Transient Stability

ELEC0447 - Analysis of electric power and energy systems

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Overview

Classification power system stability (Reminder) I

Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact. [?]
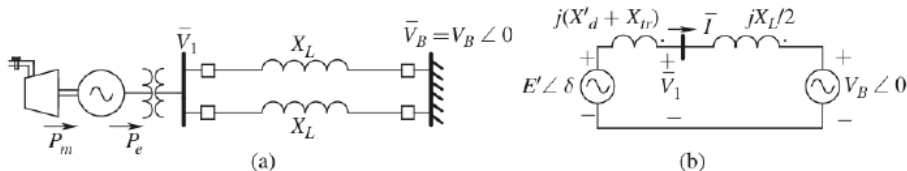
Classification power system stability (Reminder) II
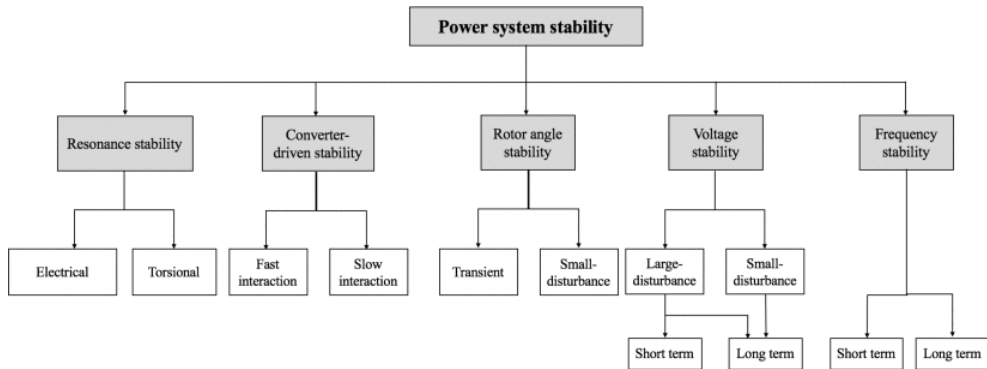
Principle of transient stability

Reminder: generator model and step up transformer I

**Figure 1:** Generator with step-up transformer connected to infinite bus through two parallel lines.

▶ Consider a generator bus connected through a transformer to a infinite bus system ($\bar{V}_B = V_B$ is an ideal voltage source).

Reminder: generator model and step up transformer II

- ▶ The mechanical power delivered through the shaft is P_m.
- ▶ The electrical power delivered to the grid is denoted P_e.
- ▶ In steady-state, the synchronous machine is modelled as a constant voltage source $E'e^{j\delta}$ behind its transient reactance X_d'.
- ▶ Finally, X_{tr} is the leackage reactance of the transformer.

Reminder: active and reactive power flow on a line I

Consider a simple radial system.
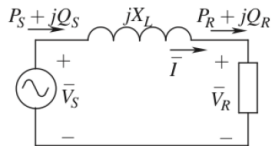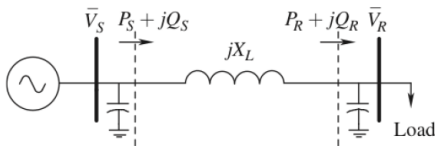
Reminder: active and reactive power flow on a line II

Assuming no transmission-line losses:

$$S_S = P_S + jQ_S = V_S e^{j\delta_S} \left(\frac{V_S e^{-j\delta_S} - V_R e^{-j\delta_R}}{X} \right) e^{j\frac{\pi}{2}}$$

If we define $\delta = \delta_S - \delta_R$, we have:

$$P_R = P_S = \frac{V_S V_R}{X_L} \sin \delta$$

$$Q_R = \frac{V_S V_R \cos \delta}{X_L} - \frac{V_R^2}{X_L}$$

$$Q_S = \frac{V_S^2}{X_L} - \frac{V_S V_R \cos \delta}{X_L}$$
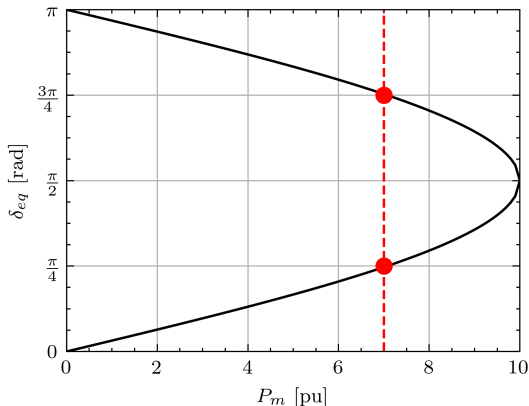
Reminder: Electrical power vs. angle

Plugging the generator and transformer models in the previous models, the electrical power P_e is expressed as follows:

$$P_e = \frac{E'V_B}{X_{tr} + X_d' + X_L/2} \sin(\delta) \tag{1}$$

At equilibrium, $P_e = P_m$. For given reactances and voltage sources, one has:

$$\delta_{eq} = \arcsin\left(P_m \frac{X_{tr} + X_d' + X_L/2}{E'V_B}\right)$$

Reminder: Electrical power vs. angle, graphically

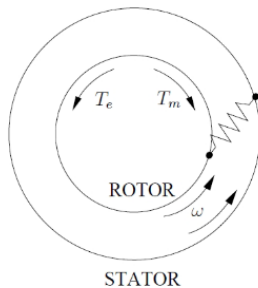- ▶ When we increase P_m, the electrical angle at equilibrium also increases.
- ▶ The maximum value is reached for $\delta_{eq} = \frac{\pi}{2}$.
- ▶ There exist
    - ▶ **TWO** equilibrium points for $P_m \in [0, 10)$
    - ▶ **ONE** for $P_m = 10$
    - ▶ **NONE** for $P_m > 10$.

Reminder: what if P_m exceeds P_e? I

(In this case, $P_m > 10$.)
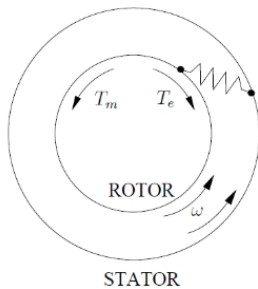
generator operation   motor operation

Reminder: what if P_m exceeds P_e? II

▶ In synchronous machines, the rotor rotates at the same angular speed as the magnetic field produced by the stator.

▶ If we denote ω_m the rotor angular speed and $\omega_s = 2\pi f_s$ the synchronous speed, then $\omega_m = \omega_s$.

▶ The field produced by the rotor and the stator tend to align.

▶ If they are not aligned, an electromechanical torque is produced.

▶ This misalignment is represented by the electrical angle: a greater angle induces a larger torque.

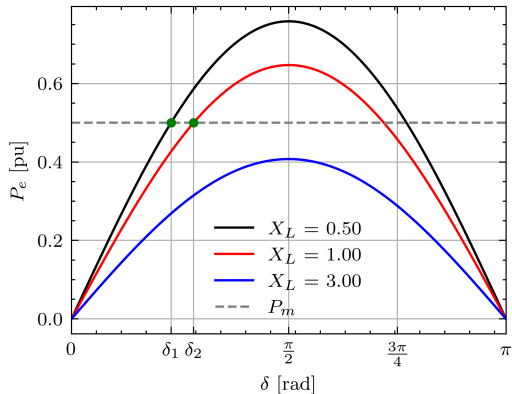▶ **BUT** if the angle is too large, the machine loses synchronism and the torque becomes 0.

Principle of transient stability

Consider equation (1) again:

$$P_e = \frac{E' V_B}{X_{tr} + X'_d + X_L/2} \sin(\delta)$$

with (all in per unit)

▶ $E' = 1.1, V_B = 1, X_{tr} = 0.8, X'_d = 0.4$

▶ various values of $X_L = [0.5, 1, 3]$ (see next slide).

Let us consider the following sequence of actions

Refer to Figure 1:

1. **Pre-fault conditions**: the impedance between the generator and the infinite bus system is $jX_L = j0.5$ pu.

2. **During-fault conditions**: one line is short-circuited, the impedance becomes $jX_L = j3$ pu.

3. **Post-fault conditions**: the short-circuit is cleared by disconnecting a line, the impedance becomes $jX_L = j1$ pu.

For the system to be stable, the system has to stabilize at $\delta = \delta_2$ after the fault occured. *Why?*

We will now look at how the system moves from the equilibrium point δ_1 to equilibrium point δ_2

The Swing equation

The swing equation I

Let us define the following quantities; J_m is the moment-of-inertia of the rotational system, T_m is the mechanical torque, T_e is the electrical torque, ω_m is the rotor speed and δ_m the rotor angle (in mechanical radians). Using Newton's Second Law, one has:

$$J_m \frac{d^2\delta_m}{dt^2} = T_m - T_e$$

Multiplying both sides by ω_m (the rotor speed), one has:

$$\omega_m J_m \frac{d^2\delta_m}{dt^2} = P_m - P_e$$

The swing equation II

We then define

$$H_{gen} = \frac{\frac{1}{2}J_m\omega_{syn,m}^2}{S_{rated,gen}} = \left[\frac{kg \cdot m^2 \times 1/s^2}{kg \cdot m^2/s^3}\right] = [s]$$

where $\omega_{syn,m}$ is the synchronous speed (in mechanical radians/s), $S_{rated,gen}$ the nominal rated size of the generator.

The swing equation III

About the inertia constant H_{gen}

The inertia constant of a machine H_{gen} is a value that quantifies how much kinetic energy is stored in the rotating mass of a machine relative to its power rating. It indicates the duration for which a machine can supply its rated power using only its stored rotational kinetic energy when the input power is suddenly removed. It takes a value in a range of

► 3 to 11s for turbo-alternators

► 1 to 2s for hydro generators.

The swing equation IV

One can write the swing equation by substituing J_m by H_{gen}:

$$\left(\frac{\omega_m}{\omega_{syn,m}^2} \right) 2H_{gen} \frac{d^2\delta_m}{dt^2} = P_{m,gen,pu} - P_{e,gen,pu}$$

where P_m and P_e are in per-unit of the generator MVA base. One can also write the equation in the system base S_{system}:

$$\left(\frac{\omega_m}{\omega_{syn,m}^2} \right) 2H \frac{d^2\delta_m}{dt^2} = P_{m,pu} - P_{e,pu}$$

The swing equation V

with

$$H = H_{gen} \left(\frac{S_{rated,gen}}{S_{system}} \right)$$

Finally, we make the reasonable assumption that $\omega_m \approx \omega_{syn,m}$, and we express everything in terms of electrical radians.

$$\left(\frac{2H}{\omega_{syn}} \right) \frac{d^2\delta}{dt^2} = P_{m,pu} - P_{e,pu} \tag{2}$$

What can we say about the swing equation?

- ▶ If the mechanical power provided at the shaft P_m is greater than the electrical power transferred to the network P_e, the machine accelerates $\frac{d^2\delta}{dt^2} = \frac{d\Delta\omega}{dt} > 0$, where $\Delta\omega$ is the deviation from the synchronous speed ω_{syn}.
- ▶ It decelerates if the electrical power is greater than the mechanical power.
- ▶ The acceleration is proportional to the machine inertia H

Question

What is the inertia of PV panels?

Transient stability using Equal-Area Criterion I

From the swing equation (2)

$$\left(\frac{2H}{\omega_{syn}}\right)\frac{d^2\delta}{dt^2} = P_{m,pu} - P_{e,pu}$$

Rearrange the terms and multiply both sides by $d\delta/dt$:

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{\omega_{syn}}{H}\left(P_{m,pu} - P_{e,pu}\right)\frac{d\delta}{dt}$$

Applying change of variables $\delta = \theta$, and integrating both sides between δ_0 and an arbitrary angle δ:

$$\int_{\delta_0}^{\delta}\left(2\frac{d\theta}{dt}\frac{d^2\theta}{dt^2}\right)dt = \frac{\omega_{syn}}{H}\int_{\delta_0}^{\delta}\left(P_{m,pu} - P_{e,pu}\right)d\theta$$

Transient stability using Equal-Area Criterion II

Assume δ_0 being an equilibrium point, *i.e.* $d\delta/dt|_{\delta=\delta_0} = 0$, we have:

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{\omega_{syn}}{H} \int_{\delta_0}^{\delta} \left(P_{m,pu} - P_{e,pu}\right) d\theta$$

The left term $\left(\frac{d\delta}{dt}\right)^2 = \Delta\omega^2$ represents the kinetic energy of the machine at an arbitrary angle δ^* (with respect to the synchronous speed ω_{syn}) $\rightarrow$ rotational kinetic energy: $\frac{1}{2}J\omega^2$ with J the moment-of-inertia and ω the angular speed. As long as $P_{m,pu} > P_{e,pu}$, the
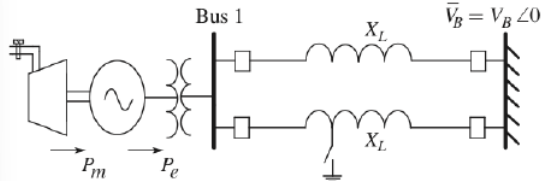
Transient stability using Equal-Area Criterion III

machine gains kinetic energy and accelerates. For the system to stabilize, it must exist an angle δ_m at which the kinetic energy becomes 0, and thus:
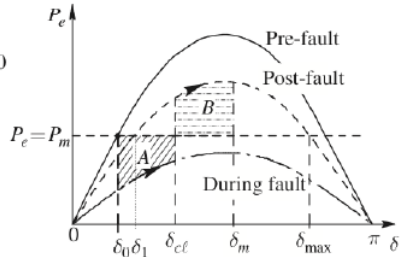
$$\int_{\delta_0}^{\delta} \left(P_{m,pu} - P_{e,pu} \right) d\theta = 0$$

Let us consider the following system; at time t and angle δ_0, a short-circuit occurs and is cleared at time $t + t_{cl}$ and angle δ_{cl} by tripping the line.

Transient stability using Equal-Area Criterion IV

Transient stability using Equal-Area Criterion V

For a stable system, we highlighted that:

$$\int_{\delta_0}^{\delta} \left(P_{m,pu} - P_{e,pu}\right) d\theta = 0$$

For our little example, one can write the same condition:

$$\underbrace{\int_{\delta_0}^{\delta_{cl}} \left(P_{m,pu} - P_{e,fault,pu}\right) d\theta}_{\text{Area A}} - \underbrace{\int_{\delta_{cl}}^{\delta_m} \left(P_{e,post-fault,pu} - P_{m,pu}\right) d\theta}_{\text{Area B}} = 0$$

During the first part (Area A), the machine accelerates. After the fault (Area B), the machine decelerates and the net acceleration becomes 0. At angle δ_m, there is still a mismatch
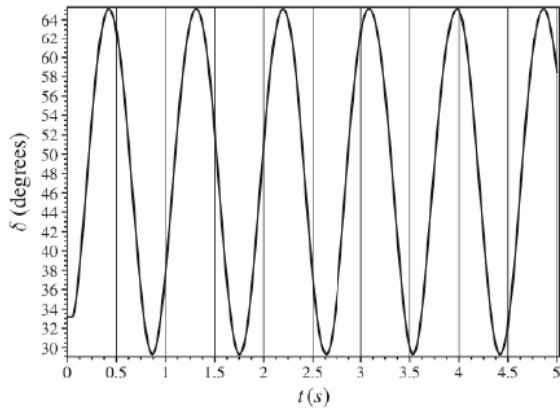
Transient stability using Equal-Area Criterion VI

between the electrical power and the mechanical power, thus the machine swings back (from δ_m to δ_2).
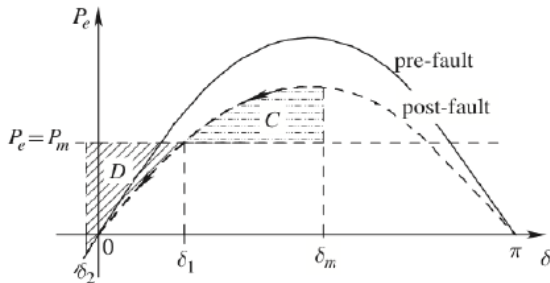
Transient stability using Equal-Area Criterion VII

Without any damping (kinetic energy losses), the system oscillates indefinitely between angle δ_2 and δ_m.

Transient stability using Equal-Area Criterion VIII

Transient stability using Equal-Area Criterion IX

However, in a real system, the damping would cause the machine to settle down at an angle δ_1 (new equilibrium point). Synchronous machines have *damper windings*. In perfect steady state, the magnetic fields produced by both the stator and the rotor are fixed relative to the rotor so there is no current in the dampers. On the other hand, if the rotor moves with respect to the magnetic field, the current induced in the dampers create a damping torque according to Lenz's law.

Critical Clearing Angle

Let us come back to our definition of stability for our little system:

$$\underbrace{\int_{\delta_0}^{\delta_{cct}} \left(P_{m,pu} - P_{e,fault,pu}\right) d\theta}_{\text{Area A}} - \underbrace{\int_{\delta_{cct}}^{\delta_{max}} \left(P_{e,post-fault,pu} - P_{m,pu}\right) d\theta}_{\text{Area B}} = 0$$

What are the conditions to ensure there exists δ_{cl} such that Area A = Area B? We introduce the concept of critical clearing angle δ_{cct}. Past this point, Area A will always be greater than Area B, such that the system cannot stabilize. We associate the critical clearing time *CCT* to the critical clearing angle, *i.e.* the maximum time allowed to clear the fault before the system gets unstable.

Critical Clearing Angle: Visual representation
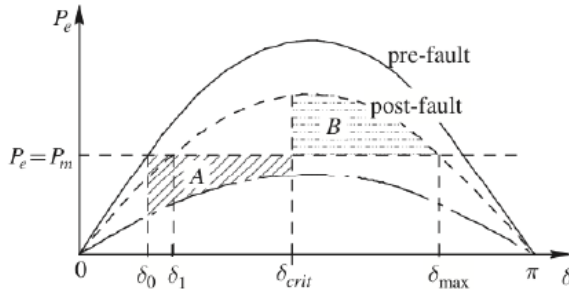
Mathematical formulation

$$\underbrace{\int_{\delta_0}^{\delta_{cct}} \left(P_{m,pu} - P_{e,fault,pu} \right) d\theta}_{\text{Area A}} - \underbrace{\int_{\delta_{cct}}^{\delta_{max}} \left(P_{e,post-fault,pu} - P_{m,pu} \right) d\theta}_{\text{Area B}} = 0$$

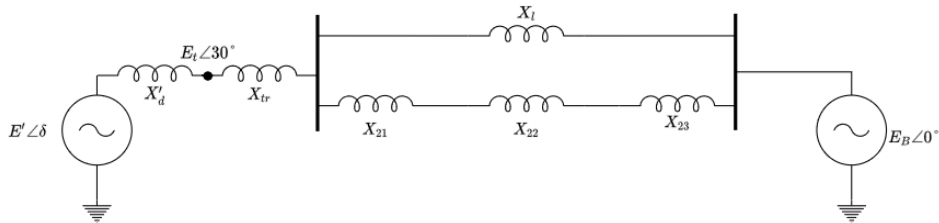One has to determine the angle δ_{max} such that $P_{e,post-fault} = P_m$ and the angle δ_0 such

Practical Example

Problem statement I

Let us consider the following system:

Problem statement II
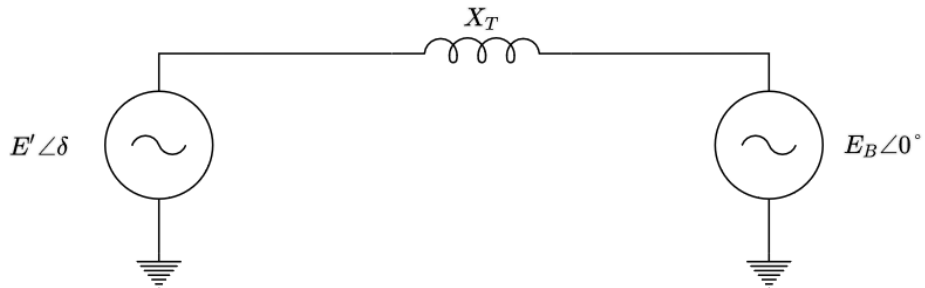
and the following data:

$$X'_d = 0.3, X_{tr} = 0.5, X_l = 1, X_{21} = 0.5, X_{22} = \frac{1}{6}, X_{23} = \frac{1}{3}$$

and

$$\bar{E}_t = 1e^{j\pi/6}, \bar{E}_B = 1e^{j0}$$

Equivalent I

We can derive the following equivalent:

Equivalent II

where

$$X_T = X_{tr} + X_d' + \frac{X_l\left(X_{21} + X_{22} + X_{23}\right)}{X_{21} + X_{22} + X_{23} + X_l} = 1.3$$

The power transfer from the machine to the infinite-bus system takes the following form:

$$P = \frac{E'E_B}{X_T}\sin\delta$$

Find $\bar{E}'$, P_e and P_m I

1. Current from $\bar{E}_t$ to $\bar{E}_B$

$$\bar{I}_{t \to B} = \frac{\bar{E}_t - \bar{E}_B}{jX_T - jX'_d}$$

2. $\bar{I}_{t \to B}$ is the same as the one from $\bar{E}'$ to $\bar{E}_t$

$$\bar{E}' = jX'_d \bar{I}_{t \to B} + \bar{E}_t = 1.05095 e^{j38.2057/180}$$

3. Find P_e

$$P_e = \frac{1.05095 \; 1 \; \sin(38.2057/180)}{1.3} = 0.5$$

Find $\bar{E}'$, P_e and P_m II

4. Initially, the system is at equilibrium.

$$P_m = P_e = 0.5$$

In the following, we will compute the maximum power outputs for three different conditions: Pre-fault, During-Fault and Post-Fault. It will allow us to determine the 3 different $P - \delta$ curves.

Pre-fault condition

We compute the maximum power output while assuming E' does not change. *If the transient stability study lasts a second or less, it is reasonable to consider, as a first-order approximation, that the exciter of the synchronous machine cannot respond in such short amount of time. Hence, E' does not change*
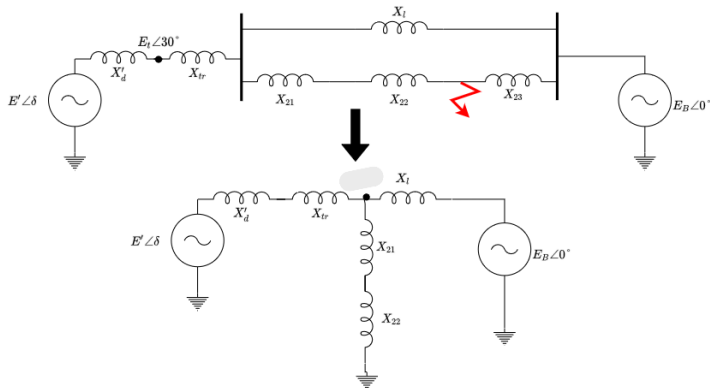

Maximum power output:

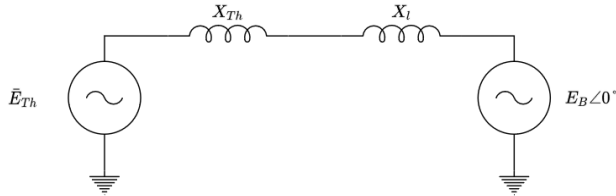$$\hat{P}_{e,bf} = \frac{E'E_B}{X_T} = 0.808$$

During-fault condition I

A short-circuit occurs between lines 22 and 23. The circuit topology changes and is depicted below.

During-fault condition II

During-fault condition: Thévenin equivalent I

We can derive the following Thevenin's equivalent

During-fault condition: Thévenin equivalent II

where

$$\bar{E}_{th} = \bar{E}' \frac{X_{21} + X_{22}}{X_d + X_{tr} + X_{21} + X_{22}} = 0.478 e^{j38.206/180}$$

and

$$X_{th} = \frac{1}{\frac{1}{X_d' + X_{tr}} + \frac{1}{X_{21} + X_{22}}} = 0.364$$

and then get the maximum power output:

$$\hat{P}_{e,df} = \frac{E_{th} E_B}{X_{th} + X_l} = 0.35$$

Post-fault condition

The line 2 is tripped to clear the short-circuit. The impedance of the path connecting the machine to the infinite-bus system becomes $X_d' + X_{tr} + X_l = 1.8$. The maximum power output is:

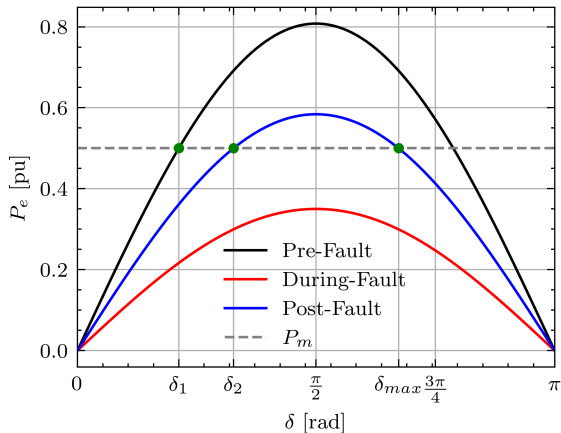$$\hat{P}_{e,pf} = \frac{E' E_B}{X_d' + X_{tr} + X_l} = 0.584$$

Practical Example

The final $P - \delta$ curves are shown here under, with

$$\delta_1 = \arcsin\left(\frac{P_m}{\hat{P}_{e,bf}}\right) = 0.667$$

$$\delta_2 = \arcsin\left(\frac{P_m}{\hat{P}_{e,pf}}\right) = 1.028$$

$$\delta_{max} = 2.114$$

Critical clearing angle δ_{cct} I

$$\underbrace{\int_{\delta_0}^{\delta_{cct}} \left(P_{m,pu} - P_{e,fault,pu}\right) d\theta}_{\text{Area A}} = \underbrace{\int_{\delta_{cct}}^{\delta_{max}} \left(P_{e,post-fault,pu} - P_{m,pu}\right) d\theta}_{\text{Area B}}$$

Area A

$$\int_{\delta_0}^{\delta_{cct}} \left(P_{m,pu} - P_{e,fault,pu}\right) d\theta = P_{m,pu} \left(\delta_{cct} - \delta_0\right) - \int_{\delta_0}^{\delta_{cct}} \hat{P}_{e,df} \sin\theta d\theta$$

$$= P_{m,pu} \left(\delta_{cct} - \delta_0\right) + \hat{P}_{e,df} \left(\cos\delta_{cct} - \cos\delta_0\right)$$

Critical clearing angle δ_{cct} II

Area B

$$\int_{\delta_{cct}}^{\delta_{max}} \left(P_{e,pf,pu} - P_{m,pu} \right) d\theta = P_{m,pu} \left(\delta_{cct} - \delta_{max} \right) - \int_{\delta_{cct}}^{\delta_{max}} \hat{P}_{e,pf} \sin \theta d\theta$$

$$= P_{m,pu} \left(\delta_{cct} - \delta_{max} \right) + \hat{P}_{e,pf} \left(\cos \delta_{cct} - \cos \delta_{max} \right)$$

Equal-area criterion

$$\text{Area A} = \text{Area B}$$

$$P_{m,pu} \left(\delta_{max} - \delta_0 \right) + \hat{P}_{e,df} \left(\cos \delta_{cct} - \cos \delta_0 \right) = \hat{P}_{e,pf} \left(\cos \delta_{cct} - \cos \delta_{max} \right)$$

Critical clearing angle δ_{cct} III

We can find out the critical clearing angle.

$$\cos \delta_{cct} = \frac{P_{m,pu}\left(\delta_0 - \delta_{max}\right) + \hat{P}_{e,df} \cos \delta_0 - \hat{P}_{e,pf} \cos \delta_{max}}{\hat{P}_{e,df} - \hat{P}_{e,pf}}$$
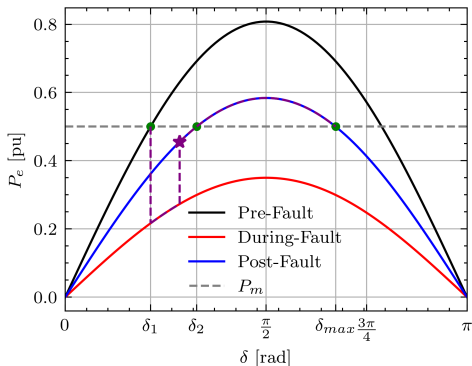
The purple star corresponds to $\delta_{cct} = 0.8938$, and the dashed purple line shows the evolution of the angle δ. Even without damping, the system stabilizes at $\delta = \delta_{max}$ since the kinetic energy reaches 0 at that point ($\Delta \omega^2 = 0$)

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Estimation of the critical clearing time *CCT* I

In order to find the critical clearing time, one would need to solve the swing equation:

$$\left(\frac{2H}{\omega_{syn}}\right)\frac{d^2\delta(t)}{dt^2} = P_{m,pu} - P_{e,pu}(\delta(t))$$

where $P_{e,pu}(\delta(t))$ is a non-linear function depending on δ; $P_{e,pu}(\delta(t)) = K(t)\sin(\delta(t))$, and with K changing non-continuously with time. We would need to solve a non-linear differential equation, which is not an easy task! Thus, let us assume that the acceleration $\frac{d^2\delta(t)}{dt^2}$ is constant over time. We have:

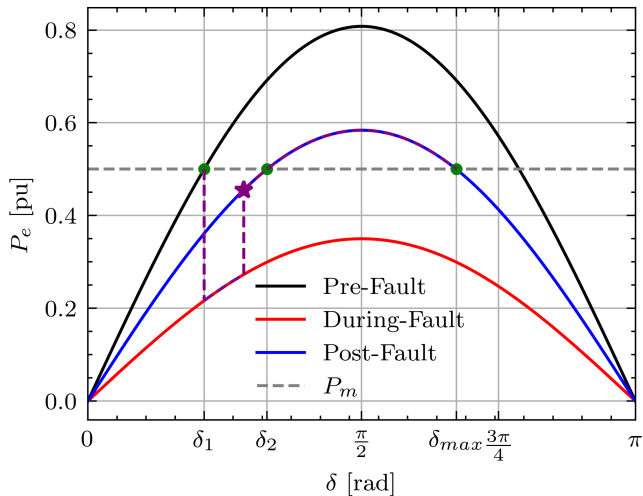$$\delta_{cct} = \delta_1 + \Delta\omega_0\,CCT + a\frac{CCT^2}{2}$$

Estimation of the critical clearing time *CCT* II

where *a* is the constant acceleration and $\Delta\omega_0$ the initial speed assumed to be 0. Let us assume two cases:

1. We pick the maximum acceleration for angles in $[\delta_1, \delta_{cct}]$.
2. We pick the minimum acceleration for angles in $[\delta_1, \delta_{cct}]$.

From the following figure

Estimation of the critical clearing time *CCT* III

Extension on dynamics

Swing equations with damping coefficient

$$\frac{d\Delta\omega(t)}{dt} = \frac{1}{2H}\left(P_m - P_{e,max}(t)\sin(\delta(t)) - D\Delta\omega(t)\right)$$
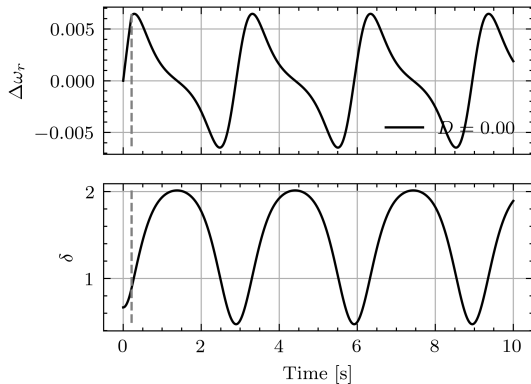
$$\frac{d\delta(t)}{dt} = \omega_0\Delta\omega(t)$$

Euler discretization

$$\Delta\omega_{t+1} = (1 - \tau\frac{D}{2H})\Delta\omega_t + \frac{\tau}{2H}\left(P_m - P_{e,max}^{k\tau}\sin(\delta_t)\right)$$

$$\delta_{t+1} = \delta_t + \tau\omega_0\Delta\omega_t$$

where τ is the time step and where we explicitly denote that $P_{e,max}^{k\tau}$ is changing over time in a discretize fashion (pre-fault→during-fault→post-fault) through the superscript $k\tau$.
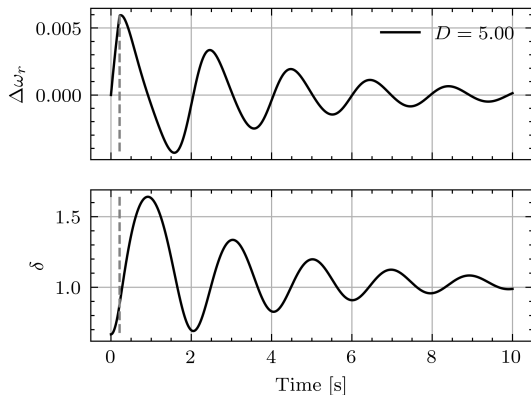
Extension on dynamics

Results without damping ($t_{cl} = 0.214s \leq CCT$)

The fault is cleared after 214 ms (lower bound on the clearing time). As expected from previous calculations, the system is safe.

Extension on dynamics

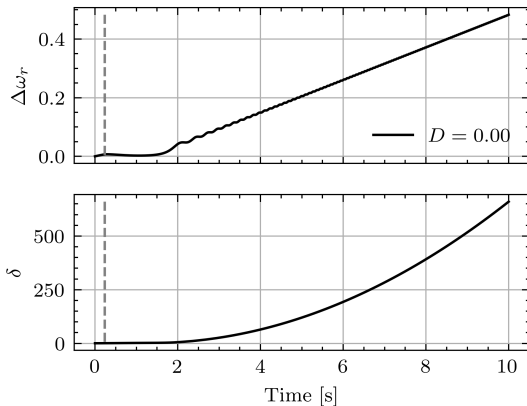**Results with damping ($t_{cl} = 0.214 \leq CCTs$)**

The fault is cleared after 214 ms (lower bound on the clearing time). As expected from previous calculations, the system is safe. The damping adds energy dissipation, which allows the system to stabilize around $\delta_2 = 1.028$
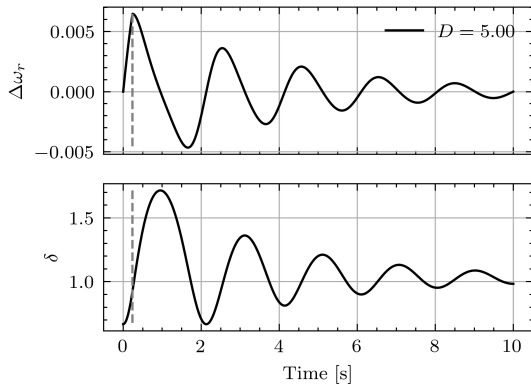
Extension on dynamics

Results without damping ($t_{cl} = 0.239s \geq CCT$)

The fault is cleared after 239 ms (upper bound on the clearing time). The machine loses synchronism.

Extension on dynamics

Results with damping ($t_{cl} = 0.239s \geq CCT$)

The fault is cleared after 239 ms (upper bound on the clearing time). The system is stable thanks to the damping effect (energy dissipation).

Dynamic Security Assessment

Dynamic Security Assessment goal

Main question that Dynamic Security Assessment (DSA) should answer:

Imagine a set of major, yet credible contingencies, can the system resist such events without jeopardizing its integrity?

If yes, then the states are determined secure $\rightarrow$ the system trajectories do not bring the states inside an unsafe set.

Dynamic Security Assessment principle

▶ Based on fast time domain contingency simulations.

▶ Study main stability issues such that: Voltage, Transient and Small-Signal (not covered throughout this course).

▶ Start from actual and future operating points.

▶ Has to be visual for the operators → show the type of instability, where it comes from and even possible solutions.

There are mainly types of DSA analysis

1. Off-line analysis

- ▶ They are subjected to forecast errors $\rightarrow$ system security cannot be taken for granted.
- ▶ But they can be performed with no constraints on time (performed day-ahead in order to set up a recommended operating schedule)
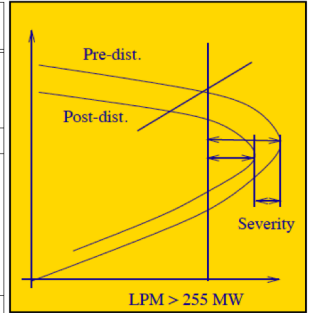
2. On-line analysis

- ▶ Not prone to forecast errors since they are based on real-time information.
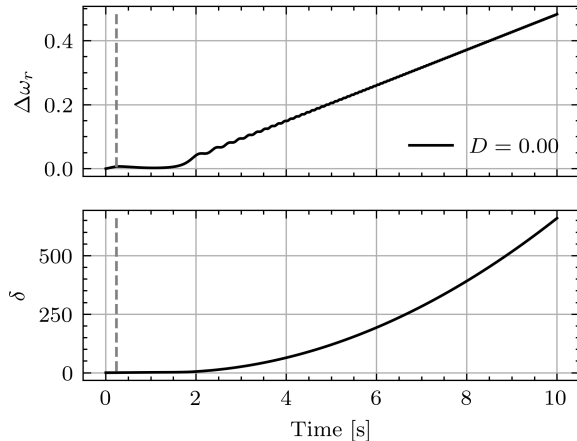- ▶ But need to be fast.

Voltage stability assessment

- ▶ Mainly static analyses.
- ▶ Goal is to ensure a sufficient load power margin.
- ▶ List of contingencies: all single-component outages.

Transient stability assessment

We want to avoid that:

- ▶ Simulations with different three-phase short-circuits at various locations.
- ▶ And with different clearing schemes!
- ▶ Very time consuming computations.

Trends

Nowadays

► Integration of large off-shore wind parks, and PV systems without reinforcing the network.

► Large power plants are decommissioned, and conventional generators are no longer accepted in urban areas.

► New units are built far away from load centers.

► Hard to build new lines because of public debates and environmental constraints.

This leads to a weaker electrical system, more prone to stability issues. There's a need for tools that can quickly perform security assessment to guarantee a secure system.

Example 1: static analysis I

► For static analyses, we rely on power flow solvers to estimate the state of the system.

► With increasing penetration of RES, probabilistic approaches are envisioned to perform risk assessment.

► But traditional power flow solvers based on Newton-Raphson methods are too slow.

► Usage of AI tools to derive approximate solutions of the power flow equations.

Donon, B., Clément, R., Donnot, B., Marot, A., Guyon, I., & Schoenauer, M. (2020). Neural networks for power flow: Graph neural solver. Electric Power Systems Research, 189, 106547.

Example 1: static analysis II
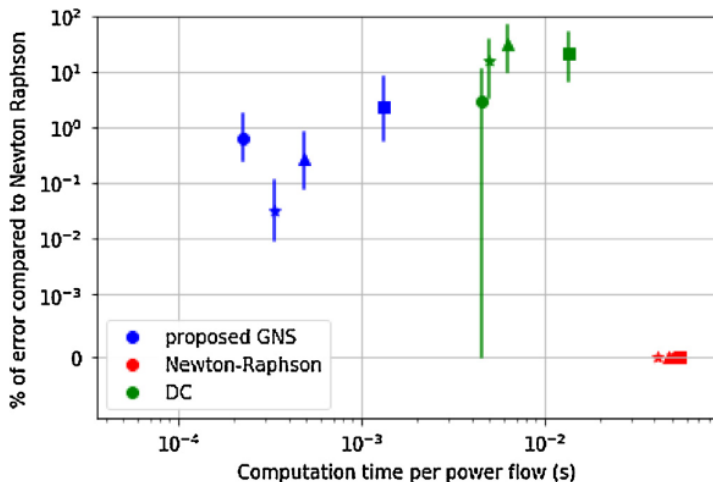
Example 1: dynamic analysis I

► For dynamic analyses, we study the time-evolution of a power system trajectory (e.g. internal angle in transient stability)

► It requires solving the differential-algebraic equations for multiple scenarios (different short-circuit locations, different clearing schemes).

► Usage of reachability analysis techniques: gives the reach set, i.e., the set that contains all possible system trajectories.

Chen, Y. C., & Dominguez-Garcia, A. D. (2012). A method to study the effect of renewable resource variability on power system dynamics. IEEE Transactions on Power Systems, 27(4), 1978-1989.

Example 1: dynamic analysis II

(a)

(b)