simpler, more effective type of nomograph. Special mention should be made of the many contributions by Professor Hardy to this development. The stimulating interest of Commander Dayton R. E. Brown during this phase of the work is gratefully acknowledged. The resulting nomographic visibility charts, presented in this paper, were constructed by Professor Douglass.