

The transmission line

ELEC0447 - Analysis of electric power and energy systems

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Overview

1. Introduction
2. Distributed model
3. Surge impedance loading
4. Lumped transmission line model
5. Line rating

Introduction

The background of the slide features a minimalist design with teal-colored geometric shapes. Two large teal triangles point upwards from the bottom corners, meeting at a central point. Below this meeting point, a smaller, darker teal triangle points downwards. The remaining space is white, providing a clean backdrop for the text.

What will we learn today?

- ▶ The transmission line
- ▶ An introduction to power flow analysis

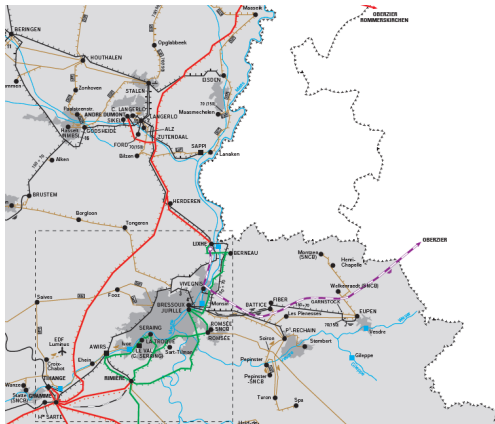
You will be able to do exercises 4.3, 4.4, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, Lab4 (power-flow in python), 5.1, 5.2, 5.5, 5.6 from the Ned Mohan's book.

Introduction video link

Definition

- ▶ An (overhead) transmission line is a set of 3 bundles of conductors corresponding to the three phases of the system.
- ▶ Commonly used voltages range from 70 kV to 380 kV in Belgium (more where distances are larger).
- ▶ Minimum distances between conductors depend on the voltage level, and thus electrical properties also depend on the voltage level.
- ▶ Underground cables are more and more used. They can be modeled in a similar way as overhead transmission lines, but they have different properties.

A part of ELIA's network



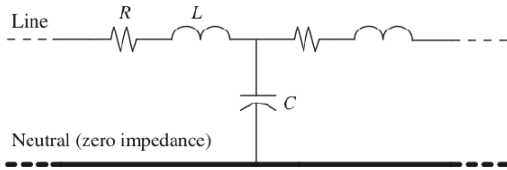
LIGNES AERIENNES		BOVENGRONDSE LIJNEN	
Tension d'exploitation 380kV 220kV 150kV 110kV 70kV		Uitvoeringspanning 380kV 220kV 150kV 110kV 70kV	
Nombre de terres previs 1 1 2 1 2 2 > 2		Aantal draadstellen voorsien 1 1 2 1 2 2 > 2	
(voir schéma de référence dans le tableau des composants)		(voir referentiepunten in de samenstellingstabel)	
en construction ou en projet 2 ^{ème} terre en construction ou en projet		in aanbouw of in ontwerp 2 ^{de} draadstel in aanbouw of in ontwerp	
lignes à 2 terres de tensions différentes tension d'exploitation inférieure à la tension de construction ligne appartenant à un tiers (T)		lijn met 2 draadstellen van verschillende spanningen uitvoeringspanning lager dan de constructiespanning lijn eigendom van een derde (T)	
Tableau des compositions des lignes à plus de 2 terres:		Samenstellingstabel van de lijnen met meer dan 2 draadstellen:	
1 2 x 150 + 1 x 70 (4 x 150)	7 2 x 300 (4 x 300)	13 1 x 70 (4 x 150)	
2 4 x 150	8 2 x 75	14 1 x 220 + 2 x 75	
3 1 x 150 + 2 x 70 (4 x 150)	9 2 x 110 (4 x 150)	15 2 x 150 (4 x 150)	
4 1 x 150 + 2 x 70 (4 x 150)	10 1 x 150 + 1 x 70 (4 x 150) + 1 x 75	16 2 x 150 + 1 x 70 (4 x 150)	
5 2 x 150 + 2 x 70 (4 x 150)	11 4 x 75	17 4 x 250	
6 2 x 150 (2 x 300 + 2 x 150)	12 2 x 150	18 1 x 300 + 2 x 150 (2 x 300 + 2 x 150)	

CABLES SOUTERRAINS		ONDERGRONDSE KABELS	
380kV 220kV 150kV 110kV 70kV HVDC		380kV 220kV 150kV 110kV 70kV HVDC	
en construction ou en projet		in aanbouw of in ontwerp	
câbles parallèles câble appartenant à un tiers (T)		parallelle kabels kabel eigendom van een derde (T)	

Source: <https://www.elia.be/fr/infrastructure-et-projets/nos-infrastructures>

Transmission line parameters

A *chunk* (a tiny piece) of transmission line can be represented as:



with R , L and C expressed **per unit of length**.

where

- ▶ R represents the series resistance, as small as possible to minimize RI^2 (influence of the frequency and skin effect)
- ▶ the series inductance L models the magnetic coupling between phases
- ▶ the shunt capacitance C models the capacitive coupling between phases
- ▶ a shunt conductance G can be added to model e.g. the leakage current through insulators

Approximate Overhead Transmission Line Parameters

For bundled conductors at 60 Hz.

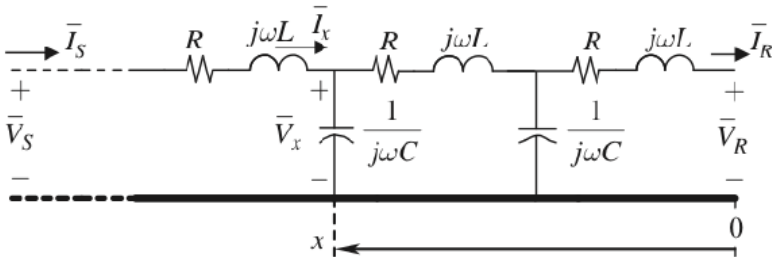
Nominal Voltage	$R(\Omega/\text{km})$	$\omega L(\Omega/\text{km})$	$\omega C(\mu\text{F}/\text{km})$
230 kV	0.06	0.5	3.4
345 kV	0.04	0.38	4.6
500 kV	0.03	0.33	5.3
765 kV	0.01	0.34	5.0

Distributed model

The background of the slide features a minimalist design with teal-colored geometric shapes. Two large teal triangles point upwards from the bottom, meeting at a central point. A smaller, darker teal triangle is positioned at the very bottom center, creating a layered effect.

Distributed parameter representation I

We consider that we are in sinusoidal steady state. On a per-phase basis, the line can be represented as many chunks connected to each other:



Distributed parameter representation II

How do voltage and current evolve as a function of the position on the line?

- ▶ As R is small, let's assume R is considered as *lumped* (a discrete resistive element on one side of the line)
- ▶ $\frac{d\bar{V}(x)}{dx} = j\omega L\bar{I}(x)$
- ▶ $\frac{d\bar{I}(x)}{dx} = j\omega C\bar{V}(x)$

Hence

$$\frac{d^2\bar{V}(x)}{dx^2} + \beta^2\bar{V}(x) = 0$$

with $\beta = \omega\sqrt{LC}$ the *propagation constant*

Solution of the ODE I

The previous equation has a solution of the type

$$\bar{V}(x) = \bar{V}_1 e^{\beta j x} + \bar{V}_2 e^{-\beta j x}.$$

By derivation, the current is

$$\bar{I}(x) = (\bar{V}_1 e^{\beta j x} - \bar{V}_2 e^{-\beta j x})/Z_c.$$

With the **surge impedance**

$$Z_c = \sqrt{\frac{L}{C}}.$$

Solution of the ODE II

The boundary conditions at $x = 0$,

$$\bar{V}(0) = \bar{V}_R = V_R \angle 0,$$

and

$$\bar{I}(0) = \bar{I}_R$$

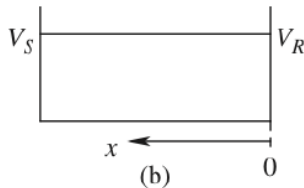
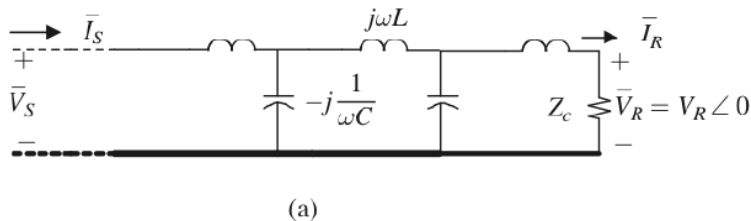
allow to determine constants \bar{V}_1 and \bar{V}_2 , and finally

$$\bar{V}(x) = \bar{V}_R \cos(\beta x) + jZ_c \bar{I}_R \sin(\beta x).$$

Surge impedance loading

Closing the line on the surge impedance Z_c

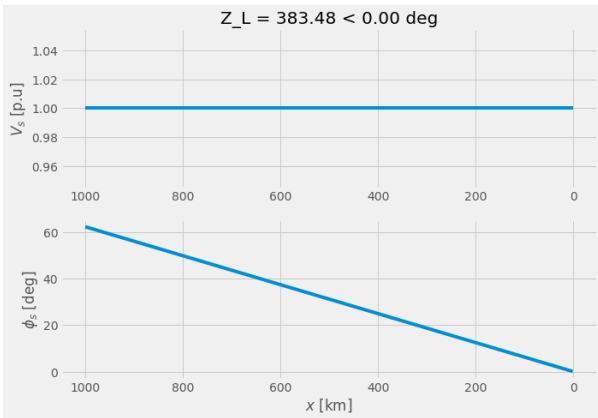
If the line is assumed lossless and we close it with Z_c , assuming $\bar{V}_R = V_R \angle 0$:



then the voltage *magnitude is constant* over the line: $\bar{V}(x) = V_R e^{j\beta x}$, and only the *angle increases with x* . Similar conclusion for $\bar{I}(x)$.

- Why? The reactive power consumed by the line is the same as the reactive power produced, everywhere.

Illustration in Python



SIL, 230 kV line params

See the [Python notebook](#).

Surge impedance loading

Z_c depends on the line characteristics/geometry and is, hence, mainly a function of the voltage level (distances between conductors, etc.).

The surge impedance loading (SIL) is the power drawn by the load Z_c , which depends on the voltage level V_{LL}

$$SIL = \frac{V_{LL}^2}{Z_c}$$

Example: for 500 kV, $SIL \approx 1020$ MW.

Line loadability

The SIL gives an idea of the loadability of a line depending on its length:

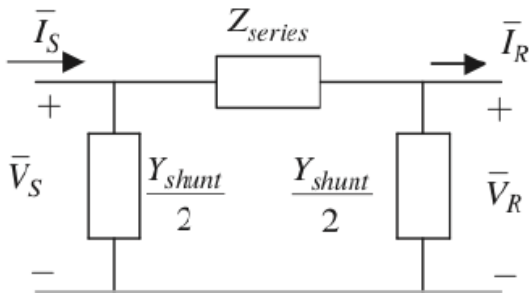
- ▶ short line, $l < 100$ km
 - ▶ load limit = $3 \times SIL$
 - ▶ thermal limit (See Section 5 on Line rating)
- ▶ Medium length line, $100 \text{ km} < l < 300 \text{ km}$
 - ▶ load limit = 1.5 to $3 \times SIL$
 - ▶ voltage drop $< 5\%$
- ▶ Long line, $l > 300 \text{ km}$
 - ▶ load limit $\approx 1 \times SIL$
 - ▶ for system stability, the angle difference between line ends should stay $< 40^\circ$, see lecture on Transient stability.

Lumped transmission line model

The background of the slide features a white upper half and a teal lower half. The teal section is composed of two large triangles meeting at a point at the bottom center, with a smaller, darker teal triangle positioned directly beneath that point.

The π model

If l is relatively small (< 300 km), we can *approximate* the line with **lumped** parameters,



with, by manipulation of the previous equations and assuming βl small,

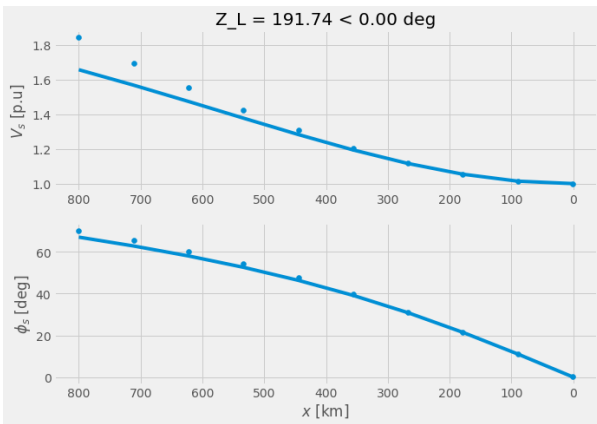
- ▶ $Z_{series} = Rl + j\omega Ll$

- ▶ $\frac{Y_{shunt}}{2} = j\frac{\omega Cl}{2}$

Remember that R , L and C are per km values.

This π model is symmetrical by design.

Illustration in Python



SIL, 230 kV line params

Dots are obtained with the π model, while plain lines are from the distributed model. The approximation error grows for $l > 300$ km.

See this [Python notebook](#).

Line rating

Static line rating I

The **Static Line Rating** is the maximum continuous current a transmission line can carry under a specific set of predefined, fixed environmental conditions.

This rating is primarily constrained by **thermal limits** to ensure the safe and reliable operation of the line.

► Conductor Thermal Limit:

- The conductor's electrical resistance generates heat (RI^2).
- This heat must be balanced by cooling from the environment.
- An excessive temperature can cause:
 1. **Increased Sag:** Conductor expansion due to heat causes the line to sag, potentially violating minimum clearance requirements to the ground, buildings, or other infrastructure.

Static line rating II

2. **Material Damage:** Prolonged high temperatures can anneal the conductor, reducing its tensile strength and lifespan.

► **Environmental Conditions:**

- The "static" nature of the rating comes from assuming a fixed set of weather parameters.
- **Ambient Air Temperature:** A baseline temperature (e.g., 40°C) is used. Higher temperatures reduce the cooling capability of the air.
- **Wind Speed:** A low, static wind speed (e.g., 2 ft/s at a 45° angle) is assumed to provide minimal convective cooling. Higher wind speeds would allow for more current.
- **Solar Radiation:** A fixed value for solar heat gain is included, assuming a specific level of direct sunlight on the conductor.

Static line rating III

Static line ratings are a conservative and fundamental safety measure. They represent the worst-case continuous scenario to prevent physical damage and clearance violations under predictable conditions.

Dynamic line rating

[Dynamic line rating video link](#)