Matlab Code

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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%% NEUROMECHANICS   %%%%%%%%%%%%%%
% (c) Francisco Valero-Cuevas
% September 2013, version 1.0
% Filename: J2D2DOF.m
% Jacobian of 2D, 2DOF linkage system

% Clear memory and screen
close all;clear all;
clc

% Define variables for symbolic analysis
syms G J         % Vector functions
syms q1 q2 x y   % Degrees of freedom
syms l1 l2       % System parameters


%Define x and y coordinates of the endpoint
%Create Matrix for Geometric Model
x = l1.*cos(q1) + l2.*cos(q1+q2);
y = l1.*sin(q1) + l2.*sin(q1+q2);
G = [x;y];

%Create Jacobian adn its permutations
J = jacobian(G,[q1,q2])
J_inv = inv(J)
J_trans = J'
J_trans_inv = inv(J')

latex(G)
latex(J)
latex(J_trans)
latex(J_inv)
latex(J_trans_inv)

% Numerical example
% Define Link Lengths (m)
l1 = 1
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l2 = 1

% Define join angles (radians)
q1 = 0     % 0 degrees
q2 = pi/2  % 90 degrees

fprintf('Evaluate the functions for these parameter values\n')
fprintf('G');subs(G)
fprintf('J');subs(J)
fprintf('J_trans');subs(J_trans)
fprintf('J_inv');subs(J_inv)
fprintf('J_trans_inv');subs(J_trans_inv)


fprintf('Example of applying a positive angular veolocity at q1 to find the endpoint velocity')
q1_dot = 1
q2_dot = 0

x_dot = subs(J*[q1_dot q2_dot]')

fprintf('Example of applying that same endpoint velocity to find the angular velocities')
q_dot = subs(J_inv*x_dot)

fprintf('Example of applying a horizontal force vector to find the resulting joint torques')
tau = subs(J_trans*[1 0]')

fprintf('Example of applying those jointtorques to find the resulting endpoint force')
f = subs(J_trans_inv*tau)
```

(1)
$$G = \begin{bmatrix} \mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2}) + \mathrm{l1}\ \cos(\mathrm{q1}) \\ \mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2}) + \mathrm{l1}\ \sin(\mathrm{q1}) \end{bmatrix}$$

(2)
$$J = \begin{bmatrix} -\mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2}) - \mathrm{l1}\ \sin(\mathrm{q1}) & -\mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2}) \\ \mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2}) + \mathrm{l1}\ \cos(\mathrm{q1}) & \mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2}) \end{bmatrix}$$

(3)
$$J^T = \begin{bmatrix} -\sin(\mathrm{q1})\ \mathrm{l1} - \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2} & \cos(\mathrm{q1})\ \mathrm{l1} + \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2} \\ -\sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2} & \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2} \end{bmatrix}$$

(4)
$$J^{-1} = \begin{bmatrix} -\frac{\cos(\mathrm{q1}+\mathrm{q2})}{\mathrm{l1}\ \cos(\mathrm{q1}+\mathrm{q2})\ \sin(\mathrm{q1}) - \mathrm{l1}\ \sin(\mathrm{q1}+\mathrm{q2})\ \cos(\mathrm{q1})} & -\frac{\sin(\mathrm{q1}+\mathrm{q2})}{\mathrm{l1}\ \cos(\mathrm{q1}+\mathrm{q2})\ \sin(\mathrm{q1}) - \mathrm{l1}\ \sin(\mathrm{q1}+\mathrm{q2})\ \cos(\mathrm{q1})} \\ \frac{\mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2}) + \mathrm{l1}\ \cos(\mathrm{q1})}{\mathrm{l1}\ \mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2})\ \sin(\mathrm{q1}) - \mathrm{l1}\ \mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2})\ \cos(\mathrm{q1})} & \frac{\mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2}) + \mathrm{l1}\ \sin(\mathrm{q1})}{\mathrm{l1}\ \mathrm{l2}\ \cos(\mathrm{q1}+\mathrm{q2})\ \sin(\mathrm{q1}) - \mathrm{l1}\ \mathrm{l2}\ \sin(\mathrm{q1}+\mathrm{q2})\ \cos(\mathrm{q1})} \end{bmatrix}$$

(5)
$$J^{-T} = \begin{bmatrix} \frac{\cos(\mathrm{q1}+\mathrm{q2})}{\cos(\mathrm{q1})\ \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1} - \sin(\mathrm{q1})\ \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}} & -\frac{\cos(\mathrm{q1})\ \mathrm{l1} + \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2}}{\cos(\mathrm{q1})\ \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}\ \mathrm{l2} - \sin(\mathrm{q1})\ \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}\ \mathrm{l2}} \\ \frac{\sin(\mathrm{q1}+\mathrm{q2})}{\cos(\mathrm{q1})\ \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1} - \sin(\mathrm{q1})\ \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}} & -\frac{\sin(\mathrm{q1})\ \mathrm{l1} + \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l2}}{\cos(\mathrm{q1})\ \sin(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}\ \mathrm{l2} - \sin(\mathrm{q1})\ \cos(\mathrm{q1}+\mathrm{q2})\ \mathrm{l1}\ \mathrm{l2}} \end{bmatrix}$$

Example with unit link lengths and the second joint flexed 90° (in radians)

$$l1 = 1$$
$$l2 = 1$$
$$q1 = 0$$
$$q2 = 1.5708$$

Evaluating these functions with those parameter values:

$$G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$J = \begin{bmatrix} -1.0000 & -1.0000 \\ 1.0000 & 0.0000 \end{bmatrix}$$
$$J^T = \begin{bmatrix} -1.0000 & 1.0000 \\ -1.0000 & 0.0000 \end{bmatrix}$$
$$J^{-1} = \begin{bmatrix} 0.0000 & 1.0000 \\ -1.0000 & -1.0000 \end{bmatrix}$$
$$J^{-T} = \begin{bmatrix} 0.0000 & -1.0000 \\ 1.0000 & -1.0000 \end{bmatrix}$$

Example of applying a positive angular velocity at q1 to find the endpoint velocity $\dot{\vec{x}}$

$$\dot{\vec{q}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$\rightarrow \dot{\vec{x}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Example of applying that same endpoint velocity $\dot{\vec{x}}$ to find the angular velocities $\dot{\vec{q}}$

$$\rightarrow \dot{\vec{q}} = \begin{bmatrix} 1.0000 \\ 0 \end{bmatrix}$$

Example of applying a horizontal endpoint force vector $\vec{f}$ to find the resulting joint torques

$$\vec{f} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\rightarrow \vec{\tau} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

Example of applying those joint torques $\vec{\tau}$ to find the resulting endpoint force $\vec{f}$

$$\rightarrow \vec{f} = \begin{bmatrix} 1.0000 \\ 0 \end{bmatrix}$$