GP 9505A Problem Set #6: Paper Review
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Submitted to Dr. R. Gerhard Pratt
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Paper under review:
Romdhane, A., Grandjean, G., Brossier, R., Virieux, J., 2011. Shallow-structure characterization by 2D elastic full-waveform inversion 76.

Abstract: Assessing the effectiveness of elastic full-waveform-inversion (FWI) algorithms when applied to shallow 2D structures in the presence of a complex topography is critically important. By using FWI, we overcome inherent limitations of con- ventional seismic methods used for near-surface prospecting (acoustic tomography and multichannel spectral analysis of surface waves). The elastic forward problem, formulated in the frequency domain, is based on a mixed finite-element P0-P1 discontinuous Galerkin method to ensure accurate model- ing of complex topography effects at a reasonable computing cost. The inversion problem uses an FWI algorithm to mini- mize the misfit between observed and calculated data. Based on results from a numerical experiment performed on a realistic landslide model inspired from the morphostructure of the Super-Sauze earthflow, we analyzed the effect of using a hier- archical preconditioning strategy, based on a simultaneous multifrequency inversion of damped data, to mitigate the strong nonlinearities coming from the surface waves. This strategy is a key point in alleviating the strong near-surface effects and avoiding convergence toward a local minimum. Using a limited-memory quasi-Newton method improved the convergence level. These findings are analogous to recent applications on large-scale domains, although limited source-receiver offset ranges, low-frequency content of the source, and domination of surface waves on the signal led to some dif- ficulties. Regarding the impact of data decimation on the inversion results, we have learned that an inversion restricted to the vertical data component can be successful without significant loss in terms of parameter imagery resolution. In our investigations of the effect of increased source spacing, we found that a sampling of 4 m (less than three times the theo- retical maximum of one half-wavelength) led to severe aliasing.

Answers to questions:

(a) The data set is a synthetic 2D land seismic, based on a realistic landslide case. The data were derived from a section of the Super-Sauze earthflow in the French Alps. The section measures 210 m wide by 60 m deep, with several proposed velocity layers. The data are thus discrete, as they have finite start and end points.

(b) The greatest source of noise reported by the authors is surface waves. The surface waves are reported to produce strong nonlinearities in the data, making the inversions difficult. The errors show up in the top layers of the velocity model images. The authors found that the surface waves could not be easily separated from the body waves because of the acquisition geometry used. The source sampling of 4 m introduced aliasing effects at the surface.

(c) The model parameters are best stated as part of the whole equation given for the forward model. The equations given are:

$$-i\omega\rho V_x = \frac{1}{\rho(\mathbf{x})}\left[\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xz}}{\partial z}\right] + F_x,$$
$$-i\omega\rho V_z = \frac{1}{\rho(\mathbf{x})}\left[\frac{\partial\sigma_{xz}}{\partial x} + \frac{\partial\sigma_{zz}}{\partial z}\right] + F_z,$$
$$-i\omega\sigma_{xx} = (\lambda(\mathbf{x}) + 2\mu(\mathbf{x})) + \lambda(\mathbf{x})\frac{\partial V_z}{\partial z},$$
$$-i\omega\sigma_{zz} = \lambda(\mathbf{x})\frac{\partial V_x}{\partial x} + (\lambda(\mathbf{x}) + 2\mu(\mathbf{x}))\frac{\partial V_z}{\partial z},$$
$$-i\omega\sigma_{xz} = \mu(\mathbf{x})\left[\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right],$$

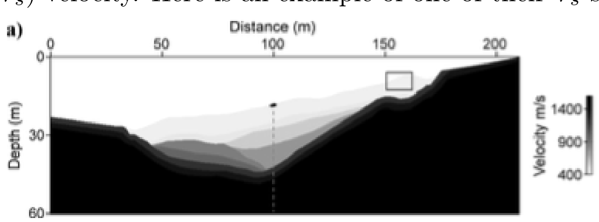
The model parameters are the velocities V_x, V_z; and the stresses σ_{xx}, σ_{zz}, and σ_{xz}. The rest of the variables in the equation are not model parameters. The model parameters are assumed to be continuous functions; otherwise, the partial derivatives would not exist!

(d) The forward problem presented in the paper is nonlinear. It is presented as the elastic wave equation in the frequency domain. The wave equation relates velocities to stresses. The derivation of the forward problem is based on a discontinuous Galerkin (DG) method. It is important to note that if the inversion problem is nonlinear, so is the forward problem.

(e) The authors do discuss the accuracy of the forward model in stating the various assumptions. In making the forward model, they assume that the seismic waves propagate through 2D, elastic, isotropic, and only very slightly heterogenous media. The authors specifically state that their techniques are based on smoothed media and are unsuitable for highly heterogeneous media. The subsurface structures of the Earth are actually never all perfectly elastic. Some deformation occurs as a result of heat loss. The solution would be to consider a visco-elastic Earth. The authors also assume a two-dimensional Earth, while we know that waves propagate in all three dimensions. The assumption of an isotropic medium is also not valid, as the direction of propagation does depend on the material (anisotropy). The authors also note at the end of the discussion that an important extension to their solution would be to incorporate attenuation parameters, which were not incorporated in their forward problem.

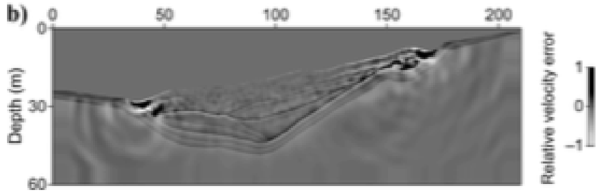
(f) The authors use previous information from the discontinous Galerkin method given in Brossier (2009) for a solution of the forward model. The authors note that there are only a few studies on waveform inversion involving near-surface (0-100 m) body and surface waves. Though not stated explicitly, the authors do use Hooke's law for sound propagation through elastic materials.

(g) The authors present the solutions graphically. They solved for P-wave velocity (V_p) and S-wave (V_s) velocity. Here is an example of one of their V_s solutions:



The above plot is a depth section through the Earth for their landslide case, showing shear-wave velocity information.

(h) The reliability of the final V_s model is actually given graphically (see below).



The above plot is a relative velocity error (ratio of the velocity error to the true velocity.

The authors noted that their final plot of V_s for the vertical component had a degradation error of 24%, while the final plot of V_s for the horizontal component had a degradation error of 102%. The authors ultimately note that the inversion of raw data failed to yield acceptable velocity images. Although the uniqueness of solutions is not discussed outright in the paper, it is clear that the multiple iterations of the inversion have all produced solutions for velocity. There are many solutions that model the data.

References

Brossier, R. 2009. Imagerie sismique a deux dimensions des milieux visco-elastiques par inversion des formes d'onde: Developements methodologiques et applications: Ph.D. dissertation, Universite de Nice-Sophia-Antipolis.