Stat 243 Final Project

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Architecture

The code was written as a set of modular functions, each of which carries out a specific task. Each of these functions is then called by the main wrapper function `ars`.

Modular Functions

The functions we added include a function for generating tangent intersections for the given density from the abscissae provided, finding the upper hull from these tangent lines, sampling from the envelope created by the upper hull, and creating the lower hull from given abscissae. We included three additional helper functions: one for approximating a derivative via the finite difference method, one to comprehensively check the inputs to the main wrapper function, and one to periodically check that log-concavity is satisfied.

Below are the descriptions for each modular function:

- `tanIntersect()` calculates the tangent intersections to a given log-density, for tangent lines at a given set of abscissae. It does this by approximating the derivatives to the log-density at the given abscissae, and then using Equation (1) from Gilks et. al. to compute the x-coordinates of the intersections. This function returns a list containing the original abscissae, the x-coordinates of the interesections, and the values of the log-density and its derivatives at these intersections.
- `upperHull()` corresponds mainly to Equation (2) from Gilks et. al., and calculates the y-values for the upper hull of the log-density at a given set of x-coordinates. To do this, it first calculates the tangent intersections for the segments forming the upper hull, using the `tanIntersect()` function. Then, it uses the values returned by this function and Equation (2) to compute the y-value on the hull corresponding to any x-value provided as an input. This is done in a segment-wise manner, exploiting the fact that the intersections returned by `tanIntersect()` correspond to the boundaries of each segment in order to partition the x-coordinates given based on which segment they fall into.
- `sampleEnv()` corresponds most closely to Equation (3) from Gilks et. al., and samples `n` values from the upper hull for the log-density. It does this by first computing tangent intersections for a given log-density. Then, it finds the normalizing constant in Equation (3) by integrating numerically (using the `integrate()` function on `exp(upperHull(...))`). From here, samples are created using the inverse CDF method applied to each segment, using numerical integration to find the value of the CDF at each tangent intersection and using the explicit equation for the inverse of the CDF (derived below). The function includes a check to remove non-finite sample values and resample from the hull as needed.

[derive the inverse CDF here]

- `lowerHull()` corresponds to Equation (4) of Gilks et. al., and computes y-values for the lower hull of the log-density. It does this by first calculating tangent intersections, and then uses Equation (4) to compute the appropriate y-values.
- `approxD()` approximates the derivative of a given input function at a specified point, using a central finite-difference method. It is capable of returning first or second-order derivatives, and uses a value

of `sqrt(.Machine$double.eps)` as its default finite difference, although it allows a user to specify a different difference value.

- `checkThat()` serves as a comprehensive check of inputs to the main wrapper function. It includes checks for appropriate parameters, sample sizes, initial abscissae, and log-concavity of the density. It also generates starting values if none are provided.
- `cavitySearch()` checks for log-concavity of a given function at a single value or a vector of values. If the log-concavity condition is satisfied, the function returns `TRUE`.

Main `ars()` Function

`ars()` takes five arguments, the details of which can be seen by calling `help(ars)` or `?ars`. These arguments give the user some flexibility. For example, the user can choose to supply some initial abscissae values, or have the function generate them instead.

Testing

We included tests for each individual modular function, as well as more comprehensive tests for the `ars()` function.

Tradeoffs

One major tradeoff in our approach is due in part to the sequential nature of the algorithm.

Individual Contributions

Brian wrote the `approxD()`, `tanInteresect()`, and `checkThat()` functions, as well as their tests. Ashish wrote the `upperHull()` and `sampleEnv()` functions, as well as tests for these and the `lowerHull()` function. Ben wrote the `lowerHull()` function and the main wrapper `ars()` function.