LIÈGE université
Sciences Appliquées

Optimal Sizing of Microgrids

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Overview

1. Optimization objective
2. Features and constraints
3. Use case
4. Representative days
5. Notation

Abstract

Properly sizing a microgrid is of paramount importance to reach an optimal operation during its entire lifetime. This presentation presents a (stochastic) mixed-integer linear optimization formulation of this sizing problem, emphasizing:

- Generation, converter, and battery storage system degradation
- Corresponding necessary reinvestments along the life of the project
- Connection costs to the public grid

Considering the evolution of energy and component prices, two scenarios are identified ("optimistic" and "pessimistic").

The resolution of this large optimization problem is facilitated by the use of representative days.

Optimization objective

What is this section for?

- Investing is always a tradeoff between costs, benefits, value, risk, etc.
- The value of money varies with time
- Investing requires a financing scheme (e.g. a constant annuity loan)
- Besides the technical constraints, it is thus key to determine
    - Which financial objective to optimize (e.g. minimize costs?)
    - Whether the investment can be funded
    - How to present results from a financial perspective

Net Present Value (NPV) I

- NPV is a metric to determine the **present value** of future cash flows.
- Assesses project profitability.
- Accounts for the **time value of money**.
- Considering annual cash flows and a microgrid lifetime of Y years, the NPV can be expressed as:

$$NPV(c,a) = \sum_{y=1}^{Y} \frac{-I_y(c) + R_y(c,a) - O_y(c,a)}{(1+d)^y} \tag{1}$$

Net Present Value (NPV) II

- The numerator terms depend on the microgrid sizing decisions c and control decisions a (e will come back to this later), representing annual costs.
- I stands for (re)investment costs R for revenues and O for overall operating costs.
- The present value of the cash flow is obtained by using a discount factor d applied over the microgrid lifetime Y.
- Positive NPV $\implies$ Project is expected to generate more value than its costs
  - Example: solar farm, revenues from electricity produced
- Negative NPV $\implies$ Project is expected to generate less value than its costs
  - Example: microgrid, primary goal is to feed the load

Net Present Value (NPV) III

  - NP is negative because we usually do not account for the value associated with electricity consumption!
  - even if negative NPV, try to find the least negative value
- The NPV becomes positive when the payback period is within the lifetime of the microgrid.
- A salvage value may be taken into account.

Capital Expenditure (CAPEX)

- Funds used to acquire, upgrade, and maintain physical assets.
- Examples: property, plants, buildings, technology, equipment.
- **Long-term** investments providing benefits over multiple years.
- Distinct from Operating Expenditure (OPEX).

Operating Expenditure (OPEX)

- Ongoing costs of running a business.
- **Day-to-day** expenses for regular operations.
- Examples:
  - Salaries
  - Rent
  - Utilities
  - Inventory costs
  - Marketing expenses

Maintenance

- Costs associated with keeping assets in good working condition.
- Includes:
  - Repairs
  - Regular servicing
  - Upkeep
- Can be CAPEX or OPEX, depending on whether it extends asset life.

Time Value of Money

- Money available now is worth more than the same amount in the future.
- Due to potential earning capacity.
- Factors:
  - Interest/returns over time.
  - Inflation eroding purchasing power.
- Fundamental principle in finance for informed investment decisions.

NPV Example: Solar Farm Project

- Project: Construction and operation of a solar farm.
- Initial Investment (CAPEX): €1,000,000
- Project lifespan: 5 years
- Discount rate: 8%

Cash Flow Breakdown I

- Year 0:
  - CAPEX: -€1,000,000 (Initial Investment)
  - Funding: €1,000,000 (Loan/Equity)
  - Net Cash Flow: €0
- Year 1:
  - Revenue: €300,000
  - OPEX: -€50,000 (Operational costs)
  - Maintenance: -€20,000 (Routine servicing)
  - Net Cash Flow: €230,000
- Year 2:
  - Revenue: €350,000

Cash Flow Breakdown II

- OPEX: -€55,000
- Maintenance: -€25,000
- Net Cash Flow: €270,000

- Year 3:
  - Revenue: €400,000
  - OPEX: -€60,000
  - Maintenance: -€30,000
  - Net Cash Flow: €310,000
- Year 4:
  - Revenue: €420,000
  - OPEX: -€62,000

Cash Flow Breakdown III

  - Maintenance: -€35,000
  - Net Cash Flow: €323,000
- Year 5:
  - Revenue: €450,000
  - OPEX: -€65,000
  - Maintenance: -€40,000
  - Residual Value: €100,000 (Sale of equipment)
  - Net Cash Flow: €445,000

NPV Calculation

- NPV = $\sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$
- Where:
  - CF_t = Cash flow at time t
  - r = Discount rate (8% or 0.08)
  - n = Project lifespan (5 years)
- NPV = $-1,000,000 + \frac{230{,}000}{(1.08)^1} + \frac{270{,}000}{(1.08)^2} + \frac{310{,}000}{(1.08)^3} + \frac{323{,}000}{(1.08)^4} + \frac{445{,}000}{(1.08)^5}$
- NPV $\approx$ €$231,000$

Sensitivity analysis on the discount rate

Years	Cashflow	Discount rate			
		4.00%	8.00%	12.00%	16.00%
0	-1.00×10^6	-1.00×10^6	-1.00×10^6	-1.00×10^6	-1.00×10^6
1	2.30×10^5	2.21×10^5	2.13×10^5	2.05×10^5	1.98×10^5
2	2.70×10^5	2.50×10^5	2.31×10^5	2.15×10^5	2.01×10^5
3	3.10×10^5	2.76×10^5	2.46×10^5	2.21×10^5	1.99×10^5
4	3.23×10^5	2.76×10^5	2.37×10^5	2.05×10^5	1.78×10^5
5	4.45×10^5	3.66×10^5	3.03×10^5	2.53×10^5	2.12×10^5
NPV	5.78×10^5	3.88×10^5	2.31×10^5	9.90×10^4	-1.22×10^4

Conclusion

- The NPV of the solar farm project is approximately €231,000.
- Since the NPV is positive, the project is considered financially viable at an 8% discount rate.
- This example demonstrates how CAPEX, OPEX, maintenance, and funding are factored into NPV calculations.

Features and constraints

Setup I

We are considering

- multi-year investment $y = 1, 2, \ldots, Y$ (initial investment at year 0)
- several scenarios $\omega \in \Omega$ with associated probabilities $P(\omega)$ to account for uncertainty
- representative days, so a number *ppy* of periods per year (same days for all scenarios) to decrease the computational burden

Setup II

Hence we are maximizing the **expected NPV**

$$\max E_{\omega\in\Omega}[NPV] = -I - \sum_{\omega\in\Omega}\left(\sum_{y=1}^{Y}\left(\frac{RI_{y,\omega} + O_{y,\omega} - R_{y,\omega}}{(1+d)^{y}}\right) - \frac{SV_{\omega}}{(1+d)^{Y}}\right)\cdot P(\omega) \qquad (2)$$

where I is the initial investment (not subject to uncertainty), and if scenario ω materializes

- $RI_{y,\omega}$ represents a reinvestment in year y
- $O_{y,\omega}$ represents the OPEX in year y
- $R_{y,\omega}$ represents the revenues in year y
- SV_{ω} represents the residual value of the assets at the final year Y.

Sizing problem I

In essence, the sizing problem can be seen as an operational planning problem extended over the lifetime of the project:

$$\begin{aligned} \max \quad & E_{\omega \in \Omega}[NPV] \\ \text{s.t.} \quad & \forall p \in \mathcal{P}, \forall y \in \mathcal{Y}, \forall \omega \in \Omega \\ & \quad \text{Planning constraints} \\ & \quad \text{Devices capacity updates and degradation} \\ & \quad \text{Definition of CAPEX and OPEX} \end{aligned}$$

Sizing problem II

We let device capacities vary (instead of fixed parameters) and account for their CAPEX in the NPV. We optimize NPV to account for the time value of money, and consider several years to scale OPEX w.r.t. CAPEX. A variation of the consumption over the years can also be included.

Sizing problem III

Example: PV installation sizing (no reinvestment)

Assuming the PV generation is maximum 10 kW at time t_1 for a PV installation of 20 KWp, we constrain the PV power produced at t_1 with

$$p_{t_1}^{PV} \leq \frac{10}{20} C^{PV} \tag{3}$$

with C^{PV} a new variable of the optimization problem, instead of $p_{t_1}^{PV} \leq 10$ in a planning problem. We duplicate constraint (3) for all the considered time steps.

C^{PV}, multiplied by the unitary cost of PV installation, enters in the NPV estimation.

The problem stays linear, but C^{PV} creates a link between all the time steps.

Device capacities I

Let us consider a device d to be sized (e.g., a generator, an inverter, or a battery).

The cost of a device per unit capacity usually decreases as its size increases. We model this through

- (non-overlapping) capacity intervals $[\underline{\Lambda_{d,c}}, \overline{\Lambda_{d,c}}]$, $c \in \Pi_d$
- defining a CAPEX per unit capacity $\pi_{d,c}^{CAPEX}$ parameter such that

$$\pi_{d,1}^{CAPEX} > \pi_{d,2}^{CAPEX} > \ldots > \pi_{d,|\Pi_d|}^{CAPEX}$$

- defining a binary variable $\beta_{d,c}$ to select one capacity interval.

Device capacities II

Then the capacity of device d in the interval c is $\lambda_{d,c}$:

$$\underline{\Lambda_{d,c}} \cdot \beta_{d,c} \leq \lambda_{d,c} \leq \overline{\Lambda_{d,c}} \cdot \beta_{d,c}, \ \forall c \in \Pi_d.$$

And the total capacity of device d is $\lambda_d = \sum_{c \in \Pi_d} \lambda_{d,c}$.

We can force the use of at most one interval with $\sum_{c \in \Pi_d} \beta_{d,c} \leq 1$.

Remark: if $|\Pi_d| = 1$, no need to define the variables $\beta_{d,c}$.

Initial investment

The initial investment cost sums up the microgrid device installation costs and the connection capacity to the public grid.

$$I = \sum_{d \in \mathcal{D}} \sum_{c \in \Pi_d} \lambda_{d,c} \cdot \pi_{d,c}^{\text{capex}} + \mu_{\text{con-cost}} \tag{4}$$

With $\mathcal{D} = \mathcal{G} \cup \mathcal{I} \cup \mathcal{B}$.

The public grid connection cost $\mu_{\text{con-cost}}$ is typically proportional to the power κ^{init} contracted with the system operator, but could also model more complex cost schemes.

Reinvestment, capacity update

We can apply a similar principle to compute reinvestment in devices, e.g. in device d in year y, $\lambda^{\mathrm{ri}}_{d,y}$, and thus to compute the total reinvestment cost in year y, RI_y.

We can also consider that some devices have a limited lifetime T_d and account for that in the total capacity at year y:

$$\lambda^{\mathrm{tot}}_{d,y} = \sum_{y'=y-T_d}^{y} \lambda^{\mathrm{ri}}_{d,y'} + (\lambda_d \text{ if } y < T_d)$$

We then use $\lambda^{\mathrm{tot}}_{d,y}$ as a bound in the constraints expressing the constraint on the power of devices in year y.

OPEX and revenues I

The costs and revenues related to the exchanges of energy with the grid during year y and for scenario ω are calculated as follows:

$$\mu_{y,\omega}^{\text{operation}} = \sum_{p=y\cdot ppy}^{(y+1)\cdot ppy} (i_p^{\text{grid}} \cdot \pi_{p,\omega}^{\text{import}}) w_p$$

$$R_{y,\omega} = \sum_{p=y\cdot ppy}^{(y+1)\cdot ppy} (e_p^{\text{grid}} \cdot \pi_{p,\omega}^{\text{export}}) w_p$$

where ppy is the number of periods per year[1], w_p the weight associated to period p, and i^{grid} and e^{grid} the energy imported from and exported to the grid.

OPEX and revenues II

The OPEX can also include

- yearly maintenance costs, e.g. as a percentage of the investment
- yearly public grid connection fees
- peak power costs.

[1] See section on representative days.

Energy balance

The **energy balance** constraint is modeled as follows:

$$
\begin{aligned}
e_p^{\text{grid}} - i_p^{\text{grid}} = & \sum_{g \in \mathcal{G}} \theta_{g,p} \Delta_t \\
& + \sum_{b \in \mathcal{B}} (\theta_{b,p}^{-} - \theta_{b,p}^{+}) \Delta_t - \zeta_p^{\text{cons}} \Delta_t \qquad \forall p \in \mathcal{P}
\end{aligned} \tag{5}
$$

where θ_g is the renewable power produced, θ_b^+ and θ_b^- the charging and discharging power of BESS b, and ζ^{cons} the overall consumption power.

The import and export are bounded by the contracted power:

- $e_p^{grid} \leq \kappa^{\text{init}} \Delta_t$
- $i_p^{grid} \leq \kappa^{\text{init}} \Delta_t$

RES generation rules I

The renewable energy generated must always be below the maximum that can be generated at a time p,

$$\theta_{g,p} \leq \theta_{g,p}^{\text{RES, max}}$$

which is the output per peak kW (per unit) at the considered location times the invested (and reinvested) capacity

$$\theta_{g,p}^{\text{RES, max}} = o_{g,y,p}^{\text{prod-pu}} \cdot \lambda_{g,y}^{\text{tot}}$$

The difference is the curtailed power

$$\theta_{g,p}^{\text{RES-curt}} = \theta_{g,p}^{\text{RES, max}} - \theta_{g,p}$$

RES generation rules II

- All this applies $\forall g \in \mathcal{G}, \forall p \in \mathcal{P}$.
- $o^{\text{prod-pu}}_{g,y,p}$ is the per unit power generated at period p for generator g invested in year y

Cost of curtailment

Curtailment leads to an **opportunity cost**, but it should not be explicitly modeled in the OPEX.

Hybrid inverter Constraints

Hybrid inverters connect batteries and PV panels, both on the DC side, to the AC bus of the microgrid. Therefore, the power generated by the PV together with the power discharged from the BESS must always be less than the usable capacity of the inverters $\lambda_{i,\omega}^{\text{us-cap}}$:

$$\sum_{g \in \mathcal{G}} \theta_{g,p} + \sum_{b \in \mathcal{B}} \theta_{b,p}^{-} \leq \sum_{i \in \mathcal{I}} \lambda_{i,y}^{\text{us-cap}} \qquad \forall p \in \mathcal{P} \tag{6}$$

The inverters usable-capacity must be at least half of the PV capacity, although in practice care should be taken so as to avoid excessive voltages and currents on the DC side, depending on the particular inverter selected and PV strings topology:

$$\sum_{i \in \mathcal{I}} \lambda_{i,y}^{\text{us-cap}} \geq \frac{\sum_{g \in \mathcal{G}} \lambda_g + \sum_{x=2}^{y} \lambda_{g,x}^{\text{ri}}}{2} \qquad \forall y \in \mathcal{Y} \tag{7}$$

Battery I

We assume the batteries can be charged or discharged at a 1C-rate[2] As the battery capacity fades with time and usage, the charging and discharging power decreases as well. This translates through the use of a new variable $\lambda_{b,p}^{\text{us-cap}}$ that represents the usable capacity of the BESS at period p, which is a function of its degradation level:

$$\theta_{b,p}^{+} \leq o_b^{+,\max} \cdot \lambda_{b,p}^{\text{us-cap}} \qquad \forall b \in \mathcal{B}, \forall p \in \mathcal{P} \tag{8}$$

$$\theta_{b,p}^{-} \leq o_b^{-,\max} \cdot \lambda_{b,p}^{\text{us-cap}} \qquad \forall b \in \mathcal{B}, \forall p \in \mathcal{P} \tag{9}$$

where $o_b^{+,\max}$ and $o_b^{-,\max}$ are the charge and discharge C-rate of the BESS.

Battery II

The evolution of the state of charge of the BESS $s_{b,p}$ is therefore governed by the following constraints:

$$s_{b,p} = s_{b,p-1} + (\theta^{+}_{b,p} \cdot \eta^{+}_{b} - \frac{\theta^{-}_{b,p}}{\eta^{-}_{b}})\Delta_t \tag{10}$$

$$s_{b,p} \leq \lambda^{\text{us-cap}}_{b,p} \tag{11}$$

$$\forall b \in \mathcal{B}, \forall p \in \mathcal{P}$$

where η^{+}_{b} and η^{-}_{b} are the charging and discharging efficiencies of the BESS.

Battery III

The initial state of charge is initialized arbitrarily:

$$s_{b,0} = \frac{\lambda_b}{2}$$

The degradation of the usable capacity of the BESS is supposed to be a linear function of the number of cycles. One complete charge/discharge cycle is assumed to occur per day.

Battery IV

When the maximum number of cycles $nc_{b,\max}$ is reached the BESS is removed from the system and can be replaced by a new one:

$$
\begin{aligned}
&\textbf{if } nc_{1,b,p} \leq nc_{b,\max} : \qquad \forall b \in \mathcal{B}, \forall p \in \mathcal{P} \\
&\quad \lambda_{b,p}^{\text{us-cap}} = \left(\frac{us_{b,\min} - 1}{nc_{b,\max}} \cdot nc_{1,b,p} + 1 \right) \lambda_b + \lambda_{b,p}^{\text{ri-us-cap}} && (12) \\
&\textbf{else } : \\
&\quad \lambda_{b,p}^{\text{us-cap}} = \lambda_{b,p}^{\text{ri-us-cap}} && (13)
\end{aligned}
$$

where $nc_{1,b,p}$ is the number of cycles at period p of the initial BESS investment b (at year 1), $us_{b,\min}$ the minimum usable capacity achieved when the number of cycles is maximum,

Battery V

and $\lambda^{\text{ri-us-cap}}_{b,p}$ the added usable capacity provided by the reinvestments. The latter is derived by adding up all the reinvested capacities from period 1 to p and computing their degradation using the same procedure as above:

$$\textbf{for } y \in \mathcal{Y}[2, y_p] : \qquad \forall b \in \mathcal{B}, \forall p \in \mathcal{P}$$

$$\textbf{if } nc_{b,y,p} \leq nc_{b,\max} :$$

$$\lambda^{\text{ri-us-cap}}_{b,p} \mathrel{+}= \left(\frac{us_{b,\min} - 1}{nc_{b,\max}} \cdot nc_{b,y,p} + 1\right) \cdot \lambda^{\text{ri}}_{b,y} \tag{14}$$

$$\textbf{else} :$$

$$\lambda^{\text{ri-us-cap}}_{b,p} \mathrel{+}= 0 \tag{15}$$

Battery VI

where y_p is the year of period p and $\lambda^{\text{ri}}_{b,y,\omega}$ the reinvestment BESS capacity at year y.

[2] They can be fully charged or discharged in 1 hour

Connection Cost

- Significant upfront investment.
- Varies with distance, voltage level, and grid infrastructure.
- Impacts project feasibility and profitability.
- Can include substation upgrades, line construction, and permitting.

Peak Power consumed or imported

- Cost of using the grid function of your actual peak injection or consumption.
- Usually smaller than grid connection contracted capacity.

Salvage Value Rule (Finite Duration)

- Residual value of assets at the end of the project's lifespan.
- Must consider component degradation and market value.
- Requires accurate estimation of asset lifespan and disposal costs.

Use case

Description I

We are sizing

- a PV installation (panels + inverter)
- a storage system (batteries + inverter)
- a generator
- a connection to the grid

We are considering an investment period of 20 years, one year replicated 20 times (no increase of costs, e.g., fuel costs, etc.)

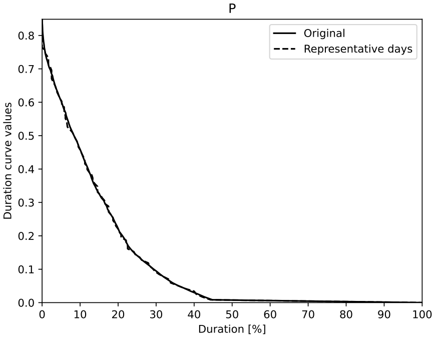
Data:

Description II

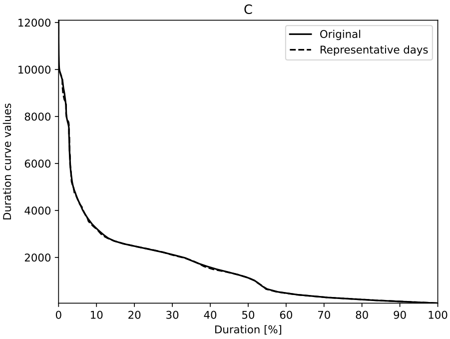
- One year (2021) of consumption data with 2 EVs, one heat pump, 4 people. Generated with `https://github.com/Diffels/ULG_Flex_Residential_Load`. The model estimates the needs of the heat pump from historical temperature records.
- One year of PV data from PVgis (2021) for a location in Huy, Belgium (`https://re.jrc.ec.europa.eu/pvg_tools/fr/`)
- Consumption statistics: min 51 W, average 1522 W, max 12097 W, energy 13.3 MWh.
- 12 representative days obtained with `https://github.com/bcornelusse/daysxtractor`

Description III

Production duration curve **for** 1 **Wp**



Load duration curve



Parameters of the base case I

```
# Time
RESOLUTION_IN_MINUTES = 10  # Time step duration [min]
# Number of operation time steps [/]
N_PERIODS_PER_DAY = 24 * 60 / RESOLUTION_IN_MINUTES
INVESTMENT_HORIZON = 20  # [Years] Investment horizon
DISCOUNT_RATE = 0.  # Discount rate [/] for computation of NPV

# Storage capacity
STORAGE_UNIT_CAPACITY = 90  # [Ah]
STORAGE_UNIT_VOLTAGE = 12  # [V]
```

Parameters of the base case II

```
STORAGE_UNIT_PRICE = 400  # [EUR]
# [/], means max current is 2 * STORAGE_UNIT_CAPACITY [A]
STORAGE_MAX_C_RATE = 2
INITIAL_SOC = 0  # Currently cot used [Wh]
CHARGE_EFFICIENCY = 0.95  # [/]
DISCHARGE_EFFICIENCY = 0.95  # [/]
BATERRY_USAGE_FEE = 0.01  # [EUR/kWh]

# Inverter capacity
INVERTER_UNIT_CAPACITY = 1000  # [VA]
INVERTER_UNIT_PRICE = 250  # [EUR]
```

Parameters of the base case III

```
# PV
PV_CAPACITY_PRICE = 0.4  # [EUR/Wp]
MAX_PV_CAPACITY = 10000  # [Wp]

# Grid connection cost
# Key: [A], value: [EUR]
GRID_CAPACITY_PRICE = {16: 1000, 32: 2500, 64: 6000}
GRID_VOLTAGE = 230  # [V]
GRID_IMPORT_PRICE = 0.28  # [EUR/kWh]
GRID_EXPORT_PRICE = 0.0  # [EUR/kWh]
```

Parameters of the base case IV

```
# Genset
GENSET_CAPACITY_PRICE = {3100: 450, 5500: 750,
                         8000: 2880}  # Key: [VA], value: [EUR]
FUEL_PRICE_COEFF = 0.19  # [EUR/kWh]
```

Results: base case

Parameter	Value	Unit
NPV (20 years, discount factor 0.0)	-43140.1	EUR
CAPEX	13200	EUR
Yearly OPEX	1471.26	EUR
Grid connection capacity	0	A
Storage capacity	14.04	kWh
Storage inverter capacity	6	kVA
PV capacity	10	kWp
PV inverter capacity	7	kVA
Genset capacity	5.5	kVA

Results: exports valorized at 10 cEUR

Parameter	Value	Unit
NPV (20 years, discount factor 0.0)	-36606.9	EUR
CAPEX	11950	EUR
Yearly OPEX	1199.37	EUR
Grid connection capacity	16	A
Storage capacity	8.64	kWh
Storage inverter capacity	4	kVA
PV capacity	10	kWp
PV inverter capacity	8	kVA
Genset capacity	5.5	kVA

Results: exports valorized at 20 cEUR

Parameter	Value	Unit
NPV (20 years, discount factor 0.0)	880.39	EUR
CAPEX	16080	EUR
Yearly OPEX	-1010.87	EUR
Grid connection capacity	64	A
Storage capacity	0	kWh
Storage inverter capacity	0	kVA
PV capacity	10	kWp
PV inverter capacity	8	kVA
Genset capacity	16.6	kVA

Base case with discount factor (0.05)

Parameter	Value	Unit
NPV (20 years, discount factor 0.05)	-32388.3	EUR
CAPEX	11900	EUR
Yearly OPEX	1524.27	EUR
Grid connection capacity	0	A
Storage capacity	11.88	kWh
Storage inverter capacity	5	kVA
PV capacity	10	kWp
PV inverter capacity	6	kVA
Genset capacity	5.5	kVA

Fuel price increase (0.1 EUR)

Parameter	Value	Unit
NPV (20 years, discount factor 0.0)	-55384.3	EUR
CAPEX	13700	EUR
Yearly OPEX	2084.22	EUR
Grid connection capacity	16	A
Storage capacity	14.04	kWh
Storage inverter capacity	7	kVA
PV capacity	10	kWp
PV inverter capacity	7	kVA
Genset capacity	0	kVA

Fuel price increase (0.1 EUR) and discount factor (0.05)

Parameter	Value	Unit
NPV (20 years, discount factor 0.05)	-40786.2	EUR
CAPEX	13050	EUR
Yearly OPEX	2119.64	EUR
Grid connection capacity	16	A
Storage capacity	12.96	kWh
Storage inverter capacity	6	kVA
PV capacity	10	kWp
PV inverter capacity	7	kVA
Genset capacity	0	kVA

Representative days

‘

Problem statement I

Let's try to give a first definition:

Representative days selection: definition 1

Out of N_{total} days of data (with a time resolution of M minutes), identify a subset of N_{repr} days that represent the original data set as well as possible.

This definition is inaccurate because we do not know what a good representation of the original dataset means. Why do we want representative days?

- To solve a problem that takes a long time to solve over N_{total} days

Problem statement II

- Overall (including the selection of the days), the time to solve the problem on the N_{repr} days should be much smaller than the time to solve it on the N_{total} days.

Representative days selection: definition 2

Let $\mathcal{P}$ be a problem to be solved over N_{total} days of data (with a time resolution of M minutes). Identify a subset of N_{repr} (and their weights) days such that the solution of $\mathcal{P}$ over the N_{repr} days has **the same solution** than the one of the original problem.

Usually, the solution to the original problem is not known, though.

Optimal selection of representative days I

Reference [2] proposes to solve the optimization problem

$$\min_{u_d, w_d} \quad \sum_{c \in \mathcal{C}} \sum_{b \in \mathcal{B}} \left| L_{c,b} - \sum_{d \subset \mathcal{D}} \frac{w_d}{N_{\text{total}}} \cdot A_{c,b,d} \right| \tag{16a}$$

$$\text{s.t.} \quad \sum_{d \in \mathcal{D}} u_d = N_{\text{repr}} \,, \tag{16b}$$

$$w_d \leq u_d \cdot N_{\text{total}} \,, \quad \forall d \in \mathcal{D}, \tag{16c}$$

$$\sum_{d \in \mathcal{D}} w_d = N_{\text{total}} \,, \tag{16d}$$

$$u_d \in \{0, 1\}, \quad w_d \in \mathbb{R}_0^+, \quad \forall d \in \mathcal{D}. \tag{16e}$$

Optimal selection of representative days II

where

- $u_d = 1$ if day d is selected
- w_d is the weight associated to day d (0 if not selected)
- $\mathcal{C}$ is the set of series considered (e.g., load and PV)
- $\mathcal{B}$ is the set of bins used: the duration curves of the series are subdivided into $|\mathcal{B}|$ bins.
- the objective minimizes the error between the number of points in the bins in the original data ($L_{c,b}$) and the selected days $A_{c,b,d}$ (weighted by the w_d variables).

Optimal selection of representative days III

How can we ensure that the problem $\mathcal{P}$ solved over the N_{repr} days gives the optimal solution?

- We cannot really
- We must select enough days (look at the sampling error)
- We can estimate costs by simulation on the complete dataset
- We can make a study on small problems [1].

Decoupling of days

Since

- the days selected are most of the time not contiguous
- the forecasts of load and generation are good for several hours to one day

it is common to decouple the days by imposing the equality of the state of the system at the beginning and end of each day day.

The state of charge of storage devices essentially represents the state:

$$s_d^{init} = s_d^{end}, \forall d \in days.$$

Bibliography I

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[2] Kris Poncelet et al. "Selecting representative days for capturing the implications of integrating intermittent renewables in generation expansion planning problems". In: *IEEE Transactions on Power Systems* 32.3 (2016), pp. 1936–1948.

Notation

Notation: sets and indices

- b Battery index
- c Capex index
- d Device index
- g Generator index
- i Inverter index
- p Period index
- ν Voltage level index
- ω Scenario index
- y Year index

- $\mathcal{B}$ Set of battery energy storage systems
- Π Set of device capex
- $\mathcal{D}$ Set of devices
- $\mathcal{G}$ Set of generators
- $\mathcal{I}$ Set of inverters
- $\mathcal{P}$ Set of periods
- $\mathcal{Y}$ Set of years (investment horizon)
- $\mathcal{V}$ Set of voltage levels
- Ω Set of scenarios

Notation: parameters I

- d Discount factor (%)
- Y Investment horizon (year)
- $\pi_{d,c}^{\text{capex}}$ Capex index c of device d (€/kW - kWh)
- $\pi_{d,c}^{\text{maint}}$ maintenance index c of device d (€/kW - kWh)
- π_{p}^{export} Export electricity price (€/kWh)
- π_{p}^{import} Import electricity price (€/kWh)
- $\underline{\lambda_{d,c}}, \overline{\lambda_{d,c}}$ Lower and upper capacity bound of the capex index c for device d
- w_p representative day weight of period p

Notation: parameters II

- $o_g^{\text{prod-pu}}$ Per unit power generated of RES generator g (kW)
- $o^{+,\text{max}}$, $o^{-,\text{max}}$ Charge and discharge C-rate of BESS (-)
- ζ_p^{cons} Overall consumption power (kW)
- η_b^+, η_b^- BESS charging and discharging efficiencies (-)
- $nc_{b,\text{max}}$ BESS Maximum number of cycle
- $nc_{b,y,p}$ Number of cycle at period p of BESS b installed at year y
- $us_{b,\text{min}}$ Minimum usable capacity
- κ^{init} Actual grid capacity contracted with the distribution system operator
- o^{min}, o^{max} Minimum stable generation and maximum output of diesel generator g

Notation: variables I

- I Initial investment cost (€)
- O Annual total operational costs (€)
- R Annual total revenues (€)
- RI Reinvestment costs (€)
- NPV Net present value (€)
- λ Device capacity (kW - kWh)
- i^{grid} Energy import from the grid (kWh)
- e^{grid} Energy export to the grid (kWh)
- θ Power generated / exchanged (kW)

Notation: variables II

- $\theta_b^+ ; \theta_b^-$ Battery charging and discharging power (kW)
- $\lambda_{b,p}^{\text{us-cap}}$ BESS usable capacity (kW)
- $s_{b,p}$ BESS state of charge (kWh)
- $\lambda_{b,p}^{\text{ri-us-cap}}$ Usable capacity provided by BESS reinvestments
- $\lambda_{b,y}^{\text{ri}}$ BESS reinvestment at year y
- $\overline{p}_y$ Peak import at year y
- μ_y^{op} Operating cost related to the energy purchase (€)
- μ_y^{peak} Peak penalty cost (€)
- $\mu_{\text{con-cost}}$ Total connection cost (€)

Notation: variables III

- $\mu_{\text{con-fee}}$ Total connection fee (€/y)
- $\mu^{\text{fix}}_{\text{con-cost}}, \mu^{\text{prop}}_{\text{con-cost}}$ Fixed, proportional connection cost (€)
- $\mu^{\text{fix}}_{\text{con-fee}}, \mu^{\text{prop}}_{\text{con-fee}}$ Fixed, proportional connection fee (€/y)
- β, k, δ Binary variables