

A photograph of a large array of solar panels installed on a grassy hillside. The panels are dark blue with white grid lines, arranged in rows. In the background, there are green trees and a blue sky with white clouds. The image is partially obscured by a white diagonal shape on the right side.

Frequency control

ELEC0447 - Analysis of electric
power and energy systems

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Overview

1. Why do we need to control power systems?
2. The link between frequency and power balance
3. Primary frequency control
4. Secondary frequency control (aka AGC)
5. Economic dispatch and optimal power flow (brief)
6. Assessing the security/reliability of an electric power system

This lecture expands on

- ▶ Lecture notes of Pr. Thierry Van Cutsem
- ▶ Chapter 12 from the Ned Mohan's book (summarized by Pr. Louis Wehenkel).
- ▶ Mohan, Ned. Electric power systems: a first course. John Wiley & Sons, 2012.

Why do we need to control power systems?

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Why to control power systems?

- ▶ Technical requirements: power system devices are designed so as to operate within well-defined "tolerance regions"
 - ▶ around nominal values of voltage V_n : 1 ± 0.1 pu in Europe
 - ▶ around nominal value of frequency f_n : 50 ± 0.2 Hz in Europe (in steady state)
 - ▶ within the P - Q capabilities of devices
 - ▶ under the current limits of lines and transformers
- ▶ Large/persistent deviations from nominal values could lead to
 - ▶ damages and safety problems (e.g. high voltage)
 - ▶ cascading phenomena
 - ▶ service interruptions

Exogeneous threats

- ▶ Sudden disturbances, such as line or generator tripping
- ▶ Fast variations of the *net load* (cf. Duck curve)
 - ▶ the net load refers to the load "seen" by the transmission system, i.e. the load minus the non-controllable dispersed generation
- ▶ weather conditions, such as storms, which can impact the generation of renewable energy sources (RES) (e.g. wind turbines' cut-out speed)

Impact of a storm on wind generation

- ▶ On November 2, 2023, a storm hit Belgium, causing a significant drop in wind generation.
- ▶ Wind generation dropped from 4.5 GW to 0.5 GW in less than 30 minutes.
- ▶ This sudden drop required immediate action to maintain the power balance and frequency stability.



From <https://www.elia.be/en/grid-data/power-generation/wind-power-generation>

Control resources

Several control resources are available to maintain the system within acceptable limits, and to restore them to nominal values after a disturbance.

- ▶ Adjust synchronous generators' field current
- ▶ Adjust synchronous generators' mechanical power
- ▶ Change transformer taps
- ▶ Change shunt compensation
- ▶ Act on topology: switch lines and transformers in/out of service
- ▶ Fast start-up generator units
- ▶ In extremis load curtailment
- ▶ Control renewable generation (e.g. PV curtailment)
- ▶ Use batteries and other energy storage systems

How to use these control resources?

- ▶ But which control resources to use, and when?
- ▶ What will have an effect on frequency and voltage?
- ▶ How to coordinate them?

Focus of the lecture

In this lecture we will focus on the frequency control, and we will see how it is related to the power balance in the system.

Later, we will address voltage control and also introduce other problems related to transient stability.

The link between frequency and power balance

The background of the slide features a white upper section and a teal lower section. The teal section is composed of two large triangles that meet at a point at the bottom center, creating a V-shape. The top edges of these triangles are diagonal, sloping upwards towards the left and right corners of the slide.

Frequency and power

The frequency of the grid is tightly linked to the **active** power balance in the system.

- ▶ Electrical energy cannot be stored (in large quantities)
- ▶ it is produced when it is requested
- ▶ in the very first instants after a disturbance, the missing (resp. excess) energy is taken from (resp. stored into) the rotating masses of the synchronous machines
- ▶ this causes a variation of their speed of rotation, and hence of frequency
- ▶ this is sensed by the speed governors (en français: régulateurs de vitesse), which adjust the steam/water/gas flow in the turbines to correct the speed deviation
- ▶ Other mechanisms are used to restore the power balance and the frequency to nominal values

Frequency correction is performed in three steps

Three reserve mechanisms are used to correct the frequency deviation and restore the power balance:

- ▶ primary reserve (in a few seconds): local/decentralized, speed control in power plants
- ▶ secondary reserve (in a few minutes): distributed: corrections of generator power setpoints to restore frequency and power exchanges to nominal values
- ▶ tertiary reserve (in a few tens of minutes): centralized: re-optimization of generation and exchange schedules, while restoring secondary control reserves

| Reserve Level | Current Name (ENTSO-E) |
|-------------------|--|
| Primary Reserve | Frequency Containment Reserve (FCR) |
| Secondary Reserve | Automatic Frequency Restoration Reserve (aFRR) |
| Tertiary Reserve | Manual Frequency Restoration Reserve (mFRR) |

System frequency evolution: theory, example, and intuition

Before understanding in more details these three levels, we will try to get intuition based on a simplified physical model of the system.

Let us consider a power system with loads and, for now, only synchronous generators.

Let

- ▶ $p_e(t)$ be the total power absorbed by the loads (incl. losses in the network and conversion losses in the generators)
 - ▶ this is thus equal to the electrical power that generators must output (Kirchhoff laws)
- ▶ $p_m(t)$ be the mechanical power input to generators

Except for the losses mentioned above, we neglect the network.

How does the frequency of the system $f(t)$ (and thus of the machines) evolve?

The swing equation I

We consider a synchronous power system that should operate at the nominal frequency f_N . To simplify, we will group all synchronous generators into a single equivalent generator with an inertia constant J .

Newton's law

$$J\alpha = T_m - T_e,$$

relates the angular acceleration α of the rotor to the difference between the mechanical torque T_m and the electrical torque T_e .

The swing equation II

Since $p = T\omega$ and $\omega = 2\pi f$, we can derive a more convenient expression linking the frequency and the active power balance in the system:

$$J \frac{df}{dt} = \frac{p_m - p_e}{4\pi^2 f}$$

We must correct this expression to include the damping effect in generators, assuming D_g is an equivalent total damping coefficient of the system:

$$J \frac{df}{dt} = \frac{p_m - p_e}{4\pi^2 f} - D_g(f - f_N)$$

What happens if ...

What happens in case there is a power imbalance?

E.g. a sudden increase of load?

What is the effect of inertia?

What is the effect of damping?

Note: Inertia constant of a generator

The inertia constant J_i of generator i can be expressed as a function of its rated power $P_{\max,i}$ and its dimensionless inertia constant H_i . The relationship is given by:

$$J_i = \frac{2P_{\max,i}H_i}{4\pi^2f_N^2}$$

where f_N is the nominal grid frequency, typically 50 or 60 Hz. This formula converts the per-unit inertia constant H_i , which represents the stored kinetic energy at rated power in seconds, into the physical moment of inertia J_i in SI units (kg m^2), assuming a synchronous speed of $2\pi f_N$ radians per second.

In the Swing equation, we have considered that $J = \sum_i J_i$.

To answer these questions, let's look at some simulations

[Link to the simulations](#)

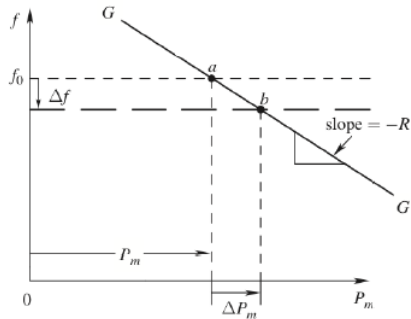
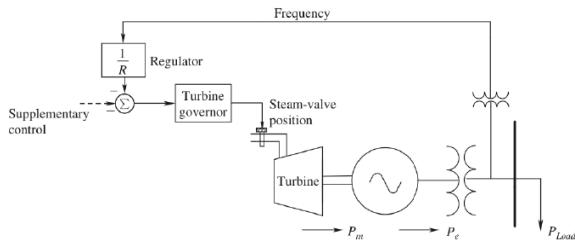
The code is organized in 3 main parts:

1. Plotting functions
2. A class encoding the power system data and the set of differential equations governing the system
3. Simulations for several cases

Let's add generator reaction to frequency change

- ▶ So far, only the generator's inertia and damping were considered.
- ▶ This is not enough to stabilize the frequency.
- ▶ Hence, generators react to frequency changes by adjusting their mechanical power output.
- ▶ The first reaction is the primary control. Let's understand how a power plant is modeled and controlled.

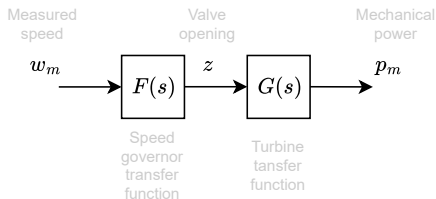
Speed governor of a synchronous generator



- ▶ Measurement of rotor speed (or stator frequency as a proxy)
- ▶ If frequency (speed) is a bit below f_0 , the governor opens a bit more the valve to increase the mechanical power P_m
- ▶ In steady state, speed and mechanical power (or equivalently, frequency and electric power) are related by a linear relationship (see diagram on the right)

Model of a power plant (mechanical part)

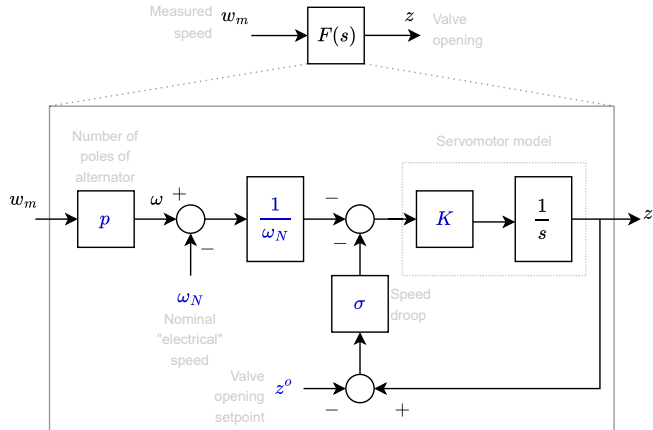
The mechanical part of a power plant is modeled by the following block diagram:



- ▶ ω_m is the rotation speed (which is not necessarily equal to the grid angular speed ω (in rad/s) depending on the number of poles of the alternator)
- ▶ z is the fraction of opening of the turbine control valves ($0 \leq z \leq 1$), to inject e.g. more steam in the turbine.

We will not delve deeper into the turbine model, but focus on the speed governor.

Speed governor transfer function



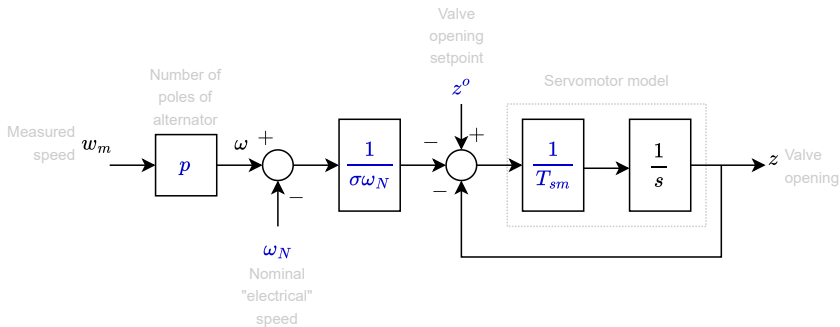
Parameters are in blue.

Speed governor transfer function (...)

- ▶ The rotation speed is scaled and compared to the nominal electrical speed to yield a speed error signal
- ▶ z^o sets the desired power production setpoint of the generator, if the frequency were at nominal value
- ▶ In the feedback branch, z is compared to z^o and scaled by the droop coefficient σ to yield a second error signal
- ▶ The error signals are then fed to the servomotor to adjust z

Equivalent block diagram

We can redraw the diagram in an equivalent manner, with $T_{sm} = 1/(K\sigma)$ the time constant of the servomotor, as



Equivalent block diagram (...)

The transfer function of the speed governor is then:

$$z = F(s) = \frac{1}{1 + sT_{sm}} \left(z^o - \frac{\omega - \omega_N}{\sigma\omega_N} \right)$$

Exercise

Obtain this transfer function from the previous diagram, and show it is equivalent to the first diagram.

Steady-state characteristic of a turbine-governor set

In steady state:

► $p_m = G(0)z$ (remember $G(s)$ models the turbine)

► $z = F(0) = z^0 - \frac{\omega - \omega_N}{\sigma \omega_N}$

For $z = 1$, $p_m = P_N$, the nominal power of turbine (MW)

Hence, $P_N = G(0)1$ and

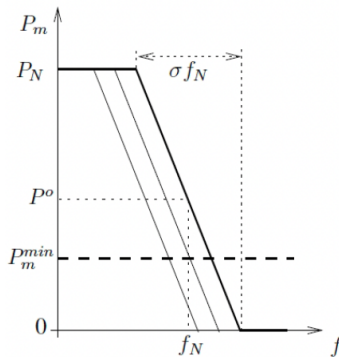
$$p_m = P_N z$$

Thus, $p_m = P_N z^0 - \frac{P_N}{\sigma} \frac{\omega - \omega_N}{\omega_N}$ or

$$p_m = P_o - \frac{P_N}{\sigma} \frac{f - f_N}{f_N}$$

where P_o is the power setpoint of the generator.

Of course, we must account for the limits of the generator.



Time domain turbine-governor model

Back to the time domain, we can write the following differential equation for the mechanical power output of the generator:

$$T_{sm} \frac{dp_m}{dt} = P^0 - p_m - \frac{P_N}{\sigma} \frac{f - f_N}{f_N}$$

Typical values are: $T_{sm} \approx 1.5$ s, $\sigma \approx 4\%$.

What is the impact of primary control and of the value of σ ?

We should not forget generator limits

$$p_m \in [P_{\min}, P_{\max}]$$

Where

- ▶ P_{\max} the maximum power available for primary control
- ▶ P_{\min} the minimum power available for primary control

What is the consequence of these limits?

About the speed droop σ (en français: le statisme) I

Speed droop σ

The speed droop is the ratio between the relative frequency deviation and the relative power deviation.

$$\sigma = \left| \frac{\Delta f / f_N}{\Delta P_m / P_N} \right| = \left| \frac{\Delta \omega / \omega_N}{\Delta P_m / P_N} \right|$$

- ▶ Δf is the frequency deviation from nominal value f_N .
- ▶ ΔP_m is the mechanical power deviation from nominal value P_N .

About the speed droop σ (en français: le statisme) II

- ▶ A frequency deviation $\Delta f = \sigma f_N = 0.04 \times 50 = 2$ Hz would result in a variation of mechanical power $\Delta P_m = P_N$.
- ▶ if $\sigma \rightarrow \infty$ the machine operates at constant power, and does not participate in frequency control.

The speed controller is of the proportional type

- ▶ it leaves a steady-state frequency error, but . . .
- ▶ this is precisely the signal allowing to share the effort over the various generators.

The power consumed by the load is frequency dependent

$$p_e = P_e^0(1 + D_l(f - f_N))$$

where:

- ▶ P_e^0 nominal (or initial) consumption
- ▶ D_l the sensitivity of consumption to frequency, assumed equal to 1%.

Not all loads are frequency dependent.

Do you see the impact on the curves?

The impact of renewable generation

Assume we now have $p_{PV}(t)$ generated by a PV farm.

Which equation does it impact?

Our model so far:

$$\begin{aligned}J \frac{df}{dt} &= \frac{p_m - p_e}{4\pi^2 f} - D_g(f - f_N) \\T_{sm} \frac{dp_m}{dt} &= P_m^0 - p_m - \frac{P_N}{\sigma} \frac{f - f_N}{f_N} \\p_m &\in [P_{\min}, P_{\max}] \\p_e &= P_e^0(1 + D_l(f - f_N))\end{aligned}$$

Let's now add renewable generation

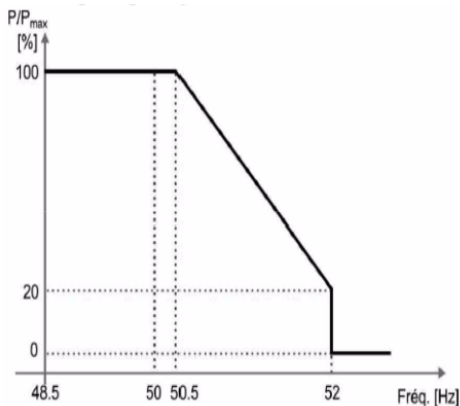
Assume we now have $p_{PV}(t)$ generated by a PV farm.

Which equation does it impact?

$$J \frac{df}{dt} = \frac{p_m - p_e + p_{PV}}{4\pi^2 f} - D_g(f - f_N)$$

What is the consequence?

Renewables also have to participate to frequency control



Over frequency curtailment of renewable generation

Let's add over frequency curtailment of renewable generation: model

if $f < 50.5\text{Hz}$:

$$p_{PV} = P_{PV}^0$$

else if $50.5 \leq f < 52\text{Hz}$:

$$p_{PV} = P_{PV}^0 \left(1 - \frac{0.8}{1.5}(f - 50.5) \right)$$

else if $f \geq 52\text{Hz}$:

$$p_{PV} = 0.0$$

where P_{PV}^0 is the maximum possible solar output (MPPT) at the initial instant of simulation.

Note: With more RES, less synchronous machine, so relatively less inertia for same demand.

Primary frequency control

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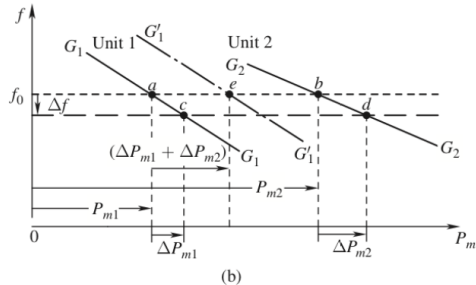
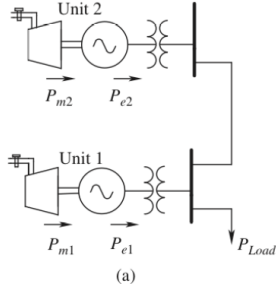
Objective and assumptions

This section aims at understanding the properties of primary frequency control, which has been introduced in the previous section.

Assumptions:

- ▶ the system has come back to steady state
- ▶ we consider several machines, and they all have the same electrical speed
- ▶ all machines participate in primary frequency control
- ▶ the network is lossless
- ▶ the mechanical power produced by the turbines is completely converted into electrical power
- ▶ load is sensitive to frequency as introduced in the previous section
- ▶ the system initially operates at the nominal frequency ($f = f_N$), for simplicity

The case of two generators helping out with a sudden increase of load I



The case of two generators helping out with a sudden increase of load II

- ▶ Imagine that suddenly load power increases by ΔP_L : because of KL, total electric power of generators will also increase, hence they start to decelerate leading to a frequency drop. Both will react, according to the governor settings of their primary frequency control loop, to increase their mechanical power
- ▶ At steady state, Δf will be such that $\Delta P_1 + \Delta P_2 = \Delta P_L$; typical time to reach steady state: 10-20s

Share of power variation among the generators I

The steady-state characteristics of the various generators can be combined into:

$$p_m = \sum_{i=1}^n p_{mi} = \sum_{i=1}^n \left(P_i^o - \frac{f - f_N}{f_N} \frac{P_{Ni}}{\sigma_i} \right)$$

Expressing that load is balanced by generation, and remembering that $\Delta f = f - f_N$:

$$\sum_{i=1}^n \left(P_i^o - \frac{\Delta f}{f_N} \frac{P_{Ni}}{\sigma_i} \right) = P_e^o (1 + D_l \Delta f)$$

In particular, at the initial operating point, the system is assumed balanced $\sum_{i=1}^n P_i^o = P_e^o$

Share of power variation among the generators II

A disturbance appears: an increase ΔP_e of consumption, the setpoints P_i^o being unchanged

$$\sum_{i=1}^n \left(P_i^o - \frac{\Delta f}{f_N} \frac{P_{Ni}}{\sigma_i} \right) = P_e^o (1 + D_l \Delta f) + \Delta P_e$$

Which can be simplified to:

$$-\frac{\Delta f}{f_N} \sum_{i=1}^n \frac{P_{Ni}}{\sigma_i} = P_e^o D_l \Delta f + \Delta P_e$$

given the relation for the initial operating point.

Share of power variation among the generators III

This can be rearranged to yield:

$$\Delta f = -\frac{\Delta P_e}{\beta}$$

with $\beta = \frac{1}{f_N} \sum_{i=1}^n \frac{P_{Ni}}{\sigma_i} + D_I P_e^o$

Stiffness of system β

β is called the *composite frequency response characteristic* [MW/Hz], also called *network power frequency characteristic* or *stiffness of system*. En français: énergie régulante. It characterizes the accuracy of primary frequency control.

Share of power variation among the generators IV

Variation of power of j-th generator

$$\Delta P_{mj} = -\frac{\Delta f}{f_N} \frac{P_{Nj}}{\sigma_j} = \frac{\Delta P_e P_{Nj}}{f_N \beta \sigma_j}$$

The steady-state frequency error allows a predictable and adjustable sharing of the power variation over the various (participating) generators

- ▶ all speed droops being fixed, the larger the nominal power of a generator, the larger its participation
- ▶ all nominal powers being fixed, the smaller the speed droop of a generator, the larger its participation
- ▶ the larger the number of generators participating in frequency control, the smaller the frequency deviation.

Primary reserve

Only a fraction of the total number of generators participate in primary frequency control.

To participate, the generator must have a primary reserve, i.e. it must produce less than its maximum power, and more than its minimum power

- ▶ but some units cannot (easily) vary their power: e.g. nuclear units
- ▶ for generators using a renewable energy source, the amount of reserve varies and is uncertain, and they can provide only downward reserve (unless they are curtailed on purpose to provide upward reserve)

Primary reserve is thus a service offered by the producer on the corresponding dedicated market. If it is selected, the generator is paid.

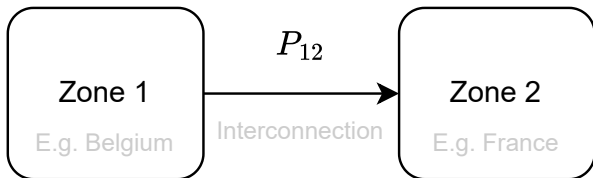
Secondary frequency control (aka AGC)

Secondary frequency control (aka AGC)

- ▶ After (gentle and successful) settlement of primary frequency control there are however some undesirable side effects:
 - ▶ Frequency has deviated from the nominal value
 - ▶ The exchanges between areas have deviated from their scheduled values
 - ▶ Part of the primary frequency control reserves have been depleted
- ▶ AGC (or secondary frequency control) aims at cancelling these side effects:
- ▶ Since AGC is an integral control, its steady state corresponds to zero Δf and zero ΔP (tie-line flows are also steered back to scheduled values)
- ▶ Once frequency is back to normal, primary control reserves are again fully available (takes 5-15 minutes)

Consider 2 interconnected zones I

Each zone contains several machines, and the two zones are interconnected by a tie-line (or several tie-lines).



Consider 2 interconnected zones II

With the same modelling assumptions as previously, the impact of primary frequency control is as follows:

Zone 1

- ▶ generators:

$$P_{m1} = \sum_{i \in 1} P_i^o - \frac{f - f_N}{f_N} \sum_{i \in 1} \frac{P_{Ni}}{\sigma_i}$$

- ▶ load: $P_{e1} = P_{e1}^o + D_{l1} P_{e1}^o (f - f_N)$

- ▶ power balance: $P_{m1} = P_{e1} + P_{12}$

Zone 2:

- ▶ generators:

$$P_{m2} = \sum_{i \in 2} P_i^o - \frac{f - f_N}{f_N} \sum_{i \in 2} \frac{P_{Ni}}{\sigma_i}$$

- ▶ load: $P_{e2} = P_{e2}^o + D_{l2} P_{e2}^o (f - f_N)$

- ▶ power balance:

$$P_{m2} = P_{e2} + P_{21} = P_{e2} - P_{12}$$

Power balance of whole system: $P_{m1} + P_{m2} = P_{e1} + P_{e2}$

Consider 2 interconnected zones III

Scenario

- ▶ the whole system operates initially at frequency f_N
- ▶ the load power in zone 1 increases by ΔP_{e1} .

Analysis I

Applying the relations of primary frequency control, by linearity, to the two zones, we have:

► zone 1: $-\beta_1 \Delta f = \Delta P_{e1} + \Delta P_{12}$

► zone 2: $-\beta_2 \Delta f = -\Delta P_{12}$

β_1 and β_2 are composite frequency response characteristics of zones 1 and 2, respectively.

Hence the frequency changes by

$$\Delta f = -\frac{\Delta P_{e1}}{\beta_1 + \beta_2}$$

Analysis II

The tie-line power changes by

$$\Delta P_{12} = -\frac{\beta_2}{\beta_1 + \beta_2} \Delta P_{e1} < 0$$

- ▶ The power flow from zone 1 to zone 2 decreases, due to the support provided to zone 1 by the generators of zone 2
- ▶ the larger β_2 with respect to β_1 , the more pronounced this effect.

Objectives and principle of secondary frequency control

- ▶ Eliminate the frequency error inherent to primary frequency control
- ▶ Bring the power exchange between zones to the desired value (contracts)
- ▶ Restore the generator primary reserves

In our two-zone example:

- ▶ increase the production of some generators in zone 1, where load increased
- ▶ the powers of the generators in zone 2 need not be adjusted!

Implementation of secondary frequency control

Acts in pre-defined control areas:

- ▶ corresponding to a country, to the zone managed by a transmission operator, etc.

Measurements are gathered in each control area:

- ▶ frequency
- ▶ sum of power flows in the tie-lines linking the area to the rest of the system

Power **set-point** corrections ΔP_i^0 are sent to dedicated generators in the area. It is very important to note that the setpoints are corrected, not the actual power outputs of the generators.

Distributed control in interconnected areas

Control distributed in the various areas:

- ▶ Measurements from one area are gathered by the control center of that area
- ▶ There is no exchange of real-time measurements between areas

Area Control Error (ACE)

Two-zone example, assuming zone 1 \equiv area 1 and zone 2 \equiv area 2.

We define the Area Control Error (ACE) as the difference between the power exchanged on the tie-line and the desired value, plus a frequency term:

► in area 1:

$$ACE_1 = P_{12} - P_{12}^0 + \lambda_1(f - f_N) = \Delta P_{12} + \lambda_1 \Delta f$$

► in area 2:

$$ACE_2 = P_{21} - P_{21}^0 + \lambda_2(f - f_N) = -\Delta P_{12} + \lambda_2 \Delta f$$

λ_1 and λ_2 are two bias factors.

Generator power correction

It is the output of a Proportional-Integral controller:

- ▶ in area 1: $\Delta P_1^o = -K_{p1}ACE_1 - K_{i1} \int ACE_1 dt, \quad K_{i1}, K_{p1} > 0$
- ▶ in area 2: $\Delta P_2^o = -K_{p2}ACE_2 - K_{i2} \int ACE_2 dt, \quad K_{i2}, K_{p2} > 0$

The corrections is then distributed over the generators participating in secondary frequency control:

- ▶ for the i-th generator of area 1: $P_i^o + \rho_i \Delta P_1^o$ with $\sum_i \rho_i = 1$
- ▶ for the j-th generator of area 2: $P_j^o + \rho_j \Delta P_2^o$ with $\sum_j \rho_j = 1$

Properties

- ▶ When the system comes back to steady state, the integral control imposes:
 - ▶ $ACE_1 = 0 \Rightarrow \Delta P_{12} + \lambda_1 \Delta f = 0$
 - ▶ $ACE_2 = 0 \Rightarrow -\Delta P_{12} + \lambda_2 \Delta f = 0$
- ▶ whose solution is: $\Delta f = 0$ and $\Delta P_{12} = 0$
- ▶ both objectives of secondary frequency control are met !

Choosing the bias factors λ_i

- ▶ They do not impact the final system state but the dynamics to reach it
- ▶ It is appropriate to choose: $\lambda_1 = \beta_1, \lambda_2 = \beta_2$
- ▶ Indeed, in the above example:
 - ▶ $ACE_2|_{\lambda_2=\beta_2} = -\Delta P_{12} + \beta_2 \Delta f = 0$ by definition of the primary frequency response characteristic β_2
 - ▶ Hence $\Delta P_2^o = 0$ all the time, since $ACE_2 = 0$
 - ▶ No adjustment of the generators in zone 2, that's what we wanted!
- ▶ the more λ_2 differs from β_2 , the more the generators in zone 2 are uselessly adjusted by the secondary frequency controller.

Choosing the K_i and K_p gains of the PI controllers

- ▶ They influence the dynamics, in particular the speed of action of secondary frequency control
- ▶ secondary frequency control must not act too promptly, in order not interfere with primary frequency control (which is the “first line of defense”)
- ▶ quite often, $K_p = 0$ (integral control only).
- ▶ ρ_i coefficients: distribute the correction signal ΔP_1^0 (or ΔP_2^0) on the participating generators, which must have secondary reserve
- ▶ for both primary and secondary frequency controls, the power variation that a participating unit commits to provide, in a given time interval, must be compatible with its maximum rate of change:
 - ▶ ' a few % P_N / min for thermal units
 - ▶ ' P_N / min for hydro units.

Economic dispatch and optimal power flow (brief)

Economic dispatch problem statement (ED)

Solve an optimization problem

- ▶ Given a total load P_L to serve, and a set of candidate generators $i = 1 \dots n$, with P_i constrained to $\underline{P}_i \dots \bar{P}_i$, and a cost function $C_i(P_i)$,
- ▶ Find optimal values P_i such that $\sum_i^n C_i(P_i)$ is minimized
 - ▶ while ensuring $\sum_i^n P_i = P_L$ (+ a loss term, possibly).
- ▶ See 12.4.1 of ref. book for some intuition about the solution to this problem

Optimal power flow problem statement (OPF)

- ▶ As above, + network constraints
- ▶ Given
 - ▶ network topology,
 - ▶ electrical parameters of lines, cables, transformers, and shunt devices,
 - ▶ current flow limits of lines and transformers, and voltage tolerance intervals at buses
- ▶ Determine the P_i such that $\sum_i^n C_i(P_i)$ is minimized
 - ▶ while modeling Kirchhoff laws and enforcing the above network-wise limits

Recap I

- ▶ *First*, primary control stabilizes frequency and power balance
 - ▶ response within 0-30s, fully local and fully automatic
- ▶ *Second*, frequency and power exchanges are brought back to nominal/contractual values, and primary control reserves are restored
 - ▶ response within 0.5-15 minutes, via area-control center (e.g. one country in Europe)
- ▶ *Third*, generation and exchange schedules are re-optimized, while restoring secondary frequency control reserves
 - ▶ response within 10-60 minutes, e.g. via intraday power exchange markets

Recap II

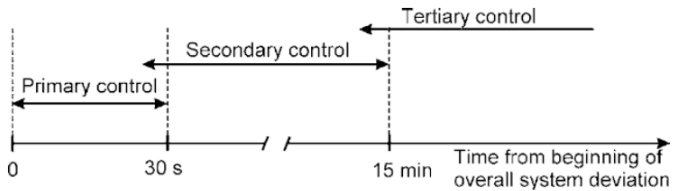


Fig from: <https://top10electrical.blogspot.com/2015/10/primary-secondary-and-tertiary.html>

Control resources

Which of these control resources are the main levers for frequency stability?

- ▶ Adjust synchronous generators' field current
- ▶ **Adjust synchronous generators' mechanical power**
- ▶ Change transformer taps
- ▶ Change shunt compensation
- ▶ Act on topology: switch lines and transformers in/out of service
- ▶ **Fast start-up generator units**
- ▶ **In extremis load curtailment**
- ▶ **Control renewable generation (e.g. PV curtailment)**
- ▶ **Use batteries and other energy storage systems**

Assessing the security/reliability of an electric power system (brief)

Assessing the security/reliability of an electric power system

- ▶ General idea of security assessment
- ▶ Standard $N-1$ static security assessment
- ▶ Voltage stability assessment (next lecture)
- ▶ Transient stability assessment (next lecture)

General idea of security/reliability assessment I

- ▶ General motivation:
 - ▶ Check the robustness of the system to a number of possible *contingencies*
 - ▶ NB: see examples of contingencies in subsequent slides
- ▶ Generic security assessment approach:
 - ▶ Given a list of contingencies and a steady-state *base-case* operating point of the system
 - ▶ Simulate the impact of each contingency if it is applied to the base case
 - ▶ For each simulation, check whether system response is acceptable
 - ▶ Summarize and display the results for all contingencies
 - ▶ If *too many contingencies lead to unacceptable response*, the base-case is *insecure*
- ▶ Remarks:

General idea of security/reliability assessment II

- ▶ Such analyses can be carried out periodically, based on real-time measurements
- ▶ They can also be applied in look-head mode, on a set of possible future system conditions
- ▶ Such analyses are extensively carried out by TSOs, and when the conclusion is negative, the TSO has to decide preventive/corrective counter-measures

Standard $N-1$ static security assessment I

- ▶ List of contingencies: all single-component outages
 - ▶ e.g. all single-line outages + all single-transformer outages
 - ▶ plus possibly all single-generator outages
- ▶ Procedure:
 - ▶ Run a power flow computation for the base case, and check pre-contingency limits
 - ▶ For each outage in the contingency list, run a power flow computation to determine post-contingency voltages and currents, and check post-contingency limits
- ▶ Precise limit values can depend on the context of security assessment:
 - ▶ e.g. in real-time operation, line/transformer permanent thermal limits and steady state voltage magnitude tolerance intervals are checked both for pre- and post-contingency

Standard *N-1* static security assessment II

- ▶ Remarks/orders of magnitude:
 - ▶ e.g. for a system like the Belgian EHV transmission system (ELIA)
 - ▶ 1000-2000 contingencies
 - ▶ 2000-4000 limits to check for each contingency
 - ▶ Need to summarize in graphical form the results to be useful for a human operator