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Secchi disk science: Visual optics of natural waters[1]

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Abstract

The Secchi disk is a circular white disk that is lowered into a natural body of water by a human observer until it disappears from view. The depth of disappearance is a visual measure of the clarity of the water. This review examines the physical and physiological basis of the Secchi disk procedure. The theory of the white disk is detailed to show the underlying assumptions and the consequent strengths and limitations of the procedure. The theory shows how to use a calibrated Secchi disk to predict illuminance levels as a function of depth. In particular it is shown how to predict the euphotic depth of a medium. Ten laws of the Secchi disk are stated verbally and in mathematical form. The laws show how variations in properties of the disk and the surrounding light field affect the depth of disappearance of the disk. Theory and examples lead to the following three main conclusions of this paper: (i) the Secchi disk reading z_{SD} (in meters) yields a quantitative estimate of a single apparent optical property ($\alpha + K$) (in meter^{-1}) of a natural hydrosol, where α is the (photopic) beam attenuation coefficient and K the (photopic) diffuse attenuation coefficient of the medium; (ii) the primary function of a Secchi disk is to provide a simple visual index of water clarity via z_{SD} or $\alpha + K$; (iii) to extend the use of the Secchi disk by auxiliary objective electronic measurements of α or of K, or both, is to risk obviating or abusing this primary function.

The Secchi disk is a device used to visually measure the clarity of natural waters. It is usually a circular white disk of 30-cm diameter that is lowered vertically into the water, disk plane horizontal, until it disappears from sight. The depth of disappearance of the disk is inversely proportional to the average amount of organic and inorganic materials along the path of sight in the water. This technique, systematically studied by the Italian physicist Angelo Secchi (1866) is, interestingly, still in use today. It is one of the few instruments remaining in the armory of modern science for which the visual sense of the human operator is an integral part of the measurement procedure. Some historical background is given by Sauberer and Ruttner (1941).

This review examines the physical and physiological basis of the Secchi disk procedure. The theory of the white disk is detailed and the various laws of its operation, informally discovered over the years, are given quantitative expression. A sensitivity analysis of Secchi depths as a function of environmental parameters is made. In this way a potential user of the technique can balance its various subjective disadvantages against its simple and inexpensive operation.

The Secchi disk procedure is valued by many aquatic biologists as a useful and informal visual index of the trophic activity of a lake or oceanic region. The accumulated listings of the depth of disappearance of the

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disk, as a function of season and location within a given lake, estuarine, or coastal region can over the years provide a readily understood and quite useful record of the growth and decay of aquatic plant life in such media. It is also useful in tracking visually the movements of suspended detritus and the migration of sediment influxes from tributary streams and rivers. In these modes of its use the white disk continues, just as it did in Secchi's day and before, to be a handy, robust, visual indicator of the variations of water clarity and biologic activity.

There has been a tendency in the last few years, however, to extend the Secchi disk technique to scientific studies of natural waters that would better be conducted with precision optical instrumentation. Certain rules of thumb, for example, which connect the disk's depth of disappearance (*Secchi depth*) with the euphotic depth of the hydrosol and rules which give the relative illuminance level at Secchi depth, are stated without confidence limits and are proffered as if they were universal connections precisely determined and holding alike in all media for all seasons. One disturbing tendency I have seen in Secchi disk papers that I have reviewed for this journal are those that attempt to use the Secchi disk readings to determine the values of the inherent optical properties of the medium, such as the spectral volume absorption function and the spectral volume attenuation function. This activity stands modern hydrologic optics on its head by using apparent optical properties to infer the inherent optical properties of a medium. This is a logically indefensible activity, as we shall see.

Now, the inherent optical properties of a water body are those defined for a narrow wavelength band and which are independent of the great variations in the directional form of the light field in the water brought about by such things as the height of the sun above the horizon, ship shadows in the water, and dancing light beams of refracted sunlight caused by surface waves. As we shall see, the Secchi disk readings are affected by all these lighting changes. Moreover, the disk readings are dependent on the visual acuity of the observer and his total physiological state at the moment of measurement. Hence Secchi disk readings are subjectively determined apparent optical properties of the water under study.

Another spectacle I have encountered in reviewing recent Secchi disk papers is not so much disturbing as it is faintly ridiculous at first sight. This is the systematic use of modern electronic light measuring equipment (such as illuminance meters and beam transmissometers), in conjunction with Secchi depth measurements, for the express purpose of establishing statistical links between Secchi depths and various inherent and apparent optical properties of the medium that can be determined by the electronic equipment. Once these statistical links have been established, the intent is to use subsequent disk readings, via the links, to infer the desired values of the inherent and apparent optical properties of the medium. This, it appears to me, is a Sisyphean activity. Such experimental endeavors are progressively frustrated because, in time, the statistical links may dissolve as the natural hydrosol undergoes its seasonal and interannual changes. These changes, while tending to be periodic, are always perturbed to some extent by random climatic and man-made events. Consider, for example, last year's Secchi disk calibrations. They may possibly no longer be trustworthy for this year's phytoplankton assay or sediment transport survey. How does one check this possibility? By calibration! And so, to be safe, a researcher recalibrates the Secchi disk anew each year by a battery of modern optical sensors and meters. Of such people I have asked: why not use the modern electronic measuring devices exclusively and be done with the disk? The best answers, invariably, are those of researchers who have found the simplicity and elegance of the method sufficiently appealing to try to place it on firmer ground.

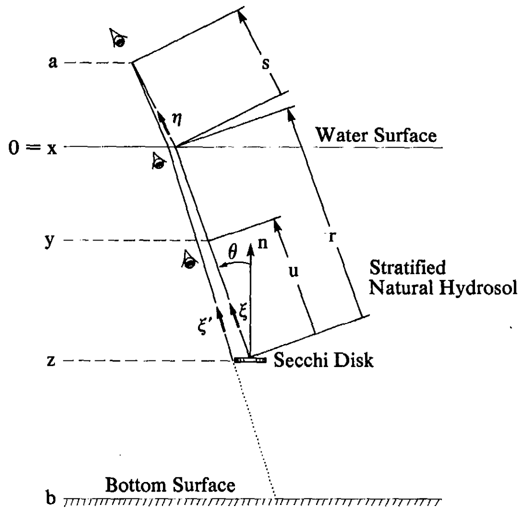
This review is intended for three types of researchers. First, it is for the scientist who is simply curious about the Secchi disk procedure, and the theoretical and experimental ground on which it rests. Second, the review is for those Secchi diskers who have energy left over to be scientists in the Baconian sense (Medawar 1979): the world is there, how does it work? For such people, there is the experimental problem of linking up the disk readings with other optical properties of a natural hydrosol. For them, the links will be obtained by carefully designed experiments and well done statistical analyses. Such workers will, as a matter of course, elicit by experiment the limits and legitimate uses of the Secchi disk procedure. The third type of researcher is the one who uses the disk as a simple visual indicator of the clarity of natural waters and wants occasionally, without making a research project out of it, to refer to the useful formulas of the subject. Before copying out any of the formulas, this researcher should at least read and underline any cautionary comments in the text on the use of the formula of interest.

Here is an outline of the study. We begin with the statement of the equation of transfer for radiance and then derive from this the radiance difference law needed for Secchi disk theory. Then we state the laws governing the upwelling and downwelling irradiances in a stratified natural hydrosol, also required in our derivations. Since Secchi disk observations are made by human eyes, we must convert the radiances and irradiances to photometric form. We shall also give the bridge from radiometry to quantal magnitudes so useful in photosynthetic studies. Once this is done we will have the photometric and quantal equivalents of the radiometric laws. From these laws we derive the photometric form of the contrast reduction equation which stands at the base of Secchi disk theory. The effects of the air-water surface's lighting will be included in the contrast reduction equation. The contrast reduction formula is then rearranged into the central equation of Secchi disk theory which, with the reader's indulgence, I will call the "binocle of eyeball optics." This equation, as in the case of a real pair of binoculars, can be looked through in two directions: the direct view yields Secchi depths given the optical properties of the medium and the contrast threshold properties of the human eye; the inverse view yields a certain single apparent optical property of the medium from a Secchi depth reading. From the binocle we go on to deduce the sensitivity of Secchi depth measurements to perturbations of the parameters of the disk, to perturbations of the environmental lighting conditions, and to perturbations of the optical state of the air-water surface. This sensitivity analysis of the binocle will yield quantitative expressions of what we shall call the "ten laws of the Secchi disk," some of which were first noted, for example, partially and informally by Secchi (1866). We then come to what I consider the active center of today's research on Secchi disk science: the problem of how to extract from the disk readings quantitative estimates, for example, of certain apparent optical properties of natural waters. This will require the use of other instruments besides the white disk, such as illuminance meters and luminous beam transmissometers. Measurements by these instruments will provide the apparent optical properties in photometric form that can be linked statistically, if perhaps only momentarily and locally, to Secchi depth readings. From such links, once established, it will be shown how one can deduce formulas for the euphotic depth of the medium and several other optical depths of practical use.

A more discursive and somewhat eclectic review of Secchi disk will be presented elsewhere (Preisendorfer 1986). To John Kirk, I extend gratitude for going over a first draft and suggesting many improvements. Niels Højerslev provided historical background and some helpful suggestions on the text. Ros Austin replaced my lost copy of the Secchi/Cialdi experiments. I also thank Yvette Edmondson for the invitation to vent opinions found in some of my Secchi disk manuscript reviews.

Equation of transfer for radiance

Let $N(y,\xi,\lambda)$ be the radiance at depth y (in meters $\equiv$ m) in a horizontally layered natural hydrosol. The radiance is associated with a packet of photons producing a flow of radiant energy (Joule per second $\equiv \mathrm{J\,s^{-1}} \equiv$ Watt $\equiv$ W) streaming within a unit solid angle (in steradian $\equiv$ sr) around and along direction ξ and crossing a unit area ($\mathrm{m^2}$)



Geometry of the Secchi Disk Sighting

Fig. 1. Geometry for the derivation of the Secchi disk formulas.

normal to ξ. The photons are of wavelength λ within a unit wavelength interval (in nanometer $\equiv$ nm). Thus $N(y,\xi,\lambda)$ has units W m^{-2} sr^{-1} nm^{-1}. The depth rate of change of radiance $N(y,\xi,\lambda)$ for fixed ξ and λ is given by the *equation of transfer*

$$-\mu \frac{d}{dy} N(y,\xi,\lambda) = -\alpha(y,\lambda)N(y,\xi,\lambda) + N_*(y,\xi,\lambda) \qquad (1)$$

where the *path function* (in W m^{-3} sr^{-1} nm^{-1}) is defined as

$$N_*(y,\xi,\lambda) \equiv \int_{\Xi} N(y,\xi',\lambda)\sigma(y;\xi';\xi,\lambda)\, d\Omega(\xi'). \qquad (2)$$

Here $\sigma(y;\xi';\xi,\lambda)$ (in m^{-1} sr^{-1}) is the *volume scattering function* which gives the amount of radiance produced per meter along ξ by scattering of photons incident along ξ' on an element of volume at depth y. In most natural media, $\sigma(y;\xi';\xi,\lambda)$ depends, for each y and λ, only on $\xi'\cdot\xi$, so that we have *isotropy* of scattering. $d\Omega(\xi')$ is an element of solid angle (in sr) along ξ', and Ξ is the set of all directions (unit vectors) ξ. Moreover, if n is the vertical upward direction then $\mu = \cos\theta = \xi\cdot n$ (Fig. 1) and $\alpha(y,\lambda)$ (in m^{-1}) is the *volume attenuation function.*

Radiance-difference law

One may formally integrate Eq. 1 along the path, of length r and direction ξ, from depth z to depth x, where $x = z - r\cos\theta$. Thus if $N_0(z,\xi,\lambda)$ is the *inherent radiance* perceived at z, then the *apparent radiance* $N_r(x,\xi,\lambda)$ perceived at x, a distance r along the photon path, is given by

$$N_r(x,\xi,\lambda) = N_0(z,\xi,\lambda)T^0{}_r(z,\xi,\lambda) + N^*{}_r(x,\xi,\lambda) \qquad (3)$$

where the *path radiance is*

$$N^*{}_r(x,\xi,\lambda) = \int_0^r N_*(y,\xi,\lambda)T^0{}_{r-u}(y,\xi,\lambda)\, du \qquad (4)$$

and the *beam transmittance,* of a path of length r along direction ξ for initial depth z in the hydrosol, such that $x \leq z$, is given by:

$$T^0{}_r(z,\xi,\lambda) \equiv \exp\left[-\int_0^r \alpha(y,\lambda)\, du\right] \qquad (5)$$

$$y = z - u\cos\theta, \qquad \cos\theta = \xi\cdot n.$$

Suppose we have two photon paths in the stratified medium having arbitrarily close directions ξ' and ξ (i.e. $\xi'\cdot\xi \simeq 1$), and going from level z to level x. (*See Fig. 1.*) Then if $\alpha(y,\lambda)$ and $\sigma(y;\xi';\xi,\lambda)$ vary continuously with depth y for fixed λ, it can be shown, by reasoning from Eq. 1, that the relations

$$T^0{}_r(z,\xi',\lambda) = T^0{}_r(z,\xi,\lambda) \qquad (6a)$$

and

$$N^*{}_r(x,\xi',\lambda) = N^*{}_r(x,\xi,\lambda) \qquad (6b)$$

are satisfied arbitrarily closely. We can write Eq. 3 for each of these directionally close paths, and on taking differences of corresponding sides, we find, by Eq. 6a, b, that

$$[N_r(x,\xi,\lambda) - N_r(x,\xi',\lambda)] = [N_0(z,\xi,\lambda) - N_0(z,\xi',\lambda)] \cdot \exp\left[-\int_0^r \alpha(y,\lambda)\, du\right] \quad (7)$$

$$x = z - r\cos\theta, \qquad \xi'\cdot\xi \simeq 1$$
$$y = z - u\cos\theta.$$

This is the *radiance-difference law.*

Irradiance equations

Photons streaming upward (+) and downward (−) through the laterally extensive horizontal plane at level y in the hydrosol produce *irradiances* $H(y,\pm,\lambda)$ (in W m^{-2} nm^{-1}) given by

$$H(y,\pm,\lambda) = \int_{\Xi_\pm} N(y,\xi,\lambda)\,|\xi\cdot n|\, d\Omega(\xi) \quad (8)$$

where $\Xi_\pm$ are the *upward* (+) and downward (−) hemispheres of directions. Also associated with these two flows are the *scalar irradiances* $h(y,\pm,\lambda)$ (in W m^{-2} nm^{-1}) given by

$$h(y,\pm,\lambda) = \int_{\Xi_\pm} N(y,\xi,\lambda)\, d\Omega(\xi). \quad (9)$$

The depth rates of change of $H(y,\pm,\lambda)$, and hence the equations governing $H(y,\pm,\lambda)$, can be derived from Eq. 1 (cf. Preisendorfer 1976, vol. 5, p. 8). The derivation proceeds by integrating each side of Eq. 1 first over Ξ_+ and then over Ξ_-. The results are simplified, using the definitions Eq. 8 and 9, to find:

$$\mp\frac{d}{dy}H(y,\pm,\lambda) = -[a(y,\pm,\lambda) + b(y,\pm,\lambda)] \cdot H(y,\pm,\lambda) + b(y,\mp,\lambda)H(y,\mp,\lambda) \quad (10)$$

where

$$a(y,\pm,\lambda) \equiv a(y,\lambda)D(y,\pm,\lambda) \qquad (\mathrm{m}^{-1})$$
$$D(y,\pm,\lambda) \equiv h(y,\pm,\lambda)/H(y,\pm,\lambda),$$

and where the *backscattering functions* $b(y,\pm,\lambda)$ are defined by

$$b(y,\pm,\lambda) = H^{-1}(y,\pm,\lambda)\int_{\Xi_\mp} d\Omega(\xi) \cdot \int_{\Xi_\pm} N(y,\xi',\lambda)\sigma(y;\xi';\xi,\lambda)\, d\Omega(\xi') \qquad (\mathrm{m}^{-1}). \quad (11)$$

Here $a(y,\lambda)$ is the *volume absorption* function (in m^{-1}). It is related to $\alpha(y,\lambda)$ in Eq. 1 via

$$\alpha(y,\lambda) = a(y,\lambda) + s(y,\lambda) \qquad (\mathrm{m}^{-1}) \quad (12)$$

where $s(y,\lambda)$ (in m^{-1}) is the *volume total scattering* function given for every ξ' by

$$s(y,\lambda) = \int_{\Xi} \sigma(y;\xi';\xi,\lambda)\, d\Omega(\xi). \quad (13)$$

Each of the three functions α, a, and s can be determined independently by modern instrumentation (Preisendorfer 1976, vol. 6, chap. 13). Therefore Eq. 12 serves as a check of these determinations; or, any two of the three functions can be measured and the third found by Eq. 12. $s(y,\lambda)$ is generally independent of ξ' since in most real media $\sigma(y;\xi';\xi,\lambda)$ is isotropic, as noted above. Since $\alpha(y,\lambda)$ in Eq. 1 is generally independent of ξ, then so, too, by virtue of the interconnection (Eq. 12), is $a(y,\lambda)$. All four functions $\alpha(y,\lambda)$, $\sigma(y;\xi';\xi,\lambda)$, $s(y,\lambda)$, and $a(y,\lambda)$ are therefore, for each wavelength λ, *inherent optical properties* at all depths y in the optical medium $X[x,z]$ stretching from depths x to z. It was shown elsewhere (Preisendorfer and Mobley 1984) how Eq. 10 under reasonably general conditions can be used to solve for $H(y,\pm,\lambda)$ given $a(y,\pm,\lambda)$, $b(y,\pm,\lambda)$; and inversely, how to find $a(y,\pm,\lambda)$ and $b(y,\pm,\lambda)$ from measurements of the irradiance quartet, $H(y,\pm,\lambda)$, $h(y,\pm,\lambda)$.

Photometric conversion formulas

We now prepare for the conversion of Eq. 10 to photometric form, the form that is adapted to the visual level of detection of photon flows, as needed for example in Secchi disk observations. For this we use the (dimensionless) photopic (daylight) luminosity function $\bar{y}(\lambda)$ defined over the *visible spectrum* from 390 to 760 nm (cf. e.g. Preisendorfer 1976, vol. 2, sec. 2.12). This function is zero below 390 nm and above 760 nm and rises smoothly and nearly symmetrically to a maximum of 1.0 at 555 nm. For each λ, $\bar{y}(\lambda)$ is proportional to the sensation of brightness (lumens $\equiv$ lm) produced in the observer's eye by incident photons of wavelength λ of some fixed radiance N_0. The constant of proportionality is K_m = 680 (lm W^{-1}). For example, suppose the radiance $N(y,\xi,450)$ is 10 W m^{-2} sr^{-1} nm^{-1} and suppose this radiance is constant over a λ interval of $\Delta\lambda$ = 10 nm centered on 490 nm. From a table of $\bar{y}(\lambda)$ values we find $\bar{y}(450)$ = 0.038. Hence the *luminance* (informally, the *brightness*) $B(y,\xi)$ associated with this radiance is, by definition,

$$\begin{aligned} B(y,\xi) &= N(y,\xi,450)[K_m\bar{y}(450)]\Delta\lambda \\ &= 10\ (\mathrm{W\ m^{-2}\ sr^{-1}\ nm^{-1}}) \\ &\quad \times 680\ (\mathrm{lm\ W^{-1}}) \\ &\quad \times (0.038) \times (10\ \mathrm{nm}) \\ &= 2{,}584\ \mathrm{lm\ m^{-2}\ sr^{-1}}. \end{aligned}$$

In general, the *luminance* $B(y,\xi)$ associated with a spectral sample of radiance $N(y,\xi,\lambda)$ over the whole electromagnetic spectrum is given by

$$B(y,\xi) = \int_\Lambda N(y,\xi,\lambda)S(\lambda)\,\mathrm{d}\lambda \qquad (\mathrm{lm\ m^{-2}\ sr^{-1}}) \qquad (14)$$

where Λ is the set of all λ from 0 to ∞, and

$$S(\lambda) \equiv K_m\bar{y}(\lambda) \qquad \mathrm{lm\ W^{-1}}.$$

The *illuminance* $E(y,\pm)$ associated with the irradiance $H(y,\pm,\lambda)$ over Λ is defined as (cf. Eq. 8)

$$\begin{aligned} E(y,\pm) &\equiv \int_\Lambda H(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda \\ &= \int_{\Xi_\pm} B(y,\xi)\,|\xi\cdot n|\,\mathrm{d}\Omega(\xi) \\ &\qquad (\mathrm{lm\ m^{-2}}), \end{aligned} \qquad (15)$$

and similarly the *hemispherical scalar illuminances* (cf. Eq. 9) are given by

$$\begin{aligned} e(y,\pm) &\equiv \int_\Lambda h(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda \\ &= \int_{\Xi_\pm} B(y,\xi)\,\mathrm{d}\Omega(\xi) \\ &\qquad (\mathrm{lm\ m^{-2}}). \end{aligned} \qquad (16)$$

Illuminance equations

The photometric counterpart to Eq. 10 is obtained by multiplying each side of Eq. 10 by $S(\lambda)$ and integrating over Λ. The result is

$$\begin{aligned} &\mp\frac{\mathrm{d}}{\mathrm{d}y}E(y,\pm) \\ &\quad = -[a(y,\pm,\Lambda) + b(y,\pm,\Lambda)]E(y,\pm) \\ &\qquad + b(y,\mp,\Lambda)E(y,\mp) \end{aligned} \qquad (17)$$

where we have written

"$a(y,\pm,\Lambda)$" for

$$\frac{\int_\Lambda a(y,\pm,\lambda)H(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda}{\int_\Lambda H(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda} \qquad (\mathrm{m^{-1}}) \qquad (18)$$

and

"$b(y,\pm,\Lambda)$" for

$$\frac{\int_\Lambda b(y,\pm,\lambda)H(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda}{\int_\Lambda H(y,\pm,\lambda)S(\lambda)\,\mathrm{d}\lambda} \qquad (\mathrm{m^{-1}}). \qquad (19)$$

These are the *photometric absorption* and *backscatter functions* for the two photon flows. Equation 17 is the result of two averaging operations applied to Eq. 1: once over direction space Ξ and once over wavelength space Λ. Much information has thereby been lost in coming to Eq. 17; yet it is one of the two equations at the foundation of Secchi disk theory. The other equation is the photometric version of the radiance-difference law, Eq. 7, to be considered below.

Equation of transfer for luminance

To arrive at the photometric counterpart to Eq. 7 we return to Eq. 1 and convert it to photometric form by multiplying each side by $S(\lambda)$ and integrating over Λ. The result is

$$-\mu \frac{d}{dy} B(y,\xi) = -\alpha(y,\xi,\Lambda) B(y,\xi) + B_*(y,\xi) \qquad (20)$$

where the *photopic volume attenuation function* is defined by writing

'$\alpha(y,\xi,\Lambda)$' for

$$\frac{\int_\Lambda N(y,\xi,\lambda)\alpha(y,\lambda)S(\lambda)\,d\lambda}{\int_\Lambda N(y,\xi,\lambda)S(\lambda)\,d\lambda} \quad (m^{-1}). \qquad (21)$$

Moreover the *photopic path function* is given by writing

'$B_*(y,\xi)$' for

$$\int_\Xi B(y,\xi')\sigma(y;\xi';\xi,\Lambda)\,d\Omega(\xi') \quad (lm\ m^{-3}\ sr^{-1}) \qquad (22)$$

and where in turn we write

'$\sigma(y;\xi';\xi,\Lambda)$' for

$$\frac{\int_\Lambda N(y,\xi',\lambda)\sigma(y;\xi';\xi,\lambda)S(\lambda)\,d\lambda}{\int_\Lambda N(y,\xi',\lambda)S(\lambda)\,d\lambda} \quad (m^{-1}\ sr^{-1}). \qquad (23)$$

Observe that $\alpha(y,\xi,\Lambda)$, unlike $\alpha(y,\lambda)$, is only an apparent optical property, since it in principle depends on ξ via weightings over Λ by the radiance along ξ at y. For similar reasons, $\sigma(y;\xi';\xi,\Lambda)$ is also not an inherent optical property of the medium. In particular σ is no longer isotropic, i.e. it no longer depends simply on $\xi' \cdot \xi$, although the departure from isotropy may be slight. (This is one of the questions the Baconian researcher, mentioned above, may be curious about.) This means that deductions from Eq. 20 about the luminance distribution $B(y,\xi)$ in a natural hydrosol are in principle conditioned by the directional distribution as well as the spectral distribution of the light field entering the medium from above. It follows that there are in principle no inherent optical properties in a photometrically measured light field. In the theory of the radiance field based on Eq. 1, however, we have the ability, through spectral filtering, to control the spectral parameters of the incident light field. Hence, as in Eq. 17, in going over from the radiometric level of Eq. 1 to the photometric level of Eq. 20, we lose important spectral information about the light field, information occasionally critical to some questions in aquatic biology.

Luminance-difference law

We can now derive the photometric counterpart to Eq. 7 starting from Eq. 20. The steps from Eq. 3 to 7 can be exactly repeated [replace $N(y,\xi,\lambda)$ by $B(y,\xi)$, and $\alpha(y,\lambda)$ by $\alpha(y,\xi,\Lambda)$] to find the *luminance-difference* law:

$$[B_r(x,\xi) - B_r(x,\xi')] = [B_0(z,\xi) - B_0(z,\xi')] \cdot \exp\left[-\int_0^r \alpha(y,\xi,\Lambda)\,du\right] \qquad (24)$$

$$x = z - r\cos\theta, \qquad \xi' \cdot \xi \simeq 1$$
$$y = z - u\cos\theta.$$

Two-flow illuminance model of the observable light field

Despite the loss in spectral information in going to Eq. 17, it serves as our point of entry into the practical theory of the Secchi disk, and we must turn it into a useful model.

First of all we can define the basic *distribution functions* $D(y,\pm)$, *reflectance functions* $R(y,\pm)$, and *diffuse attenuation functions* $K(y,\pm)$ for the photometric case by setting

$$D(y,\pm,\Lambda) \equiv e(y,\pm)/E(y,\pm), \qquad (25)$$

$$R(y,\pm,\Lambda) \equiv E(y,\mp)/E(y,\pm), \qquad (26)$$

and

$$K(y,\pm,\Lambda) \equiv -E^{-1}(y,\pm) \cdot dE(y,\pm)/dy \quad (m^{-1}). \qquad (27)$$

All the observable optical properties of these functions in the radiometric (spectral) context (cf. Preisendorfer 1976, vol. 5, sec. 9.2) hold, mutatis mutandis, in the present photometric context. In particular we find directly from Eq. 17 the following exact expressions:

$$\mp K(y,\pm,\Lambda) = a(y,\pm,\Lambda) + b(y,\pm,\Lambda) - b(y,\mp,\Lambda)R(y,\pm,\Lambda) \quad (28)$$

and

$$R(y,-,\Lambda) = \frac{K(y,-,\Lambda) - a(y,-,\Lambda)}{K(y,+,\Lambda) + a(y,+,\Lambda)}. \quad (29)$$

Next, observe that in general

$$E(z,\pm) = E(x,\pm) \cdot \exp\left[-\int_x^z K(y,\pm,\Lambda)\, dy\right] \quad (30)$$

for any two depths $x \leq z$ in a stratified medium. Unlike the spectral theory, where $K(y,\pm,\lambda)$ is essentially constant with depth y for each λ in deep hydrosols (Preisendorfer 1976, vol. 1, p. 67), the $K(y,\pm,\Lambda)$ in Eq. 28 can be noticeably nonconstant with depth y; cf. Jerlov (1976, p. 139). In early explorations of the photometric case, Duntley and Preisendorfer (1952) found that the departure from linearity of $\ln[E(y,\pm)/E(x,\pm)]$, when plotted as a function of y, can be ignored in some practically oriented investigations, certainly, e.g. in the present Secchi disk context. At any rate, Eq. 30 can be written rigorously as

$$E(z,\pm) = E(x,\pm)\exp[-K_\pm(z - x)] \quad (31a)$$

where

$$K_\pm \equiv (z - x)^{-1} \int_x^z K(y,\pm,\Lambda)\, dy \quad (31b)$$

and where $K_\pm$, as shown, are the averages over the depth interval $[x,z]$, $x < z$ of $K(y,+,\Lambda)$ in Eq. 28. These averages of course depend on x and z, but only in a relatively mild manner, if x and z are not too far apart.

For the purposes of establishing a workable Secchi disk theory, we will assume that in an arbitrary medium $X[x,z]$, $K_\pm$, and the other optical properties comprising them are independent of depth over each of a set of sufficiently thin sublayers $X[y_0,y_1]$, . . . , $X[y_{n-1},y_n]$ of $X[x,z]$, where $y_0 = x$ and $y_n = z$. Then, following the procedure of Preisendorfer and Mobley (1984), we can write Eq. 17 in the form:

$$\mp\frac{d}{dy}E(y,\pm) = -[a_\pm + b_\pm]E(y,\pm) + b_\mp E(y,\mp) \quad (32)$$

where

$$a_\pm \equiv D_\pm \bar{a} \quad (\mathrm{m}^{-1})$$
$$b_\pm \equiv D_\pm \bar{b} \quad (\mathrm{m}^{-1})$$

and where in each homogeneous subinterval $X[y_{i-1},y_i]$ we have set

$$\bar{a} \equiv a(y,\pm,\Lambda)/D(y,\pm,\Lambda), \quad D_\pm = D(y,\pm,\Lambda)$$
$$\bar{b} \equiv b(y,\pm,\Lambda)/D(y,\pm,\Lambda), \quad y = \tfrac{1}{2}[y_{i-1} + y_i], \quad (33)$$
$$i = 1, \ldots, n.$$

Hence $\bar{b}$ is exactly analogous to the mean backscatter coefficient $\bar{b}$ in Preisendorfer and Mobley's (1984) equation 11, and now $\bar{a}$ assumes a similar structure, i.e. it is a *mean absorption coefficient*. Equation 32 is now in a form exactly parallel to Preisendorfer and Mobley's equation 15, and both the direct and inverse solution procedures in that paper are therefore available for the *illuminance two-flow model* Eq. 32. We can now deduce the mean values $\bar{a}$ and $\bar{b}$ in $X[y_{i-1},y_i]$, $i = 1, \ldots, n$ and thereby estimate the depth dependence of $a(y,\pm,\Lambda)$ and $b(y,\pm,\Lambda)$ arbitrarily closely throughout any natural hydrosol $X[x,z]$, provided that one has measured the illuminance quartet $[E(y,\pm),e(y,\pm)]$ throughout $X[x,z]$.

Quantal two-flow models of the light field

Aquatic biologists in recent years have learned much about the molecular details of photosynthesis by working on the quantal level of light (Clayton 1980). In modern research on photosynthetically active radiation in natural waters it is necessary to know the numbers of photons (i.e. particles, or quanta of the electromagnetic field) available to plant cells at each depth y and for each wavelength λ. We shall show the connection between radiometric and quantal measures of light and point up an important observation relative to Secchi disk science.

The connection between radiometric radiance $N(y,\xi,\lambda)$ (J s^{-1} m^{-2} sr^{-1} nm^{-1}) and *quantal radiance* $n(y,\xi,\lambda)$ (photon s^{-1} m^{-2} sr^{-1} nm^{-1}) is

$$n(y,\xi,\lambda) = N(y,\xi,\lambda)p(\lambda) \quad (34)$$

where the conversion factor to photon counts is

$$p(\lambda) = \lambda/h_0 v = 5.03 \times 10^{15}\ \lambda \text{ photon J}^{-1}$$

where λ is in nm, Planck's constant h_0 = 6.63×10^{-34} Js photon^{-1} (action per photon), and v is the observable speed of photons at the point of radiant flux measurement, e.g. 3.00×10^8 m s^{-1} in air (within a watertight instrument).

Knowledge of $n(y,\xi,\lambda)$ is used to determine molecular construction rates by plant cells as follows. Let $\eta(\lambda)$ be the *quantum efficiency* of the photosynthetic activity of a particular phytoplankton cell. That is,

$$\eta(\lambda) = \frac{\text{molecules } ^{14}\text{C fixed s}^{-1}\text{ m}^{-3}}{\lambda\text{-photons absorbed s}^{-1}\text{ m}^{-3}}. \quad (35)$$

Clearly $\eta(\lambda)$ is a dimensionless number. Further, let $a(y,\lambda)$ (cf. Eq. 10) be the volume absorption function at depth y for wavelength λ of a source-free nonfluorescing phytoplankton suspension [so that $a(y,\lambda)$ is the true absorption function of the suspension; cf. Preisendorfer 1965, p. 65, where processes v and vi there are not operating]. Next, on recalling Eq. 10, observe that

$$\begin{aligned} h(y,\lambda) &\equiv h(y,+,\lambda) + h(y,-,\lambda) \\ &= D(y,+,\lambda)H(y,+,\lambda) \\ &\quad + D(y,-,\lambda)H(y,-,\lambda) \\ &= \int_{\Xi} N(y,\xi,\lambda)\, d\Omega(\xi) \\ &\qquad (\text{J s}^{-1}\text{ m}^{-2}\text{ nm}^{-1}) \quad (36) \end{aligned}$$

is the spectral scalar irradiance at depth y. Then $h(y,\lambda)a(y,\lambda)$ is the number of J s^{-1} m^{-3} nm^{-1} of λ-wavelength radiant flux absorbed at depth y by the suspension. Hence by Eq. 34 $h(y,\lambda) \times a(y,\lambda) \times p(\lambda)$ is the number of λ-wavelength photon s^{-1} m^{-3} nm^{-1} absorbed at depth y by the suspension. Thus, finally by Eq. 35 we compute

$$m(y) \equiv \int_{\Lambda} h(y,\lambda)a(y,\lambda)p(\lambda)\eta(\lambda)\, d\lambda \quad (37)$$

as the number of ^{14}C molecules fixed s^{-1} m^{-3} at depth y by the suspension. The restriction to nonfluorescing material can be removed by monitoring the spectral output of the suspension; this need not be considered here.

In this way we see that the two-flow irradiance model, Eq. 10 (in both its direct and inverse modes), is an appropriate model of the light field in natural waters to determine the photosynthetic activity index of a substance, by way of Eq. 37. This points up once again the observation that we must not use the photometric two-flow model (Eq. 17) in any way to infer photosynthetic activity in natural waters, for the reason that all of the spectral information in $h(y,\lambda)$ defined in Eq. 36 is lost on going over into the photometric (human visual) domain—the domain of the Secchi disk.

Derivation of the contrast reduction equation

We now have assembled enough concepts to derive the basic equation of Secchi disk science, namely Eq. 48, below. The setting is a *homogeneous infinitely deep* water body were x is any level below the air-water surface, and $z > x$ is any depth below x. Under such conditions Eq. 32, with constant $a_\pm$, $b_\pm$, can be shown to have the solution

$$E(z,\pm) = E(x,\pm)\exp[-K(z - x)] \quad (38)$$

where, from Eq. 31b, on dropping the bR term in Eq. 28 as being relatively small, we find that

$$K_+ = K_- \equiv K = D_-[\bar{a} + \bar{b}]. \quad (39)$$

Moreover, from Eq. 29 it follows, with the help of Eq. 39, that for all depths y,

$$\begin{aligned} R(y,-,\Lambda) &\equiv R_\infty \\ &= (\bar{b}/\bar{a})/[1 + (D_+/D_-) \\ &\quad + \bar{b}/\bar{a}] \\ &\simeq \bar{b}/3\bar{a} \quad (40) \end{aligned}$$

where the last approximation comes from dropping $\bar{b}/\bar{a}$ in the denominator of Eq. 40 and setting $D_+/D_- = 2$. From Eq. 26 and 40, we find

$$E(y,+) = E(y,-)R_\infty \quad (41)$$

for all depths y.

In the present simple model we shall assume that the upward inherent luminance $B_\infty(z,n')$ of the water medium just to one side of the disk and the upward inherent luminance $B(z,n)$ of the center of the disk (cf. Fig. 1) are given by

$$B_\infty(z,n') = E(z,-)R_\infty/\pi = E(z,+)/\pi \quad (42)$$

$$B(z,n) = E(z,-)R/\pi \quad (43)$$

for every z, where R is the reflectance of the matte surface of the disk in water. Thus we assume that the homogeneous water body below each level z is a matte reflector, just as in the case of the Secchi disk surface.

The *inherent contrast* of the disk's luminance $B(z,n)$ against the background inherent luminance $B_\infty(z,n')$ is by definition

$$C_0 \equiv \frac{B(z,n) - B_\infty(z,n')}{B_\infty(z,n')} = \frac{R - R_\infty}{R_\infty} \quad (44)$$

where Eq. 44 has been simplified by using Eq. 42 and 43. Observe that C_0 is independent of z. The *apparent contrast* of the disk against its background, as seen from level x, is

$$C_{z-x} = \frac{B(x,n) - B_\infty(x,n')}{B_\infty(x,n')}. \quad (45)$$

This apparent contrast can be determined as a function of C_0 and the water's apparent optical properties, as follows. From what we have just observed in Eq. 42 and 43 we can write the luminance difference in the numerator of Eq. 44 as

$$\begin{aligned} B(z,n) - B_\infty(z,n') &= E(z,-)(R - R_\infty)/\pi \\ &= \pi^{-1}E(x,-)(R - R_\infty) \\ &\quad \cdot \exp[-K(z - x)] \end{aligned}$$

where Eq. 38 was also used. This luminance difference can be transferred to level x by means of the luminance-difference law (Eq. 24), with $\theta = 0$, $\xi' = n'$, and $\xi = n$. The result is

$$\begin{aligned} B(x,n) - B_\infty(x,n') = \pi^{-1}E(x,-)(R - R_\infty) \\ \cdot \exp[-(\alpha + K) \\ \cdot (z - x)] \quad (46a) \end{aligned}$$

where

$$\alpha \equiv (z - x)^{-1} \int_0^r \alpha(y,n,\Lambda)\, dy. \quad (46b)$$

Dividing each side of Eq. 46a by $B_\infty(x,n')$ and recalling Eq. 42, 44, and 45, we find

$$C_{z-x} = C_0 \exp[-(\alpha + K)(z - x)]. \quad (47)$$

This holds for all underwater depths $x < z$. For Secchi disk work we set $x = 0$, the level just under the surface (Fig. 1), to obtain

$$C_z = C_0 \exp[-(\alpha + K)z] \quad (48)$$

which is the desired *photopic contrast reduction* formula for a vertical path of sight in a homogeneous medium. The derivation makes it clear that α and K are photopic quantities, namely the *photopic beam attenuation* and *photopic diffuse attenuation* coefficients of the medium. The quantity $\alpha + K$ is therefore a spectrally averaged and depth-averaged apparent optical property of the medium; cf. Eq. 46b, and Eq. 31b.

Before going on to the next step, we observe that a completely general derivation, for what it is worth, can be given for C_z in terms of C_0 and the apparent optical properties of the medium (cf. Preisendorfer 1986). However, to make such a general formula useful, approximations will have to be introduced once again, yielding essentially Eq. 48 once more, but for separable media (a term defined in the reference cited). Since Secchi disk science is basically a visual science to be used under rugged conditions, it is not in the spirit of the subject to make second-order refinements of Eq. 48 of the kind shown by Preisendorfer (1986). If such refinements are indeed needed to find the optical properties of the medium, then the Secchi disk should be set aside and recourse to precision optical instrumentation is recommended. One would then use the appropriate theories built up from Eq. 10 (using a radiometric approach) or Eq. 17

(using a photometric approach). We note in passing still another way to Secchi disk theory. This can be found in Levin (1980). The distinguishing feature of the latter work is its attempt to handle the occasionally observed halo around the disk, which is a second-order effect on the disk's depth of disappearance.

Air-water surface effects

We now examine the effects on Secchi disk visibility of the refractive and reflective effects of the air-water surface.

Consider a water surface freshly crinkled by capillary waves produced by a passing breeze. We now view the disk from just above the surface. It can be shown (Preisendorfer 1976, vol. 6, p. 256) that the contrast transmittance factor $\overline{\mathscr{T}}_0$, by which the apparent contrast C_z in Eq. 48 is reduced on average in passing upward through an ensemble of crinkled surfaces, is

$$\overline{\mathscr{T}}_0 = 1 - \exp[-\tan^2\phi/2\sigma^2], \qquad (49)$$

where $\phi = 4\phi'$, and ϕ' is the angular subtense (to first-order in radians) of the disk's radius as seen from just below the surface. Moreover, σ^2 is the *optical state of the surface,* i.e. the *variance* of the slopes of the capillary waves and is given by

$$\sigma^2 = wU, \quad w = 2.54 \times 10^{-3}\ \mathrm{s\ m^{-1}}. \qquad (50)$$

Here U is wind speed measured at "mast height," 19 m above mean sea level. For example, in the original Secchi (1866) experiment, his 43-cm-diameter disk disappeared at 16.5 m in calm water. Hence we have a disk radius of $r = 21.5$ cm and the Secchi depth $z_{SD} = 1{,}650$ cm. The subtense of the disk radius at this depth is $\phi' = 0.0130$ radians and so $\phi = 4\phi' = 0.052$ radians. Suppose a fresh breeze of $U = 1\ \mathrm{m\ s^{-1}}$ sprang up during the experiment and maintained a set of capillary waves, so that $\sigma^2 = 2.54 \times 10^{-3}$. Then $\tan^2\phi/2\sigma^2 = 0.533$, whence by Eq. 50, $\overline{\mathscr{T}}_0 = 0.413$. Thus the apparent contrast of the disk would on average have been reduced by 41% and it would on average have disappeared at a smaller depth were the experiment repeated several times during the capillary episode. The amount of the decrease of the depth of disappearance can be determined by law 2 of the Secchi disk, below.

The apparent contrast C_z in Eq. 48 can also be reduced by reflection of skylight on a flat calm surface. The contrast transmittance $\mathscr{T}_0$ for this effect is given by

$$\mathscr{T}_0 = \frac{B^0 t}{B^0 t + B_0 r m^2} \qquad (51a)$$

where B^0 is the vertically upward luminance just below the surface, and B_0 is the vertically downward luminance just above the water (cf. Preisendorfer 1976, vol. 6, p. 42). The Fresnel reflectance r and transmittance $t = 1 - r$ of the surface are $r = 0.02$ and $t = 0.98$ over the visible spectrum, and the index of refraction m of water is very nearly 4/3 over the visible λ range. Suppose for example that $B_0/B^0 \cong 50$, which is often the case for overcast skies. Then from Eq. 51a,

$$\mathscr{T}_0 = 1/[1 + (B_0/B^0)(r/t)m^2] \simeq 0.355. \qquad (51b)$$

This is a considerable loss of transmittance due to sky reflections in the water surface. When both factors $\overline{\mathscr{T}}_0$ and $\mathscr{T}_0$ are simultaneously in force, we can estimate their combined effect by the product

$$\mathscr{T} = \overline{\mathscr{T}}_0 \mathscr{T}_0. \qquad (52)$$

An advanced discussion of this factor is given elsewhere (Preisendorfer 1986, equation 5). In this way we arrive at the following general *contrast reduction formula.* Equation 48 for the Secchi disk with surface contrast transmittance factor $\mathscr{T}$ becomes

$$C_z = \mathscr{T} C_0 \exp[-(\alpha + K)z]. \qquad (53)$$

The binocle of eyeball optics

We now convert Eq. 53 into three very useful formulas in visual—or eyeball—optics. Solving Eq. 53 for the product $(\alpha + K)z$, we obtain

$$[\text{binocle}] \quad (\alpha + K)z = \ln[\mathscr{T} C_0/C_z] \qquad (54)$$

which we will call the *binocle* of visual optics of natural waters. This nomenclature draws attention to an important feature of Eq. 54, namely that, like its material counterparts, the binoculars or field glasses, it can be looked through from either end. The manufacturer of a real binocular expects its user to look through the small end so as to see objects magnified. When Eq. 54 is used this way, one can find, as its own inventors intended (cf. Duntley and Preisendorfer 1952, p. 14), the depth z_{SD} of disappearance of the Secchi disk by means of the so-called *direct ocle*:

$$[\text{direct ocle}] \quad z_{SD} = \frac{\ln[\mathscr{T}C_0/C_T]}{\alpha + K} \equiv \frac{\Gamma}{\alpha + K} \quad (\text{m}). \qquad (55)$$

Here $\Gamma \equiv \ln[\mathscr{T}C_0/C_T]$ forms the heart of the ocle and is called the *coupling constant*. Incidentally, the word "ocle" is a neologism which, with the reader's indulgence, can be used to keep in mind the fact that the apparent contrast C_z in Eq. 54 has been replaced in Eq. 55 by C_T, the *threshold* (or *liminal*) *contrast* of the disk. This threshold contrast is subtended by the luminance $B(x,n)$ as perceived by a normal human eye viewing the disk at depth x against a watery background of luminance $B_\infty(x,n')$. *See Fig. 1, with* $\theta = 0$.

To go from Eq. 54 to 55 requires more than simple mathematics. One must combine the physics of the hydrosol (using α, K, and C_0) with the psychophysiology of the human eye-brain system (using C_T). Blackwell (1946) reported experimental determinations of C_T in terms of the adaptation luminance B_0 [our $B_\infty(x,n')$] of the observer's eye and the angular subtense ψ of the diameter of a luminous circular target. The contrast C_T of a disk is said to be *threshold* if an observer, on repeated attempts under identical conditions to decide that the disk is seen, is correct 50% of the time.

Table 1 gives some representative samples of C_T for various choices of B_0 and ψ. In reading the table it is useful to know that the disks of the sun and moon subtend about 30 minutes of arc. Secchi's 43-cm disk subtended at disappearance depth 16.5 m about 45 minutes of arc as seen from just below the surface. The luminance level (in footlambert ≡ ftL) on a day with highly variable clouds can vary from 3,000 to 300 ftL (1 ftL = π^{-1} lm ft^{-2} sr^{-1} = 3.426 lm m^{-2} sr^{-1}). Twilight has an adaptation luminance level of about 1 ftL. A clear night with a full moon has an adaptation level of about 10^{-1} ftL. Observe from Table 1 how C_T increases as ψ decreases for a given B_0.

To arrive at Eq. 55 we must in Eq. 53 simultaneously watch C_z and C_T change as z increases. By Eq. 53, C_z decreases, and by Table 1, C_T increases as z increases. When C_z equals C_T, we obtain z_{SD}. Convenient nomographic solutions of Eq. 53 for z_{SD} in the case of underwater Secchi disk work (so that $\mathscr{T} = 1$) can be found elsewhere (Preisendorfer 1976, vol. 1, sec. 1.9). The nomographs yield *sighting range* v. The required z_{SD} is then given by $z_{SD} = 2v$. This amounts to having the coupling constant Γ in Eq. 56 be of order 8–9.6 (*see* Preisendorfer 1976, vol. 1, p. 194). The difference between sighting range v and Secchi depth z_{SD} arises from the fact that v is designed for swimmers making visual searches for an underwater object whose whereabouts in the visual field is unknown. A Secchi disker, however, keeps his eye on the disk as it descends, and thereby can track it deeper than the sighting range before the disk disappears. [If one uses the nomographs in the cited reference, observe that there are two complete sets of nomographs in vol. 1 of Preisendorfer 1976, namely the low clarity and high clarity nomographs. The *high clarity* nomographs are in figure 1.85 and figures 1.89–1.97 with the range of $\alpha - K\cos\theta$ being 0.01–0.14 ft^{-1}. The *low clarity* nomographs are in figure 1.84 and figures 1.98–1.106 with the range of $\alpha - K\cos\theta$ being 0.10–1.40 ft^{-1}, rather than 0.01–0.14 ft^{-1} as drawn.]

Once we have Eq. 55, we can invert it to estimate $\alpha + K$ from the observation of a Secchi depth z_{SD}:

$$[\text{inverse ocle}] \quad \alpha + K = \frac{\ln[\mathscr{T}C_0/C_T]}{z_{SD}} = \frac{\Gamma}{z_{SD}} \quad (\text{m}^{-1}). \qquad (56)$$

We call Eq. 56 the *inverse ocle* of eyeball optics, since it uses the binocle to infer an optical property from a depth measurement z_{SD}. From our examples above we see that $\mathscr{T}$ can affect the size of z_{SD}. Good Secchi

Table 1. Samples of threshold contrast C_T.

Angular subtense ψ (min arc)	Adaptation luminance B_0 (in ftL) (and in lm m^{-2} sr^{-1}) 1,000 (3,426)	1 (3.426)	10^{-1} (0.3426)
360.00	0.0027	0.0033	0.0053
55.20	0.0028	0.0037	0.0074
9.68	0.0046	0.0089	0.0213

Table 2. Values of $\Gamma = \ln[C_0/C_T]$ where $C_0 = (R - R_\infty)/R_\infty$ with $R = 0.85$.

	R_∞					
C_T	0.015	0.02	0.03	0.05	0.07	0.10
0.005	9.32	9.02	8.61	8.07	7.71	7.31
0.010	8.63	8.33	7.90	7.38	7.02	6.62
0.020	7.93	7.64	7.22	6.69	6.32	5.93

disk practice will eliminate $\mathscr{T}$ and set it to 1.0. This may be done, for example, by working with a black umbrella overhead to blot out the zenith reflections in a still-water surface while keeping to the sunny side of the boat and trying not to produce ripples in the water. A hooded glass-bottomed boat will yield better z_{SD} readings (cf. Preisendorfer 1976, vol. 1, p. 46). Even Secchi (1866) and his helpers instinctively used hats and umbrellas to make $\mathscr{T} = 1$. Another way to eliminate the $\mathscr{T}$ effect is to use a pair of vertically suspended submerged disks which are moved closer or farther apart until one achieves between them a visual match of their apparent luminances (Preisendorfer 1976, vol. 1, p. 100).

Table 2 shows values of $\Gamma = \ln[C_0/C_T]$ where $C_0 = (R - R_\infty)/R_\infty$ is computed for a submerged disk reflectance $R = 0.85$. Tables 1 and 2 together suggest that reasonable estimates of Γ are found in the neighborhood of 8 and 9. It is possible to do fieldwork with the disappearing disk and thereby obtain fresh estimates of C_T beyond Blackwell's (1946) tables. For example, Holmes (1970) made the first of such estimates and found $C_T = 0.0014 \pm 0.0013$. Højerslev (1986) found $C_T = 0.0070 \pm 0.0003$ and observed that Holmes' C_T may have been affected by bottom luminance. When doing such experiments, one must document all parameters entering into the coupling constant Γ, as well as α, K, and z_{SD}, along with the depth of the medium. It is also possible to extend Table 2 on one's own. For this, one need only choose suitable values of R, R_∞, C_T, and even $\mathscr{T}$, if desired.

The size of the coupling constant Γ can be estimated after an individual experiment has been run. For this, one determines R for the submerged disk [note that dry disk reflectances generally differ from submerged disk reflectances, cf. Preisendorfer (1976, vol. 1, p. 171)], measures R_∞ of the medium, estimates the adaptation luminance B_0 at the time of the experiment, and notes the Secchi depth z_{SD}. The last two numbers will allow an estimate of C_T with Blackwell's (1946) tables. In the absence of knowledge of R_∞ for the medium, one can take $R_\infty = 0.02$.

It should be noted that the Secchi disk theory, as given here, is incomplete in the following sense. If one hasn't measured R_∞, but wishes to estimate R_∞ more closely than just setting it equal to 0.02, one could use Eq. 39 and 40 to obtain the formula $K = D_-\bar{a}[1 + 3R_\infty]$. Then if one knew D_-, $\bar{a}$, and K of the medium, one could estimate R_∞. But K is part of the estimate $\alpha + K$ to be obtained from the inverse ocle, Eq. 56. Therefore, in this sense, one must know part of the sum $\alpha + K$ before one can rigorously estimate the sum! What saves the Secchi disk theory from becoming useless at this point is the relative insensitivity of Γ to the various departures of the photopic R_∞ from 0.02 in many natural hydrosols. We shall now consider this matter of sensitivity in more detail.

The sensitivity formula for the binocle

There are about a dozen factors within the direct ocle, Eq. 55, that can give rise to perturbations of the Secchi depth z_{SD}. Most of these perturbations can be examined quantitatively by taking the relative differential of each side of the binocle, to find

$$\begin{aligned}\frac{\delta z}{z} + \frac{\delta(\alpha + K)}{\alpha + K} &= \frac{\delta\Gamma}{\Gamma}\\ &= \Gamma^{-1}[(\delta\mathscr{T}_0/\mathscr{T}_0) + (\delta\mathscr{T}_0/\mathscr{T}_0)\\ &\quad + (\delta C_0/C_0)\\ &\quad - (\delta C_T/C_T)]. \qquad (57)\end{aligned}$$

From our examination of Table 1, we can see the effects of varying disk diameter and adaptation luminance. These effects arise through perturbations of the threshold contrast in the form

$$-\delta C_T/C_T = c_1(\delta\psi/\psi) + c_2(\delta B_\infty/B_\infty), \quad c_1,c_2 > 0 \tag{58}$$

where c_1,c_2 can be estimated from Blackwell's (1946) tables. The relative effects on C_0 of changing disk reflectance and medium reflectance are given through Eq. 44 by

$$\delta C_0/C_0 = (\delta R/R) - (\delta R_\infty/R_\infty). \tag{59}$$

From Eq. 39 and 40 we can estimate the effects of environmental lighting and water optical properties on R_∞. Moreover, we know the functional forms of $\bar{\mathscr{T}}_0$ and $\mathscr{T}_0$ in Eq. 49 and 51a, and thereby see how they depend on wind speed U, and the two luminances B_0, B^0.

In what follows we shall sort out the major effects on z_{SD} arising from variations of these parameters. Observe that the presence of Γ^{-1} in Eq. 57 tends to keep the relative change $\delta z_{SD}/z_{SD}$ about an order of magnitude less than any of its primary causes in the brackets of Eq. 57. For once something (the sluggishness of the log function) is working in our favor in this complex theory.

Ten laws of the Secchi disk

In his original experiment, Secchi (1866) discerned several modes of behavior of the depth of disappearance z_{SD} as a function of disk size, disk reflectance, sun altitude, etc. These modes of behavior have likely been observed and noted by many experimenters both before and after Secchi. It is possible to rigorously deduce several such "laws of behavior" of z_{SD} from the sensitivity formula Eq. 57. The ten most salient laws are listed below with the appropriate part of Eq. 57, when applicable, giving the mathematical statement of the law. Each statement is preceded by the implicit proviso: "for the present factor within its prescribed bounds and all other factors held fixed." The observer, in all these laws, is just above the surface.

1. The depth of disappearance of the disk varies inversely with the average amount of attenuating material between the surface and the disk [$\delta z_{SD}/z_{SD} = -\delta(\alpha + K)/(\alpha + K)$].
2. The depth of disappearance of the disk varies inversely with the optical state of the sea surface [$\delta z_{SD}/z_{SD} = \Gamma^{-1}\,\delta\bar{\mathscr{T}}_0/\bar{\mathscr{T}}_0$ and Eq. 49].
3. The depth of disappearance of the disk varies inversely with the relative amount of reflected luminance of sky in the sea surface compared to the luminance transmitted upward from below the surface [$\delta z_{SD}/z_{SD} = \Gamma^{-1}\,\delta\mathscr{T}_0/\mathscr{T}_0$ and Eq. 51a, b].
4. The depth of disappearance of the disk varies inversely with the reflectance of the water body [$\delta z_{SD}/z_{SD} = -\Gamma^{-1}\,\delta R_\infty/R_\infty$].
5. The depth of disappearance of the disk varies directly with its reflectance [$\delta z_{SD}/z_{SD} = \Gamma^{-1}\,\delta R/R$].
6. The depth of disappearance of the disk varies directly with its diameter [$\delta z_{SD}/z_{SD} = c_1\Gamma^{-1}\,\delta\psi/\psi$].
7. The depth of disappearance of the disk varies directly with sun altitude [$\delta z_{SD}/z_{SD} = -\Gamma^{-1}\,\delta R_\infty/R_\infty$ and Eq. 40].
8. The depth of disappearance of the disk varies inversely with the immediate height of the observer above the sea surface [$\delta z_{SD}/z_{SD} = \Gamma^{-1}\,\delta\mathscr{T}_0/\mathscr{T}_0$ and Eq. 51a, b].
9. The depth of disappearance of the disk varies directly with the adaptation luminance [$\delta z_{SD}/z_{SD} = c_2\Gamma^{-1}\,\delta B_\infty/B_\infty$].
10. The depth of disappearance of the disk is larger if the water path of sight between disk and observer is more shadowed; it is larger if the water path beyond the disk is less shadowed.

In law 8, when the observer's face is right at the surface, B_0 in Eq. 51a is effectively zero and $\mathscr{T} = 1$; when the observer rises from the surface, B_0 creeps back into full magnitude and $\mathscr{T}$ decreases from 1. In law 10, the assertion is proved when one recalls that C_z/C_0 is a contrast transmittance, and as such is subject to the laws of contrast transmittance given by Preisendorfer (1976, vol. 5, p. 174). However, law 10 is intuitively clear if one remembers how the contrast of an object increases if path luminance (the "veiling light") between observer and object is diminished, or if path luminance beyond the object is increased without increasing intervening path luminance. To understand this phenomenon, note that path luminance

$B^*_r(y,\xi)$ is defined analogously to path radiance $N^*_r(y,\xi,\lambda)$ in Eq. 4. Then the luminous contrast transmittance $\mathcal{T}_r(z,\xi,\Lambda)$ of a path of sight starting at z and extending a distance r along ξ to x is by definition $1 - [B^*_r(x,\xi)/B_r(x,\xi)]$, where $B_r(x,\xi)$ is the apparent luminance, i.e. the luminous counterpart to $N_r(x,\xi,\lambda)$ in Eq. 3. An alternate and equivalent statement of law 10 replaces the two occurrences of "larger" by "smaller" and the two occurrences of "shadowed" by "illuminated." Here, shadows may be those of the observer or his boat; the illumination may come from sunbeams or searchlight beams being scattered in the water.

How large are the effects described in these ten laws? For the most part they are relatively small. From 5, a 10% increase of R will increase z_{SD} by about 1% (using $\Gamma = 9$). From 6, increasing the diameter of the disk from 43 to 60 cm (Secchi's experiment) will increase z_{SD} by about 1% when $\Gamma = 8$, namely from 16.5 to 16.7 m. In Secchi's experiment he found the new depth to be 24.5 m—which is much too large. From 1, we see that the relative changes in $\alpha + K$ and z_{SD} are of the same magnitude but of opposite sign. Note that if one wants to estimate C_T from field measurements (as Holmes 1970 did) then Eq. 57 says $\delta C_T/C_T = -\Gamma(\delta z_{SD}/z_{SD})$ with all other factors fixed. Therefore, if $\Gamma = 9$, and one wants C_T determined to within 10%, repeated experiments must keep z_{SD} to within 1%.

Extracting α and K from Secchi disk estimates of α + K

There is a tendency by Secchi diskers to try in various ways to separate α and K from the sum $\alpha + K$ yielded up by the inverse ocle, Eq. 56. There is no royal road to such a sundering—one must earn the right to do so by expending extra work of the empirical or theoretical kind; and even then the result may not be worth the effort. The following four approaches have been tried. The first two are excellent; the third is of limited use; the fourth is for the desperate.

i—Measure z_{SD} by Secchi disk and let $\hat{K}$ be the estimate of K by a downward illuminance meter (cf. Eq. 31b). Let $(\alpha + K)\hat{}$ be the inverse ocle's estimate of $\alpha + K$. Then the estimate $\hat{\alpha}$ of α is given by $\hat{\alpha} = (\alpha + K)\hat{} - \hat{K}$.

ii—Measure z_{SD} by Secchi disk and α by luminous beam transmissometer (cf. Eq. 46b). Let $\hat{\alpha}$ be the resulting estimate. Let $(\alpha + K)\hat{}$ be the inverse ocle's estimate of $\alpha + K$. Then $\hat{K} = (\alpha + K)\hat{} - \hat{\alpha}$ is the appropriate estimate of K.

iii—Measure K by means of a downward illuminance meter and measure α by means of a luminous beam transmissometer. Establish a scatter plot of K vs. α. Obtain a linear regression line $K = \epsilon\alpha + r$. This yields the slope ϵ and residual r. For example, Holmes (1970) found that $\epsilon = 0.136$, $r = 0.114\ \mathrm{m}^{-1}$. [*See also* Tyler (1968, figure 1) for the range of possible ϵ values.] Such empirical attempts at estimates of ϵ are not unreasonable. The ratio K/α is a relatively stable one. [*See* Preisendorfer (1976, vol. 1, p. 137) and Gordon and Wouters (1978).] For recent attempts to use regression techniques combined with Secchi disk procedures, *see* Carlson (1977) and comments on this by Lorenzen (1980) and Megard et al. (1980), and finally Carlson (1980). Caution: The regression line results are functions of season, location, local recent meteorological events, and ephemeral lighting conditions, among other things. The regression line is for your own immediate and local use and is unlikely to be widely applicable by others in faraway places and much later times. In practice if one has found ϵ this way, then one would use $K = \epsilon\alpha$ in the inverse ocle sum $\alpha + K$ so: $\alpha + K$ becomes $\alpha(1 + \epsilon)$ with ϵ known. Let $(\alpha + K)\hat{}$ be the inverse ocle's estimate of $\alpha + K$. Then the estimate of α is $\hat{\alpha} \equiv (\alpha + K)\hat{}/(1 + \epsilon)$, whence $\hat{K} \equiv (\alpha + K)\hat{} - \hat{\alpha}$.

iv—A theoretical estimate of $\epsilon = K/\alpha$ can be obtained as the smallest eigenvalue of certain theoretical luminance distribution models, knowing the shape of the volume scattering function of the medium (cf. Preisendorfer 1976, vol. 5, p. 245). Such an approach uses the asymptotic radiance hypothesis. Timofeeva (1974, table 2) has produced empirical tabulations, but σ was not specified. Such results should generally be used with caution, for even if σ is known, the $\epsilon = K/\alpha$ asymptotic ratio is valid only for depths ($\sim 20/\alpha$) far below the usual z_{SD}

range. In the absence of well documented theoretical listings of $\epsilon = K/\alpha$, we can proceed as follows. A simple rule of thumb for K/α can be derived from Eq. 39. First, observe that $\alpha = \bar{a} + \bar{s}$. This follows from Eq. 12 averaged over Λ for the case of a vertical line of sight, so that α, by Eq. 21, is $\alpha(y,n,\Lambda)$ and this is imagined averaged over depth, as in Eq. 46b. Next, define the dimensionless quantity $\omega = \bar{s}/\alpha$ (the *scattering-attenuation ratio*). Return to Eq. 39, drop $\bar{b}$ as small relative to $\bar{a}$, and find $\epsilon = K/\alpha = D_-\bar{a}/\alpha = D_-[1 - \omega]$, the required rule of thumb for ϵ. Of course to use this rule, one must know or estimate D_- and ω. D_- is usually of the order of 1.3; but ω can vary widely (milky media have ω near 1, clear media have ω near 0).

Critical observations and major conclusions

The observations we reach at this point of the study are: (a) The Secchi disk procedure is limited to quantitative estimates of $\alpha + K$ only. (b) If one wishes to estimate α or K, then additional experiments, beyond the Secchi disk procedure, for finding α, K, or K/α must be performed. (c) The moment one uses additional equipment [namely $E(y,\pm)$-meters, $e(y,\pm)$-meters, and α-meters)] to supplement Secchi readings, one potentially obviates the need for the Secchi readings. (d) If one attempts to establish long term statistical links between z_{SD} and the K, R, $D_\pm$ quantities defined in the two-flow illuminance theory (Eq. 25, 26, 27) via $E(y,\pm)$-, $e(y,\pm)$-, and α-meters, one is faced with the possibility that these links are subject to decay over time and over space; in other words the links may be ephemeral and local. Such a procedure can at first obviate the need for a Secchi disk (why use more instruments when one can use better and fewer?) and leads ultimately to its probable abuse (by using outdated statistical links).

The net conclusion of the preceding observations is that: *(i) the Secchi disk reading z_{SD} yields a quantitative estimate of the apparent optical property α + K of a medium. (ii) The primary function of a Secchi disk is to provide a simple visual index of water clarity via z_{SD} or α + K. (iii) To extend the use of the disk with auxiliary objective measurements (of α or* K, *or both) is to risk obviating or abusing this primary function.*

In the early years of photoelectric measurements of the light field in natural waters (cf. Poole and Atkins 1929) coupling z_{SD} to K was occasionally of interest or simply an expedient procedure. It could facilitate estimates of the illuminance levels in such media by use of Secchi readings. Today the justification for such a practice no longer exists, and we return to the above primary function of the white disk as its only legitimate raison d'être.

Three basic measures of optical depth

In the course of developing the theory of the binocle of eyeball optics, namely Eq. 54, we encountered three types of transmittance for the underwater transfer of photons, namely upward *beam transmittance* for luminance (cf. Eq. 5) with the bottom of the air-water surface located at $x = 0$:

$$B_0(0,n)/B_0(z,n) = \exp\left[-\int_0^z \alpha(y,n,\Lambda)\,dy\right] \equiv \exp[-\alpha z]; \qquad (60)$$

downward *diffuse transmittance* for illuminance (cf. Eq. 30)

$$E(z,-)/E(0,-) = \exp\left[-\int_0^z K(y,-,\Lambda)\,dy\right]; \equiv \exp[-Kz] \qquad (61)$$

and upward *contrast transmittance* (cf. Eq. 53 with $\mathcal{T} = 1$)

$$C_z/C_0 = \exp[-(\alpha + K)z]. \qquad (62)$$

We can then see that contrast transmittance is the product of the beam and diffuse transmittances. Physically, $\exp[-Kz]$ represents the illuminance that diffuses, by way of scattering, downward from just below the surface to level z, while $\exp[-\alpha z]$ gives the residual amount of image-forming light (concerning the disk and its background luminance) that propagates without scattering or absorption back up to just below the surface. It is generally the case that $K < \alpha$, so that diffusing light decays with distance at a slower rate than image-forming light.

The products αz, Kz, and $(\alpha + K)z$ in Eq.

60, 61, and 62 are *optical depths* associated with the geometric depth z, and give measures of the amount of beaming and diffusing activities of the photons in the water over the distance z. The optical depth associated with the Secchi depth is therefore a *contrast transmittance depth* and is, by Eq. 55,

$$(\alpha + K)z_{SD} = \Gamma. \tag{63}$$

Moreover, from Eq. 63 we find that the associated *beam* and *diffuse optical depths* are

$$\alpha z_{SD} = \Gamma_\alpha \equiv \frac{\Gamma}{1 + (K/\alpha)} \tag{64}$$

$$K z_{SD} = \Gamma_K \equiv \frac{\Gamma}{1 + (\alpha/K)}. \tag{65}$$

In order to find Γ_α and Γ_K we need to know for example the ratio $\epsilon = K/\alpha$. As discussed above, this entails going beyond the classic Secchi disk procedures and there is accordingly potentially a danger of obviating or abusing the procedure. If one nevertheless calibrates the disk in a given medium and uses the determined ratio $\epsilon = K/\alpha$ before it has changed with the medium through the natural course of events, then one can use the Secchi depth z_{SD} to estimate downward illuminance levels at every depth z in that medium. From Eq. 61 and 65, we have

$$\begin{aligned} E(z,-) &= E(0,-)\exp[-Kz] \\ &= E(0,-)\exp[-\Gamma_K z/z_{SD}]. \end{aligned} \tag{66}$$

From this we see that the downward illuminance at Secchi depth is, for example,

$$\begin{aligned} E(z_{SD},-) &= E(0,-)\exp[-\Gamma_K] \\ &= 0.076E(0,-) \end{aligned} \tag{67}$$

when $\Gamma = 9$ and $\alpha/K = 2.5$, so that $\Gamma_K = 2.57$. Hence in this example, the downward illuminance level at Secchi depth is about 7.6% of that just below the surface. It should be noted, however, that Γ_K is subject to change from one locale to another, and over time. If we had used $\Gamma = 8$ with $\alpha/K = 2.5$ then $\Gamma_K = 2.29$ and the downward illuminance at Secchi depth would be 10% of that just below the surface.

Another quantity of interest is the *euphotic depth,* the depth z_{eu} where $E(z_{eu},-)/E(0,-)$ is 0.01. What relation holds between z_{eu} and z_{SD}? More generally, let z_γ be the depth at which

$$\begin{aligned} E(z_\gamma,-)/E(0,-) &= \gamma \\ &= \exp[-Kz_\gamma] \le 1. \end{aligned} \tag{68}$$

By this and Eq. 65,

$$\gamma = \exp[-\Gamma_K z_\gamma/z_{SD}]$$

whence

$$z_\gamma = -(\Gamma_K^{-1}\ln \gamma)z_{SD}. \tag{69}$$

For example, consider the euphotic-depth case. Then $\gamma = 0.01$ and $z_\gamma = z_{eu}$. Let $\Gamma = 9$ and $\alpha/K = 2.5$. Then

$$z_{eu} = 1.79z_{SD} \tag{70}$$

i.e. euphotic depth in this case 1.79 times the Secchi depth. Since Γ_K is involved, Eq. 70, just as Eq. 67, is subject to variations in time and space. If, for example, $\Gamma = 8$, with $\alpha/K = 2.5$, then $z_{eu} = 2.0z_{SD}$.

Further discussion of Γ_K and Γ_α is given by Gordon and Wouters (1978) and Højerslev (1986). The latter, after some careful experimental work, has shown that under a very broad set of conditions, $\alpha z_{SD} = \Gamma_\alpha = 6.0$. This roughly agrees with the use of the rules of thumb that $\Gamma = 8$ and $K/\alpha = 0.4$, which yields $\Gamma_\alpha = 5.7$. If we use $\Gamma = 9$ instead and retain $K/\alpha = 0.4$, then $\Gamma_\alpha = 6.4$. And so on, ad infinitum.

In conclusion, we have developed the theory of the Secchi disk so that three types of researchers may draw from it what they wish: the researcher who, for example, wants without any preamble a simple formula for euphotic depth, say, has Eq. 69; the researcher wishing to connect up α and K empirically to z_{SD}, has for consideration the four procedures discussed in i–iv in a preceding section; and finally, for the merely curious scientists we have laid out the various levels of theory on which rests the performance of the white disk in natural waters under various environmental lighting conditions and properties of the disk. These performances are summarized in the ten laws of the disk.

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