

The background of the slide features a photograph of a large array of solar panels installed on a grassy hillside. The panels are arranged in neat rows and reflect the bright sunlight. In the background, there are several tall evergreen trees and a clear blue sky with a few white clouds. The image is partially obscured by a white diagonal shape on the right side, which contains the text.

Densys immersive week

Real-time optimization in
microgrids

Bertrand Cornélusse

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Overview

1. Introduction
2. Off-grid microgrid use case illustration
3. General formulation of the optimization-based controller

Introduction

The background of the slide features a minimalist design with teal-colored geometric shapes. Two large teal triangles point upwards from the bottom corners, meeting at a central point. Below this meeting point, a smaller, darker teal triangle points downwards. The remaining space is white, creating a clean, modern aesthetic.

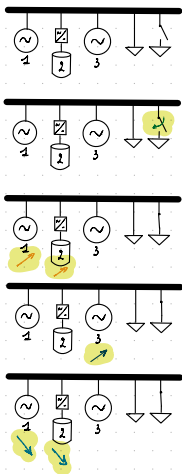
Lecture objectives

- ▶ Formulate the real-time optimization problem
- ▶ Understand it's connection with other control problems
- ▶ Practice linear programming modeling and gain intuition in the solutions' properties
- ▶ Understand the benefits of optimization with respect to simple rules

What is real-time optimization

- ▶ Real-time optimization is a secondary control layer that sends reference power set-points, or targets, to devices to
 - ▶ minimize the system's operation cost
 - ▶ restore margins for droop controllers (primary control layer).
- ▶ Power set-points are mainly for active power
 - ▶ but may also be reactive power set-points in case voltage must be regulated (but this is out of the scope of this lecture).
- ▶ Lower-level controllers receive these set points and regulate currents and voltages to satisfy them. They may, however, leave an error because of instantaneous changes in the environment and slow system dynamics that do not allow frequent set point changes.

Frequency control in one schematic



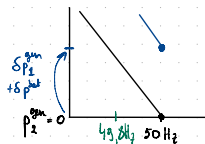
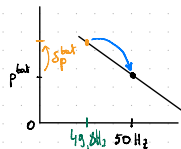
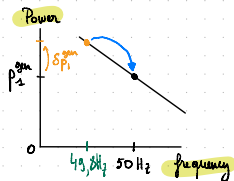
System works at 50 Hz.
Load is balanced by generation.

Load increases and frequency drops.

Primary control of generator 1
and battery act to contain
frequency at 49.8 Hz.

Secondary control
decides to start generator 2

Primary control of generator 1
and battery are relieved
Frequency back at 50 Hz!



Methods

- ▶ Real-time optimization can be accomplished through different methods.
- ▶ Many controllers use basic if-then rules, i.e., **rule-based controllers**.
- ▶ Although these rules are simple and easy to understand, they are not ideal for microgrids with numerous devices or complex grid interaction mechanisms.
- ▶ In such cases, **optimization-based control** is better suited. We will compare the two approaches in a simple example.

Control time scale

Real-time operation means that decisions are taken frequently, every Δt seconds.

- ▶ An upper bound on Δt : renewable generation that can vary significantly within a few seconds, or abrupt load changes
- ▶ A lower bound on Δt : lower-level control loops (e.g., power point tracking) must have the time to converge to avoid interference between control levels

We will assume that $\Delta t \approx 1\text{s}$.

Notation

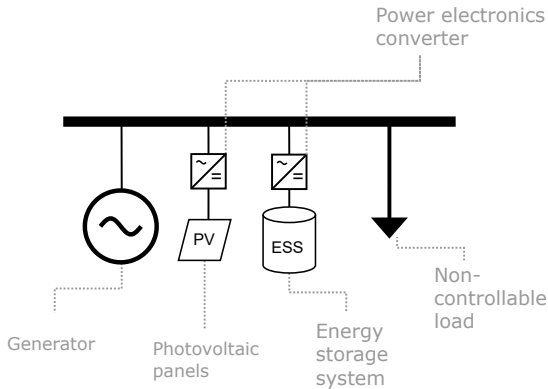
- ▶ Lowercase symbols are control variables
- ▶ Uppercase symbols and Greek letters are fixed values (parameters or measurements)
- ▶ Subscript t is a time index, sometimes omitted to simplify reading
- ▶ Subscript d is a device index, omitted if there is only one device
- ▶ Superscript PV refers to a photovoltaic system
- ▶ Superscript bat refers to a battery storage system
- ▶ Superscript $load$ refers to a load
- ▶ Superscript gen refers to a generator

Off-grid microgrid use case illustration

The background of the slide features a white upper section and a teal lower section. The teal section is composed of two large triangles that meet at a point at the bottom center, creating a V-shape. The top of the teal section is a horizontal line that spans the width of the slide.

Off-grid microgrid

Let us consider an off-grid microgrid with photovoltaic panels with an installed capacity of C^{PV} , a stationary battery, a backup diesel generator, and an uncontrollable load. For now, we are neglecting the electrical grid connecting these devices.



Microgrid state

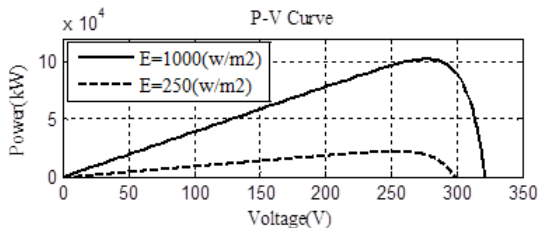
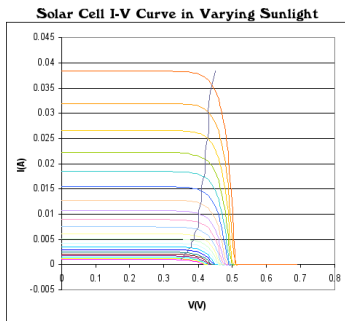
At time t , the state of the microgrid can be described by

1. the power consumed by the load P_t^{load}
2. the maximum power that the PV panels can produce $\bar{P}_t^{PV} \leq C^{PV}$.
3. the state of charge of the battery, S_t
4. the maximum power the battery can deliver \bar{P}_t^{bat} or consume \underline{P}_t^{bat} .
5. (the ON/OFF state of the generator and the duration in that state.)

This state must thus be partly measured and partly estimated.

About the Maximum Power Point \bar{P}_t^{PV}

This quantity is usually unknown and is approached when the inverter is working at its maximum power point (MPP). As soon as a curtailment instruction is given, i.e., a set point below the maximum power point, the actual MPP is unknown and must be estimated.



The maximum battery power

The maximum power the battery can deliver \bar{P}_t^{bat} or consume \underline{P}_t^{bat} varies with time since it is a function of the state of charge (SoC), the temperature, etc. This information comes from the battery management system (BMS).

Generator

A generator may need a nonnegligible minimum time to start up, and too frequent starts and stops should be avoided to limit wear and tear. There is a minimum stable generation level and ramping constraints.

We will consider a simple model in this lecture.

Control decisions and objective

Since the load is not controllable, the **control decisions** are

- ▶ the battery's charging or discharging power p_t^{bat}
- ▶ the generator's power p_t^{gen}
- ▶ if needed, a curtailment instruction for the PV panels p_t^{PV}

Let

$$p_t = (p_t^{bat}, p_t^{gen}, p_t^{PV}).$$

The **objective** is to maximize the energy harvested from the PV panels and use the diesel generator as little as possible.

Rule-based controller

Input: the microgrid state.

Compute the imbalance between the load and the maximum PV generation

$$\delta_t = p_t^{load} - \bar{p}_t^{PV}$$

if $\delta_t > 0$ then

▷ Excess load, discharge first, then use the generator

$$p_t^{PV} \leftarrow \bar{p}_t^{PV}$$

▷ Produce the maximum with PV generation

if $\bar{p}_t^{bat} > \delta_t$ then

$$p_t^{bat} \leftarrow \delta_t$$

$$p_t^{gen} \leftarrow 0$$

$$\delta_t \leftarrow 0$$

else

$$p_t^{bat} \leftarrow \bar{p}_t^{bat}$$

▷ Discharge the battery at full power

$$\delta_t \leftarrow \delta_t - p_t^{bat}$$

▷ Use the generator to cover the remaining load

if $\bar{p}_t^{gen} > \delta_t$ then

$$p_t^{gen} \leftarrow \delta_t$$

else

▷ Run the generator at full power

$$p_t^{gen} \leftarrow \bar{p}_t^{gen}$$

$\delta_t \leftarrow \delta_t - p_t^{gen}$ ▷ The load exceeds the maximum generation capacity

else

▷ Excess PV, charge first then curtail

$$p_t^{gen} \leftarrow 0$$

▷ Shut down the diesel generator

if $\bar{p}_t^{bat} > -\delta_t$ then

$$p_t^{bat} \leftarrow \delta_t$$

$$p_t^{PV} \leftarrow \bar{p}_t^{PV}$$

▷ No need to curtail

else

$$p_t^{bat} \leftarrow -\bar{p}_t^{bat}$$

▷ Charge the battery at full power

$$\delta_t \leftarrow \delta_t - p_t^{bat}$$

$$p_t^{PV} \leftarrow \bar{p}_t^{PV} + \delta_t$$

▷ Curtail excess PV generation

$$\delta_t \leftarrow 0$$

$$p_t \leftarrow (p_t^{bat}, p_t^{gen}, p_t^{PV}) \text{ return } p_t, \delta_t$$

Comments

A positive δ_t returned value indicates an emergency measure should be taken to shed some load.

This should, however, not happen if the system and the microgrid's electric protections are properly designed.

Graphical view

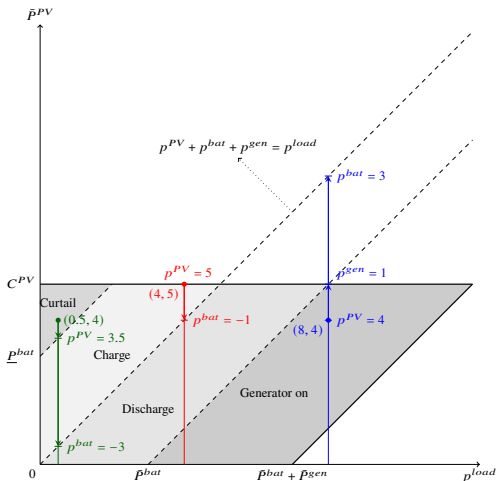


Illustration of the rule-based controller when $C^{PV} > \underline{p}_t^{bat} > 0$ and $\bar{P}_t^{bat} > 0$ (index t omitted in the graph). Three (p^{load}, \bar{p}^{PV}) operating points are highlighted in green (0.5, 4), red (4, 5) and blue (8, 4), with the corresponding setpoints for p^{PV} , p^{bat} , and p^{gen} .

Optimization-based controller I

An alternative way of solving this problem is to state the problem as a linear program.

$$\min \quad \pi^{gen} p^{gen} + \epsilon p^{bat} \quad (1a)$$

$$\text{s.t.} \quad p^{PV} + p^{bat} + p^{gen} = p^{load} \quad (1b)$$

$$p^{PV} \leq \bar{p}^{PV} \quad (1c)$$

$$-\underline{p}^{bat} \leq p^{bat} \leq \bar{p}^{bat} \quad (1d)$$

$$p^{gen} \leq \bar{p}^{gen} \quad (1e)$$

$$p^{gen}, p^{PV} \geq 0 \quad (1f)$$

Optimization-based controller II

With $\epsilon < \pi^{gen}$, the objective function favors PV generation (free), then the battery, then the generator.

- ▶ The coefficient ϵ is somewhat virtual but could also model a battery usage cost or a value degradation when $p^{bat} > 0$.
- ▶ In case of excess PV, charging the battery is preferred over curtailment since having $p^{bat} < 0$ decreases the objective.
- ▶ Without this coefficient, the optimization problem would consider charging the battery or curtailing renewable generation as equivalent solutions.

Optimization-based controller III

While the rule-based controller algorithm is relatively concise, Problem (1) is even more compact. However,

- ▶ It needs a solver. Nowadays, open-source and commercial solvers are very efficient and easy to interface with mainstream programming languages.
- ▶ Such small optimization problems are solved in a few milliseconds.
- ▶ wrong parameters may cause infeasibility.

Handling infeasibility

For instance, if the load exceeds the maximum generation capacity, the rule-based algorithm outputs the best possible solution and a non-zero δ_t . In contrast, Problem (1) returns no solution and an error message.

It is, however, easy to make the problem more robust:

- ▶ add slack variables to the power balance constraint: let $p^+ \geq 0$ and $p^- \geq 0$ represent an excess of generation and a shortage of generation, respectively.
- ▶ replace the power balance constraint by $p^{PV} + p^{bat} + p^{gen} + p^- = p^{load} + p^+$
- ▶ add a term $V(p^+ + p^-)$ to the objective function, with a parameter V large.

This leads to Problem (2).

Formulation as a feasibility problem

$$\min \quad \pi^{gen} p^{gen} + \epsilon p^{bat} + V(p^+ + p^-) \quad (2a)$$

$$\text{s.t.} \quad p^{PV} + p^{bat} + p^{gen} + p^- = p^{load} + p^+ \quad (2b)$$

$$p^{PV} \leq \bar{p}^{PV} \quad (2c)$$

$$-\underline{p}^{bat} \leq p^{bat} \leq \bar{p}^{bat} \quad (2d)$$

$$p^{gen} \leq \bar{p}^{gen} \quad (2e)$$

$$p^{gen}, p^{PV} \geq 0 \quad (2f)$$

General formulation of the optimization-based controller

The background of the slide features a white upper section and a teal lower section. The teal section is composed of two large triangles that meet at a point at the bottom center, creating a V-shape. The text is positioned in the white area above this teal section.

Battery limits and SoC update I

The battery management system (BMS) usually provides up-to-date limits to the battery power p^{bat} that reflect not only the inverter capability but also the SoC of the battery, the impact of the battery temperature, etc.

As a security, the SoC s^{bat} can be tracked with

$$s_{t+\Delta t}^{bat} = s_t^{bat} + p_t^{bat} \Delta t$$

and bounded by SoC limits

$$\underline{s}^{bat} \leq s_{t+\Delta t}^{bat} \leq \bar{s}^{bat}.$$

Battery limits and SoC update II

Since Δt is small, the amount of energy charged or discharged is also small compared to the storage size but may become significant close to the SoC limits. This type of constraint is optional here but mandatory in a planning stage where the time step is on the order of a minute.

However, this state evolution constraint is inaccurate when accounting for charge and discharge efficiencies.

Charge and discharge efficiencies I

To account for charge and discharge efficiencies, we can subdivide the battery power into charge and discharge powers as

$$p^{bat} = p^{bat,charge} - p^{bat,discharge}$$

with $p^{bat,charge} \geq 0$ and $p^{bat,discharge} \geq 0$.

Then, the SoC update can be rewritten as

$$s_{t+\Delta t}^{bat} = s_t^{bat} + p_t^{bat,charge} \eta^{charge} \Delta t - p_t^{bat,discharge} / \eta^{discharge} \Delta t$$

with $\eta^{charge} \in [0, 1]$ and $\eta^{discharge} \in [0, 1]$.

Charge and discharge efficiencies II

- ▶ Multiplying p^{bat} by an average efficiency $\eta \in [0, 1]$ would be fine for charging, but discharging the battery would lead to a problem in the energy balance.

Example

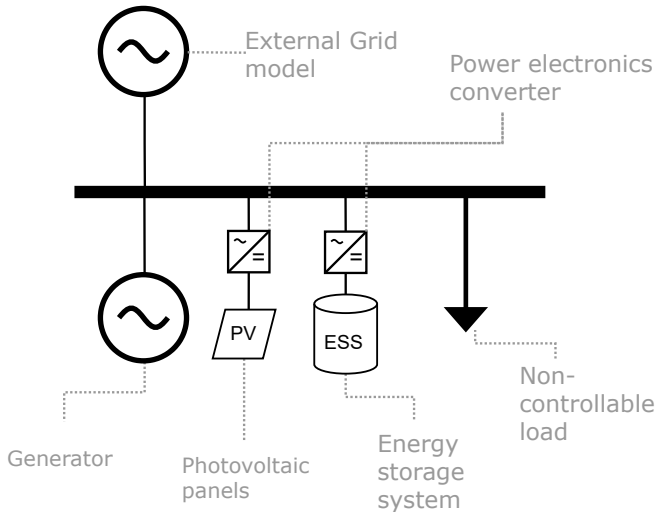
Assume $p^{bat} = -10kW$, although the battery generates 10 kW over Δt , it will be discharged by only $10kW\Delta t \times \eta$.

- ▶ The optimizer may lead to a solution with simultaneous charge and discharge. This can be avoided by adding constraints, such as $p^{bat,charge} \text{ times } p^{bat,discharge} = 0$, or introducing binary variables. The problem becomes either a non-linear program (NLP) or a mixed-integer linear program (MILP).
- ▶ Non-linear efficiencies.

Grid connection I

A grid connection allows importing p_t^{imp} or exporting p_t^{exp} power to the grid but also increases the system's inertia, offers the possibility to provide ancillary services, etc.

Grid connection II



Sets of devices I

In the general case, we can have an arbitrary number of devices of each type.

Example

Instead of one PV inverter, we now have a set \mathcal{D}^{PV} of PV inverters.

Straightforward adaptations:

- ▶ Sum over all these devices in the constraints and objective
- ▶ Express device-level constraints for all devices

Sets of devices II

Example

- ▶ Replace p^{PV} by $\sum_{d \in \mathcal{D}^{PV}} p_d^{PV}$ in the balance equation.
- ▶ $p_d^{PV} \leq \bar{p}_d^{PV} \quad \forall d \in \mathcal{D}^{PV}$.

A less obvious adaptation is related to the power-sharing among devices of the same type.

Open question: do you see other adaptations?

Power sharing between devices of the same type I

Devices of the same type may *differ* by some parameters such as efficiencies and costs. In that case, the optimizer will decide to use the devices by **merit order** (lowest cost first) unless some technical constraints lead to other solutions.

If devices are *perfectly equivalent*, then **multiple solutions** may be optimal. It is then up to the user to decide whether he wants to share the power equally or rather shut down a maximum of devices, or ...

Example

Share the curtailment between two equivalent PV installations (same peak power, orientation, and inverter).

Power sharing between devices of the same type II

A possibility is to solve the optimization problem first and then to fix all decisions except those under consideration. Then rerun a second problem by forcing the original objective to take the optimal value and changing the objective to reflect the sharing goal. For our example a new objective could be $\sum_{d \in \mathcal{D}^{PV}} p_d^{PV2}$.

Let

$$z^{(2),\star} = \min (2a) \quad \text{s.t.} (2b) - (2f)$$

and

$$p^{(2),\star} = \arg \min (2a) \quad \text{s.t.} (2b) - (2f).$$

Power sharing between devices of the same type III

$$\min \sum_{d \in \mathcal{D}^{PV}} p_d^{PV^2} \quad (3a)$$

$$\text{s.t.} \quad (2b) - (2f) \quad (3b)$$

$$\pi^{gen} p^{gen} + \epsilon p^{bat} + V(p^+ + p^-) = z_2^* \quad (3c)$$

$$(p^{gen} = p^{gen,(2),*}) \quad (3d)$$

$$(p^{bat} = p^{bat,(2),*}) \quad (3e)$$

Device availability

A device may be temporarily under maintenance, or, for an electric car, it may not be connected or not be available for recharging. This can be handled either:

- ▶ by removing explicitly the related variables from the problem formulation
- ▶ by forcing them to an adequate value (e.g. $\bar{P}_t^{charge} = 0$)

Long-term targets I

Let's assume we have a storage device, and for some reason, we want to have it charged in a given amount of time.

Example

It is 9 AM, a car is charged at 50%, and we want it to be charged at 80% at 5 PM.

Include a penalty in the objective function on the difference between the SoC $s_{t+\Delta t}^{bat}$ and the target SoC $s^{bat,\dagger}$:

$$W(s_{t+\Delta t}^{bat} - s^{bat,\dagger})$$

with W a coefficient wisely chosen.

Long-term targets II

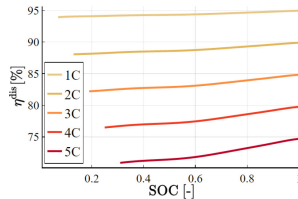
Question: what will be the effect of this term?

A more involved method is proposed in

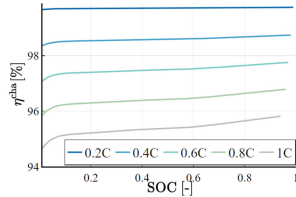
Dumas, J., Dakir, S., Liu, C., & Cornélusse, B. (2021). Coordination of operational planning and real-time optimization in microgrids. Electric Power Systems Research, 190, 106634.

where a value function is extracted from a planning problem (optimization over several hours) and integrated into the objective function to reach a trade-off between instantaneous revenues and long-term rewards.

Non-linear efficiency I



(a) Discharge efficiency



(b) Charge efficiency

Fig. 3. Discharging and charging efficiencies vs. the SOC and discharging and charging power.

From the reference cited on the next slide.

Non-linear efficiency II

It is possible to approximate non-linear efficiency and also maximal charge and discharge power using linear models. They depend on the SoC and the power.

In the article below, a convex envelope of the operating region is employed to obtain a linear model with a moderate increase in the number of variables and solution time.

Gonzalez-Castellanos, A. J., Pozo, D., & Bischi, A. (2019). Non-ideal linear operation model for li-ion batteries. IEEE Transactions on Power Systems, 35(1), 672-682.

Remark: converter inefficiency may also lead to a low charge or discharge efficiency at very low power.

Avoiding multiple solutions

Besides having sets of similar devices that may lead to several equivalent solutions, other less intuitive reasons may also lead to multiple solutions.

We thus need rules for lifting indeterminacies, and we can follow a similar approach to the one of presented for handling several equivalent devices.

Avoid oscillations and large set point changes

Let $P \leftarrow p_{t-\Delta t}$.

To avoid large variations of the control actions, we can for instance impose new constraints

$$|p_t - P| \leq \Delta P \Delta t$$

with ΔP , a parameter to limit variations expressed in power units per second.

Question: How do we model the absolute value in a linear program?