Mathematical programming application

Optimal power flow in a DC microgrid

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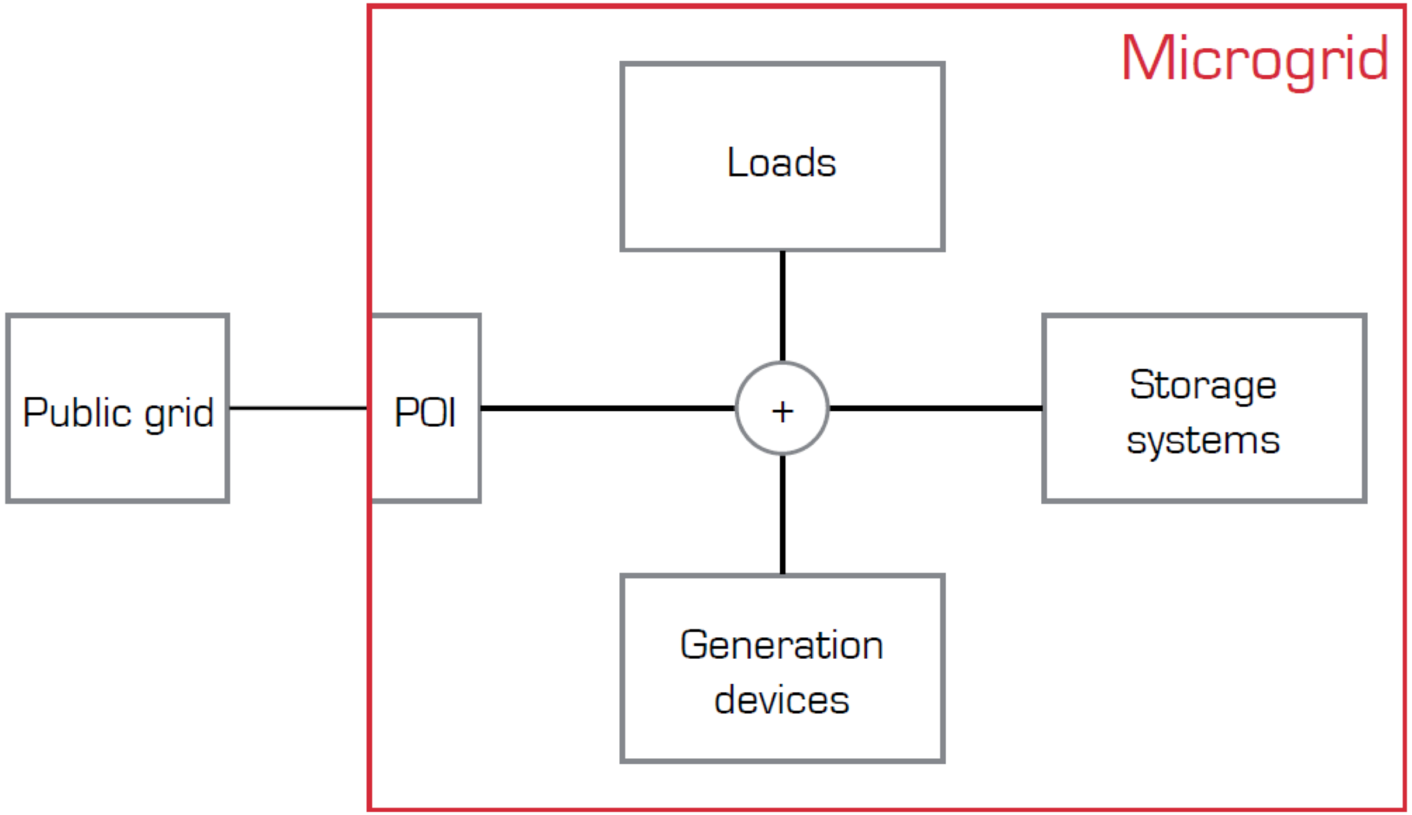
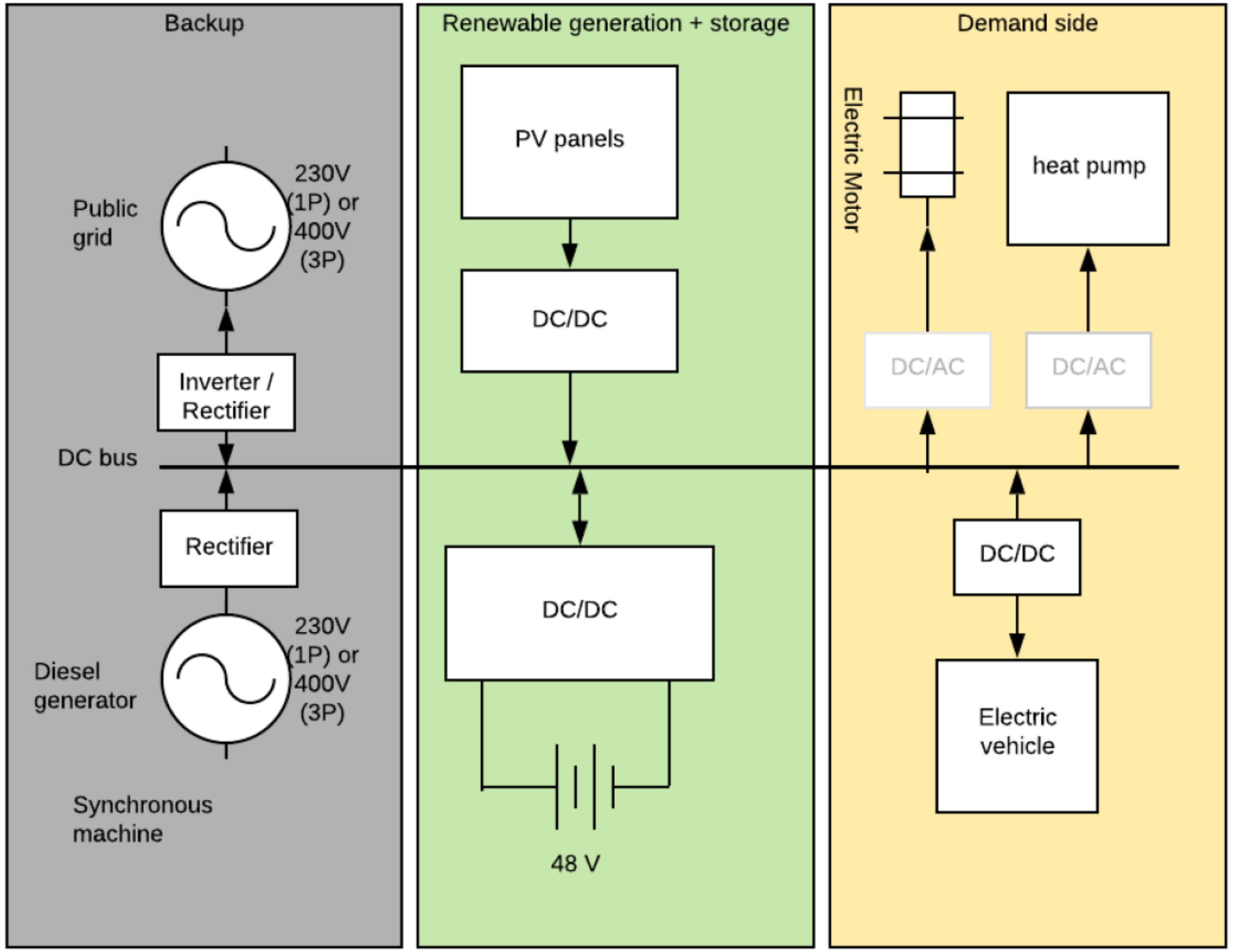
February 2021

Introduction

What is a microgrid



What is a DC microgrid



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Why DC microgrids?

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- DC systems enable a simpler integration of distributed energy resources (DERs*), since many of them are either DC by nature or require a DC interface anyway
- Fewer conversion losses
- Parallel distributed architectures are simpler to realize in DC:
  - unnecessary frequency control and phase synchronization
- Frequency control is not necessary in DC systems
  - unwanted harmonic content may by easier to filter too

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- Autonomous distributed control harder in DC than in AC because no information carried through the signal (frequency, phase)
- There may be stability issues due to DC-DC conversion stages
- It is more difficult to clear fault currents: the signal “does not go through zero”. Hence protections are more costly and harder to set up.

Control levels

- A microgrid controller is some software sensing the microgrid (currents, voltages, frequency, etc.) and taking control actions so as to operate safely, reliably and optimally the microgrid.

- In practice, a microgrid is run by multiple controllers, because there are several levels of control, which differ by their spatial and temporal scopes.

- Next to technological advances in production, consumption and storage, controllers are key elements for advanced microgrids.

Level	Function	Examples
1	Device level control	BSS control, reactive control, MPPT
2	Local area control	Frequency regulation, fast load shedding
3	Supervisory control	Forecasting, operational planning
4	Public Grid interaction	Ancillary services, energy markets

We will focus mainly on levels 1 and 2

Agenda

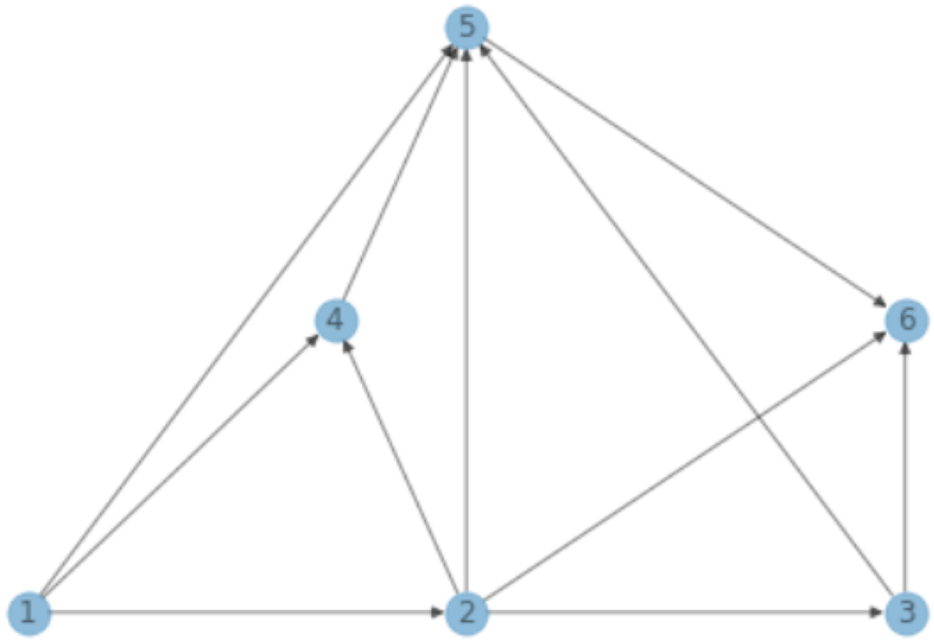
- The formulation of optimal power flow OPF in a DC microgrid as a NLP
    - Relaxation as a SOCP
- Hands on session with Python and Pyomo to model the OPF formulations

The optimal power flow problem

Optimal prower flow model

- We are at a given moment (static model)
- The DC grid is modelled as a graph (N, E) where
  - $N = \{0, \ldots, n\}$ is the set of nodes,
  - E is the set of edges.
- Let's denote $\{k, l\} \in E$ as $k \sim l$

Optimal power flow



- At buses, there are:
  - *loads* drawing power
  - *generators* injecting power
  - *Storages* doing both
- *Branches* have a resistance
- *OPF*: Optimize an objective while statisfying Kirchhoff laws and devices constraints

Electrical quantities

- We are interested in knowing
  - currents and powers in all the branches: i_{kl} and p_{kl}
  - currents and powers absorbed at buses: i_k and p_k
  - voltages at all the nodes: v_k
- Measurements or variables? some of these quantities are measured, some are not. This is really case dependent.
  - In DC grids, it is easy to measure currents and voltages, or to determine currents from power and voltage measurements

Fundamental laws relate these quantities

- Ohm (Y_{kl} is the conductance of the branch):

$$i_{kl} = Y_{kl}(v_k - v_l) \text{ for } \{k, l\} \in E$$

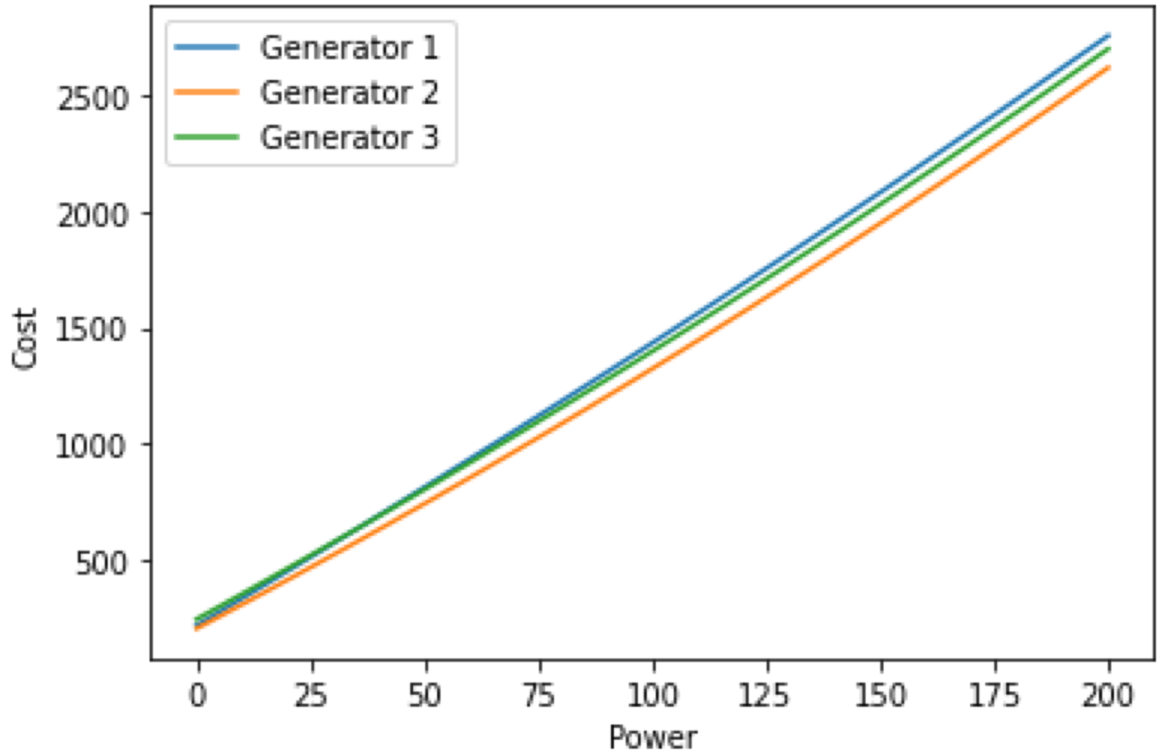
- Kirchhoff Current Law:

$$i_k = \sum_{l:k\sim l} i_{lk} \text{ for } k \in N$$

- Power withdrawn at buses:

$$p_k = v_k i_k \text{ for } k \in N$$

Generator model



- Cost function
- Minimum generation level

Note : DC-DC converter model

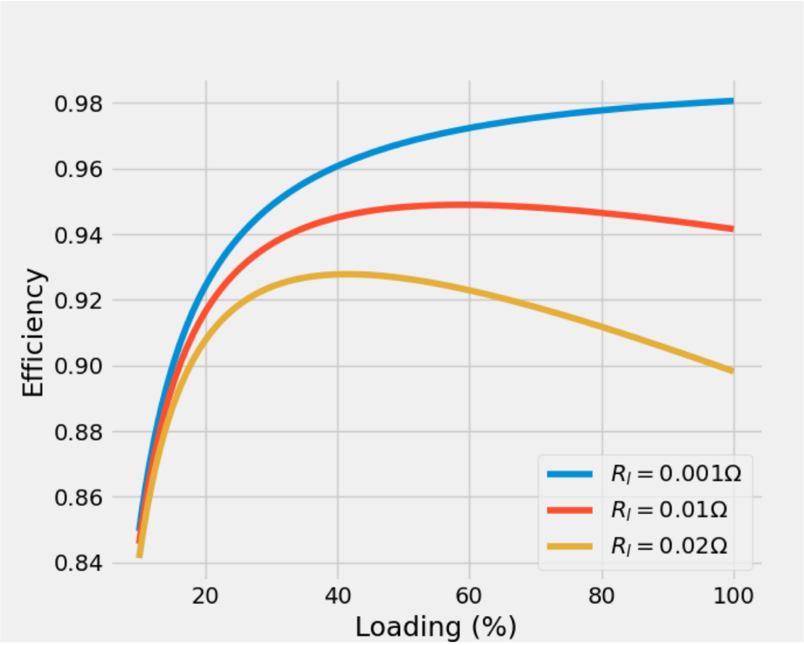
Let's simply use power conservation

$$p_{in} = p_{out}$$

Losses in a series resistance R_l

« Standby losses » can be added but requires

1. Binary variable
2. Determining the direction of the flow (in case the converter is bidirectional)



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Solving this set of equations gives you the electrical state of the system

Depending on the type of information available and the behavior of components, the system is linear or not :

- Only power and some voltage measurements:
  - $p_k = v_k \sum_{l:k\sim l} Y_{kl}(v_k - v_l)$ for $k \in N$
  - Non-linear problem
- A sufficient number of current and voltage measurements: linear problem

Remark: introducing dynamics in the OPF problem

- A full dynamical model requires modeling the state evolution as a set of non-linear differential equations, taking into account converters switching etc. -> e.g. Typhoon HIL type model

- In our context of power/energy management, the goal is to devise a discrete time model to predict what's gonna be the **state** of the system in $\Delta t \in [0.1, 900]$ seconds given its current state and some control actions that we can take

  - assuming average models for converters

  - assuming transients are much faster than Δt

At this time scale, batteries state of charges are the main state variables

Real battery model:

- voltage source function of the state of charge (and temperature, current, etc. *)
- Internal resistance

Battery State of Charge evolves with the number of *Ah* that come in or out:

$$s_{t+\Delta t} = S_t + i_t \Delta t \text{ [Ah]}$$

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* See for instance https://nl.mathworks.com/help/physmod/sps/powersys/ref/battery.html

Problem formulation

Gan, L., & Low, S. H. (2014). Optimal power flow in direct current networks. *IEEE Transactions on Power Systems*, *29*(6), 2892–2904.

$$
\begin{aligned}
\mathbf{OPF}:\ \min\ & \sum_{i \in \mathcal{N}} f_i(p_i) \\
\text{over}\ & p, V \\
\text{s.t.}\ & p_i = V_i \sum_{j: j \sim i} (V_i - V_j) y_{ij}, \quad i \in \mathcal{N}; \\
& p_i \in \mathcal{P}_i, \quad i \in \mathcal{N}^+; \\
& V_0 = V_0^{\text{ref}}; \\
& \underline{V}_i \leq V_i \leq \overline{V}_i, \quad i \in \mathcal{N}^+.
\end{aligned}
$$

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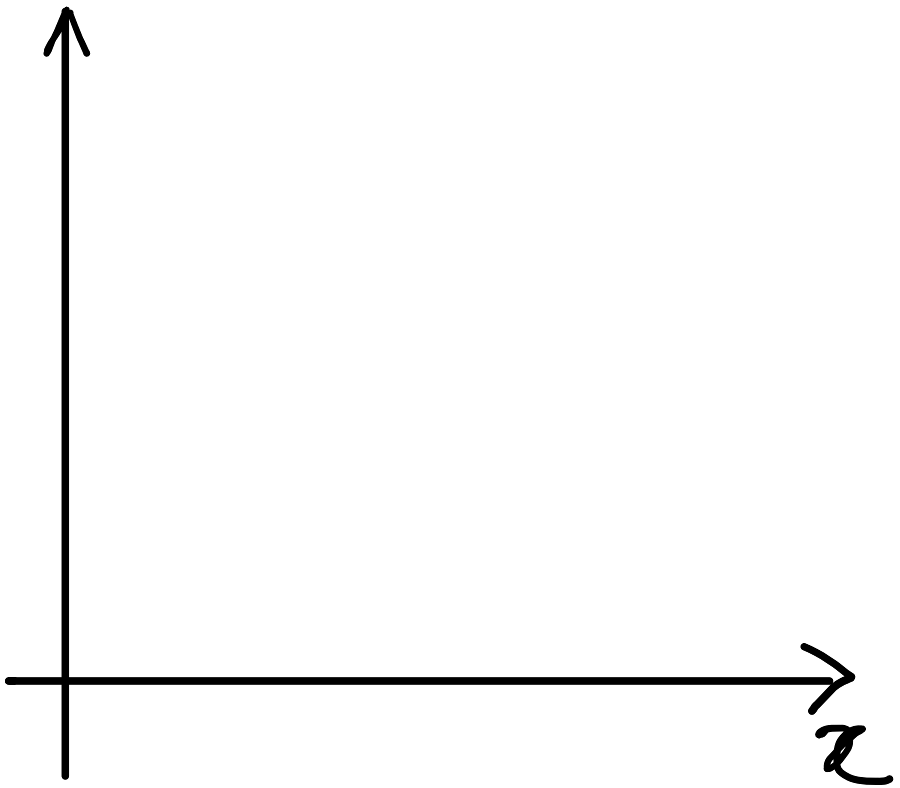
Nonlinear programming

- In a general non-linear (non-convex) problem, there is no guarantee that a locally optimal solution is also a global solution

- When there are no discrete variable, you can solve them (locally) with <u>interior point methods</u>

    - E.g. solver: ipopt



Convex programming

See the book "Stephen P. Boyd, and Lieven Vandenberghe. Convex optimization. Cambridge university press, 2004."
Freely available, you can read the introduction section

- We minimize a convex function over a convex set

- Convexity of a function f means
  - $f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1] + (1-\lambda) f(x_2)$
    $\forall \lambda \in [0,1]$

- Convexity of a set F means
  - If $x \in F$ and $y \in F$
  - then $\lambda x + (1-\lambda) y \in F, \ \forall \lambda \in [0,1]$

Why convex optimization

- Convexity guarantees that a local optimal solution is also global
- For some special cases, very efficient algorithms exist
- Use it to solve nonconvex problems
  - For initialization
  - To get some bounds for global optimization
    - Convex relaxation

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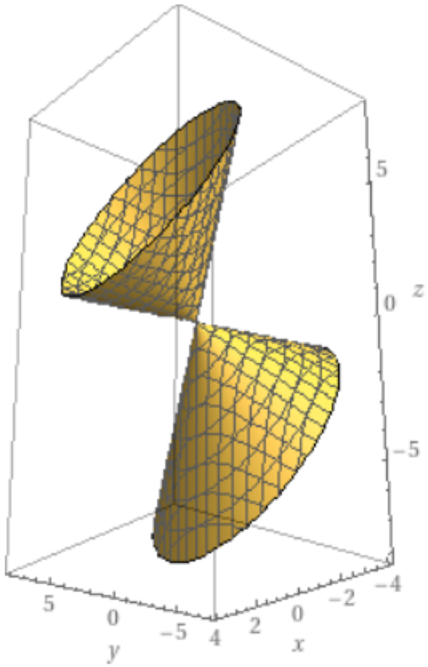
Second-Order Cone Programming

- A class of convex problems that is more general than LP but for which efficient algotihms exist

- In this problem, some constraints define cones:

    - A set C is called a cone if for every $x \in C$ and $\theta \geq 0$ we have $\theta x \in C$.

Examples



- $\{(x, y, z) \mid x^2 \leq yz\}$
- $\{(x_1, x_2, t) \mid (x_1^2 + x_2^2)^{1/2} \leq t\}$

Convex formulation

$$
\begin{aligned}
\mathbf{OPF}': \ \min \ & \sum_{i \in \mathcal{N}} f_i(p_i) \\
\text{over } & p_i \in \mathbb{R}, v_i \in \mathbb{R} \text{ for } i \in \mathcal{N}; \\
& W_{ij} \in \mathbb{R}^+ \text{ for } i \sim j, \\
\text{s.t. } & p_i = \sum_{j: j \sim i} (v_i - W_{ij}) y_{ij}, \quad i \in \mathcal{N}; && \text{(6a)} \\
& p_i \in \mathcal{P}_i, \quad i \in \mathcal{N}^+; && \text{(6b)} \\
& v_0 = \left[V_0^{\text{ref}}\right]^2; && \text{(6c)} \\
& \underline{V}_i^2 \le v_i \le \overline{V}_i^2, \quad i \in \mathcal{N}^+; && \text{(6d)} \\
& W_{ij} = W_{ji}, \quad i \to j; && \text{(6e)} \\
& \begin{bmatrix} v_i & W_{ij} \\ W_{ji} & v_j \end{bmatrix} \succeq 0, \quad i \to j; && \text{(6f)} \\
& \operatorname{rank} \begin{bmatrix} v_i & W_{ij} \\ W_{ji} & v_j \end{bmatrix} = 1, \quad i \to j. && \text{(6g)}
\end{aligned}
$$

Exact SOCP Relaxation

- If an optimal SOCP solution (p, v, W)satisfies (6g), then (p, v, W) also solves OPF'.
- Furthermore, compute V as
  - $V_i = \sqrt{v_i}$
- then it can be shown that (p, v)solves OPF

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Hands on session

- Code the OPF problem as a NLP
- Relax the problem as a QP (quadratic objective, linear constraints).
  - What happens?
- Relax the problem as a SOCP

- https://colab.research.google.com/drive/1Nr06HZMWQRHXIuOJGBnVHKV7-8j_cpDu?usp=sharing

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