LIÈGE université Sciences Appliquées

The transmission line

ELEC0447 - Analysis of electric power and energy systems

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October 2, 2025

Overview

1. Introduction

2. Distributed model

3. Surge impedance loading

4. Lumped transmission line model

5. Line rating

Introduction

What will we learn today?

- The transmission line
- An introduction to power flow analysis

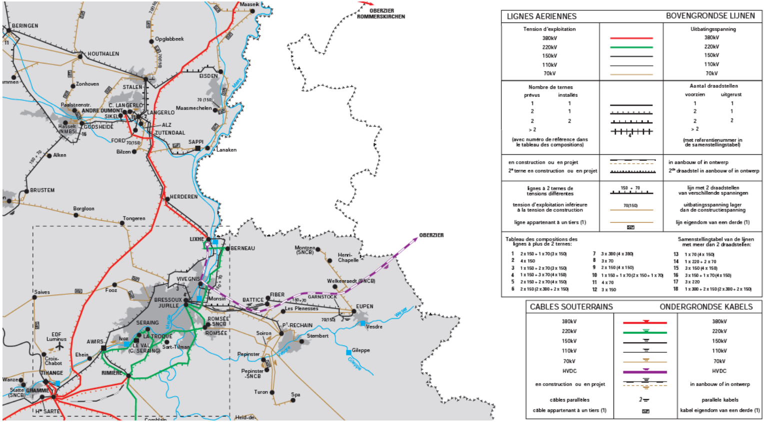
You will be able to do exercises 4.3, 4.4, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, Lab4 (power-flow in python), 5.1, 5.2, 5.5, 5.6 from the Ned Mohan's book.

Introduction video link

Definition

- An (overhead) transmission line is a set of 3 bundles of conductors corresponding to the three phases of the system.
- Commonly used voltages range from 70 kV to 380 kV in Belgium (more where distances are larger).
- Minimum distances between conductors depend on the voltage level, and thus electrical properties also depend on the voltage level.
- Underground cables are more and more used. They can be modeled in a similar way as overhead transmission lines, but hey have different properties.

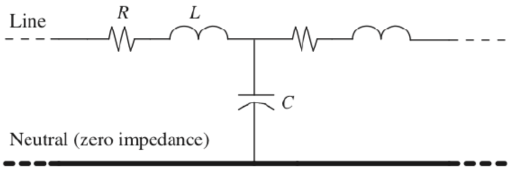
A part of ELIA's network



Source: https://www.elia.be/fr/infrastructure-et-projets/nos-infrastructures

Transmission line parameters

A *chunk* (a tiny piece) of transmission line can be represented as:



with R, L and C expressed **per unit of length**.

where

- R represents the series resistance, as small as possible to minimize RI^2 (influence of the frequency and skin effect)
- the series inductance L models the magnetic coupling between phases
- the shunt capacitance C models the capacitive coupling between phases
- a shunt conductance G can be added to model e.g. the leakage current through insulators

Typical cross section of a high voltage overhead conductor

Skin depth (detph at which decay of current density is $1/e$ of surface density):

Material	Frequency [Hz]	depth [mm]
Copper	50	9.4
Copper	60	8.6
Aluminum	50	12.0
Aluminum	60	10.9

Approximate Overhead Transmission Line Parameters

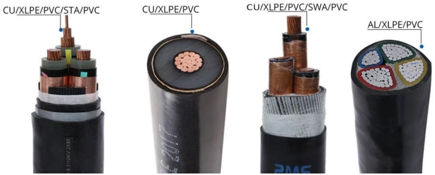
For bundled conductors at 60 Hz.

Nominal Voltage	$\boldsymbol{R}$ $(\mathbf{\Omega/km})$	ωL $(\mathbf{\Omega/km})$	$\omega \boldsymbol{C}$ $(\boldsymbol{\mu}\mho\mathbf{/km})$
230 kV	0.06	0.50	3.4
345 kV	0.04	0.38	4.6
500 kV	0.03	0.33	5.3
765 kV	0.01	0.34	5.0

Approximate underground cable parameters

For conductors at 60 Hz.

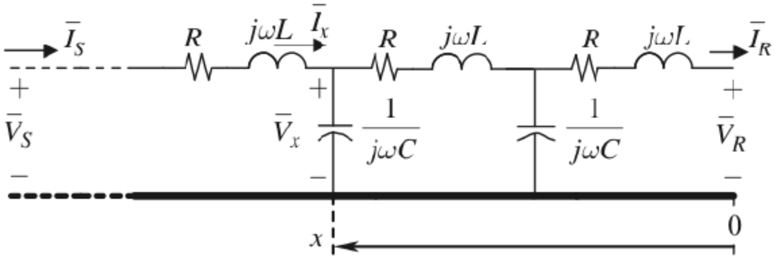
Nominal Voltage	R (Ω/km)	ωL (Ω/km)	ωC $(\mu\mho/\text{km})$
110 kV	0.05	0.030	95
220 kV	0.04	0.025	115
330 kV	0.03	0.020	130
400 kV	0.02	0.018	150



Distributed model

Distributed parameter representation I

We consider that we are in sinusoidal steady state. On a per-phase basis, the line can be represented as many chunks connected to each other:



Distributed parameter representation II

How do voltage and current evolve as a function of the position on the line?

- As R is small, let's assume R is considered as *lumped* (a discrete resistive element on one side of the line)
- $\frac{d\bar{V}(x)}{dx} = j\omega L\bar{I}(x)$
- $\frac{d\bar{I}(x)}{dx} = j\omega C\bar{V}(x)$

Hence

$$\frac{d^2\bar{V}(x)}{dx^2} + \beta^2\bar{V}(x) = 0$$

with $\beta = \omega\sqrt{LC}$ the *propagation constant*

Solution of the ODE I

The previous equation has a solution of the type

$$\bar{V}(x) = \bar{V}_1 e^{\beta j x} + \bar{V}_2 e^{-\beta j x}.$$

By derivation, the current is

$$\bar{I}(x) = (\bar{V}_1 e^{\beta j x} - \bar{V}_2 e^{-\beta j x})/Z_c.$$

With the **surge impedance**

$$Z_c = \sqrt{\frac{L}{C}}.$$

Solution of the ODE II

The boundary conditions at $x = 0$,

$$\bar{V}(0) = \bar{V}_R = V_R \angle 0,$$

and

$$\bar{I}(0) = \bar{I}_R$$

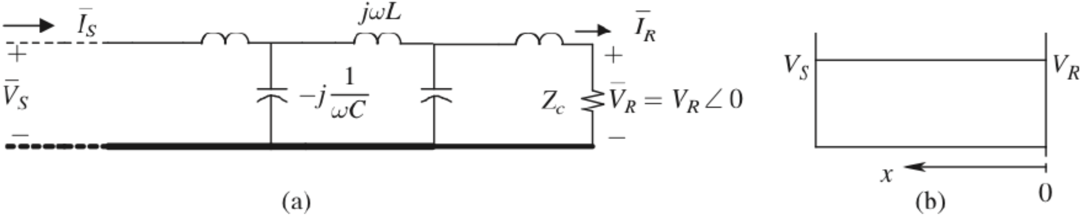
allow to determine constants $\bar{V}_1$ and $\bar{V}_2$, and finally

$$\bar{V}(x) = \bar{V}_R \cos(\beta x) + j Z_c \bar{I}_R \sin(\beta x).$$

Surge impedance loading

Closing the line on the surge impedance Z_c

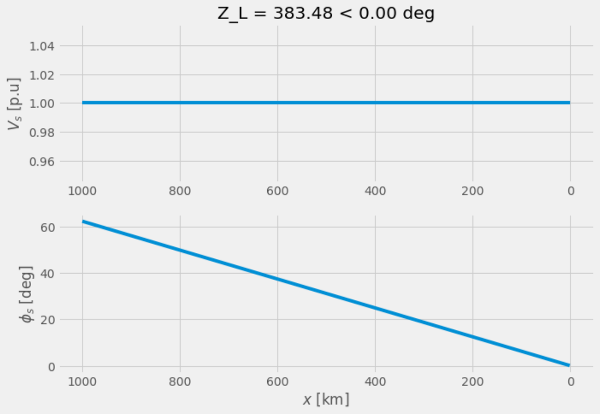
If the line is assumed lossless and *we close it with* Z_c, assuming $\bar{V}_R = V_R\angle 0$:



then the voltage *magnitude is constant* over the line: $\bar{V}(x) = V_R e^{j\beta x}$, and only the *angle increases with* x. Similar conclusion for $\bar{I}(x)$.

- Why? The reactive power consumed by the line is the same as the reactive power produced, everywhere.

Illustration in Python



SIL, 230 kV line params

See the Python notebook.

Surge impedance loading

Z_c depends on the line characteristics/geometry and is, hence, mainly a function of the voltage level (distances between conductors, etc.).

The surge impedance loading (SIL) is the power drawn by the load Z_c, which depends on the voltage level V_{LL}

$$SIL = \frac{V_{LL}^2}{Z_c}$$

Example: for 500 kV, $SIL \approx 1020$ MW.

Line loadability

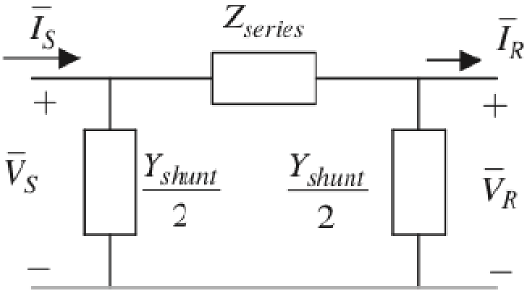
The SIL gives an idea of the loadability of a line depending on its length:

- short line, $l < 100$ km
  - load limit $= 3 \times SIL$
  - thermal limit (See Section 5 on Line rating)
- Medium length line, 100 km $< l < 300$ km
  - load limit $= 1.5$ to $3 \times SIL$
  - voltage drop $< 5\%$
- Long line, $l > 300$ km
  - load limit $\approx 1 \times SIL$
  - for system stability, the angle difference between line ends should stay $< 40°$, see lecture on Transient stability.

Lumped transmission line model

The π model

If l is relatively small (< 300 km), we can *approximate* the line with **lumped** parameters,



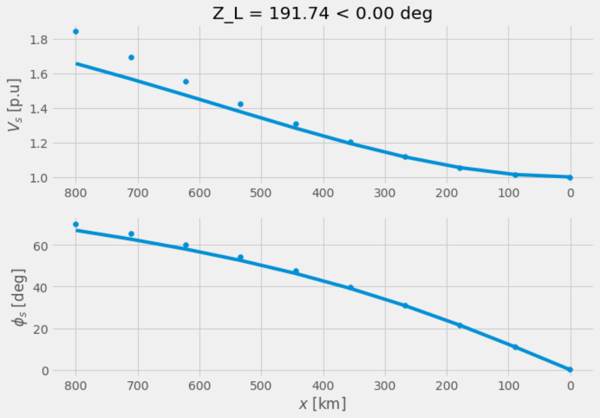
with, by manipulation of the previous equations and assuming βl small,

- $Z_{\text{series}} = Rl + j\omega Ll$
- $\frac{Y_{\text{shunt}}}{2} = j\frac{\omega Cl}{2}$

Remember that R, L and C are per km values.

This π model is symmetrical by design.

Illustration in Python



SIL, 230 kV line params

Dots are obtained with the π model, while plain lines are from the distributed model. The approximation error grows for $l > 300$ km.

See this Python notebook.

Line rating

Static line rating I

The **Static Line Rating** is the maximum continuous current a transmission line can carry under a specific set of predefined, fixed environmental conditions.

This rating is primarily constrained by **thermal limits** to ensure the safe and reliable operation of the line.

- **Conductor Thermal Limit:**
  - The conductor's electrical resistance generates heat (RI^2).
  - This heat must be balanced by cooling from the environment.
  - An excessive temperature can cause:
    1. **Increased Sag:** Conductor expansion due to heat causes the line to sag, potentially violating minimum clearance requirements to the ground, buildings, or other infrastructure.

Static line rating II

2. **Material Damage:** Prolonged high temperatures can anneal the conductor, reducing its tensile strength and lifespan.

- **Environmental Conditions:**
  - The "static" nature of the rating comes from assuming a fixed set of weather parameters.
  - **Ambient Air Temperature:** A baseline temperature (e.g., $40°C$) is used. Higher temperatures reduce the cooling capability of the air.
  - **Wind Speed:** A low, static wind speed (e.g., 2 ft/s at a $45°$ angle) is assumed to provide minimal convective cooling. Higher wind speeds would allow for more current.
  - **Solar Radiation:** A fixed value for solar heat gain is included, assuming a specific level of direct sunlight on the conductor.

Static line rating III

Static line ratings are a conservative and fundamental safety measure. They represent the worst-case continuous scenario to prevent physical damage and clearance violations under predictable conditions.

Dynamic line rating

Dynamic line rating video link