LIÈGE université
Sciences Appliquées

Voltage regulation and voltage instability

ELEC0447 - Analysis of electric power and energy systems

Bertrand Cornélusse, Antonin Colot

September 3, 2025

Overview

1. The concept of stability

2. Why do we need stability?

3. Voltage instability and voltage collapse

4. Some counter measures

5. Impact of renewable energy resources (RES)

6. Recap

Why to control power systems? (recap)

- Technical requirements: power system devices are designed so as to operate within well-defined "tolerance regions"
  - **around nominal values of voltage V_n: 1 ± 0.1 pu in Europe**
  - around nominal value of frequency f_n: 50 ± 0.2 Hz in Europe (in steady state)
  - **within the P-Q capabilities of devices**
  - under the current limits of lines and transformers
- Large/persistent deviations from nominal values could lead to
  - damages and safety problems (e.g. high voltage)
  - cascading phenomena
  - service interruptions

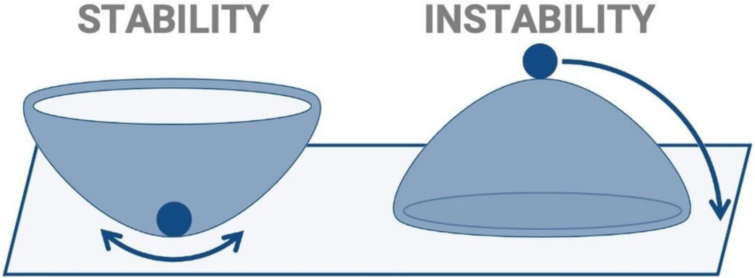
Exogeneous threats

- Sudden disturbances, such as line or generator tripping
- Fast variations of the *net load* (cf. Duck curve)
  - the net load refers to the load "seen" by the transmission system, i.e. the load minus the non-controllable dispersed generation
- weather conditions, such as storms, which can impact the generation of renewable energy sources (RES) (e.g. wind turbines’ cut-out speed)

The concept of stability

General concept

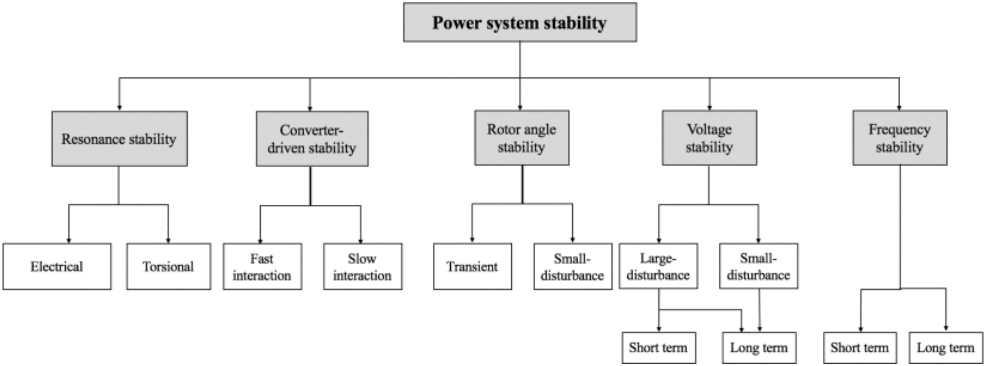
If a system has the property that it will get back into the equilibrium state again after moving away from its equilibrium state, then it is stable. [1]



In power systems I

> ***Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact.*** *[2]*

In power systems II



Why do we need stability?

Key points

- We often take electricity as a simple commodity.
- But the electric power system is one of the most complex and largest man-made system.
- The chances of system failures are very high taking into account the impact of external factors and rapid changes in system's state.
- However, power systems are very reliable (operated 24h/24h 7d/7d and only a few hours of power outages per year!).
- But when instabilities occur, it can lead to blackouts with huge financial and societal consequences.

Tokyo 1987

- Unexpected load increase and presence of constant power devices (air conditioners) led to a voltage collapse.
- **8 GW lost** and **2.8M customers impacted** [3, 4]

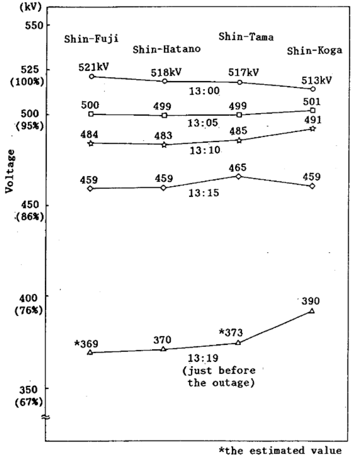
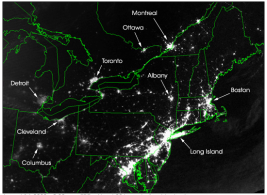


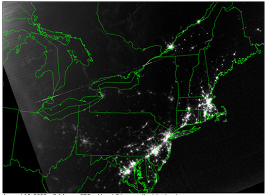
Fig.3 Voltage Drops at Main Substations

Canada/Northeast USA 2003

- Initial problem: not enough reactive power reserve. Hot weather and large consumption led to transmission lines overloading and eventually sagging into trees, further deteriorating the initial problem.
- A cascading event caused the tripping of hundreds of lines and generating units.
- **63 GW lost** and **50M customers impacted** and **Estimated cost above $5 Billion**
- Watch `https://www.youtube.com/watch?v=nd3teNgUq8E`, `https://www.youtube.com/watch?v=KciAzYfXNwU`



August 14, 2003 • 9:29 p.m. EDT • About 20 hours before blackout

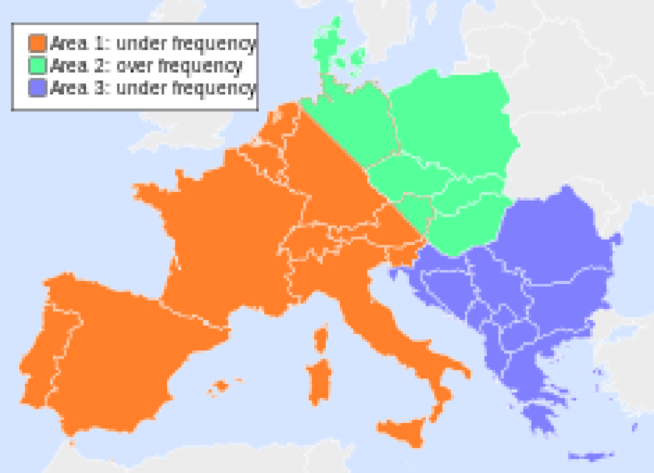


August 15, 2003 • 9:14 p.m. EDT • About 7 hours after blackout

Europe 2006 I

- Disconnection of a transmission line in Germany for the transport of a ship approved by the local TSO.
- The local TSO approved to advance the disconnection later that day, but the commercial flows remained unchanged.
- Some lines were critically loaded because of the line disconnection and a fast increase of load consumption led to a cascading event.
- European interconnected network has been split into 3 islands.
- Watch `https://www.youtube.com/watch?v=A30DdnsICuw`
- **14.5 GW lost** and **15M customers impacted** [5]

Europe 2006 II



Brazil 2023

- False operation of a relay protection system led to a 500kV line disconnection.
- The Energy Management System did not operate properly.
- **19 GW lost**

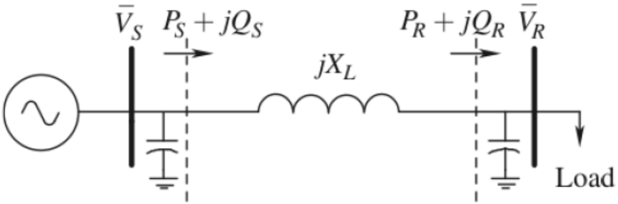
Conclusion

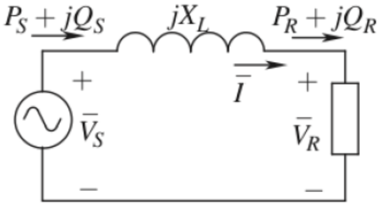
- Every time a black-out happened, lessons have been learned, and new rules have been put in place.
- Nonetheless, due to the system complexity, new complex phenomena occur that power system engineers try to understand.
- In the following, we'll dive into the mechanisms of voltage instability.

Voltage instability and voltage collapse

Impact of power flows on voltages I

Consider a simple radial system.





Impact of power flows on voltages II

Assuming no transmission-line losses:

$$S_S = P_S + jQ_S = V_S e^{j\delta_S} \left(\frac{V_S e^{-j\delta_S} - V_R e^{-j\delta_R}}{X} \right) e^{j\frac{\pi}{2}}$$

$$S_R = P_R + jQ_R = V_R e^{j\delta_R} \left(\frac{V_S e^{-j\delta_S} - V_R e^{-j\delta_R}}{X} \right) e^{j\frac{\pi}{2}}$$

If we define $\delta = \delta_S - \delta_R$, we have:

$$P_R = P_S = \frac{V_S V_R}{X_L} \sin \delta$$

Impact of power flows on voltages III

$$Q_R = \frac{V_S V_R \cos\delta}{X_L} - \frac{V_R^2}{X_L}$$

$$Q_S = \frac{V_S^2}{X_L} - \frac{V_S V_R \cos\delta}{X_L}$$

Question

What is the sign of Q_S if $V_S > V_R$. What about Q_R?

From the expression of Q_R, dividing both sides by $\frac{V_R^2}{X_L}$, we get:

$$\frac{V_R}{V_S} = \cos\delta \left(\frac{1}{1 + \frac{Q_R}{V_R^2/X_L}} \right)$$

Impact of power flows on voltages IV

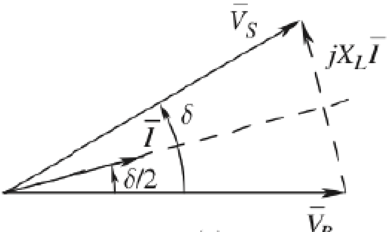
Assuming $P_S >> 0$ and $V_R \approx V_S \approx 1$, it leads to $\delta >> 0 \rightarrow \cos\delta << 1$.

Question

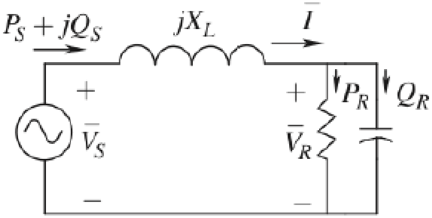
What would be the sign of Q_R to keep $\frac{V_R}{V_S} \approx 1$ as you increase P_S?

Consider the following radial system, and the associated phasor diagram for which we consider $V_R = 1e^{j0}, V_S = 1e^{j\delta}$.

Impact of power flows on voltages V



(a)



(b)

Impact of power flows on voltages VI

Considering Kirchhoff's Laws, one has:

$$\bar{V}_S = \bar{V}_R + jX_L\bar{I} \Rightarrow \bar{I} = \frac{e^{j\delta} - 1}{jX_L} = \frac{2\sin(\delta/2)}{X_L} e^{j(\delta/2)}$$

As $\bar{I}$ is leading $\bar{V}_R$, $Q_R = |\bar{V}_R||\bar{I}|\sin(-\delta/2)$ is negative. Q_S is positive and one can write:

$$Q_S = -Q_R$$

The larger $\delta \rightarrow P_{s\rightarrow r}$, the larger Q_R and Q_S to maintain $V_R = V_S = 1$

Question

Where does the reactive power go?

Impact of power flows on voltages VII

- Total reactive power loss:

$$Q_S - (-Q_S) = 2Q_S$$

- Line current:

$$\bar{I} = \frac{2\sin(\delta/2)}{X_L} e^{j(\delta/2)}$$

- Reactive power consumed by the line:

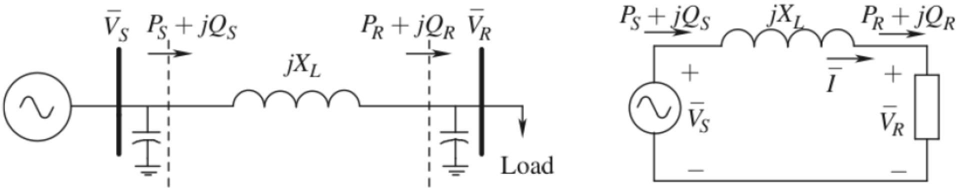
$$X_L|\bar{I}|^2 = \frac{4}{X_L}\sin(\delta/2)^2 = 2Q_S$$

Key points

- For HV systems, we usually assume: $P_{s\to r} \propto (\delta_s - \delta_r)$
- And $Q_{s\to r} \propto (V_s - V_r)$
- Lines are mostly inductive so they consume reactive power.
- Therefore, voltage control is done locally to avoid transferring reactive power over long distances.

Concept of nose curve I

Consider again the following simple radial system.



Concept of nose curve II

Consider $Q_R = 0 \Rightarrow \frac{V_S V_R \cos\delta}{X_L} - \frac{V_R^2}{X_L} = 0 \Rightarrow V_S \cos\delta = V_R$.

We know $P_R = \frac{V_S V_R}{X_L} \sin\delta$, substituing the previous results in the expression of P_R gives:

$$P_R = \frac{V_S^2}{X_L} \sin\delta \cos\delta = \frac{V_S^2}{2X_L} \sin(2\delta)$$

We can determine the maximum transmissible power by setting the partial derivative to 0:

$$\frac{\partial P_R}{\partial \delta}(\delta^{max}) = \frac{V_S^2}{X_L} \cos(2\delta^{max}) = 0 \Rightarrow \delta^{max} = \frac{\pi}{4}$$

Concept of nose curve III

Replacing δ by δ^* in the equation of P_R, we have:

$$P_R^{max} = \frac{V_S^2}{2X_L}$$

and

$$V_R \approx 0.7V_S$$

question

How can we increase the maximum transmissible power through a line?

Concept of nose curve IV

One can derive a relationship such that $\frac{V_R}{V_S} = f(\frac{P_R X_L}{V_S^2})$. Consider $y = \frac{V_R}{V_S}$ and $x = \frac{P_R X_L}{V_S^2}$, one has (trust me):

$$y = \sqrt{\frac{1}{2} \pm \sqrt{\frac{1}{4} - x^2}}$$

We verify that y has a unique solution when $x = \frac{1}{2} \Rightarrow P_R = \frac{V_S^2}{2X_L}$, which is the nose of the curve. There is no solution for $P_R > \frac{V_S^2}{2X_L}$ and two solutions for $P_R < \frac{V_S^2}{2X_L}$.

Let us consider different load characteristics:

$$P_{constant} \Rightarrow P_R = C_P \Rightarrow x = c_P$$

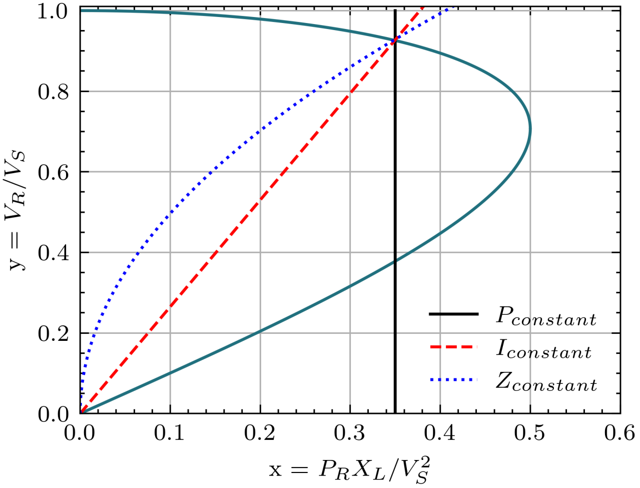
Concept of nose curve V

$$I_{constant} \Rightarrow \frac{P_R}{V_R} = C_I \Rightarrow y = c_I x$$

$$Z_{constant} \Rightarrow \frac{P_R}{V_R^2} = C_Z \Rightarrow y = c_Z \sqrt{x}$$

The operating point is where the load characteristic crosses the *PV curve*.

Concept of nose curve VI



Where does that shape come from? I

question

Where does that shape come from?

$$P_R = V_R I \cos\phi = \frac{V_S V_R}{X_L} \sin\delta$$

Let us consider $\cos\phi = 1$ (unity power factor), and $X_L = 1$ as well as $V_S = 1$.

$$I = \frac{V_S}{X_L} \sin\delta = \sin\delta$$

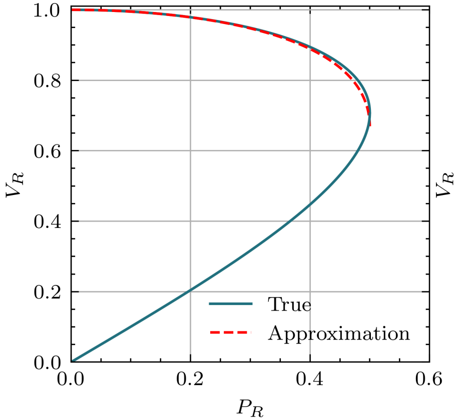
$$Q_S = \frac{V_S^2}{X_L} - \frac{V_S V_R \cos\delta}{X_L} = 1 - V_R \cos\delta$$

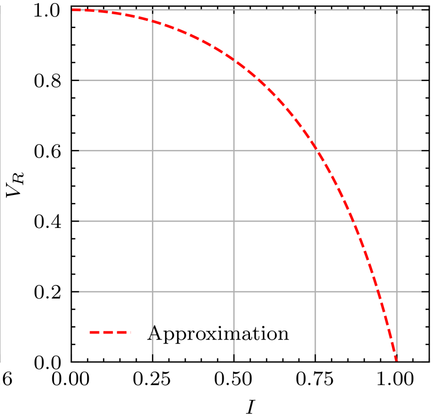
Where does that shape come from? II

Since $Q_R = 0$, you know that $Q_S = X_L I^2 = I^2 \Rightarrow \frac{-I^2+1}{\cos\delta} = V_R$. Consider the Taylor expansion of $\sin\delta \approx \delta$ and $\cos\delta \approx 1 - \frac{\delta^2}{2}$ for $\delta \approx 0$, we have

$$V_R \approx \frac{(1 - I^2)}{(1 - I^2) + I^2/2}$$

Where does that shape come from? III





Where does that shape come from? IV

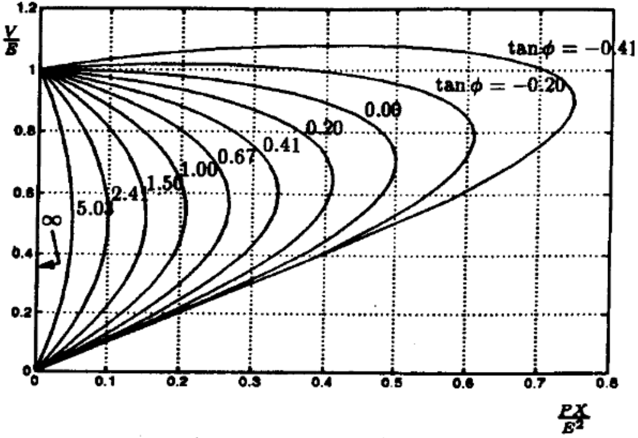
Since $I = \sin\delta \leq 1$, which implies that $(1 - I^2) \geq 0$ and therefore V_R decreases when I increases. But after a given value of I, V_R decreases faster than I increases. Since $P_R = V_R I \cos\phi$, there is a maximum power transmissible. It does so because the line is inductive, and thus consumes reactive power. The larger the current, the larger the reactive power consumed by the line, which pulls the voltage down.

question

Is that everything we need to know about nose curves?

NO When you consider $\cos\phi \neq 1$ (the load consumes or produces reactive power), you get more complex nose curves [**?**].

Where does that shape come from? V



The ideas previously developed still stand, except that now the load can:

Key points

- There's a maximum transmissible power ($P_R^{max} = \frac{V_S^2}{2X_L}$ for load with $\cos\phi = 1$).
- Inductive character of the line which consumes reactive power (influence of X_L)

How to increase P_R^{max}?

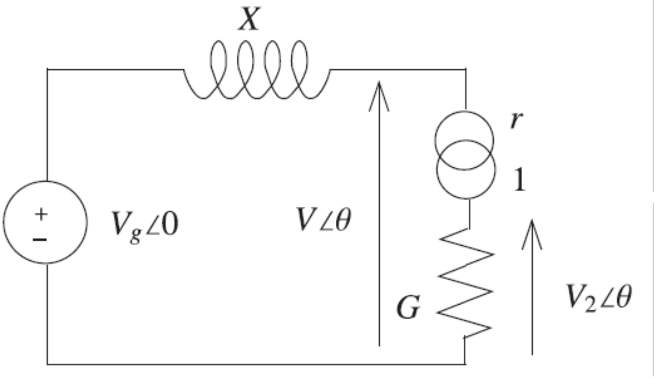
- By increasing the source voltage V_S,
- By decreasing X_L (adding lines in parallel),
- By producing reactive power at the load side to compensate for the reactive power consumed by the line.

Examples of voltage instabilities: Long-term instabilities I

On-load tap changers (OLTCs) change the turn ratio of the transformers feeding the distribution systems to keep the voltages on the secondary side as close as possible to a given setpoint.

Let us consider the following circuit, where the primary side of the transformer is the high voltage network, and the secondary side is the medium voltage network. The load on the secondary side is represented by a constant conductance G, consuming active power. The voltage on the primary side is controlled by a synchronous generator. V_g is kept constant as long as the reactive power limits of the generator are not reached.

Examples of voltage instabilities: Long-term instabilities II



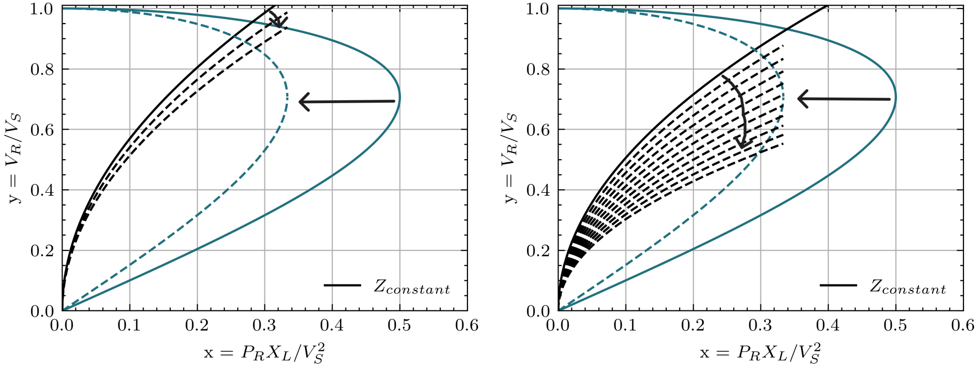
Examples of voltage instabilities: Long-term instabilities III

We assume an ideal transformer: $\frac{V}{V_2} = r$, $\frac{I_2}{I} = r$. The load characteristic seen from the primary side becomes $P_G = G\left(\frac{V}{r}\right)^2$, with P_G the power consumed by the conductance G. Now, imagine one wants to keep $V_2 = V_2^o$, if $V \searrow \Rightarrow r \searrow$. Indirectly, by decreasing r, the OLTC tries to restore the load (since it increases V_2 and $P_G = GV_2^2 = G\left(\frac{V}{r}\right)^2$). Two different scenarii:

- 1) $\frac{V}{r}$ converges towards V_2^o, the load is restored.
- 2) $\frac{V}{r}$ never converges towards V_2^o and V collapses.

Imagine a disturbance leading to a decrease in the maximum transmissible power.

Examples of voltage instabilities: Long-term instabilities IV



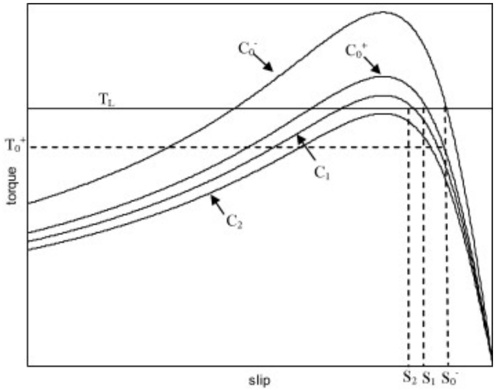
Examples of voltage instabilities: Long-term instabilities V

- 1) Left figure shows P_G is recovered after two tap changes.
- 2) Right figure shows P_G is never recovered, and the voltage collapses.

Scenario 2) is a typical case of **Long-term Voltage Stability**. The disturbance can be the tripping of a line (as it is the case here), which leads to $X_L \nearrow$ or hitting the reactive power limits of a generator (V_S is no longer maintained).

Examples of voltage instabilities: *short-term instabilities* I

Consider an induction motor. The torque-speed curve is given below.



- T_L is the mechanical torque.
- C curves correspond to different electromechanical torques depending on the slip $s = \frac{\omega_s - \omega_r}{\omega_s}$.
- The initial curve is C_0^- and the machine operates on operating point S_0.

Examples of voltage instabilities: *short-term instabilities* II

If the voltage drops

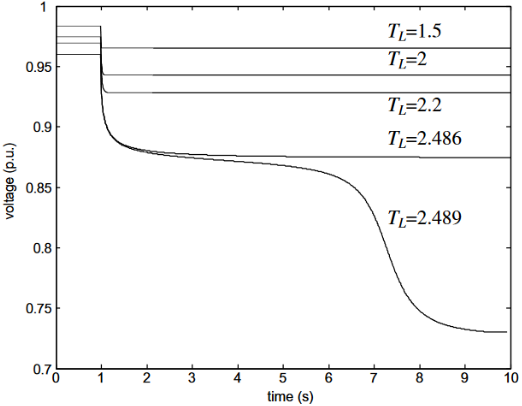
- the new curve becomes C_0^+
- the motor speed ω_r is reduced (thus s increases), and we reach a new operating point S_1
- this increase in s leads to an increase in motor current and the further decrease in voltage and electrical torque
- if the rate of decreasing voltage is slower than that of increasing slip, the voltage settles down.

If now the voltage drops faster

Examples of voltage instabilities: *short-term instabilities* III

- e.g. the new curve becomes C_2,
- the motor speed is reduced until it completely stops (since there is no intersection between C_2 and T_L).
- The induction motor acts as a large inductance, drawing reactive power.
- This is considered as a **Short-term Voltage Instability** as this phenomenon is much quicker than what we have with OLTCs (it takes several seconds to change tap positions).
- OLTCs are not able to restore the voltage.

Examples of voltage instabilities: *short-term instabilities* IV



Some counter measures

Network Reinforcement

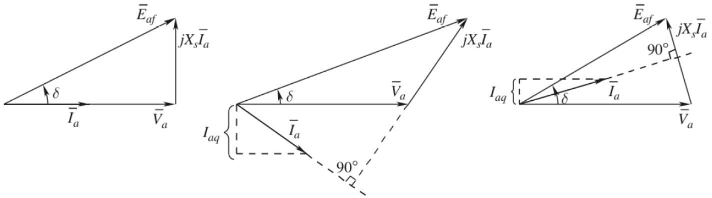
We saw that the main issues for voltage problems are:

- Large values of X_L
- Not enough reactive power compensation

To mitigate voltage problems, one solution is to reinforce the network by adding new lines and capacitor banks, static var compensators or synchronous condensers.

Voltage regulation I

The second solution is to use the reactive power reserve already available. In transmission system, we have synchronous machines. They can provide or consume reactive power.



(a) (b) (c)

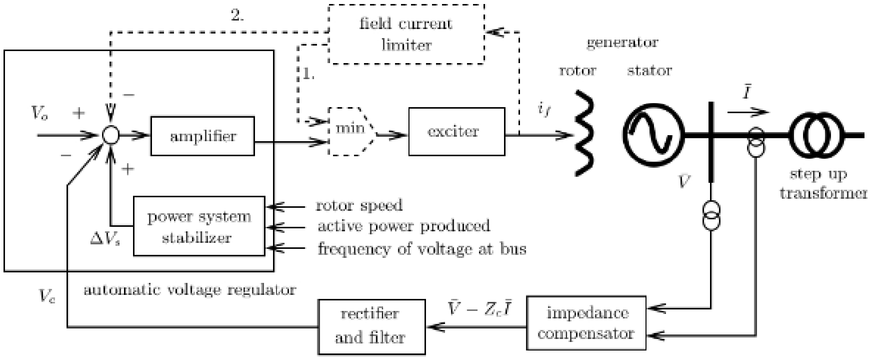
Voltage regulation II

In this figure E_{af} is the induced emf, V_a the stator voltage, I_a the stator current and X_s the synchronous reactance. In case (a), there is no reactive power transfer. In case (b), the synchronous machine **provides reactive power** and in case (c), the machine **consumes reactive power**.

- When the machine is overexcited, it produces reactive power (because of the larger induced emf).
- When it is underexcited, it consumes reactive power.

A regulator is responsible for controlling the excitation current in a synchronous generator.

Voltage regulation III



Automatic Voltage Regulator (AVR)

The voltage setpoints V_0 are dispatched by the TSO to ensure a safe and reliable grid.

Primary, secondary and tertiary voltage control I

- When a disturbance occurs, or subsequently to following the change in load (cf. ’duck curve’), the *primary* voltage control loops maintain suitable voltage levels close to the large power plants equipped with AVRs.
  - However, voltages at other buses may move out of tolerance intervals (in either direction), and reactive power reserves may not be shared in an even way among generators.
- *Secondary* voltage control loops can be used at the zonal level, to adjust the set-points of AVRs so as to control the voltage at ’pilot nodes’ in the network while distributing the required reactive power evenly among generators.

Primary, secondary and tertiary voltage control II

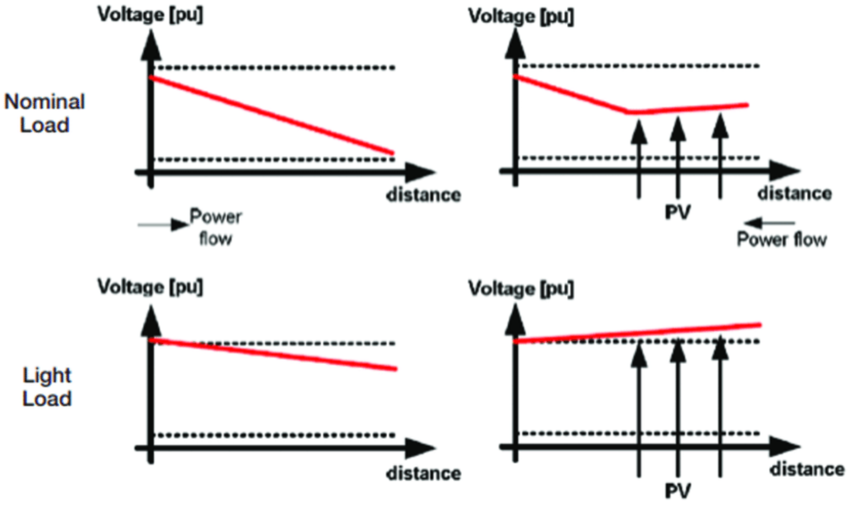
  - Secondary voltage control loops can also be used to switch shunt reactive compensation devices (capacitors/inductors) in order to increase reactive power generation margins in their zone (among a few large power plants).
- *Tertiary* voltage control uses OPF solvers to calculate set-points at pilot nodes and possibly adjust some transformer ratios, so as to minimize losses and maximize MVar reserves at the entire system level.
- Response times of different levels of voltage control
  - *Primary:* 1-3 seconds ; *Secondary:* 30 seconds -3 minutes ; *Tertiary:* 10-15 minutes

Impact of renewable energy resources (RES)

Reverse power flows in distribution systems I

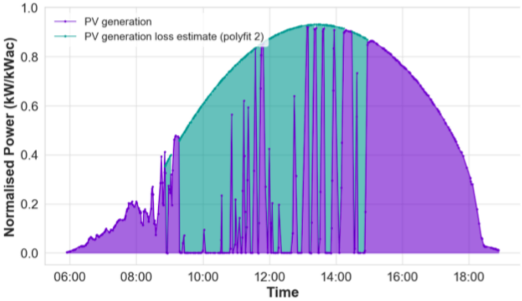
- Distribution networks are radial networks (they are built as meshed networks, but operated radially)
- Before the venue of distributed energy resources (*PV panels, batteries,...*), the power was flowing from the substation node, down to the end of the feeders.
- Increasing penetration of DERs led to reverse power flows.

Reverse power flows in distribution systems II



Reverse power flows in distribution systems

- During hours of high production and low consumption (at midday on a summer day), overvoltages can occur.
- It leads to the disconnection of PV inverters $\Rightarrow$ loss of production.



Reverse power flows in distribution systems

- In distribution systems, the $X/R < 1$. Active power flows have actually a greater influence on voltage magnitudes than reactive power flows.
- One way to mitigate overvoltages in distribution networks with high penetration of solar inverters is to do active power curtailment and reactive power compensation.

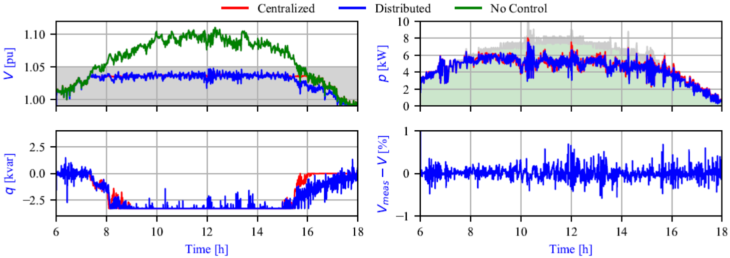
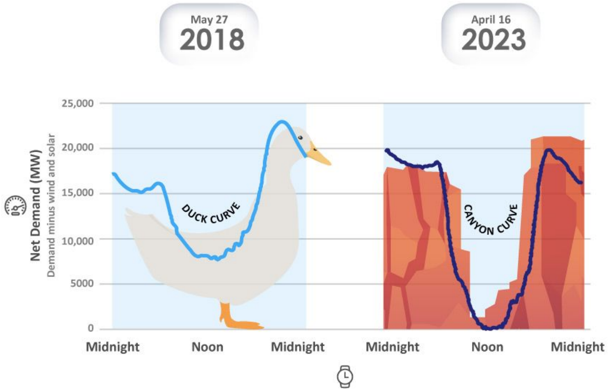


Fig. 3. Results for one specific bus equipped with a PV inverter.

Duck Curve I

- As we further increase the penetration of RES, changes appear in the demand profile (concept of Duck Curve, now becoming Canyon Curve)
- Needs for flexibility to ensure power balance: shut down of flexible production plants when RES start to produce, activate them again when they stop producing
- There exist various solutions to *fill the curve*: Demand Side Management, Energy Buffers (e.g. batteries), increasing import/export capacities.

Duck Curve II



Recap

What did we learn?

- In transmission systems, because lines are inductive $X/R > 1$, reactive power flows impact voltage magnitudes, and active power flows impact voltage angles.
- There is a maximum transmissible power through a line.
- Different voltage instabilities, caused by slow acting devices like OLTC (Long-term instabilities), or by faster dynamics like stalling of induction motors (Short-term instabilities).
- To prevent voltage instabilities, one can reinforce the grid by adding new lines or new devices, or enforce voltage regulation (local voltage control tracking voltage setpoints given by the system operator).
- The increasing penetration of RES creates overvoltage problems in distribution networks and challenges for balancing the system.

Principle of Automatic Voltage Control

- *The main tool:* primary voltage control via *Automatic Voltage Regulators (AVRs)* of large synchronous generators and synchronous condensers
- Secondary voltage control and automatic switching of reactive compensation devices and transformer taps
- Tertiary voltage control and voltage profile optimization

Control resources

Which of these control resources are the main levers for voltage stability?

- ☐ Adjust synchronous generators' field current
- ☐ Adjust synchronous generators' mechanical power
- ☐ Change transformer taps
- ☐ Change shunt compensation
- ☐ Act on topology: switch lines and transformers in/out of service
- ☐ Fast start-up generator units
- ☐ In extremis load curtailment
- ☐ Control renewable generation (e.g. PV curtailment)
- ☐ Use batteries and other energy storage systems

References I

- L. Keviczky, R. Bars, J. Hetthéssy, and C. Bányász, “Stability of linear control systems,” in *Control Engineering*, pp. 197–239, Springer, 2018.
- N. Hatziargyriou, J. Milanovic, C. Rahmann, V. Ajjarapu, C. Canizares, I. Erlich, D. Hill, I. Hiskens, I. Kamwa, B. Pal, *et al.*, “Definition and classification of power system stability–revisited & extended,” *IEEE Transactions on Power Systems*, vol. 36, no. 4, pp. 3271–3281, 2020.
- T. Ohno and S. Imai, “The 1987 tokyo blackout,” in *2006 IEEE PES power systems conference and exposition*, pp. 314–318, IEEE, 2006.

References II

- A. Kurita and T. Sakurai, “The power system failure on july 23, 1987 in tokyo,” in *Proceedings of the 27th IEEE Conference on Decision and Control*, pp. 2093–2097, IEEE, 1988.
- C. Li, Y. Sun, and X. Chen, “Analysis of the blackout in europe on november 4, 2006,” in *2007 International Power Engineering Conference (IPEC 2007)*, pp. 939–944, IEEE, 2007.