LIÈGE université
Sciences Appliquées

Transformers, power flow analysis part 2

ELEC0447 - Analysis of electric power and energy systems

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Overview I

1. The Transformer

2. Transformers in the power flow analysis

What will we learn today?

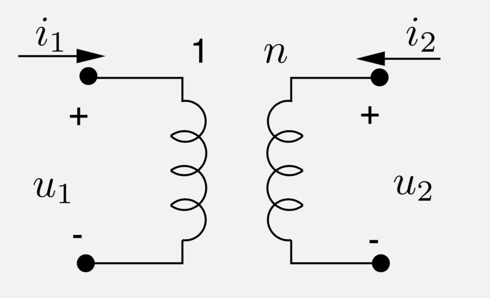
- The power transformer
- The next part of power flow analysis: how to include transformers and transformers with tap changers

You will be able to do exercises 6.2, 6.3, 6.4 from Ned Mohan's book.

The Transformer

Basic model of a transformer

A (single phase) transformer is made of two magnetically coupled **coils** or **windings**. An ideal transformer is a two-port represented as



with

$$u_2 = n u_1$$
$$i_2 = -\frac{1}{n} i_1$$

Usages

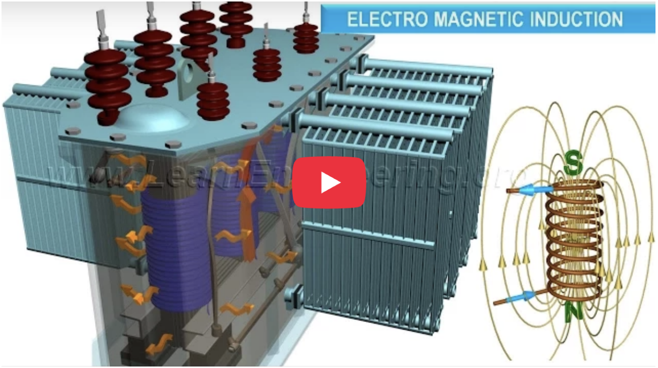
In power systems, transformers are mainly used to transmit power over long distances by changing the voltage level, thus decreasing the current for a given power level. The voltage level of a synchronous generator is around 20kV.

Voltage is changed around five times between generation and load.

It is also used to

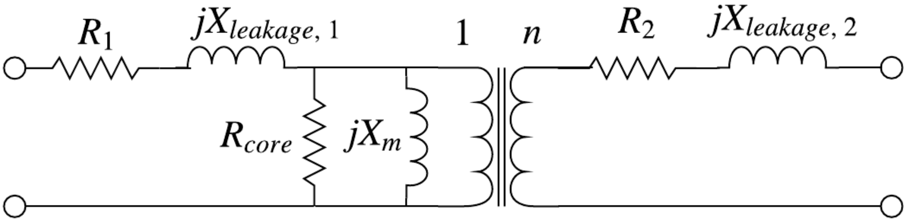
- **measure** currents and voltages,
- electrically **isolate** parts of a circuit (not the auto-transformer we will see),
- and **match** impedances.

How does a transformer work?



[Link to the video]

Non-ideal model I



The ideal model is complemented by elements

Non-ideal model II

- X_m that models the magnetizing inductance
- $X_{leakage,i}$ that models the flux not captured by the core on side i
- R_{core} that models eddy current and hysteresis losses, i.e., losses in the iron core
- R_1 and R_2 that model (coil) copper losses

Parameters are either given in the datasheet or obtained by open-circuit and short-circuit tests.

The core is ususally **laminated** to decrease losses.

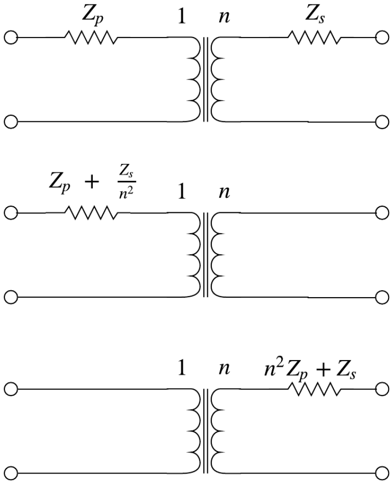
Simplification

The excitation current, the sum of the currents in R_{core} and X_m, is often neglected, leading to a simpler non-ideal model, and the series impedances can be transferred from one side to the other, with

$$Z_p = R_1 + jX_{leakage,1}$$

and

$$Z_s = R_2 + jX_{leakage,2}$$



Per unit representation I

Let's consider the rated voltages and currents on both sides of the (ideal) transformer as base values. As

$$V_{s,base} = nV_{p,base}$$

and

$$I_{p,base} = nI_{s,base},$$

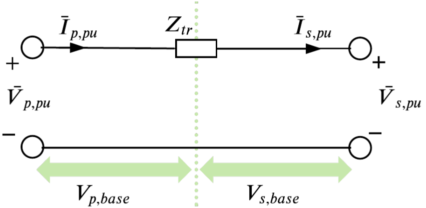
the **MVA base is the same on both sides**, and thus

$$Z_{s,base} = n^2 Z_{p,base}$$

Per unit representation II

Hence, **in per unit, the transformer can be replaced by a single impedance**

$$Z_{tr} = \frac{Z_p}{Z_{p,base}} + \frac{Z_s}{Z_{s,base}}.$$



Per unit representation III

Thus we have also that

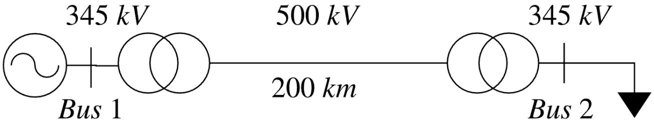
$$
\begin{aligned}
Z_{tr} &= \frac{Z_p + Z_s/n^2}{Z_{p,base}} \\
&= \frac{n^2 Z_p + Z_s}{Z_{s,base}}
\end{aligned}
$$

i.e. **the impedance is the same whether we see it from the primary or the secondary side, although the voltage bases differ**.

Also, if the three-phase transformer is **wye-delta** connected, a **30° phase shift** must be applied (more on this later).

Illustration of Per-Unit normalization I

This is Example 6.1 from the reference book. Consider the one-line diagram



with

- a 200 km line with $R = 0.029\Omega/km$, $X = 0.326\Omega/km$, neglected shunt impedances
- two transformers with a leakage reactance of $0.2pu$ in the (500 kV, 1000 MVA) base, and losses neglected.

Illustration of Per-Unit normalization II

What is the equivalent model in a (345 kV, 100 MVA) base?

In the (500 kV, 1000 MVA) base:

- $Z_{line,pu} = 200 \times (0.029 + j0.326)/(500^2/1000) = 0.0232 + j0.2608pu$
- hence, the total impedance between buses 1 and 2 is

$$Z_{12} = 0.0232 + j0.2608 + 2 * j0.2pu = 0.0232 + j0.6608pu$$

In the (345 kV, 100 MVA) base

- the pu value of the impedance is the same in the (500 kV, 1000 MVA) and (345 kV, 1000 MVA) bases!

Illustration of Per-Unit normalization III

- Why? since we can transfer the impedance from one side of each transformer to the other, cf. a previous remark.
- if we now change the MVA base to 100 MVA,

$$Z_{12} = (0.0232 + j0.6608) \times (100/1000)pu = 0.00232 + j0.06608pu$$

since the base impedance is proportional to the inverse of the MVA base.

Efficiency

The efficiency expressed in % is

$$100 \times \frac{P_{output}}{P_{input}} = 100 \times \left(1 - \frac{P_{losses}}{P_{input}}\right)$$

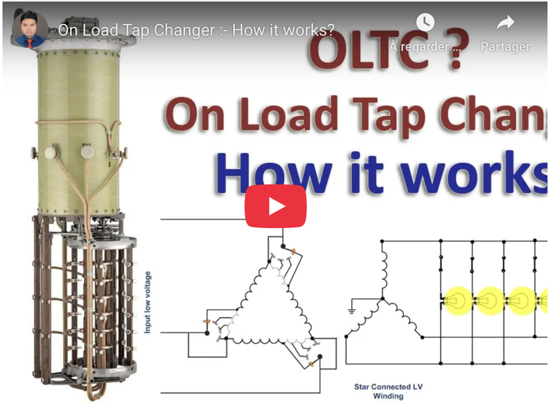
Remarks:

- maximal when loaded such that copper losses = iron losses (cancel derivative of efficiency w.r.t current)
- Around **99.5 %** in large power transformers at full load.

Tap changers

- Some transformers are equipped with a system allowing to change the $1 : n$ ratio
- The ability to change the tap under load is called load tap changer (LTC) or on-load tap changer (OLTC)
- This is mainly used for voltage control
- It is usually implemented using auto-transformers
- We will see later on how to include this in the power flow analysis

Tap changing transformers



Link to the video

Auto-transformers

The two windings (of the same phase) are connected in series, without galvanic insulation.

They are commonly used when the ratio is limited.

Advantages:

- Physically smaller
- less costly (less copper)
- higher efficiency
- easy to implement tap changes
- "solid" earth grounding

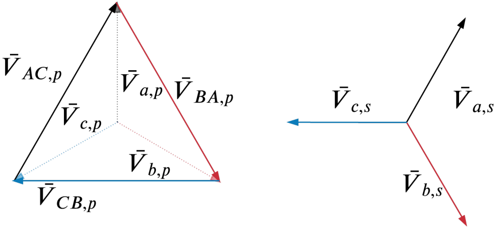
Disadvantages:

- no electrical insulation
- higher short circuit current
- full voltage at secondary if it breaks (in case of a step down)

Phase shift in delta-star transformers

The star part has n times the number of turns of the delta part (primary side). Let's reason on phase a,

- Voltage $\bar{V}_{a,s}$ is on the same core as $\bar{V}_{AC,p} = \sqrt{3}\bar{V}_{a,p}\angle{-30^\circ}$ where $\bar{V}_{a,p}$ is the (virtual) phase-neutral voltage on the primary side.
- Since $\bar{V}_{a,s} = n\bar{V}_{AC,p}$, $\bar{V}_{a,s} = n\sqrt{3}\bar{V}_{a,p}\angle{-30^\circ}$

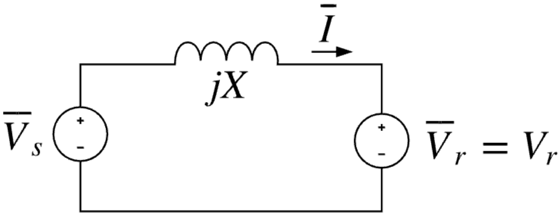


We **gain a $\sqrt{3}$ factor** in the amplification, and a **lagging phase shift of 30°**.

The same reasoning holds for phases b and c.

Power flow regulation by phase shifting I

We have seen that active power flows are dictated by the voltage magnitudes but also the sine of the angle difference between buses:



$$S_r = \bar{V}_r \bar{I}^* = V_r \left(\frac{V_s \angle - \delta - V_r}{-jX} \right)$$

$$= \frac{V_s V_r \sin\delta}{X} + j\,\frac{V_s V_r \cos\delta - V_r^2}{X}$$

δ is the angle between $\bar{V}_r$ and $\bar{V}_s$

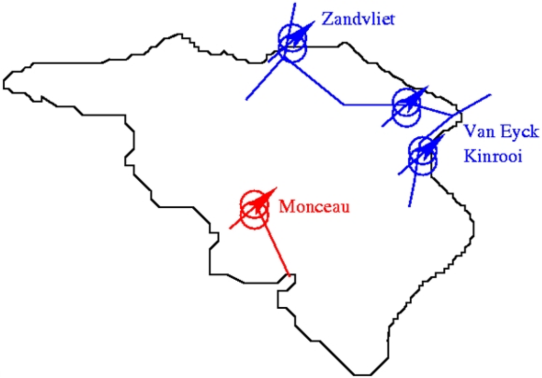
Power flow regulation by phase shifting II

If we have a device that can generate an adjustable phase shift, we can control the power flows. This is the purpose of **phase-shifting transformers**.

In practice phase shifting is achieved by "combining the signal with a fraction of itself shifted by 90°". For the details of how it is implemented or modeled, see

- Wikipedia
- Section 5.7. of the Weedy or ELEC0014.
- ENTSO-E - Phase Shift Transformers Modelling, Version 1.0.0, May 2015

Example: phase shifting transformers on the borders of Belgium I



380/380 kV, in series with:

1. line Zandvliet (B) - Borssele (NL) and Zandvliet (B) - Geertruidenberg (NL)
2. line Meerhout (B) - Maasbracht (NL)
3. line Gramme (B) - Maasbracht (NL)
   - nominal power 3VN Imax = 1400 MVA
   - phase shift adjustment: 35 positions, $+17/-17 \times 1.5$ (at no load)

Example: phase shifting transformers on the borders of Belgium II

220/150 kV :

- in series with the Chooz (F) - Monceau (B) line nominal power: 400 MVA
- in-phase adjustment: 21 positions, $+10/-10 \times 1.5\%$
- quadrature adjustment: 21 positions, $+10/-10 \times 1.2$

Remarks

In **three-phase operation**,

- either there are three separate single-phase transformers (easier to fix when there is a problem on a phase, more modular)
- or a **three-phase transformer**, that is a single core with three auto-transformers on it, cf. the video at the beginning of this presentation (cheaper, lighter core and less copper).

Some transformers called **three-winding transformers** are equipped with a third winding (not to be confused with a three-phase transformer) that is used for auxiliary purposes (feeding auxiliary devices e.g., fans, providing reactive power support, ...).

Transformers in the power flow analysis

Transformer without regulation

A transformer, in the per-unit representation, can thus be represented

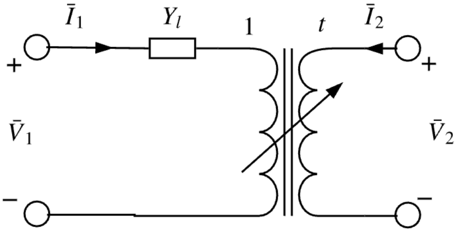
- as a two-port if the shunt admittance is considered
- as a simple series leakage impedance if the shunt admittance is neglected

It is a bit harder to model controllable taps for voltage and flow control.

Representing taps and phase shifts I

Let Y_l be the leakage admittance and t be the off-nominal turns ratio:

- if $0 < t \leq 1$, this corresponds to a simple tap-changer
- if $0 < |t| \leq 1$ but is complex, then this is a phase-shifter ($\angle t < \pi/2$)



We have

$$\bar{I}_1 = \left(\bar{V}_1 - \frac{\bar{V}_2}{t}\right) Y_l$$

Representing taps and phase shifts II

and since $\frac{\bar{V}_2}{t}\bar{I}_1^\star = -\bar{V}_2\bar{I}_2^\star$ by energy conservation

$$\bar{I}_2 = -\frac{\bar{I}_1}{t^\star} = -\bar{V}_1\frac{Y_l}{t^\star} + \bar{V}_2\frac{Y_l}{|t|^2}$$

Thus tap and phase shift can be represented by the admittance matrix

$$\left[\begin{array}{c} \bar{I}_1 \\ \bar{I}_2 \end{array}\right] = \left[\begin{array}{cc} Y_l & -\frac{Y_l}{t} \\ -\frac{Y_l}{t^\star} & \frac{Y_l}{|t|^2} \end{array}\right]\left[\begin{array}{c} \bar{V}_1 \\ \bar{V}_2 \end{array}\right]$$

Representing taps and phase shifts III

- if $0 < t \leq 1$, this can be represented as a π two-port
- if $0 < |t| \leq 1$ but is complex, this is not the case

In the power flow analysis you must **pay attention to this when constructing the system-wide admittance matrix.** As an exercise, let's add a phase-shifting transformer to our 3-bus example.

- Link to the example in PandaPower
- Link to the Newton-Raphson example