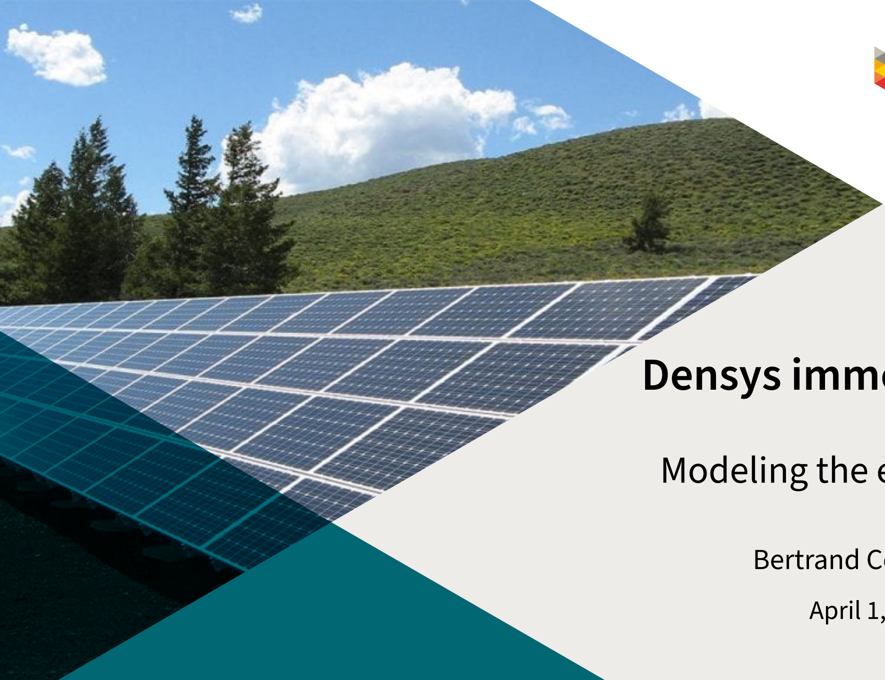

LIÈGE université
Sciences Appliquées

Densys immersive week

Modeling the electrical grid

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Lecture overview

Nowadays, the vast majority of microgrids are AC.

The goal of this lecture is to remind you how an AC network can be modeled in sinusoidal steady-state analysis using phasors and impedances. Then we derive the power flow equations of a network as a function of bus voltages and power injections.

Overview

1. Basics and conventions

2. Sinusoidal steady state analysis

3. Power flow equations

4. Hands-on session B
AC optimal power flow model

Basics and conventions

Power and energy I

- Power measures the rate of use of energy
- It is expressed in Watt [W]: 1 W = 1 Joule/second
- In an electric system,

$$p(t) = u(t) \times i(t)$$

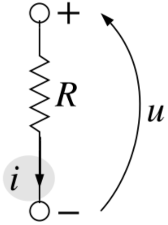
- $u(t)$ is the voltage measured in volt [V], the line integral of the electric field between two points.
- $i(t)$ is the current measured in amps [A]
- t is the time

Power and energy II

- To measure energy in power systems, we use units ranging from a kWh (a microgrid) to a TWh (a country)
- Devices have power ratings ranging from W to GW (although we generally speak in VA for ratings)

Motor convention (or standard reference)

When using the motor convention to direct u w.r.t. i, $p(t)$ represents the power **consumed** by a device (here a resistor):



- The power consumed can be < 0, = 0, or > 0 depending on the device
- E.g. for a resistor we always have $p(t) \geq 0$
- The **opposite** convention is the **generator convention**
- We will sometimes use a mix of both conventions based on intuition, so that, in general, we have few negative numbers: pay attention to the orientations!

Sinusoidal steady state analysis

Sinusoidal signals and phasor representation

Unless otherwise specified, we will always work with sinusoidal signals and in steady state:

$$y(t) = \sqrt{2}Y\cos(\omega t + \phi_y).$$

Y is the *rms* value of the signal, ϕ_y its phase and ω its angular frequency.

At a specific frequency $f = \frac{\omega}{2\pi}$, the signal can be represented as a phasor

$$\bar{Y} = Y\angle\phi_y = Ye^{j\phi_y}$$

Phasors allow working in the frequency domain, which is much nicer for computations.

How do you get the time expression from the phasor?

See https://en.wikipedia.org/wiki/Phasor

Impedance I

Let $u(t)$ and $i(t)$ be the voltage and current across a one-port, respectively, in sinusoidal steady state and with the motor convention.

- For a resistor, $u(t) = Ri(t)$ hence
$$\bar{U} = R\bar{I}$$
- For an inductor, $u(t) = L\frac{di(t)}{dt}$ hence
$$\bar{U} = j\omega L\bar{I}$$
- For a capacitor, $i(t) = C\frac{du(t)}{dt}$ hence
$$\bar{I} = j\omega C\bar{U}$$

Impedance II

The *impedance*, a complex number, generalizes this notion

$$Z = R + jX \ \ [\Omega]$$

such that $\bar{U} = Z\bar{I}$ with

- for a resistor, $Z = R$
- for a self, $Z = jX = j\omega L$
- for a capacitor, $Z = jX = -j\frac{1}{\omega C}$

Impedance, admittance, etc.

The imaginary part of the impedance, X, is called reactance

The admittance Y is the inverse of the impedance, expressed in Siemens [S]:

$$Y = G + jB$$

- G is the conductance
- B is the susceptance

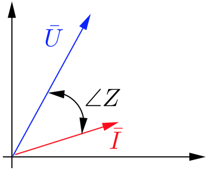
Complex calculus

$$|Z| = \sqrt{R^2 + X^2}$$

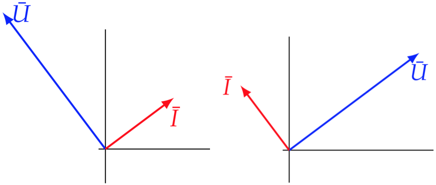
$$\angle Z = \arctan \frac{X}{R}$$

$$Z = \frac{\bar{U}}{\bar{I}} = \frac{U}{I}\angle(\phi_u - \phi_i)$$

Phasor diagrams



Plot the phasors in the complex plane!



Inductive or capacitive? Which is which?

The notions of power I

The complex power is defined as

$$S = \bar{U}\bar{I}^*$$

Let

$$\phi = \phi_u - \phi_i$$

then

$$S = UIe^{j\phi} = P + jQ$$

- $P = UI\cos\phi$ is the active power, measured in watt
- $Q = UI\sin\phi$ is the reactive power, measured in var
- $\cos\phi$ is the power factor

Reactive power is, in general, undesirable.

The apparent power is $|S| = UI$, measured in VA

Useful formulas

$$P = RI^2 = \frac{U^2}{R}$$

$$Q = XI^2 = \frac{U^2}{X}$$

$$\tan \phi = \frac{Q}{P}$$

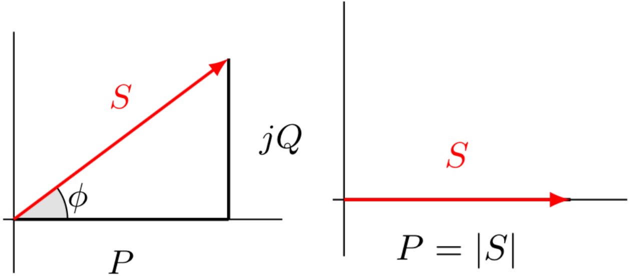
$$\cos \phi = \frac{P}{|S|}$$

The power factor does not tell you whether the system is leading or lagging

- in an inductive system, $u(t)$ precedes $i(t)$, $i(t)$ is lagging, thus $Q > 0$ (motor convention)
- in a capacitive system, this is the opposite (leading).

Power factor compensation

Produce some Q to cancel out ϕ:



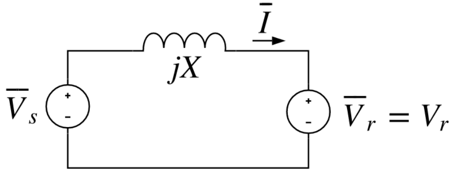
Example:

A 120V voltage source at 60 Hz that feeds an R-L load $1858.4 + j1031.4$ *VA*

Power flow equations

Power transfer between AC systems I

Consider the following simple system



We have

$$\bar{I} = \frac{\bar{V}_s - \bar{V}_r}{jX}$$

Let δ be the angle between $\bar{V}_r$ and $\bar{V}_s$, then

$$S_r = \bar{V}_r \bar{I}^* = V_r \left(\frac{V_s \angle - \delta - V_r}{-jX} \right) = \frac{V_s V_r \sin \delta}{X} + j \frac{V_s V_r \cos \delta - V_r^2}{X}$$

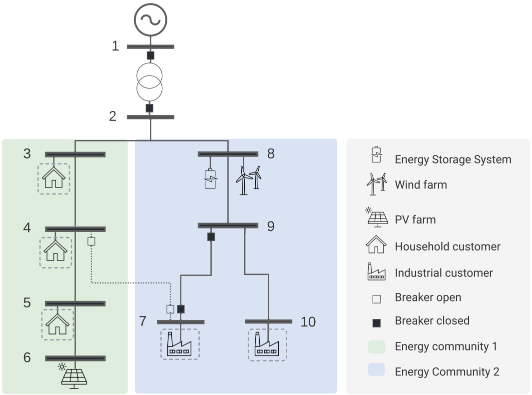
Power transfer between AC systems II

Let's remember two things:

- The **active** power is highly sensitive to δ
- The **reactive** power acts on the **voltage magnitude** (look at what happens for $\delta = 0$)

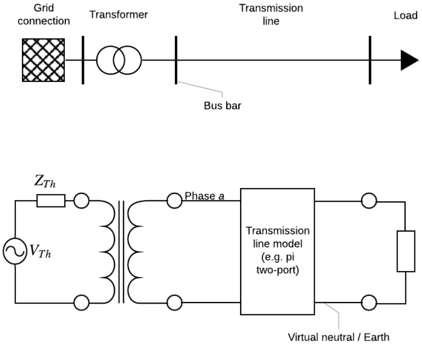
One-line diagram I

Remember, we represented a microgrid as



One-line diagram II

Looking, e.g., at the top portion of this schematic, it actually means

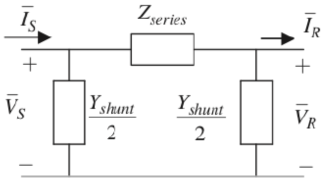


This is a simplification since most AC networks are three-phase. Here we assume the three phases are perfectly balanced to be able to represent it as a single-line equivalent.

Lumped transmission line model in steady state:

The π model

If the length *l* [km] of a transmission line is relatively small ($< 300km$, obviously true in a microgrid), we can **approximate** the line with lumped parameters.



with,

- $Z_{series} = Rl + j\omega Ll$
- $\frac{Y_{shunt}}{2} = j\frac{\omega Cl}{2}$

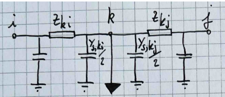
R, *L* and *C* are per km values.

This π model is symmetrical by design.

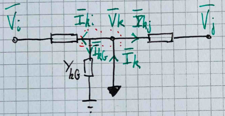
The power flow equations I

- Let $\mathcal{N}$ be the set of buses of the network
- Some buses are interconnected by transmission lines, given by their π models
- Let Y_{kG} be the sum of admittances connected between node k and the ground:
  - the shunt admittances of the lines incident to k,
  - the admittances of the devices connected at node k if any
  - but not the admittance related to the injection or withdrawal at node k, which is usually unknown.
- For two nodes k and m, let Z_{km} be the series impedance of the line connecting them and $Y_{km} = Z_{km}^{-1}$ ($Y_{km} = 0$ if there is no line)

The power flow equations II



$$Y_{kG} = \frac{Y_{s,ki}}{2} + \frac{Y_{s,kj}}{2}$$



$$\text{KCL}: \quad \bar{I}_k = \bar{I}_{kG} + \sum_{m \in \mathcal{N}(k)} \bar{I}_{km} \quad , \mathcal{N}(k) = \{i,j\}.$$

$$\left\{\begin{aligned} \bar{I}_{kG} &= \bar{V}_k Y_{kG} \\ \bar{I}_{ki} &= (\bar{V}_k - \bar{V}_i) Y_{ki} \\ \bar{I}_{kj} &= (\bar{V}_k - V_j) Y_{kj} \end{aligned}\right. \quad \Leftrightarrow \quad \bar{I}_k = \bar{V}_k Y_{kG} + \sum_{m \in \mathcal{N}(k)} (\bar{V}_k - \bar{V}_m) Y_{km}.$$

The power flow equations III

The current injection at node k is

$$\bar{I}_k = Y_{kG}\bar{V}_k + \sum_{m \in \mathcal{N} \setminus k} (\bar{V}_k - \bar{V}_m) Y_{km}$$

This last equation can be rewritten as

$$\bar{I}_k = \left(Y_{kG} + \sum_{m \in \mathcal{N} \setminus k} Y_{km} \right) \bar{V}_k - \sum_{m \in \mathcal{N} \setminus k} Y_{km} \bar{V}_m$$

The power flow equations IV

The complex power injected at bus k is

$$S_k = \bar{V}_k \bar{I}_k^{\star},$$

we develop this relation and separate the real and imaginary parts:

$$P_k = G_{kk} V_k^2 \qquad + V_k \sum_{m \in \mathcal{N} \setminus k} V_m (G_{km} \cos \theta_{km} + B_{km} \sin \theta_{km})$$

$$Q_k = -B_{kk} V_k^2 \quad + V_k \sum_{m \in \mathcal{N} \setminus k} V_m (G_{km} \sin \theta_{km} - B_{km} \cos \theta_{km})$$

with

The power flow equations V

- $Y_{km} = G_{km} + jB_{km}$
- $Y_{kk} = G_{kk} + jB_{kk}$ the sum of all the admittances connected to bus k
- $\theta_{km} = \theta_k - \theta_m$ the phase difference between voltages at nodes k and m

The equations are non-linear.

Hands-on session B: AC optimal power flow model

AC optimal power flow model I

Starting from the network flow model of the hands-on session A, consider now that

- the network lines are specified by their π model (we will neglect the shunt susceptances).
- power flows in the network according to the equations we have established in Section "Power flow equations" (You may need to reformulate the equations to avoid having trigonometric functions. How can you do this?)
- the absolute value of the current in each line is limited to avoid damaging the lines (line rating).

AC optimal power flow model II

- voltages must stay within a range around a nominal voltage value to guarantee that grid devices can function properly (the voltage at the PCC sets the reference)
- loads absorb active and reactive power you cannot control
- generators can, within some limits, generate active power and generate or consume reactive power.

You are asked to

- Extend the network flow model to include all these aspects.
- Minimize the total generation costs.

AC optimal power flow model III

- Compare the solution to the solution of the problem modeled as a network flow problem.
- Compute the losses in the system.
- Is the solution that minimizes the costs also minimizing the losses?

In practice:

- Use **the template Google Colab available here.**
- It uses the same Python libraries and solvers as the first problem.